Anyon braiding and statistical angle in the fractional quantum Hall effect

CAPRI 2026

(Capri 2023 anyon scaling dimension)
(Capri 2014 HOM interferometry fermions)
(Capri 2010 Entanglement from BCS)
(Capri 2008 Non chiral Luttinger)

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Collaboration: CPT Nanophysics team: N. Demazure and F. Ronetti…

Ecole Normale Paris, ISSP University of Tokyo

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Fractional Statistics and the Quantum Hall Effect

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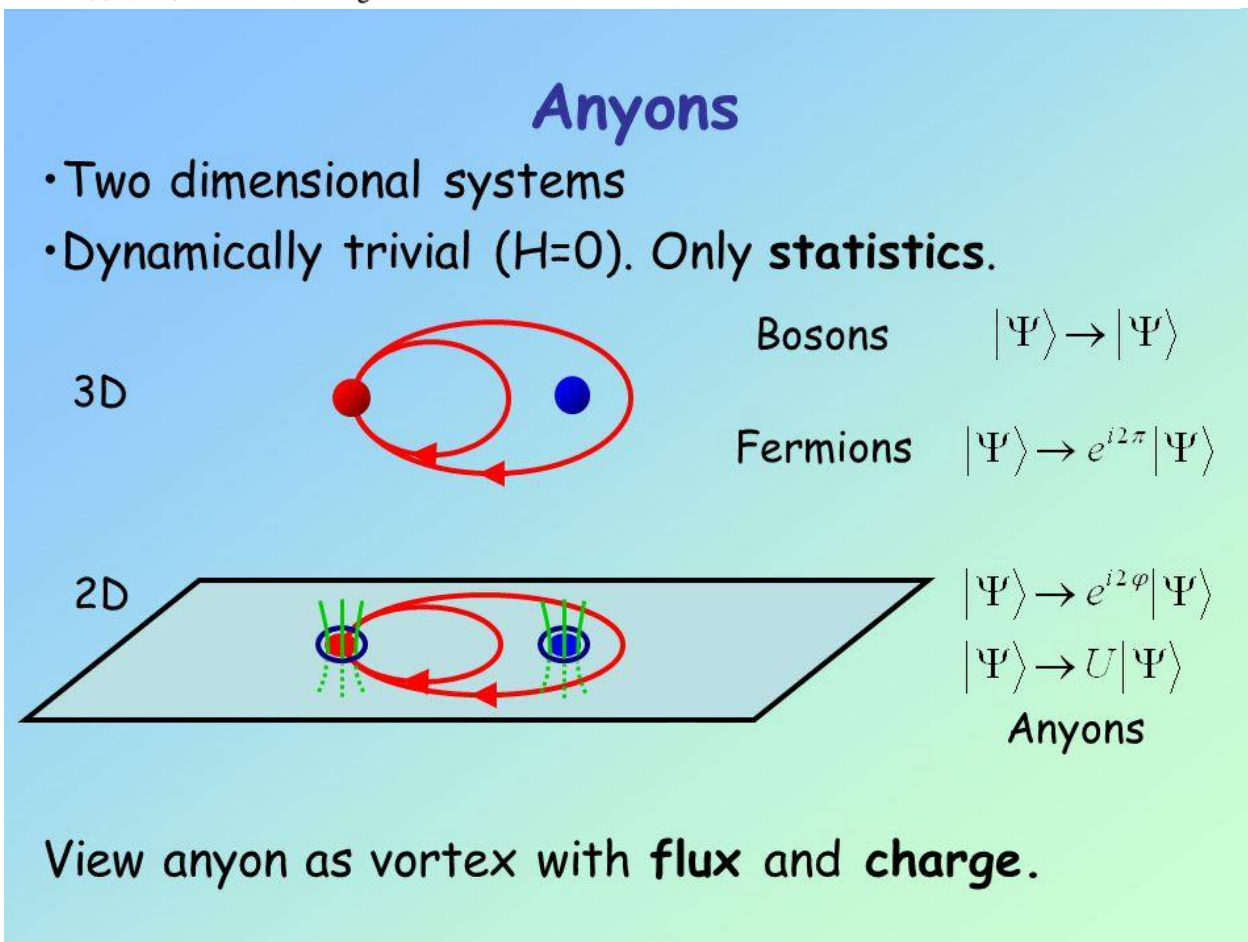
(Received 18 May 1984)

The statistics of quasiparticles entering the quantum Hall effect are deduced from the adiabatic theorem. These excitations are found to obey fractional statistics, a result closely related to their fractional charge.



PRL 116, 156802 (2016) PHYSICAL REVIEW LETTERS week ending 15 APRIL 2016

Current Correlations from a Mesoscopic Anyon Collider

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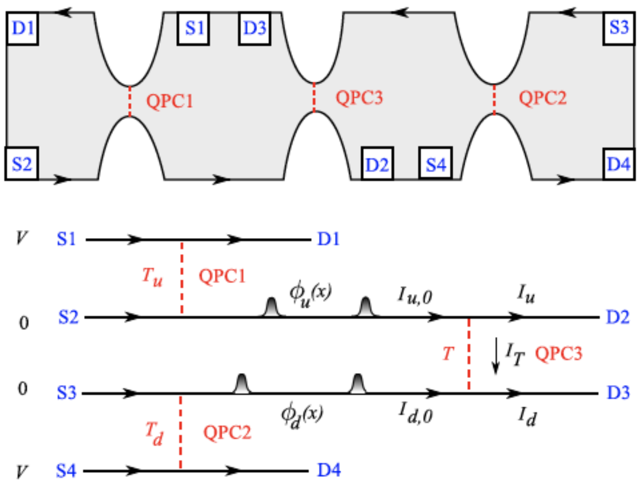
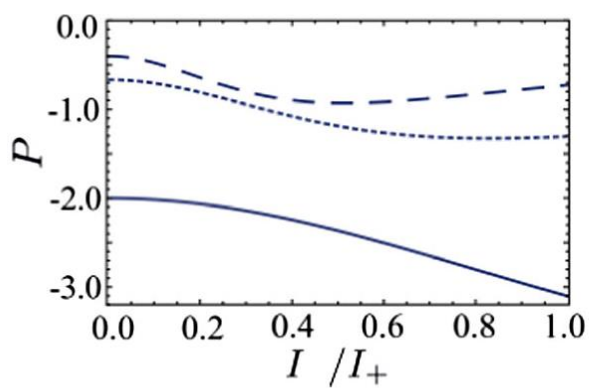


Generalized Fano Factor:

$$P(I_-/I_+) = \frac{\langle \delta I_d \delta I_u \rangle_{\omega=0}}{e^\star I_+ \frac{\partial}{\partial I_-} \langle I_T \rangle \big|_{I_-=0}}$$

$$P(0) = 1 - \frac{\tan \pi\lambda}{\tan \pi\delta} \frac{1}{1-2\delta} \underset{\lambda=\frac{1}{m},\, \delta=\frac{1}{m}}{\longrightarrow} \frac{-2}{m-2}$$



Anyons Hong Ou Mandel « collision » ?

or anyon « braiding » λ (statistical angle) and δ (scaling dimension) enter the Fano factor

$\lambda = \delta = \nu = 1/3$

MESOSCOPIC PHYSICS

Fractional statistics in anyon collisions

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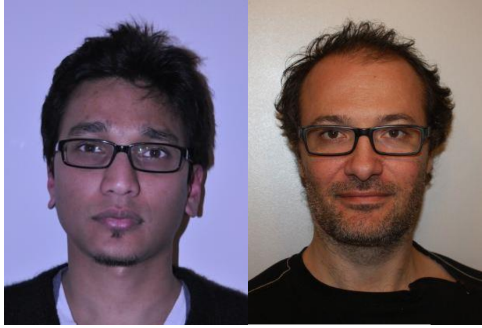


Collision of Laughlin quasiparticules in the FQHE at the location of a quantum point contact

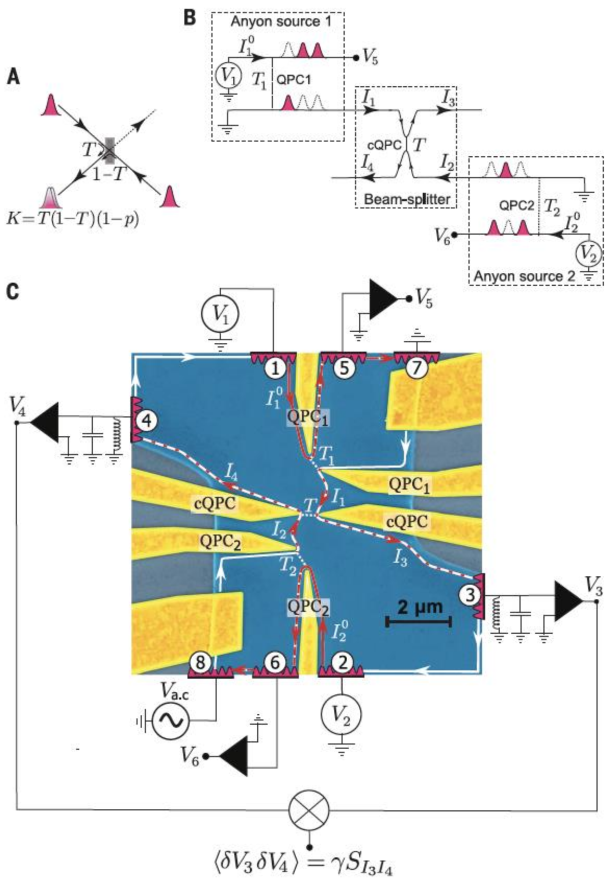
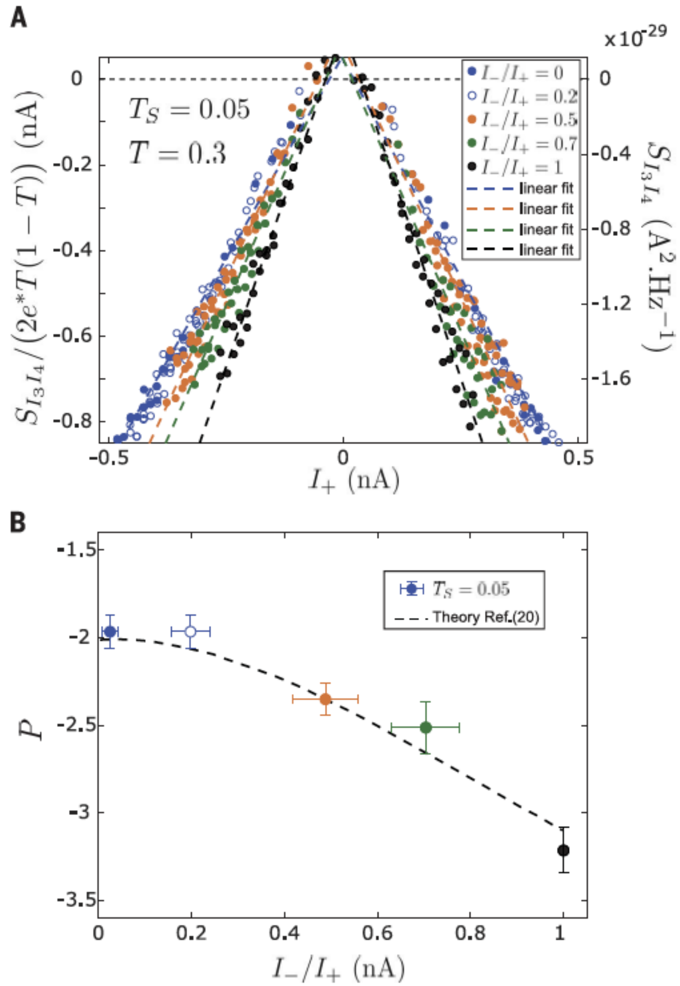


Fig. 4. Experimental test of the quantum mechanical description of an anyon collision. (**A**) $S_{I_3 I_4}$ as a function of I_+ for various values of the ratio I_-/I_+. The dashed lines are linear fits of $S_{I_3 I_4}$. (**B**) Generalized Fano factor P extracted from the slope of the linear fits in (A), plotted as a function of the ratio I_-/I_+. The colors correspond to the ratios I_-/I_+ plotted in (A). The horizontal error bars correspond to the standard deviation of the ratio I_-/I_+ of the data. The vertical error bars are given by the uncertainties of the linear fits. The dashed line is the prediction extracted from (20) for the quantum description of anyon collisions with $\phi = \pi/3$.



Agrees with

λ =1/3

δ = 1/3 (or 2/3 !!!)

RESEARCH ARTICLE SUMMARY

CPT ENS collaboration
Science (2025)

MESOSCOPIC PHYSICS

Time-domain braiding of anyons

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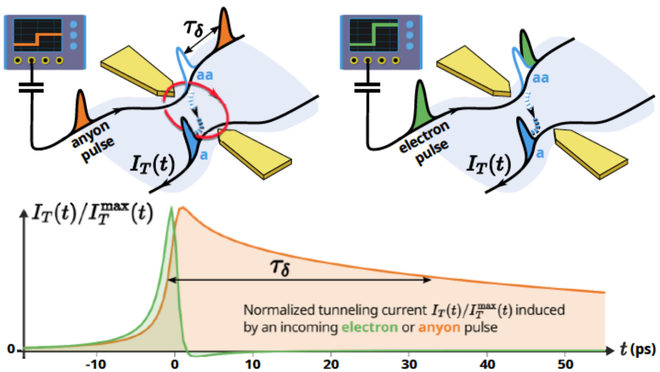
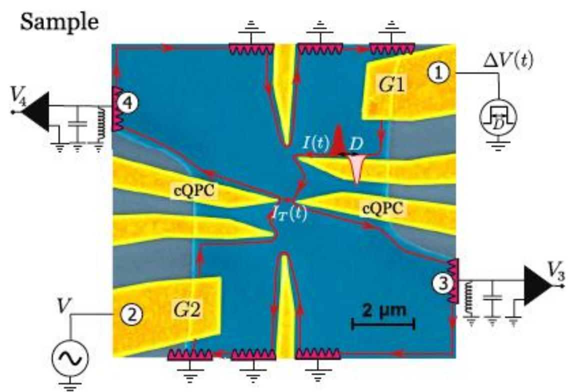
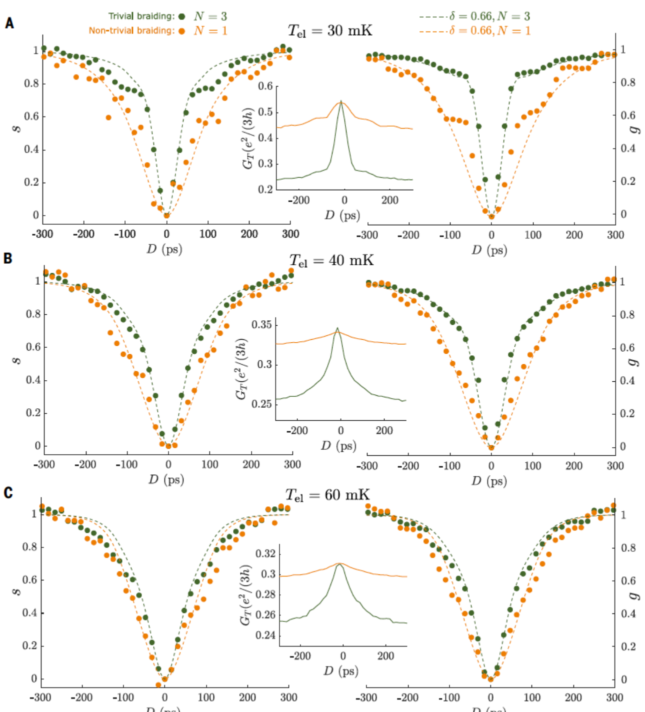
Full article and list of author affiliations:
https://doi.org/10.1126/science.adm7695







PHYSICAL REVIEW LETTERS **130,** 186203 (2023)

Anyonic Statistics Revealed by the Hong-Ou-Mandel Dip for Fractional Excitations

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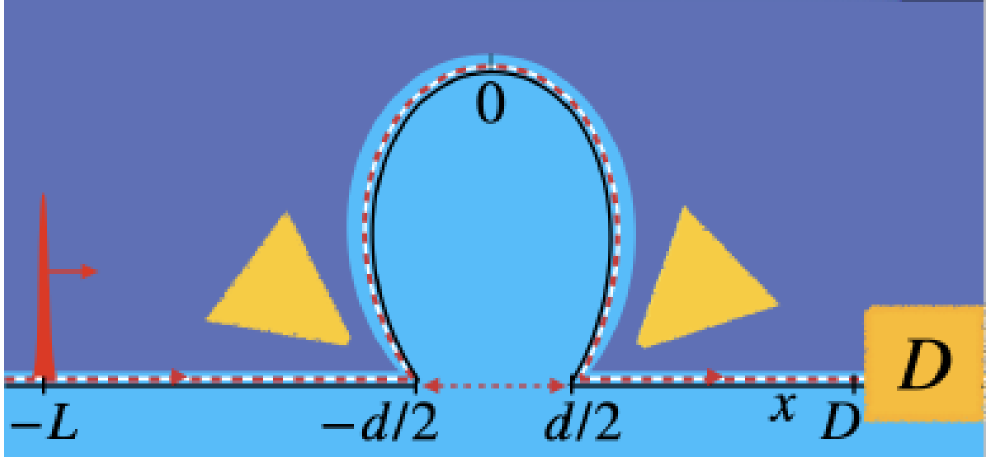
$$S_{\mathrm{HOM}}(\delta t) = \frac{1}{2S_{\mathrm{HBT}}} \int_{-\infty}^{\infty} dt dt' \mathcal{G}(t'-t)^2 \times \{\cos[2\pi\nu f_{\delta t}(t,t')] - 1\},$$

The « Omega project »



Basic idea: work with a single edge which travels through a open quantum dot.

Inject an anyon upstream.

When the anyon reaches the dot, braiding with happen at –d/2, and subsequently another braiding will occur at d/2.

This generates two opposite current « pulses »

However, it is not realistic to measure a time dependent current, and to our knowledge with trains of anyons the current averges to zero.

We thus invent a noise based protocol with a « Rosenow type » poissonian anyon source impinging on the open quantum dot, and measure the correlation of the reference current with that at the output of the open quantum dot.

This allows to have a direct measurement of the statistical/braiding angle(in opposition with previous protocols whci mix statistical angle with scaling dimension measurements.

$$H_0 = \frac{u}{4\pi}\int dx\ [\partial_x\phi(x)]^2 \qquad [\phi(x,t),\phi(x',t')] = i\pi\mathrm{sign}\,[x-x'-u\,(t-t')]$$

$$H_T = \Lambda e^{i\kappa}\Psi^\dagger(d/2)\Psi(-d/2) + \mathrm{H.c.} \qquad \Psi(x) = \frac{\mathcal{F}}{\sqrt{2\pi a}}e^{-i\sqrt{\nu}\phi(x)}$$

Anyon correlation function: note that « delta » (scaling dimension) is assumed to be different from « lambda » (statistical angle)

$$\left\langle T_K\left[e^{i\sqrt{\nu}\phi(x,t\eta)}\ \ e^{-i\sqrt{\nu}\phi\left(x',t'\eta'\right)}\right]\right\rangle = e^{\delta\mathcal{G}\left(t-t'-\frac{x-x'}{u}\right)} \times e^{-\frac{i\lambda\pi}{2}\sigma_{tt'}^{\eta\eta'}\mathrm{sign}\left(t-t'-\frac{x-x'}{u}\right)}$$

(sigma depends on Keldysh indices and the two times PRL2023)

In the canonical CLL theoretical picture « delta » and $\lambda = \upsilon$ (LLL filling factor)

« Reasonable » conjecture: δ is **non universal** (affected by environement)

λ is a **universal property** of the bulk.

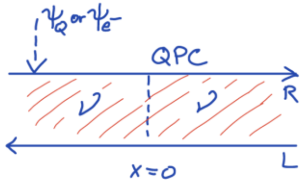
Justification: the real part of the Green's function is likely to be affected by an EM environment yet (reasonable conjecture) the imaginary part of the bosonic Green's function remains undressed b y the EM environment.

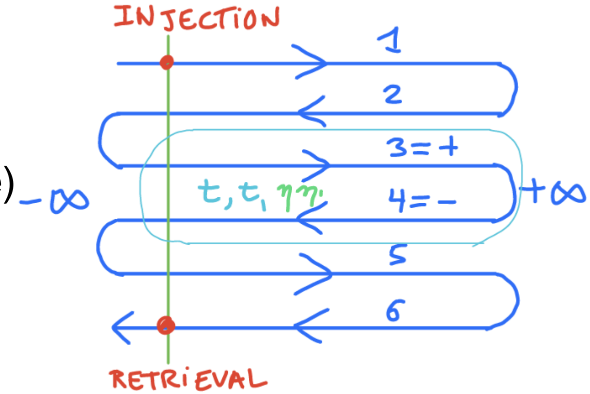
$$G^{\eta\eta'}(x,t;x',t') = \mathcal{G}\left(t - t' - \frac{x-x'}{u}\right) - \frac{i\pi}{2}\sigma_{tt'}^{\eta\eta'}\,\mathrm{sign}\left(t - t' - \frac{x-x'}{u}\right)$$

$$\mathcal{G}(t) = -\ln\left|\frac{\sinh\left(i\frac{\pi}{\beta}\tau_0\right)}{\sinh\left[\frac{\pi}{\beta}(t+i\tau_0)\right]}\right|$$

Seule la partie réelle, qui ne dépend pas des indices Keldysh, contribue à la dimension d'échelle.

La partie imaginaire contribue au braiding et à la statistique





Anyon injection upstream from the Quantum Dot (prepared state)

$$|\Phi\rangle = \frac{1}{\mathcal{N}} \prod_{\mu=1}^{M} \Psi^\dagger(-L, -T-\tau_\mu)\,|0\rangle$$

Current operator along the edge

$$I(x,t) = e\frac{u\sqrt{\nu}}{2\pi}\left[\partial_x \phi(x,t)\right] \qquad I(x,t) = \lim_{\gamma\to 0} -\frac{iue\sqrt{\nu}}{2\pi\gamma}\left[\partial_x e^{i\gamma\phi(x,t)}\right]$$

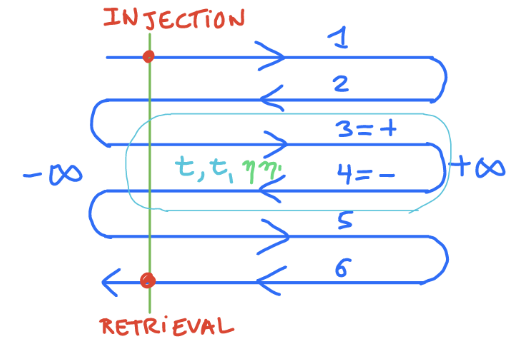
« 0 » order current

$$\langle I(x,t)\rangle^{(0)} = \frac{eu\sqrt{\nu}}{2\pi}\,\langle T_K\left[\partial_x\phi(x,t_-)\right]\rangle_\Phi$$

$$K_\gamma^{(0)}(x,t) = \left\langle T_K\left[\prod_\mu e^{-i\sqrt{\nu}\phi(-L,-T-\tau_{\mu_4})} e^{i\gamma\phi(x,t_2)} \prod_{\mu'} e^{i\sqrt{\nu}\phi\left(-L,-T-\tau_{\mu'_1}\right)}\right]\right\rangle$$

$$\langle I(x,t)\rangle^{(0)} = -\frac{ieu\sqrt{\nu}}{2\pi\gamma}\lim_{\gamma\to 0}\partial_x K_\gamma^{(0)}(x,t)$$

On utilise le contour Keldysh à 4 branches, pour que l'injection soit sur les plus petits temps, et le « retrait » de l'injection soit sur les plus grands temps (TM notes de 2010)



Result is expectedly a sum of deltas

$$\langle I(x,t)\rangle^{(0)} = -\nu e \sum_{\mu} \delta\left(\tau_\mu + t - \frac{x}{u}\right) = -\nu e u \sum_{\mu} \delta\left[x - u\left(\tau_\mu + t\right)\right]$$

Explanation:

One calculates the « K » correlator with the equivalent of a Wick's theorem, one thus obtains a exponentials of sums of Green's

Derive K with respect to « x » to obtain the current: derivatives of Greens functions are generated.

The current thus contains derivatives of difference of G-+ whose real part vanishes, only the imaginary part is left.

Yet the derivatives of sign functions gives delta functions (braiding associated terms)

First order current contribution (in the tunneling) : contrary to 2 or 3 edge states devices, quasiparticle conservation is already insured to 1st order in tunneling

$$\langle I(x,t)\rangle^{(1)} = \frac{eu\sqrt{\nu}}{2\pi}\frac{\Lambda}{i2\pi a}\sum_{\substack{\eta=\pm\\ \epsilon=\pm}} \eta e^{i\epsilon\kappa}\left\langle T_K\left\{\partial_x\phi(x,t_-)\int_{-\infty}^{+\infty} dt_1\; e^{i\sqrt{\nu}\phi(\epsilon\frac{d}{2},t_1\eta)}e^{-i\sqrt{\nu}\phi(-\epsilon\frac{d}{2},t_1\eta)}\right\}\right\rangle_\Phi$$

New correlator K which contains the Keldysh evolution to 1st order

$$K^{(1)}_{\substack{\gamma\\ \epsilon\eta}}(x,t) = \left\langle T_K\left[\prod_\mu e^{-i\sqrt{\nu}\phi(-L,-T-\tau_{\mu_4})}e^{i\gamma\phi(x,t_-)}\int dt_1\; e^{i\sqrt{\nu}\phi(\frac{\epsilon d}{2},t_1\eta)}e^{-i\sqrt{\nu}\phi(-\frac{\epsilon d}{2},t_1\eta)}\prod_{\mu'} e^{i\sqrt{\nu}\phi(-L,-T-\tau_{\mu'_1})}\right]\right\rangle$$

$$K^{(1)}_{\substack{\gamma\\ \epsilon\eta}}(x,x';t,t') = e^{\delta\; \mathcal{G}(\frac{d}{u})}\mathcal{B}_\epsilon(t_1)e^{i\pi\gamma\sqrt{\nu}\sum_\mu \mathrm{sign}(t+\tau_\mu-\frac{x}{u})}\int dt_1\; e^{-\gamma\sqrt{\nu}\epsilon[G^{-\eta}(x,\frac{d}{2};t,t_1)-G^{-\eta}(x,-\frac{d}{2};t,t_1)]}$$

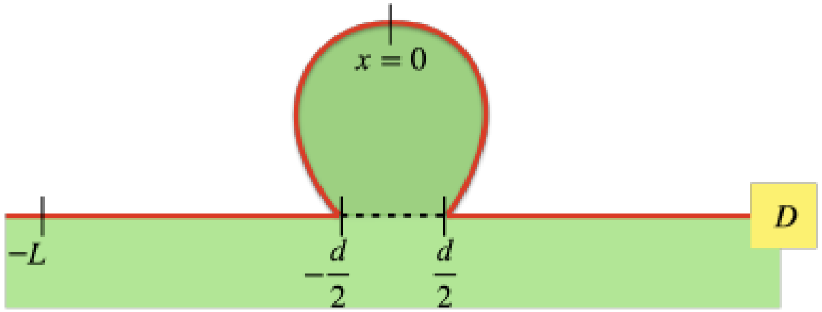
$$\mathcal{B}_\epsilon(t_1) = e^{\nu\epsilon\sum_\mu[G(-\frac{d}{2},-t_1-\tau_\mu)-G(\frac{d}{2},-t_1-\tau_\mu)+G(-\frac{d}{2},t_1+\tau_\mu)-G(\frac{d}{2},t_1+\tau_\mu)]}$$

This contains **braiding terms** which only involve the **imaginary part** of the bosonic Green's function.
The real part of the Green's function – thus the scaling dimension - appears only as a prefactor which reflect the **thermal suppression** of the current

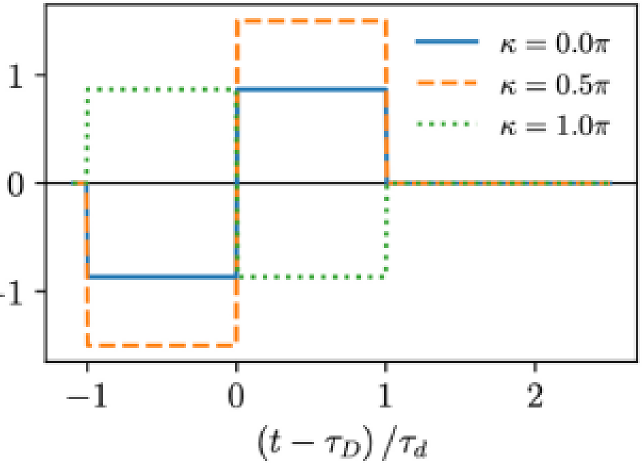
simpler alternative to extract the statistics?

Omega-shaped junction

- single edge deformed into a droplet
- tunneling Hamiltonian
  $H_T = \Lambda e^{i\kappa}\Psi^\dagger(d/2)\Psi(-d/2) + \text{H.c.}$



Time-dependent current response



- single δ-like anyon injected
- First order contrib. to the current ($I_0 \propto \Lambda$)

$$\langle I(t)\rangle^{(1)} = -I_0(\delta, T)\sin\left[2\pi\lambda\ N\left(t-\frac{\tau_d}{2}\right)+\kappa\right] + I_0(\delta, T)\sin\left[2\pi\lambda\ N\left(t+\frac{\tau_d}{2}\right)+\kappa\right]$$

λ statistical/braiding angle
δ scaling dimension

Main differences with the collider geometry

- first order response
- overlap of equal-time processes $\longrightarrow$ no integral over time
- scaling dimension and temperature factorize!

Problem: Measuring time-dependent current is notoriously difficult

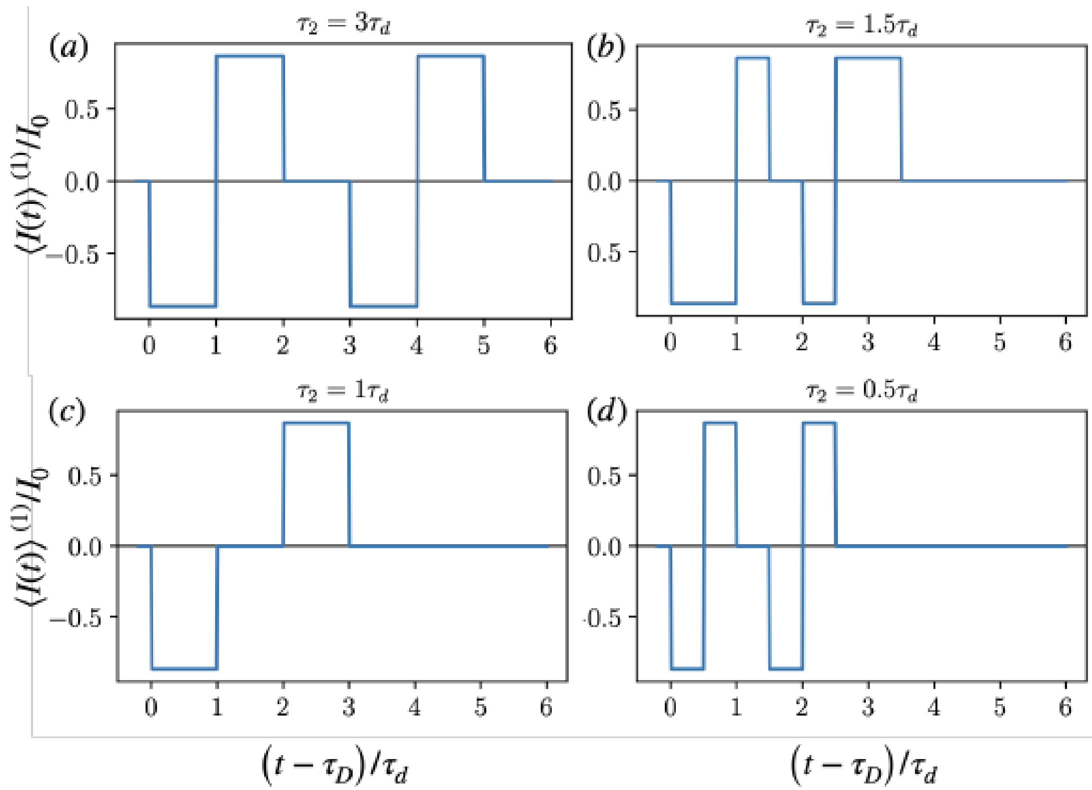
$$\langle I(t)\rangle^{(1)} = I_0 \sum_{\xi} \xi \sin\left\{\kappa + 2\pi\lambda N\left(t+\xi\frac{d}{2u}\right)\right\},$$

$$I_0 = e\nu\frac{\Lambda e^{\delta}\,\mathcal{G}\left(\frac{d}{u}\right)}{\pi a},$$

N « braiding term » in sine is unaffected by scaling dimension

$$N\left(t+\xi\frac{d}{2u}\right) = \frac{1}{2}\sum_{\mu,\chi}\chi\left[\mathrm{sign}\left(t+\tau_\mu-\frac{D}{u}+d\frac{\xi-\chi}{2u}\right)\right] = \sum_{\mu}\int_{t+\tau_\mu-\frac{D}{u}+d\frac{\xi+1}{2u}}^{t+\tau_\mu-\frac{D}{u}+d\frac{\xi-1}{2u}} dt'\,\delta\left(t'-\tau_\mu\right)$$

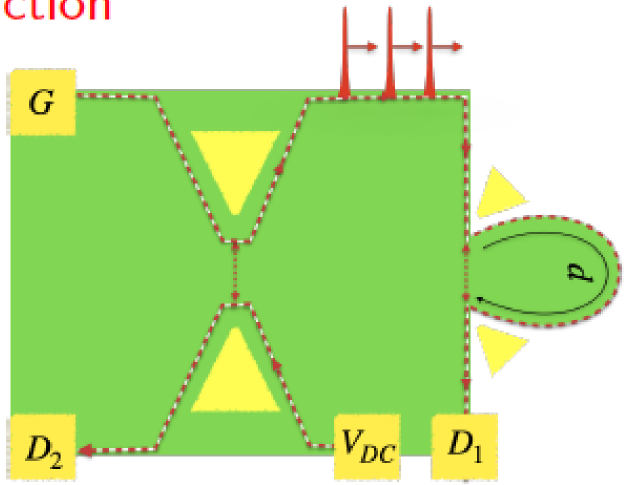
Case of 2 successive pulses
Interference between 2 anyon braiding events



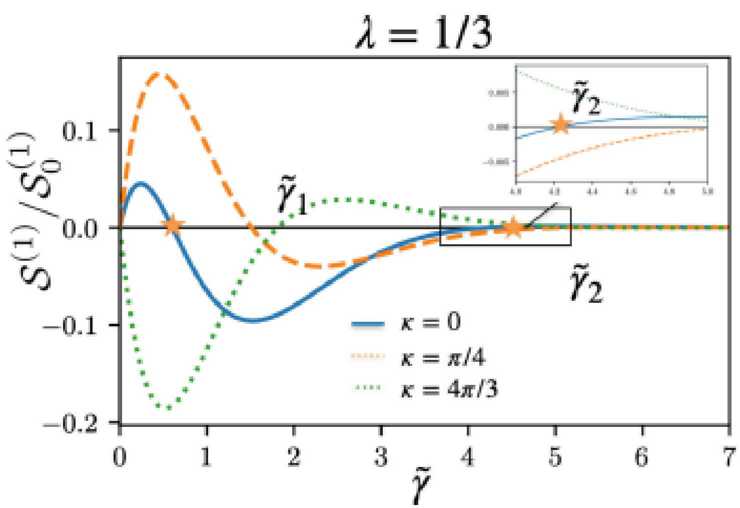
(result bencharked by an independent ν=1 scattering theory calculation)

Cross-correlation detection protocol

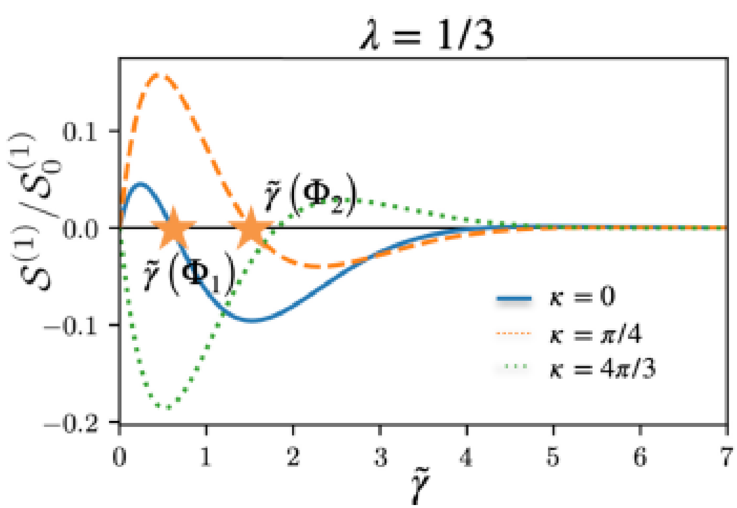
- Solution: periodic drive? $\longrightarrow$ all average quantities vanish...
- Proposed device : source QPC + Omega junction
  - measure cross-correlations
    $\mathcal{S}(t-t') = \langle \delta I_1(t)\ \delta I_2(t') \rangle$
  - First order contrib. at finite freq.
    $$\mathcal{S}^{(1)}(\omega) = \mathcal{S}_0^{(1)}(\omega)\sin(\pi\lambda)\tilde{\gamma}e^{-\tilde{\gamma}[1-\cos(2\pi\lambda)]} \times \cos\left[\tilde{\gamma}\sin(2\pi\lambda) + \pi\lambda - \kappa(\Phi)\right]$$



- Extracting the statistical angle $\phi = \pi\lambda$



from 2 consecutive zeroes of the low-frequency cross-correlations



from the evolution of a given zero with magnetic flux

PHYSICAL REVIEW B **74**, 115323 (2006)

Finite-frequency noise cross correlations of a mesoscopic circuit: A measurement method using a resonant circuit

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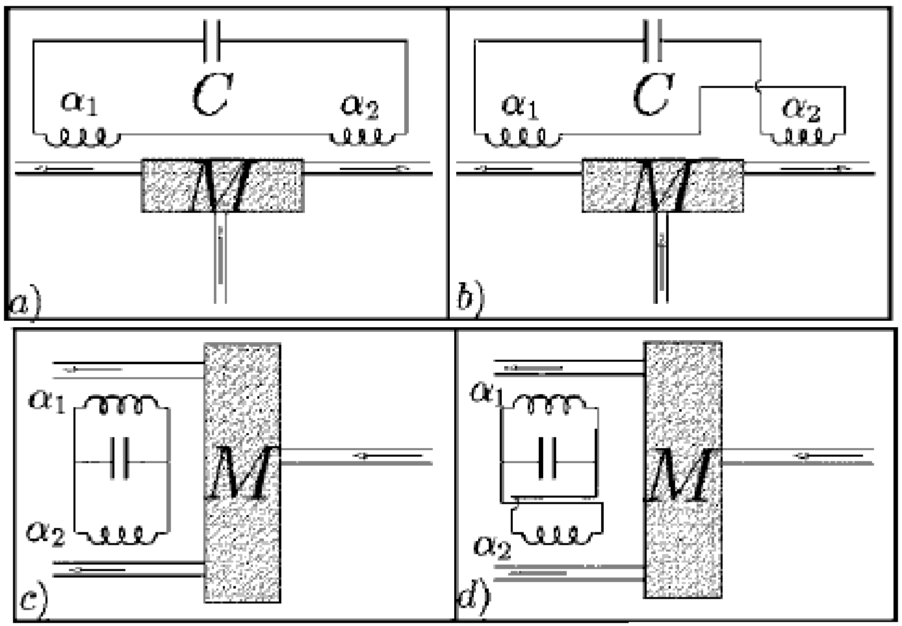

$$\langle x^2(0)\rangle = \frac{\alpha_1\alpha_2}{(4M)^2}\int_{-\infty}^{+\infty} dt \int_{-\infty}^{0} dT e^{\eta T}\{(e^{i\Omega t} + e^{-i\Omega T sign(t)})$$

$$\times [(N(\Omega)+1)(\langle I_2(0)I_1(t)\rangle + \langle I_1(0)I_2(t)\rangle)$$

$$- N(\Omega)(\langle I_1(t)I_2(0)\rangle + \langle I_2(t)I_1(0)\rangle)]\}.$$

Conclusions

- New configuration involving a deformed single edge into an Omega-shaped junction
- Non-vanishing time-dependent current at first order in tunneling, where the effects of temperature and scaling dimension factorize
- Measurement protocol requires a source QPC along with the Omega junction, and involves measuring low-frequency current cross-correlations
  $\rightarrow$ direct access to the statistical angle, without independent measurement of non-universal parameters
- Within range of current experimental realization

Anyon braiding on the single edge of a fractional quantum Hall state
F. Ronetti et al.
Phys. Rev. Lett. **135**, 146601 (2025)

Probing anyon statistics on a single-edge loop in the FQH regime
F. Ronetti et al.
Phys. Rev. B **112**, 125166 (2025)

Perspectives:

the dot cannot be too large because the overall signal decays exponentially with temperature
→ Reduce the dot size but include charging effects (exaclty)

What happens with hierachical states or non-abelian states ?

Capacitance effects on anyon braiding detection in a single-edge interferometer in the fractional quantum Hall regime

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General Abelian theory for anyon braiding on a single edge

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Abstract

In these notes we compute charge current and finite-frequency noise for a single edge of a general Abelian quantum Hall state where anyons are injected from a source QPC.