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# Quantifying robustness and locality of Majorana bound states in interacting systems

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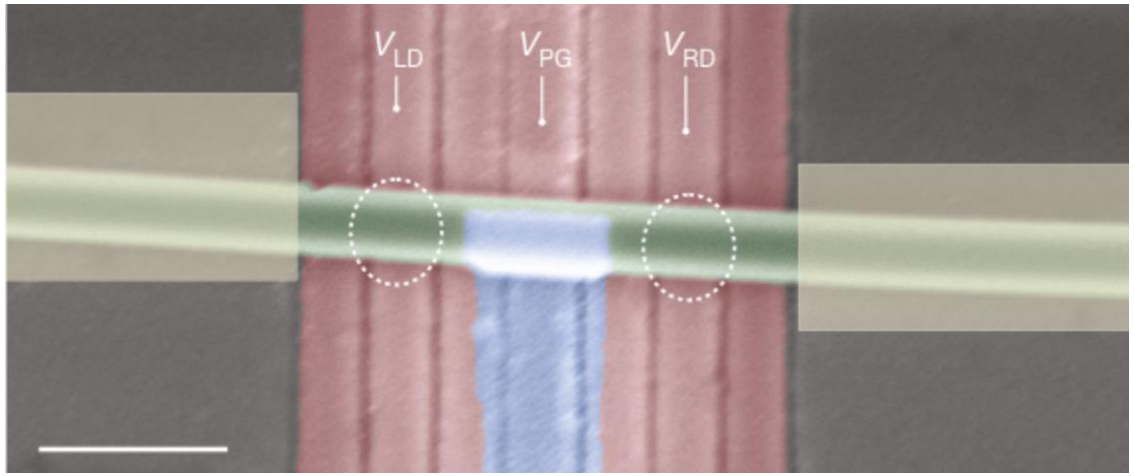
arXiv:2510.20538, Accepted in PRX Quantum

# Outline

- Motivation
- Majoranas in interacting systems,
- … their locality,
- … and their robustness.

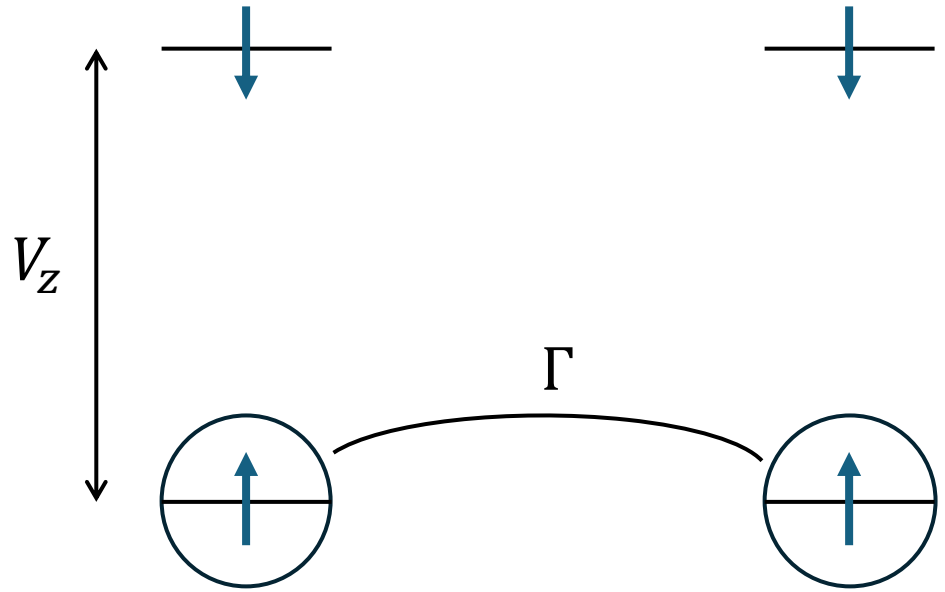
$$H_K = \sum_j \varepsilon_j n_j + (t_j c_j c_{j+1}^\dagger + \Delta_j c_j c_{j+1} + hc)$$

J.D Sau & S. Das Sarma, Nat. Commun. **3** (2012)
M. Leijnse & K. Flensberg, PRB **86** (2012)



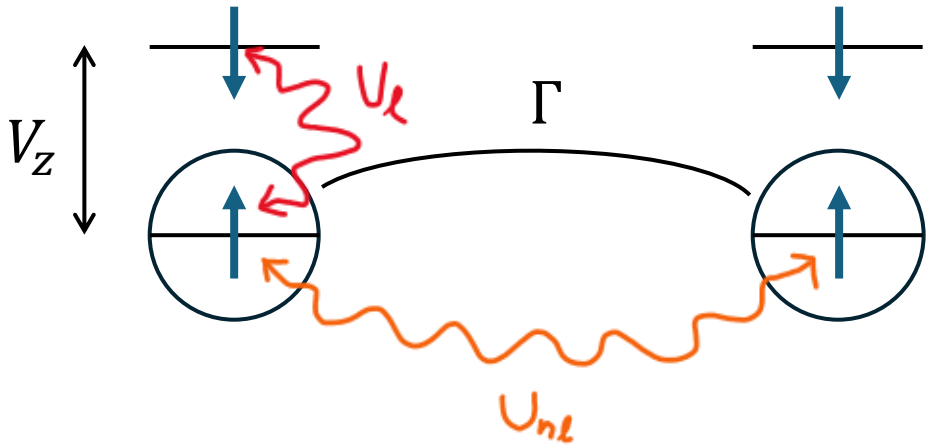

T. Dvir, *et al.*, Nature **614** (2023)
+ many more

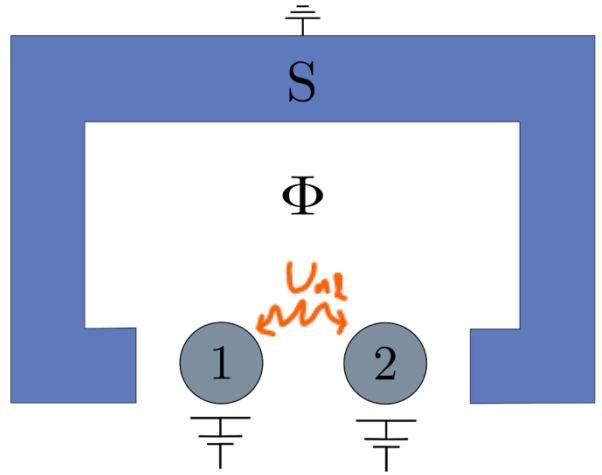
# Kitaev chains in quantum dots




Kitaev limit $V_z \gg \Gamma$

# ”Kitaev chains” in quantum dots



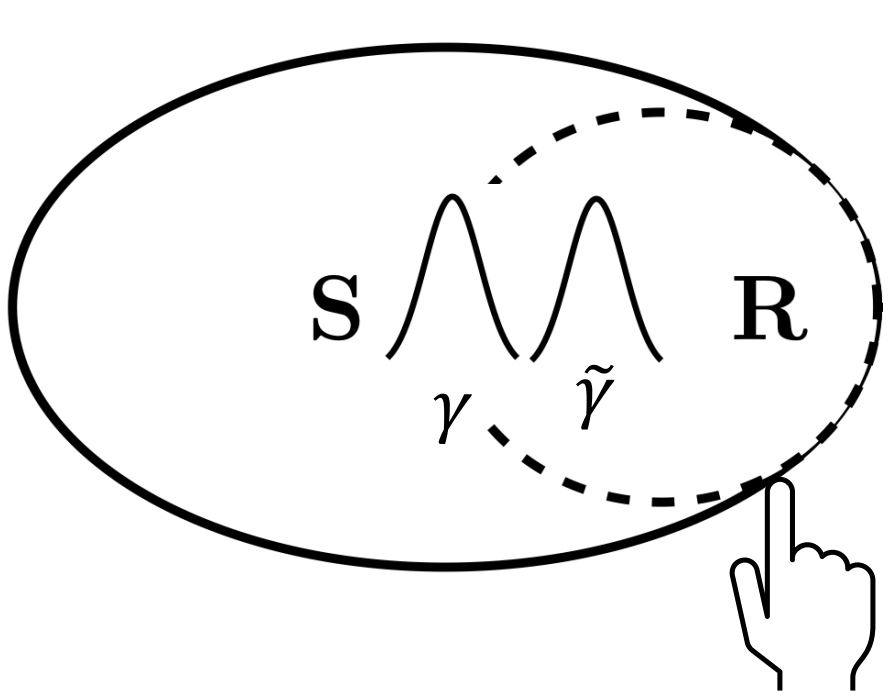

**WS**, V. Svensson, M. Leijnse, PRB **109**, (2024)

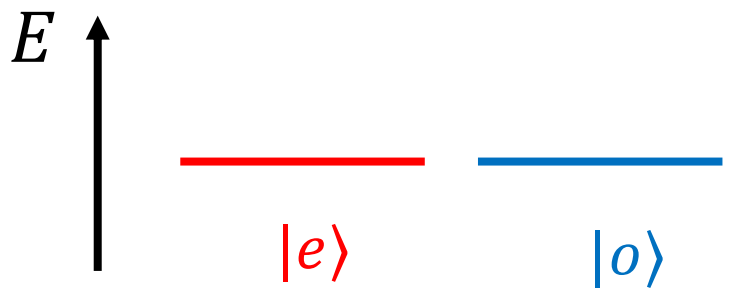
# Goal

In a general interacting setting

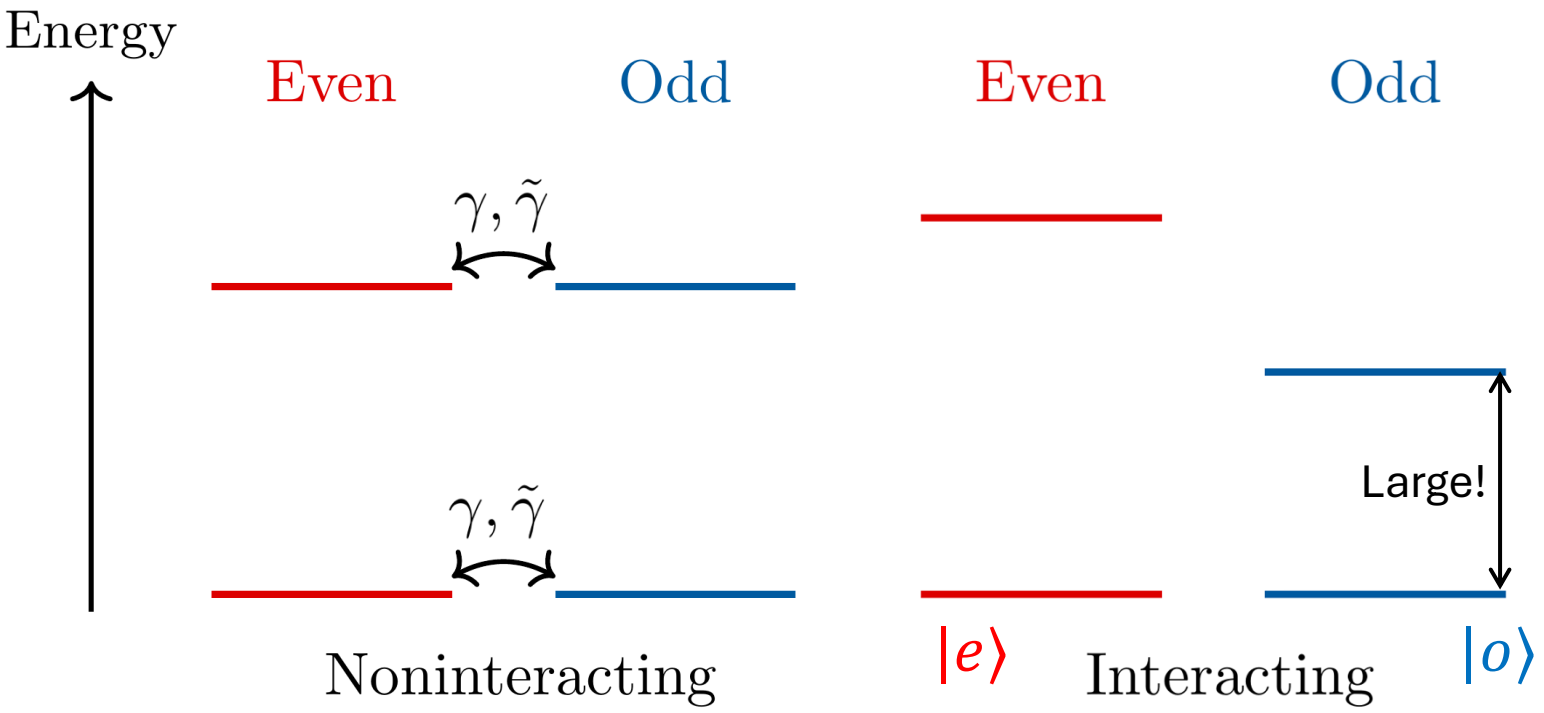
- How to define Majoranas?
- How to quantify their locality?
- … in a useful way!

# Goal



# Majoranas in interacting systems



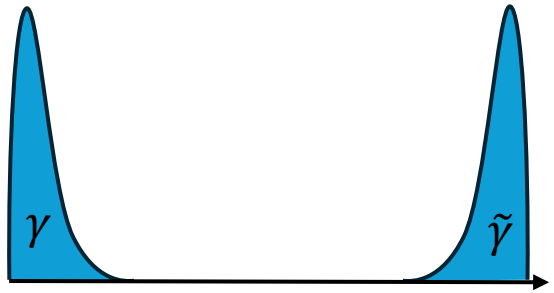

BdG-Majoranas

Ground-state Majoranas

$$\gamma = |e\rangle\langle o| + |o\rangle\langle e|$$

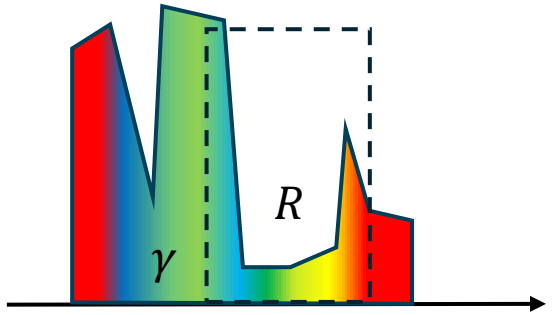
$$\tilde{\gamma} = i(|e\rangle\langle o| - |o\rangle\langle e|)$$

# Locality of ground-state Majoranas




Noninteracting

BdG-Majoranas

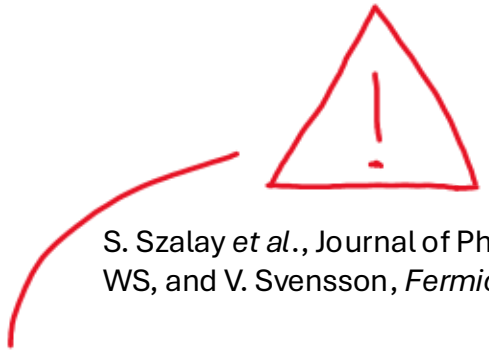



Interacting

$$\gamma_R = \mathrm{Tr}_{\bar{R}}(\gamma)$$

$$\gamma = |e\rangle\langle o| + |o\rangle\langle e|$$
$$\tilde{\gamma} = i(|e\rangle\langle o| - |o\rangle\langle e|)$$

# Ground-state Majoranas



S. Szalay *et al.*, Journal of Physics A (2021)
WS, and V. Svensson, *FermionicHilbertSpaces.jl*

$$\gamma = |e\rangle\langle o| + |o\rangle\langle e|$$
$$\tilde{\gamma} = i(|e\rangle\langle o| - |o\rangle\langle e|)$$
$$i\gamma\tilde{\gamma} = |e\rangle\langle e| - |o\rangle\langle o|$$

$$\gamma_R = \mathrm{Tr}_{\bar{R}}(\gamma)$$
$$(i\gamma\tilde{\gamma})_R = \rho_R^e - \rho_R^o$$

~ local distinguishability

V. Svensson and M. Leijnse, PRB **110**, (2024)

$\|\gamma_R\|, \|(i\gamma\tilde{\gamma})_R\|$ are small ⇒ Protection in R?

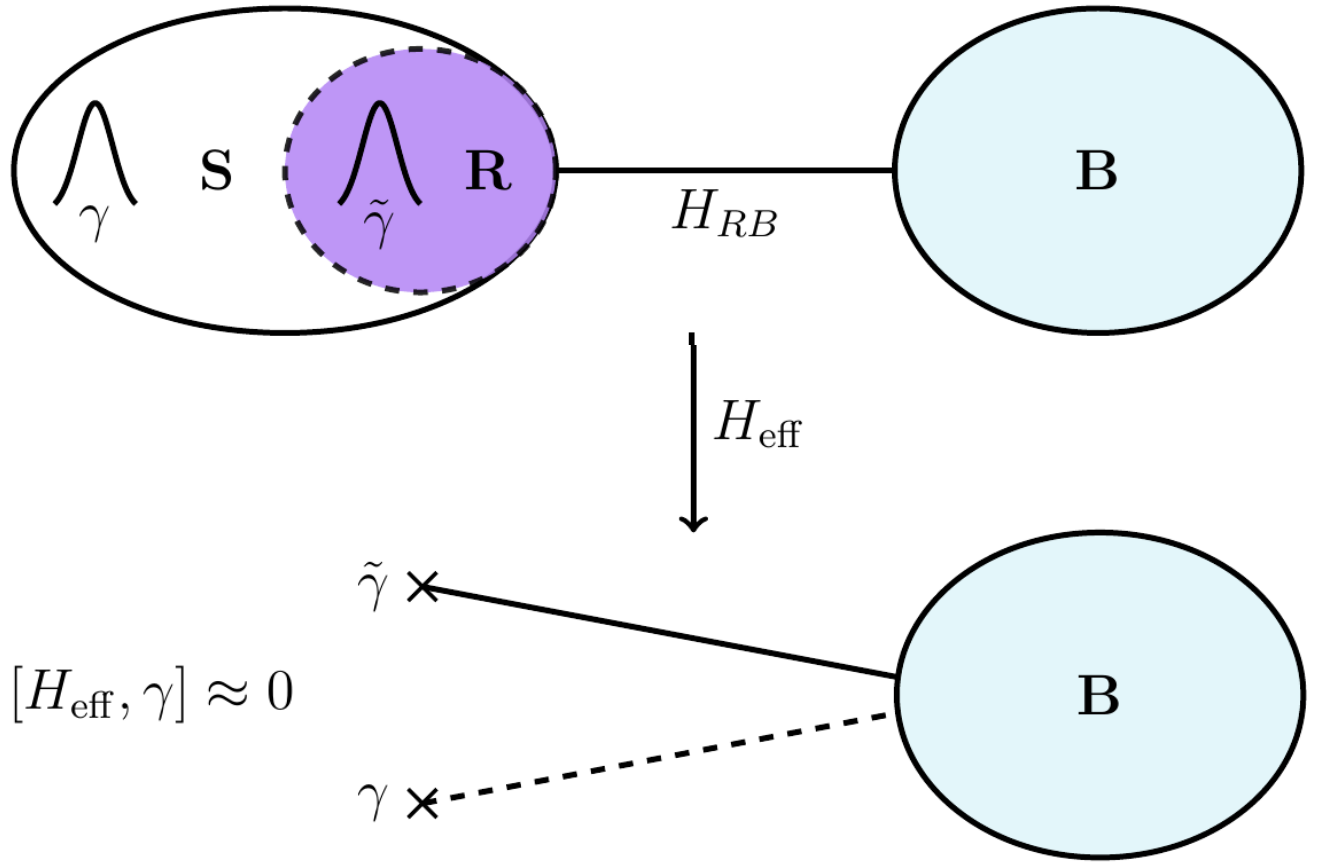
# Robustness

$$H = H_S + H_{RB} + H_B,$$

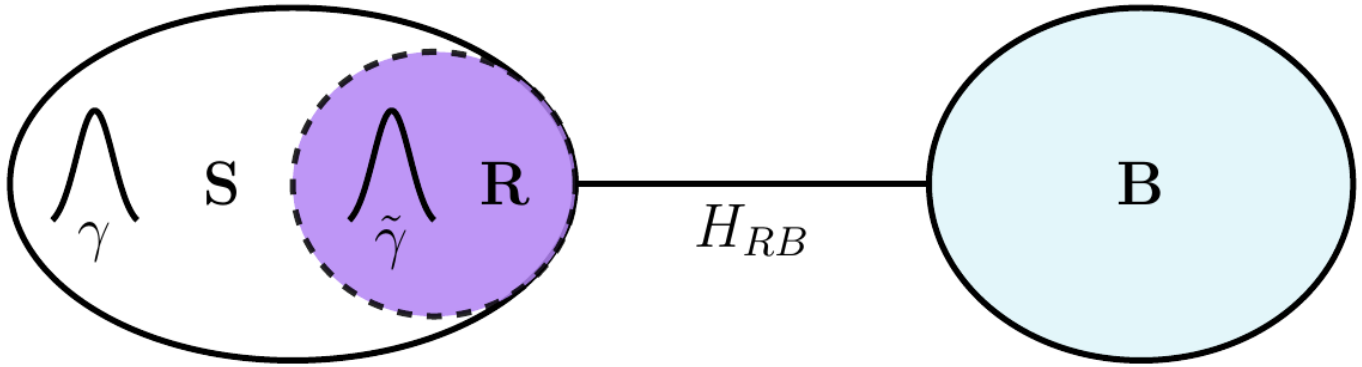
$$H_{RB}^e + H_{RB}^o.$$

E.g., Coulomb or chemical potential shifts

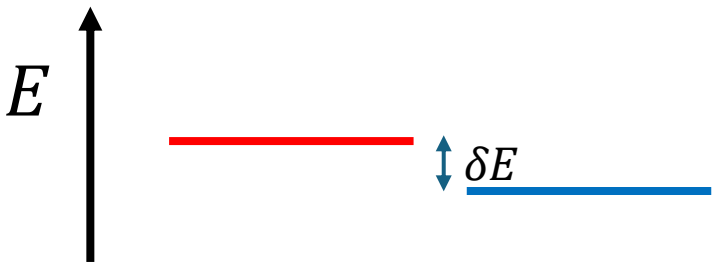
E.g., tunneling



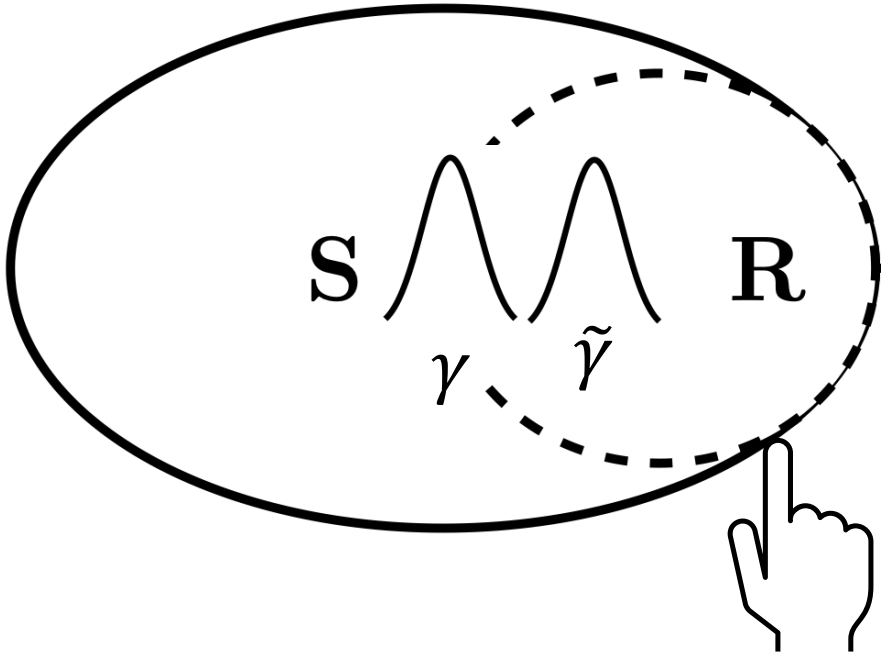
# Robustness

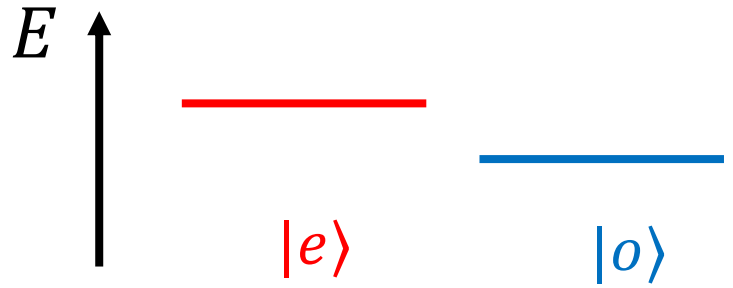



$$|\delta E| \leq \sqrt{d_B}\left(\|\gamma_R\|\|H^o_{RB}\| + \|(i\gamma\tilde{\gamma})_R\|\|H^e_{RB}\|\right)$$



# Main message



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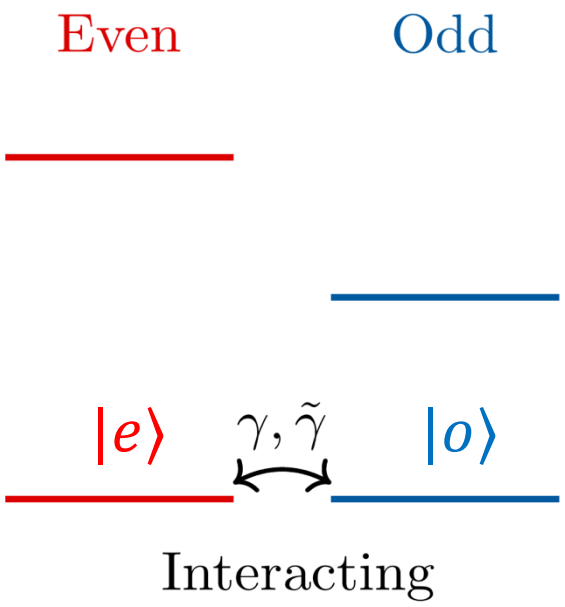


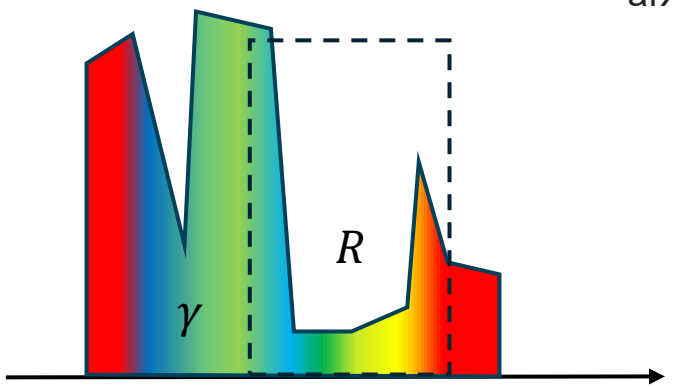


$$\gamma_R = \mathrm{Tr}_{\bar{R}}(\gamma)$$

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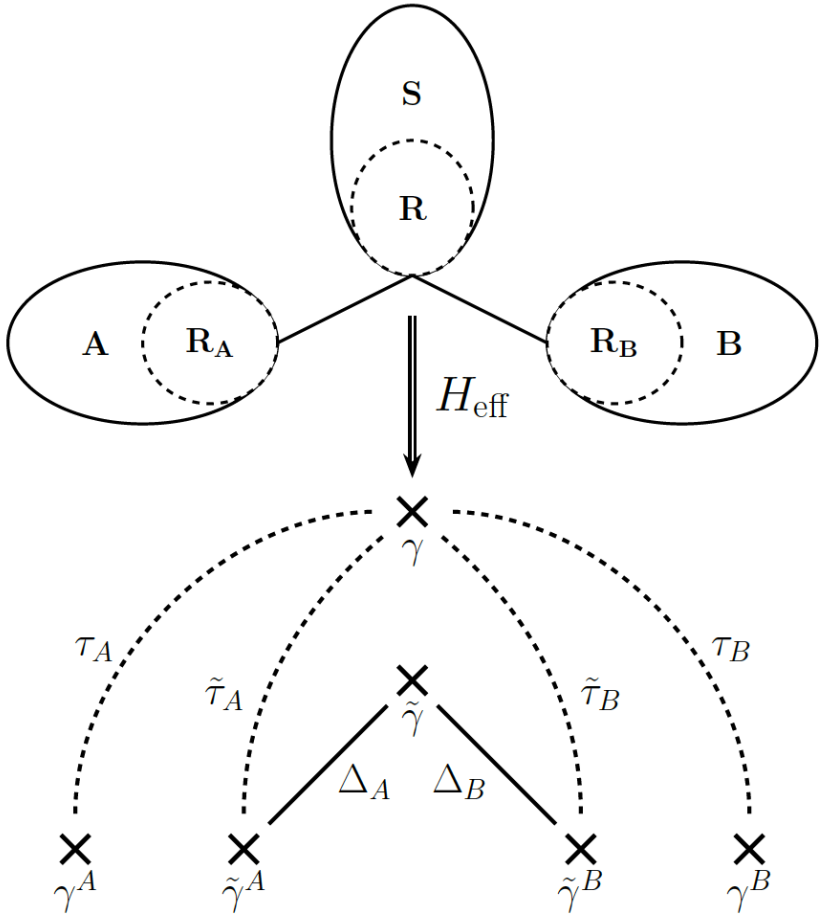
# Extra: Braiding




$$|\tau_X| \leq \left\| H^o_{RR_X} \right\|_p \|\gamma_R\|_q \|(\gamma^X)_{R_X}\|_q,$$

# Extra: Interacting Kitaev chain

$$H_K = \sum_{j=1}^{N} \mu_j n_j + \sum_{j=1}^{N-1} (t f_j^\dagger f_{j+1} + \Delta f_j^\dagger f_{j+1}^\dagger + \text{h.c.}) + U n_j n_{j+1},$$

$$t = \Delta = U/2,$$

$$\mu_j \approx \begin{cases} -2.73t \text{ at the edges,} \\ -5.46t \text{ in the bulk,} \end{cases}$$

$$N = 8 \text{ sites}$$

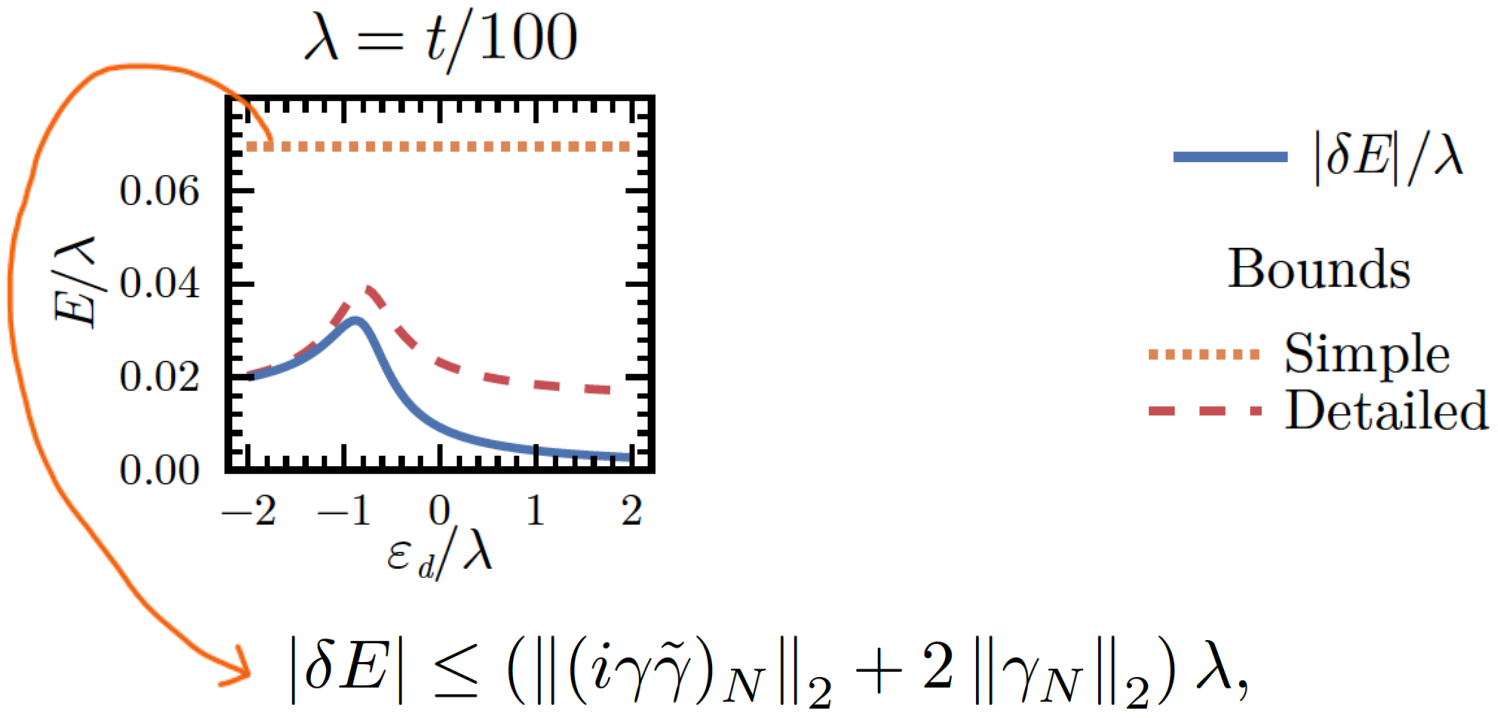
# Testing our bounds – Coupling to QD


$$H = H_K + (t_c f_N f_B^\dagger + \Delta_c f_N f_B + \text{h.c.}) + U_c n_N n_B + \varepsilon_d n_B,$$

$$t_c = \lambda \cos\phi,$$
$$\Delta_c = \lambda \sin\phi,$$
$$U_c = \lambda,$$

$$|\delta E| \leq \left( \left\| (i\gamma\tilde{\gamma})_N \right\|_2 + 2 \left\| \gamma_N \right\|_2 \right) \lambda,$$

# Testing our bounds – Coupling to QD


$$|\delta E| \leq \left( \left\| (i\gamma\tilde{\gamma})_N \right\|_2 + 2 \left\| \gamma_N \right\|_2 \right) \lambda,$$