spontaneous time-reversal symmetry breaking in the quantum spin Hall effect

1

PRL **96**, 106401 (2006) PHYSICAL REVIEW LETTERS

also Xu & Moore PRB 73, 045322 (2006)

Helical Liquid and the Edge of Quantum Spin Hall Systems

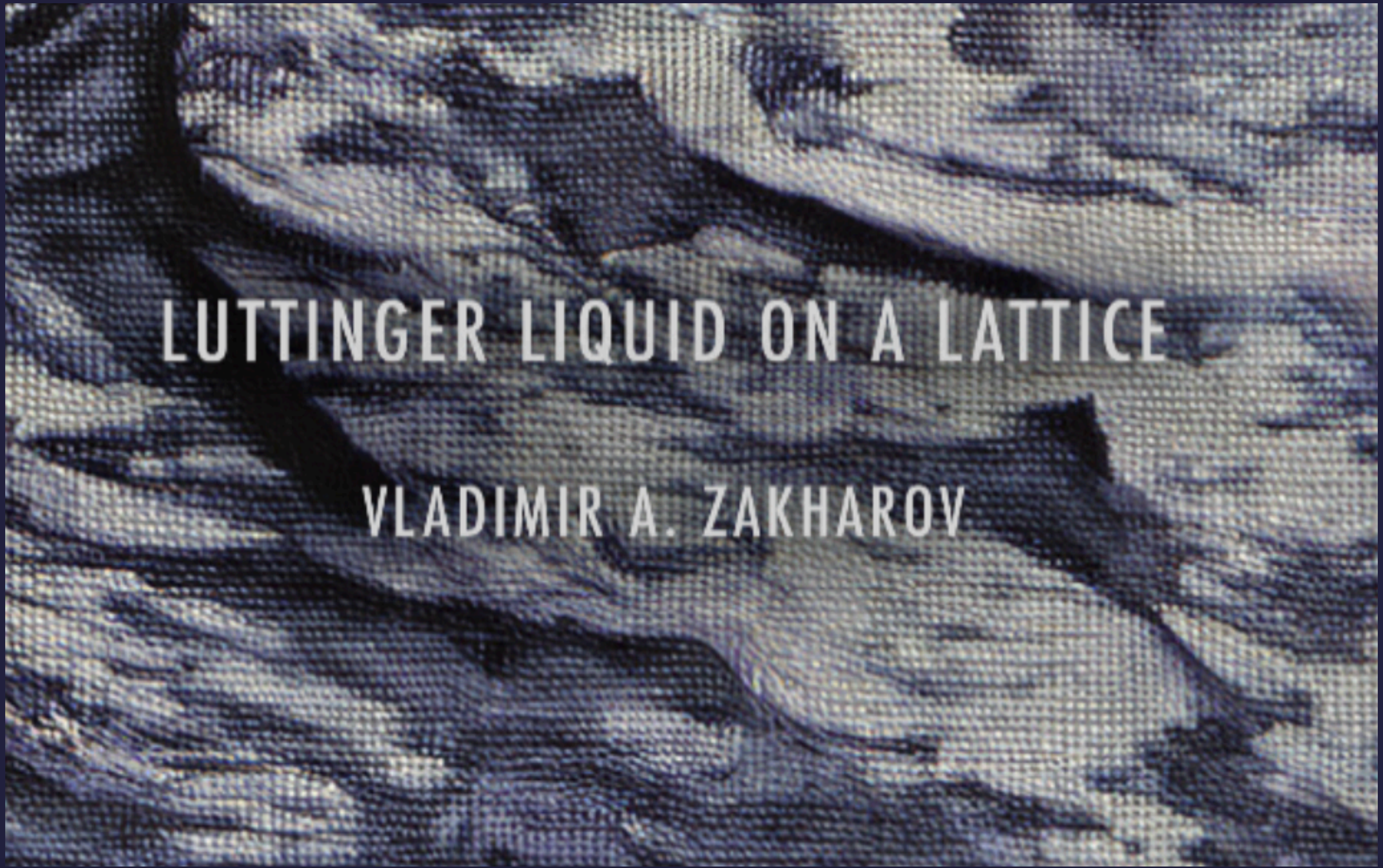
Congjun Wu,[1,2] B. Andrei Bernevig,[1] and Shou-Cheng Zhang[1]

The edge states of the recently proposed quantum spin Hall systems constitute a new symmetry class of one-dimensional liquids dubbed the "helical liquid," where the spin orientation is determined by the direction of electron motion. We prove a no-go theorem which states that a helical liquid with an odd number of components cannot be constructed in a purely 1D lattice system. In a helical liquid with an odd number of components, a uniform gap in the ground state can appear when the time-reversal symmetry is spontaneously broken by interactions.

tangent fermions allow access to this effect on a 1D lattice

Tangent fermions:

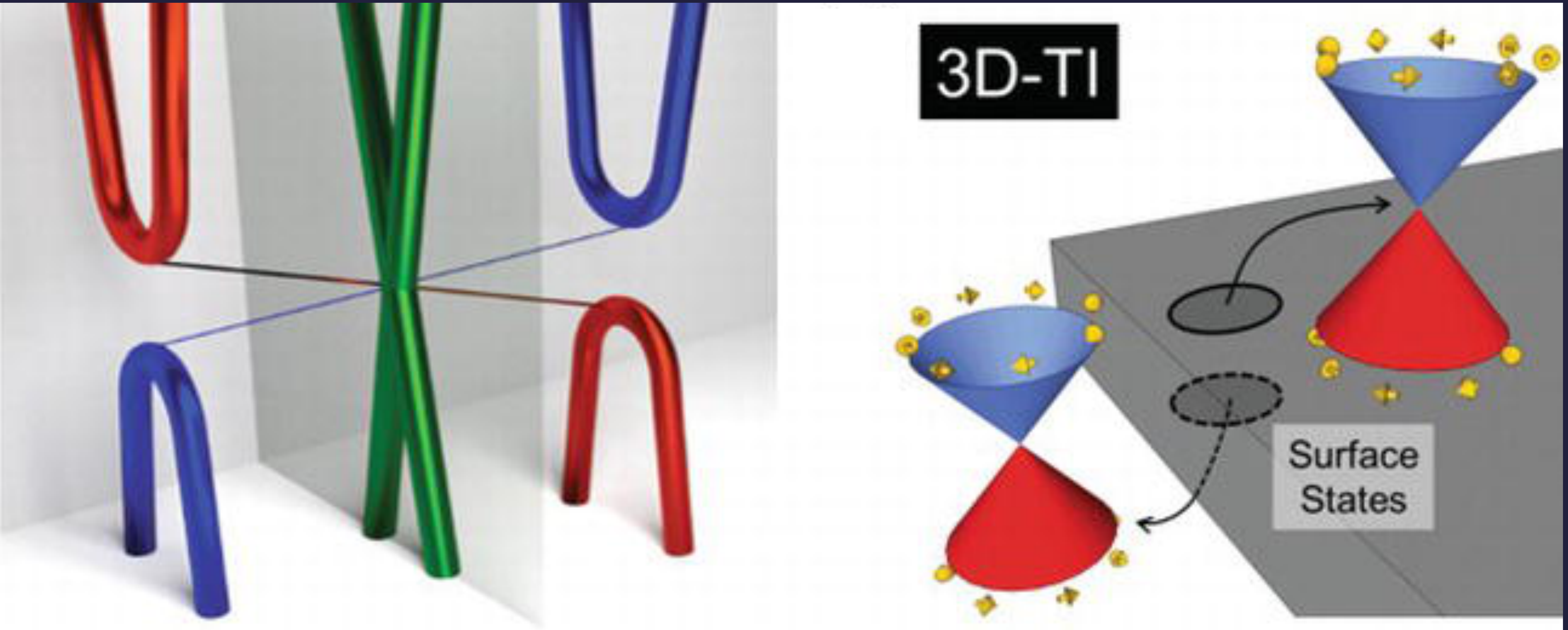
lattice fermions with a topologically protected Dirac cone



Instituut-Lorentz

3D topological insulator

DOI: 10.5772/intechopen.76152



insulating bulk with gapless Dirac cones on the surface, protected from localisation by time-reversal symmetry

$$H = p_x \sigma_x + p_y \sigma_y + V(x, y)$$

can we discretize H without gapping out the Dirac cone?

Fermion doubling

a classic problem in high-energy physics (lattice QCD)

4

Fermion doubling

From Wikipedia, the free encyclopedia

The **fermion doubling problem** is a problem that is encountered when naively trying to put fermionic fields on a lattice. It consists in the appearance of spurious states, such that one ends up having 2^d fermionic particles (with d the number of discretized dimensions) for each original fermion. In order to solve this problem, several strategies are in use, such as Wilson fermions and staggered fermions.

Susskind fermions

Contents [hide]

both these methods break the symplectic TRS

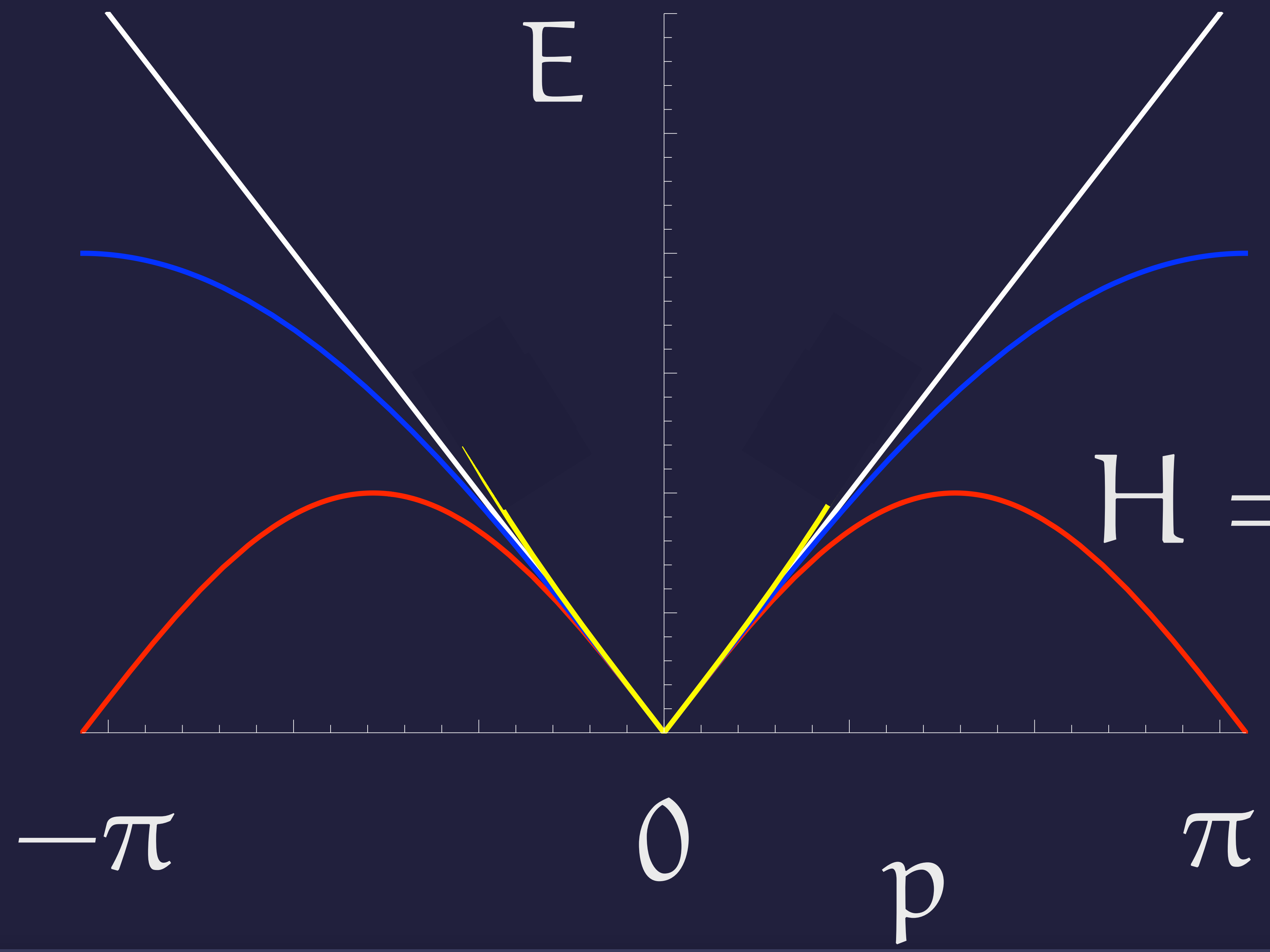
How to avoid fermion doubling

5

on a lattice p➞a[-1]sin pa (periodic in 2π/a), producing a Dirac cone at p=0 and at p=π/a

avoid the doublers by making them massive:

$$H = 2\sigma_x \tan(p/2)$$



$$H = p\sigma_x$$

$$H = \sigma_x \sin p + \sigma_z(1 - \cos p)$$

Wilson or Susskind fermions – TRS broken

$$H = \sigma_x \sin p$$

tangent dispersion (Stacey, 1982) preserves TRS,

with a *nonlocal* hopping: $t_{nm} \propto (-1)^{n-m}$

Restoring locality

$$2\tan(p/2) = \frac{\sin p}{\frac{1}{2}(1+\cos p)}$$

6

arXiv:2103.15615

the tangent dispersion E=2tan(p/2) can be obtained from a *local* generalised eigenproblem

$$H\psi = EP\Psi, \quad H = \sigma_x \sin p, \quad P = \tfrac{1}{2}(1+\cos p)$$

2D generalisation: $E = 2\sqrt{\tan^2(p_x/2) + \tan^2(p_y/2)}$

$$H = \tfrac{1}{2}\sigma_x \sin p_x (1+\cos p_y) + \tfrac{1}{2}\sigma_y \sin p_y (1+\cos p_x)$$

$$P = \tfrac{1}{4}(1+\cos p_x)(1+\cos p_y)$$

tangent fermion tensor network

Matrix-Product-Operator (MPO) representation of the Hamiltonian

$$H = \sum_{n>m=1}^{N} (t_{nm} c_n^\dagger c_m + \text{H.c.}) = M_1 M_2 \ldots M_N$$

$$t_{nm} \propto (-1)^{n-m}$$

matrices M_n depend only on site n

Q: is this possible with a dimension of M_n that does not grow with N?

arXiv:2407.06713 **YES** any dispersion of the form $P(k)\psi = Q(k)E\psi$ with P and Q polynomials in e^{ik}, has an MPO representation of fixed bond dimension

helical Luttinger liquid with Hubbard interaction

$$H = \sum_{n>m=1}^{N} \left[t_{nm} \left(c_{n\uparrow}^\dagger c_{m\uparrow} - c_{n\downarrow}^\dagger c_{m\downarrow} \right) + \text{H.c.} \right] + \sum_{n=1}^{N} U_n$$

$$t_{nm} = 2it_0(-1)^{n-m}$$ tangent dispersion:

$$M_n = \begin{pmatrix} 1 & c_{n\uparrow} & c_{n\uparrow}^\dagger & c_{n\downarrow} & c_{n\downarrow}^\dagger & (2it_0)^{-1}U_n \\ 0 & -1 & 0 & 0 & 0 & c_{n\uparrow}^\dagger \\ 0 & 0 & -1 & 0 & 0 & c_{n\uparrow} \\ 0 & 0 & 0 & -1 & 0 & -c_{n\downarrow}^\dagger \\ 0 & 0 & 0 & 0 & -1 & -c_{n\downarrow} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

exact MPO representation with *fixed* bond dimension 6

spontaneous time-reversal symmetry breaking in the quantum spin Hall effect

PRL **96,** 106401 (2006) PHYSICAL REVIEW LETTERS

also Xu & Moore PRB 73, 045322 (2006)

Helical Liquid and the Edge of Quantum Spin Hall Systems

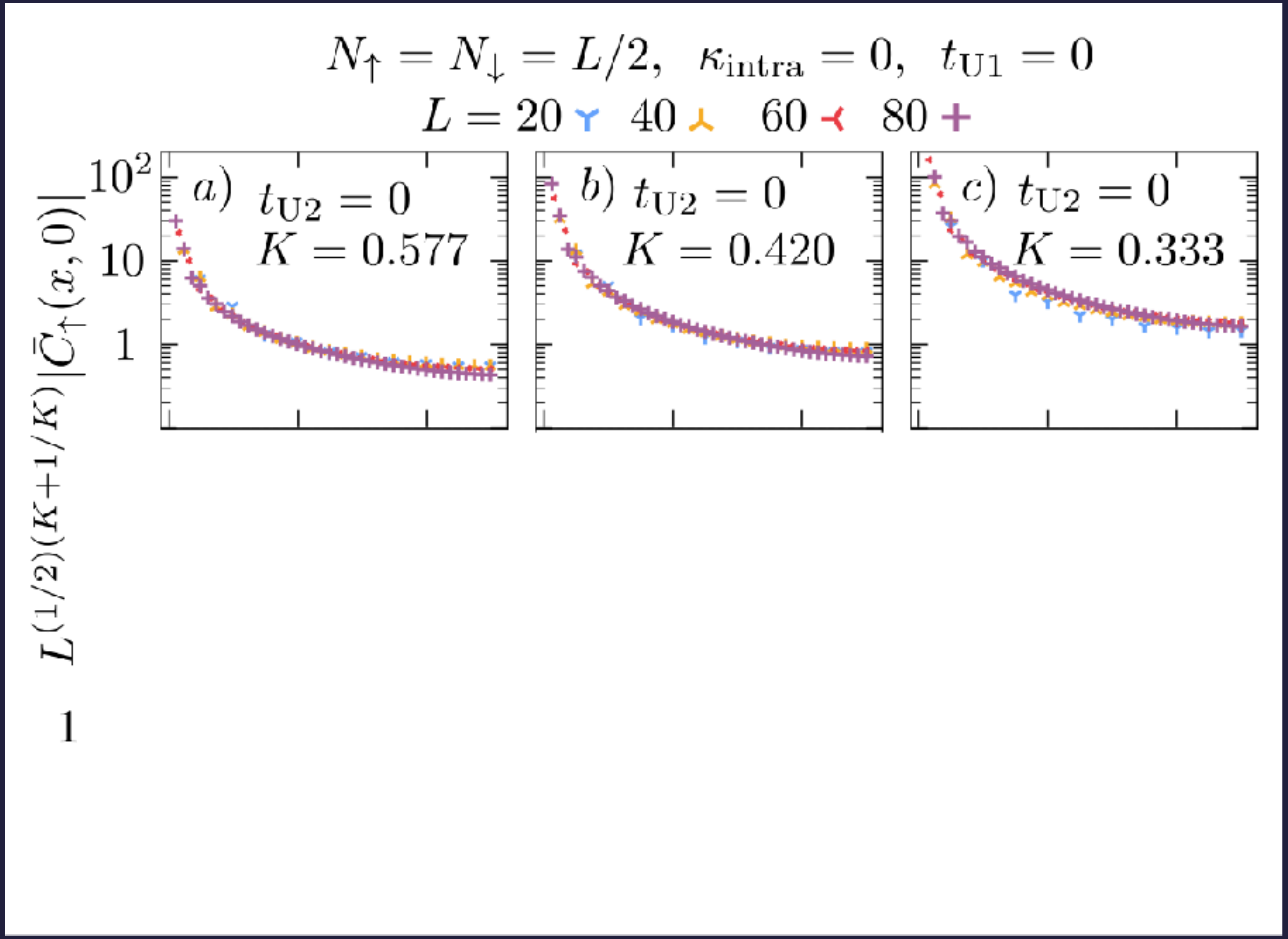
Congjun Wu,[1,2] B. Andrei Bernevig,[1] and Shou-Cheng Zhang[1]

The edge states of the recently proposed quantum spin Hall systems constitute a new symmetry class of one-dimensional liquids dubbed the "helical liquid," where the spin orientation is determined by the direction of electron motion. We prove a no-go theorem which states that a helical liquid with an odd number of components cannot be constructed in a purely 1D lattice system. In a helical liquid with an odd number of components, a uniform gap in the ground state can appear when the time-reversal symmetry is spontaneously broken by interactions.

tangent fermions allow access to this effect on a 1D lattice

spontaneous time-reversal symmetry breaking in the quantum spin Hall effect

propagator



power law scaling

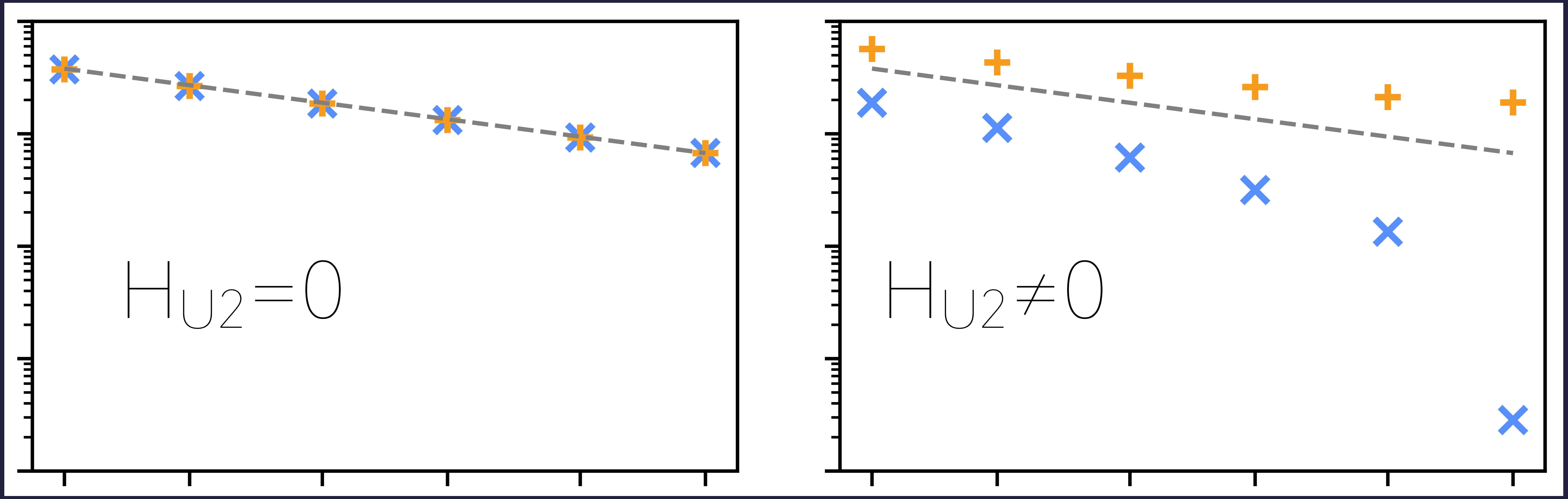
$$C_\sigma(x,0) = L^{-(1/2)(K+1/K)} f_C(x/L)$$

is broken by two-particle backscattering if K<1/2

$$H_{\rm U2} = t_{\rm U2} \sum_n [c^\dagger_{n\uparrow} c_{n\downarrow} c^\dagger_{n+1,\uparrow} c_{n+1,\downarrow} + \text{H.c.}]$$

spontaneous time-reversal symmetry breaking in the quantum spin Hall effect

first two excited states as a function of system size



ground state becomes doubly degenerate

Summary:

- *tangent fermions:* space-time discretization of the Dirac equation without fermion doubling and without zone boundary singularities
- gapless Dirac cone protected by time-reversal & chiral symmetry
- efficient quantum Monte Carlo and tensor network implementation
- vector potential can be included in a gauge invariant way
- applications: Casimir effect, Klein tunneling, Majorana metal, **QSHE**

REVIEW

annalen der physik
www.ann-phys.org

Tangent Fermions: Dirac or Majorana Fermions on a Lattice Without Fermion Doubling

C. W. J. Beenakker, A. Donís Vela, G. Lemut, M. J. Pacholski, and J. Tworzydło*

arXiv:2302.12793

13

Lattice fermion simulation of spontaneous time-reversal symmetry breaking in a helical Luttinger liquid

V. A. Zakharov, J. Sánchez Fernán, and C. W. J. Beenakker
Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
(Dated: January 2026)

arXiv:2601.09563

The Capri Spring School on Transport in Nanostructures

Capri, April 3 - 10, 2005.

thank you so much for 20 years of memorable meetings

Lecturers

- **Carlo Beenakker**: Shot noise and full counting statistics
- **Gerrit Bauer**: Spintronics
- **Carlo Di Castro**: Low dimensional disordered conductors
- **Karsten Flensberg**: Molecular electronics
- **Andrei Komnik**: Quantum impurity problems in one-dimensional metals
- **Allan MacDonald**: Quantum Hall effect
- **Francesco Petruccione** *: Quantum information processing with nanostructures
- **Hugues Pothier**: Dephasing and relaxation in disordered nanowires
- **Arturo Tagliacozzo**: Kondo and Fano resonances
- **Amir Yacoby**: Experimental study of electrons confined to one dimension