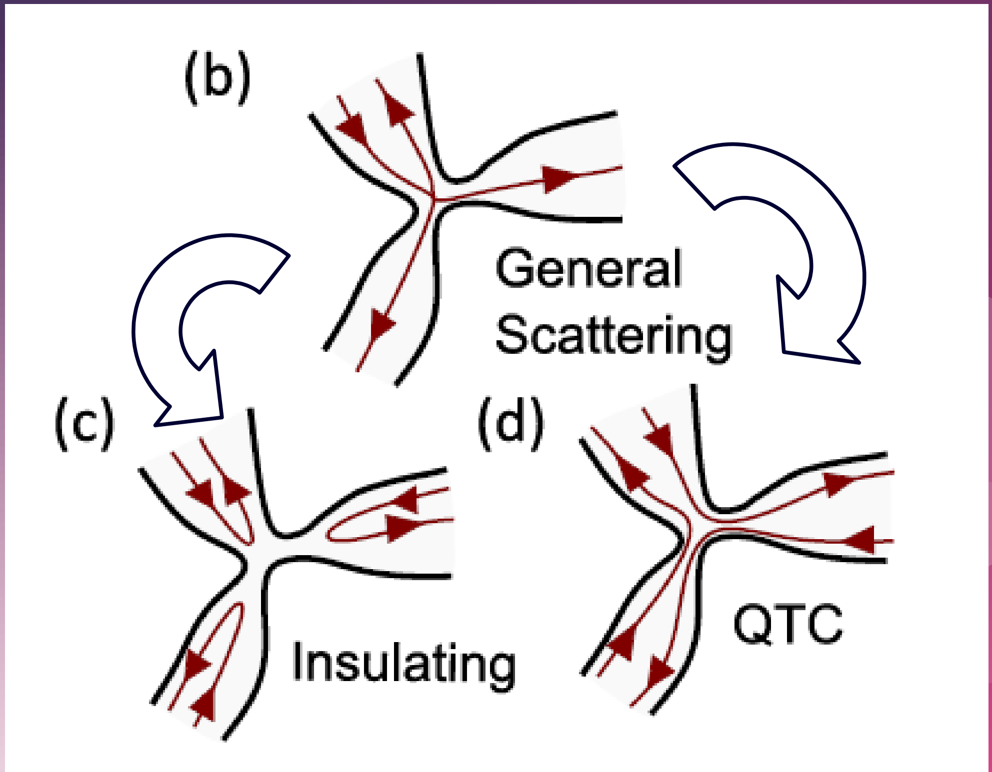
Quantized transconductance emerging from non-symmetric quantum fluctuations

Klaiv Mertiri, Yuli V. Nazarov
Delft University of Technology

Outline

- Quantum transport in the presence of electomagnetic environment = quantum fluctuations
- History => isolation
  - Tunnel junctions
  - Arbitraty transparency
  - General circuits
- New page
  - Multi-terminal
  - QHE phenomenology from asymmetric quantum fluctuations
  - Making an environment?
  - Some new results



Long history

- 1987(?)
- Environment affects the transport properties
- No locality!
- Amusing!

MEASURING THE TIME SPENT TRAVERSING THE BARRIER WHILE TUNNELING

John M. MARTINIS, Michel H. DEVORET, Daniel ESTEVE and Cristian URBINA

Service de Physique du Solide et de Résonance Magnétique, CEA-Saclay, 91191 Gif-sur-Yvette Cedex, France

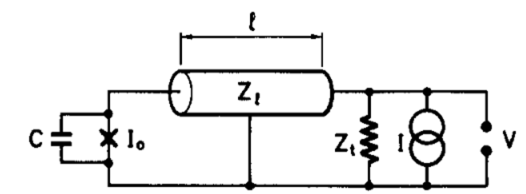
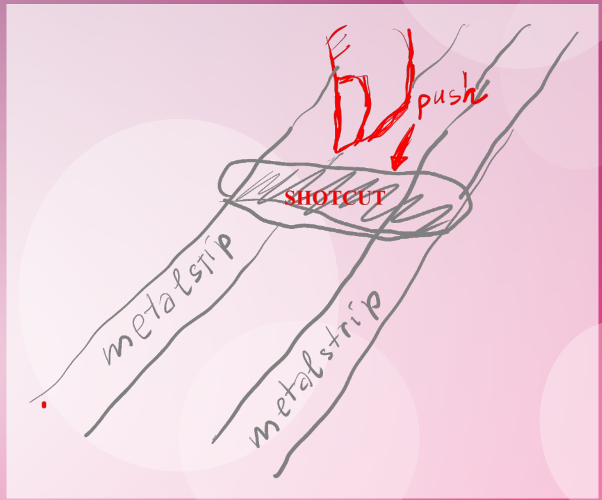


Fig. 1. Schematic representation of a current-biased Josephson junction connected to a transmission line of adjustable length l.



Tunnel junction: Anderson catastrophe

- Tunneling: electrons go one by one

S. M. Girvin, L. I. Glazman, M. Jonson, D. R. Penn, and M. D. Stiles, Quantum fluctuations and the single-junction coulomb blockade, Phys. Rev. Lett. **64**, 3183 (1990).

Y. V. Nazarov, Anomalies of current voltage characteristics of tunnel-junctions, Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki **95**, 975 (1989).

Effect of the Electromagnetic Environment on the Coulomb Blockade in Ultrasmall Tunnel Junctions

M. H. Devoret,[1] D. Esteve,[1] H. Grabert,[2] G.-L. Ingold,[2] H. Pothier,[1] and C. Urbina[1]

[1]*Service de Physique du Solide et de Résonance Magnétique, Centre d'Études Nucléaires de Saclay, 91191 Gif-sur-Yvette, France*

[2]*Fachbereich Physik, Universität–Gesamthochschule Essen, 4300 Essen, Federal Republic of Germany*

(Received 4 December 1989)

[Submitted on 28 Jan 1995 (v1), last revised 11 Jul 1995 (this version, v2)]

Coulomb Zero-Bias Anomaly: A Semiclassical Calculation

L. S. Levitov, A. V. Shytov

- Exact solution for an arbitrary environment $Z(\omega)$
- Let's keep it simple:

transmission amplitude: $t \to t e^{i\varphi}$ $\quad \varphi \to$ **voltage difference**

averaged over fluctuations $t\langle e^{i\delta\varphi}\rangle = t\exp\left(-\frac{\langle(\delta\varphi)^2\rangle}{2}\right)$ **reduced**

ohmic impedance

fluctuation $\langle(\delta\varphi)^2\rangle = \frac{e^2}{\hbar}\int\frac{d\omega}{\omega} Z \to \infty \to$ **insulator -> Anderson catastrophe**

finite E: cuts the integral $T = T_0 \exp\left(2Z \ln\left(\frac{E_{high}}{E_0}\right)\right)$ **convenience ->renormalization**

$\xi \equiv \ln\frac{E_{high}}{E}$;

$$\boxed{\frac{dT}{d\xi} = -2ZT}$$

Arbitrary transmission: the same?

- Not always tunneling: finite transmission T
- Several levels more complex problem
- Answer at small impedance z

$$\frac{dT}{d\xi} = -2zT(1-T)$$

L. W. Molenkamp, K. Flensberg, and M. Kemerink, Scaling of the coulomb energy due to quantum fluctuations in the charge on a quantum dot, Phys. Rev. Lett. **75**, 4282 (1995).

Coulomb Blockade without Tunnel Junctions 1999

Yuli V. Nazarov

Faculty of Applied Physics and DIMES, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands

- Same insulation at low energy

Complex nanostructures: same insulation

- Any complex nanostructure= scattering matrix
- For 2 terminals: separated into transport channels

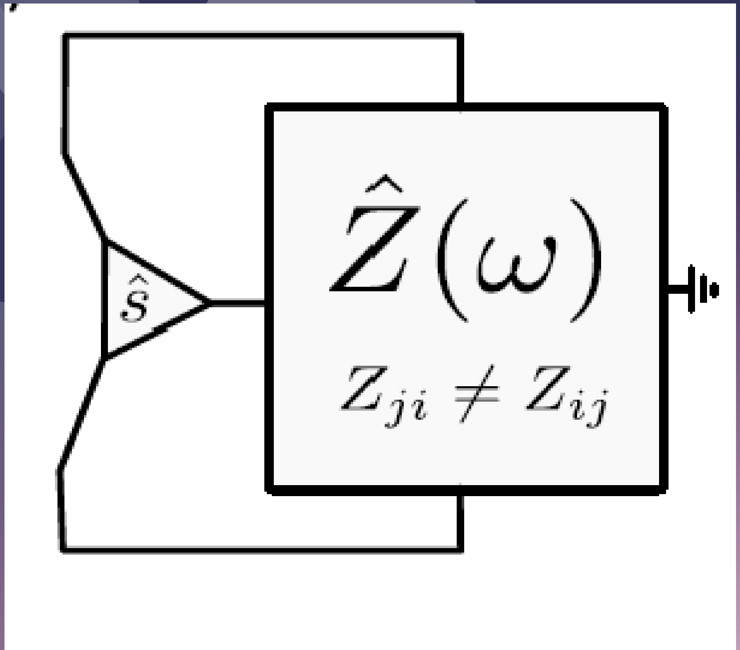
T_n

M. Kindermann and Y. V. Nazarov, Interaction effects on counting statistics and the transmission distribution, Phys. Rev. Lett. **91**, 136802 (2003).

$$\frac{dT_n(E)}{d\ln E} = 2z T_n(E)[1 - T_n(E)].$$

- Two alternative low-energy scenarios:
  - Single tunnel junction (from tunnel-like nanostructures)
  - Double tunnel junction (from diffusive conductor)

New page: multiterminal junction and (multi-terminal) environment



- Goal: to derive a renormalization equation
  - Small impedance z
  - Arbitrary s
  - Did not expect anything fancy
- Method
  - Full counting statistics
  - Interaction correction to it

Klaiv Mertiri,
Yuli V. Nazarov
arXiv:2512.05813

Renormalization equation

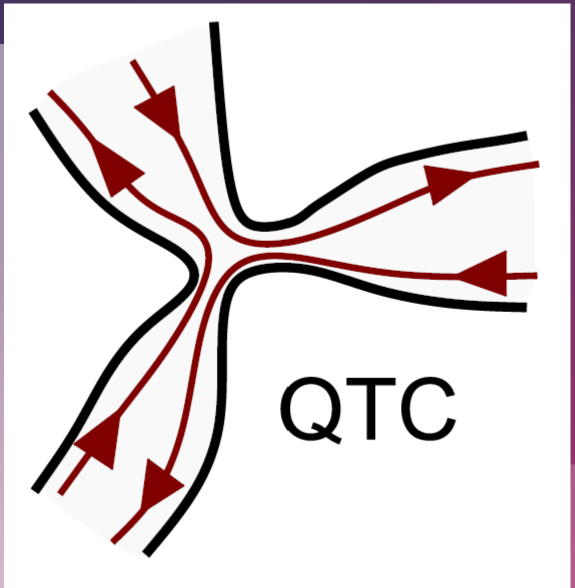
$$\frac{ds_{ij}}{d\xi} = \overbrace{z_{ji}s_{ij}}^{\text{already seen}} - \sum_{k,l} \underbrace{z_{kl}s_{ik}(s^\dagger)_{kl}s_{lj}}_{\text{to keep unitarity}}.$$

- Fixed points
  - Insulating: diagonal s
  - Stable:

$$z_{ii} + z_{jj} - z_{ij} - z_{ji} > 0 \text{ for any } i, j$$

positive dissipation in the env.

Permutation fixed points



- Alternative:
  - S is a permutation matrix
  - Implies quantized transconductance
  - Stable if

$$(\hat{z}\hat{P})_{ii} + (\hat{z}\hat{P})_{jj} - (\hat{z}\hat{P})_{ij} - (\hat{z}\hat{P})_{ji} > 0 \text{ for any } i, j$$

  - Some intuition: renormalization is like a friction motion
  - In pseudopotential ε:

$$\frac{d\hat{s}}{d\xi} = -\frac{\delta\varepsilon}{\delta\hat{s}^\dagger}; \quad \varepsilon(\hat{s}) = -\sum_{i,j} z_{ji}|s_{ij}|^2,$$

  - Large asymmetries
    - Minimum lower than the insulating point

Related work in Luttinger liquids:
Junctions of Three Quantum Wires and the Dissipative Hofstadter Model
Claudio Chamon, Masaki Oshikawa, and Ian Affleck
Phys. Rev. Lett. 91, 206403 – Published 13 November, 2003

3 terminals: criterion

a single parameter $Z_a = Z_{21}^A + Z_{32}^A + Z_{13}^A$, Z^A being the anti-symmetric part of Z, and read

$$P_{\pm 1} \text{ is stable provided } \rightarrow \pm Z_a > X_1, X_2, X_3$$

Here $X_1 = Z_{11}^S + Z_{23}^S - Z_{13}^S - Z_{12}^S$, Z_S is the symmetric part of $\hat{Z}$, and X_2, X_3 are obtained by index permutation.

Stable at sufficiently asymmetric matrices

We made some random choice: $\hat{z} = z(\hat{S} + c\hat{A})$

$$\hat{S} = \begin{bmatrix} 0.4 & 0.4 & -0.0 \\ 0.4 & 0.8 & -0.6 \\ -0.0 & -0.6 & 1.6 \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} 0 & -1.0 & 0.4 \\ 1.0 & 0 & -0.8 \\ -0.4 & 0.8 & 0 \end{bmatrix}$$

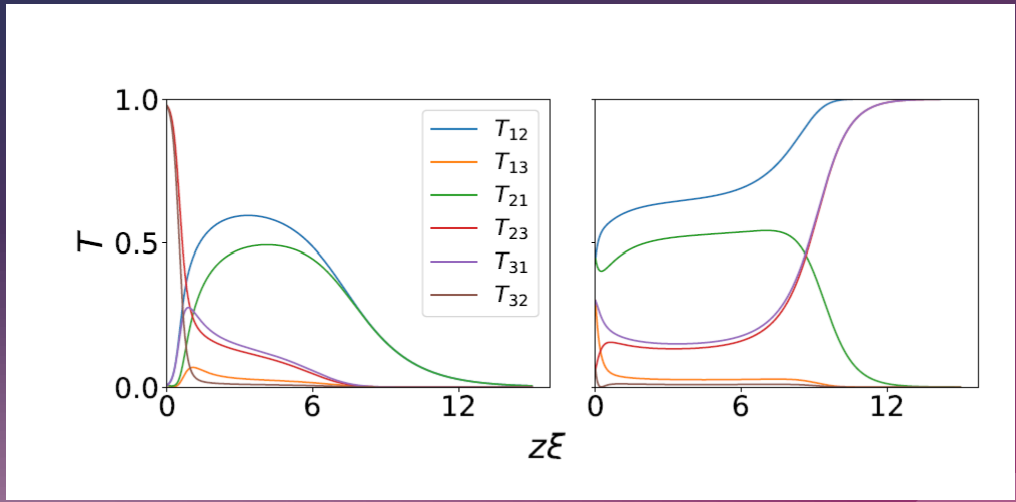
For this choice:

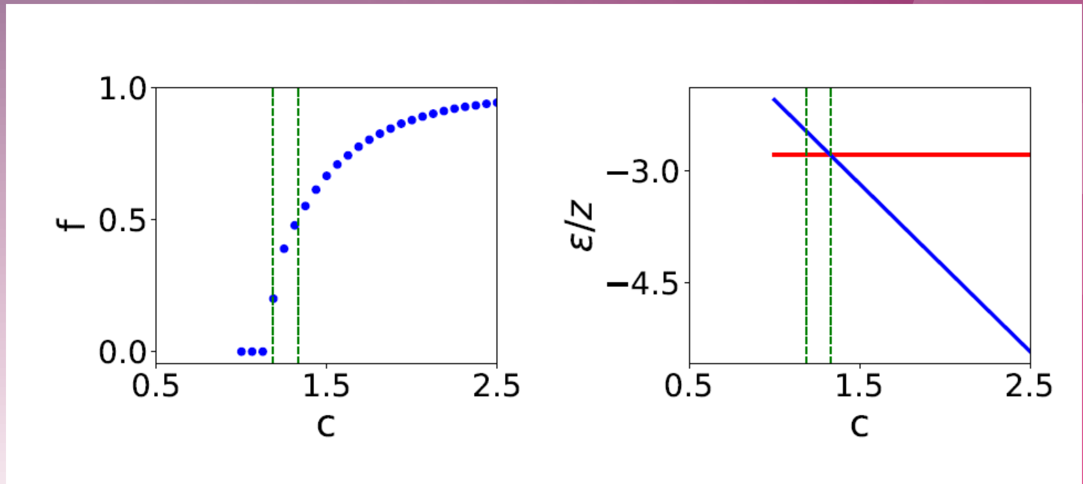
Threshold : c = 1.19

Absolute minimum:
c = 1.33

3 terminals: plots

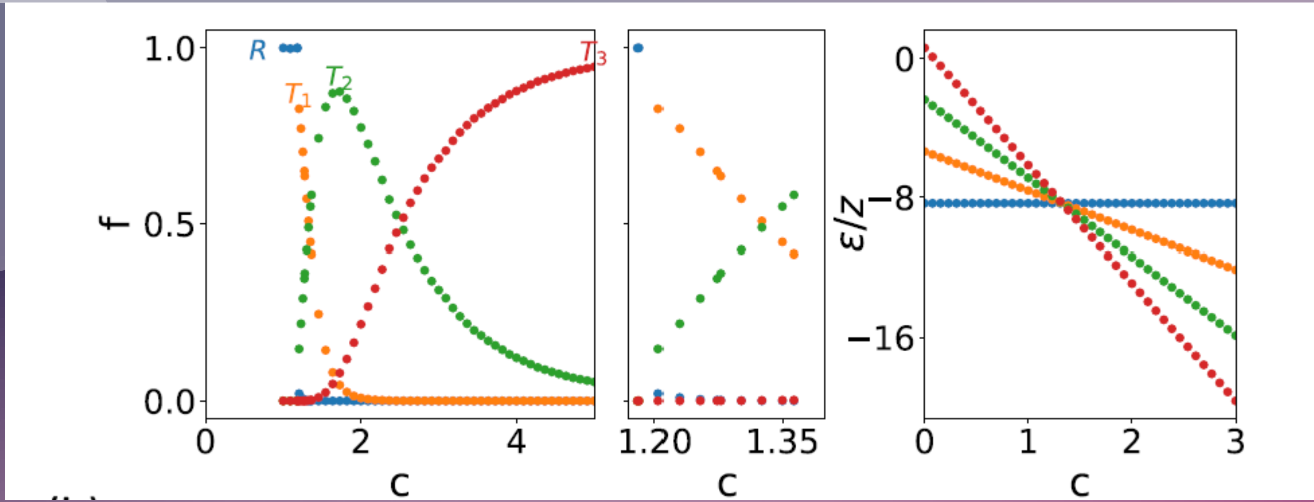
- Renormalization trace: Depends on initial conditions (c=1.5)
-
- Fraction of random matrices going to the permutation point



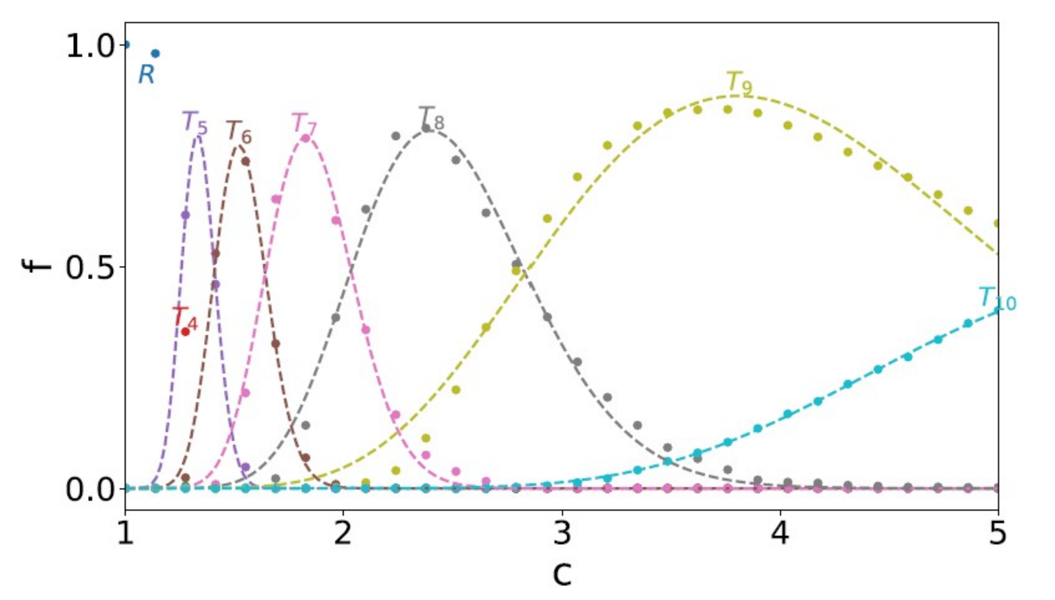


3 terminals: more channels

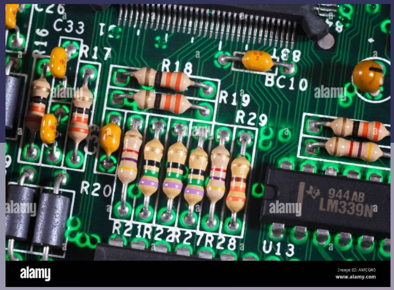
- Three channels per terminal: bigger transconductance



- 10 channels per terminal: gradually changing transconductance



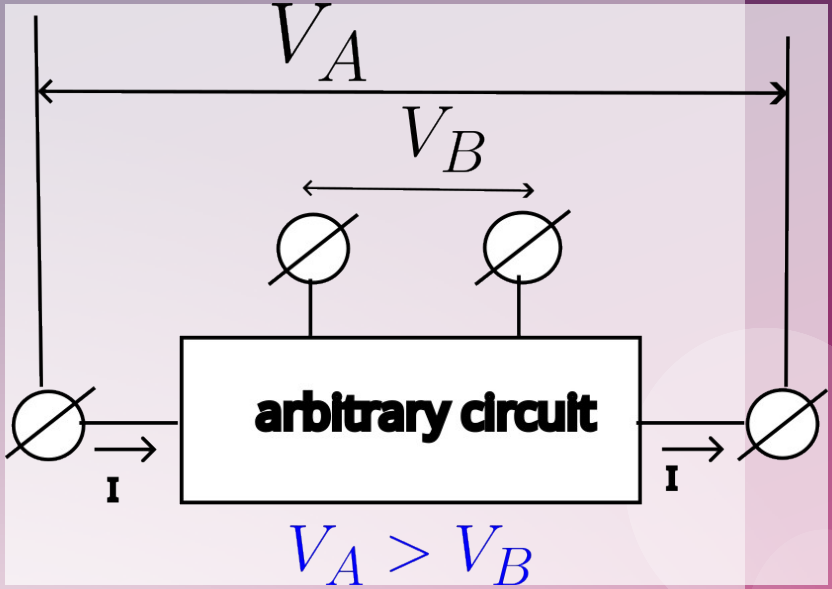
How to make such environment?



- Why bother? **Solder**

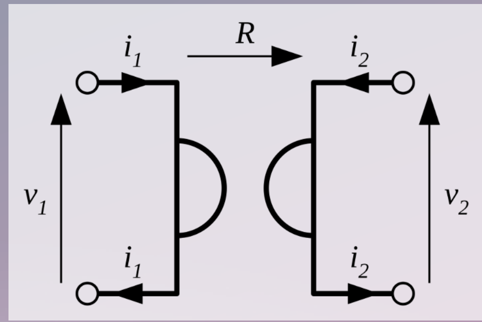
- The problem: **Non-amplification property**
- Not for any circuit, but…
-

- **Incompatible with permutation f.p. !**
- What for environment we need?
  - Passive (peace of cake if active, but …)
  - Broad-band (since renormalization)
  - Amplifying
  - Asymmetric(Hall elements)

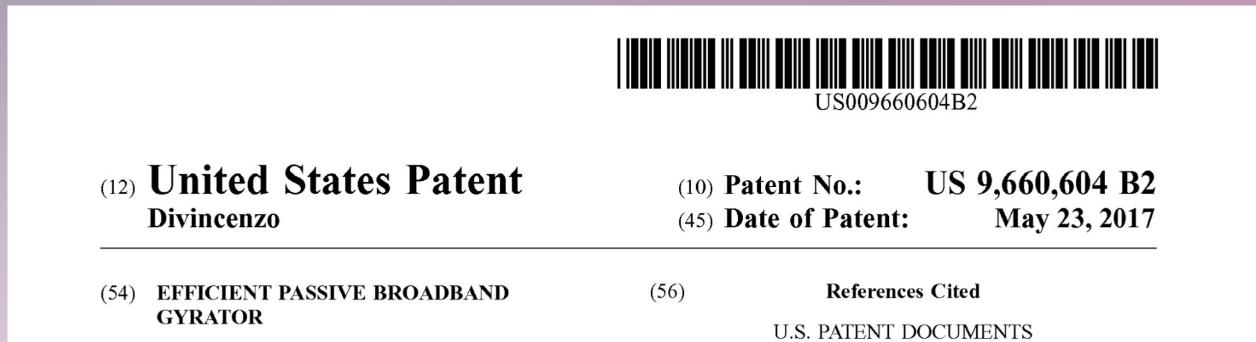


Gyrators

- **Gyrator (TU Delft, 1948)**
  - Antisymmetric
  - Passive
  - Amplifying
  - Would work
  - Unavailable :(



$$\check{Z} = R\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$


US009660604B2

(12) United States Patent
Divincenzo

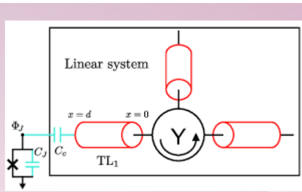
(10) Patent No.: US 9,660,604 B2
(45) Date of Patent: May 23, 2017

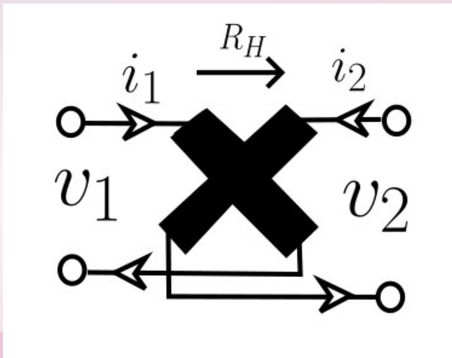
(54) EFFICIENT PASSIVE BROADBAND GYRATOR

(56) References Cited

U.S. PATENT DOCUMENTS

- **Avalilable:**
  - Transmission lines, circulators
    - not broad-band
  - Good Hall elements
    - Not amplifying

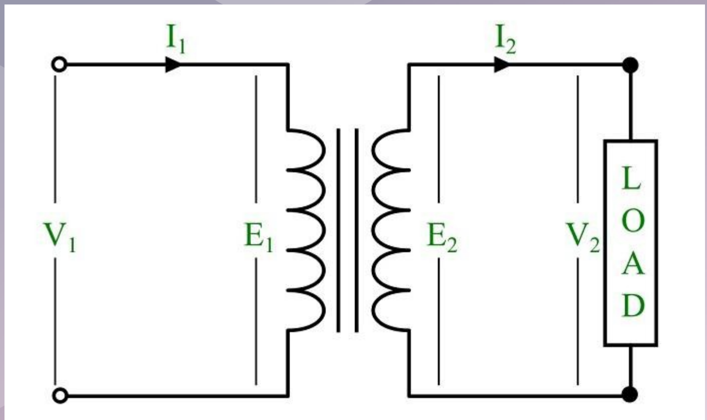




$$\check{Z} = R_H\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$$

Inductance dividers <=>transformers

- Broad-band (above R/L)
- Beats up non-amplification property

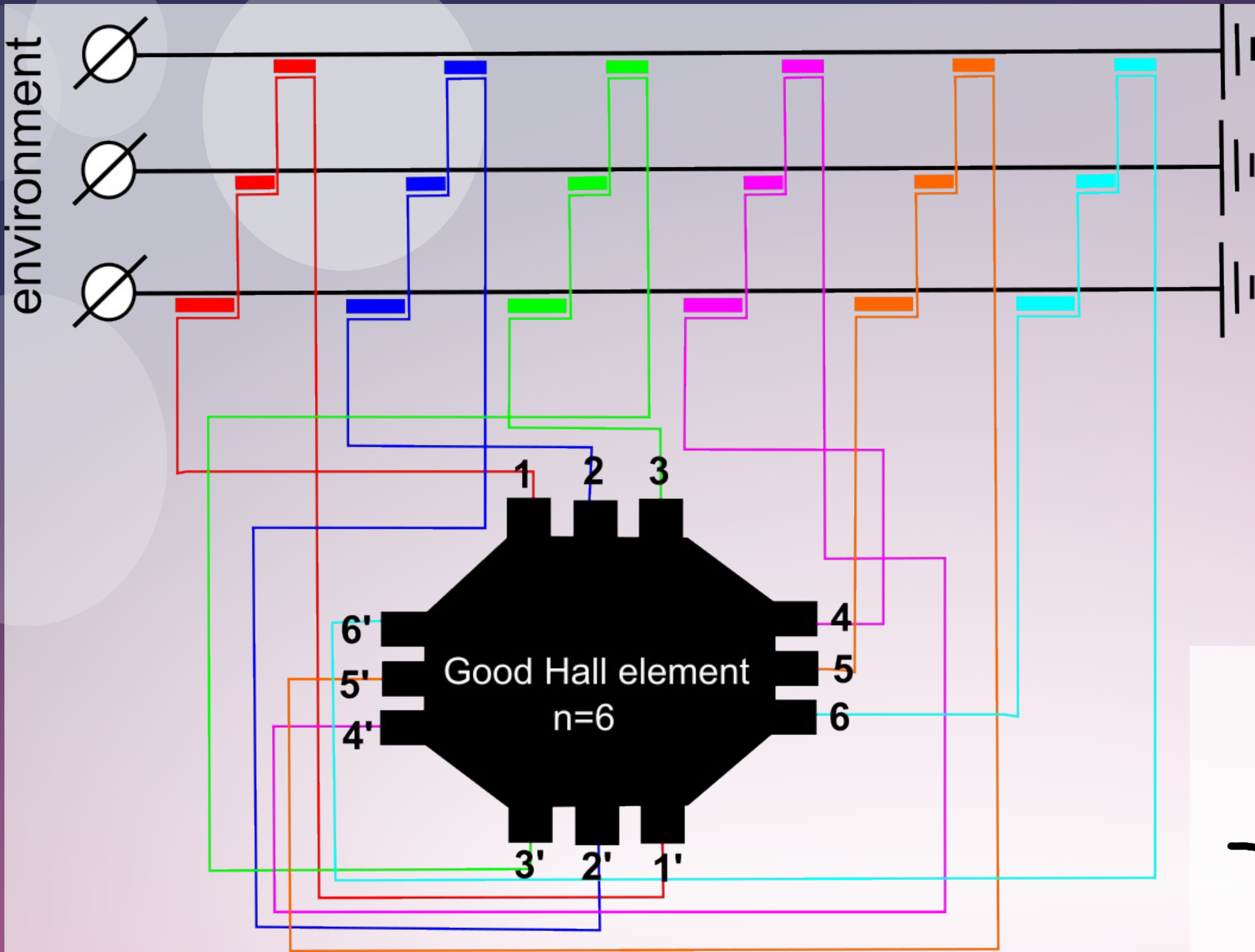


$$Z_1 = \frac{L_1}{L_2} Z_{load}$$

$$\underbrace{\left(\frac{\check{Z}^A}{\check{Z}_s}\right)}_{\text{after transformation}} \simeq \underbrace{\left(\frac{\check{R}_A}{\check{R}_s}\right)}_{\text{before transformation}} \simeq 1$$

- Does not selectively amplify antisymmetric part of the impedance

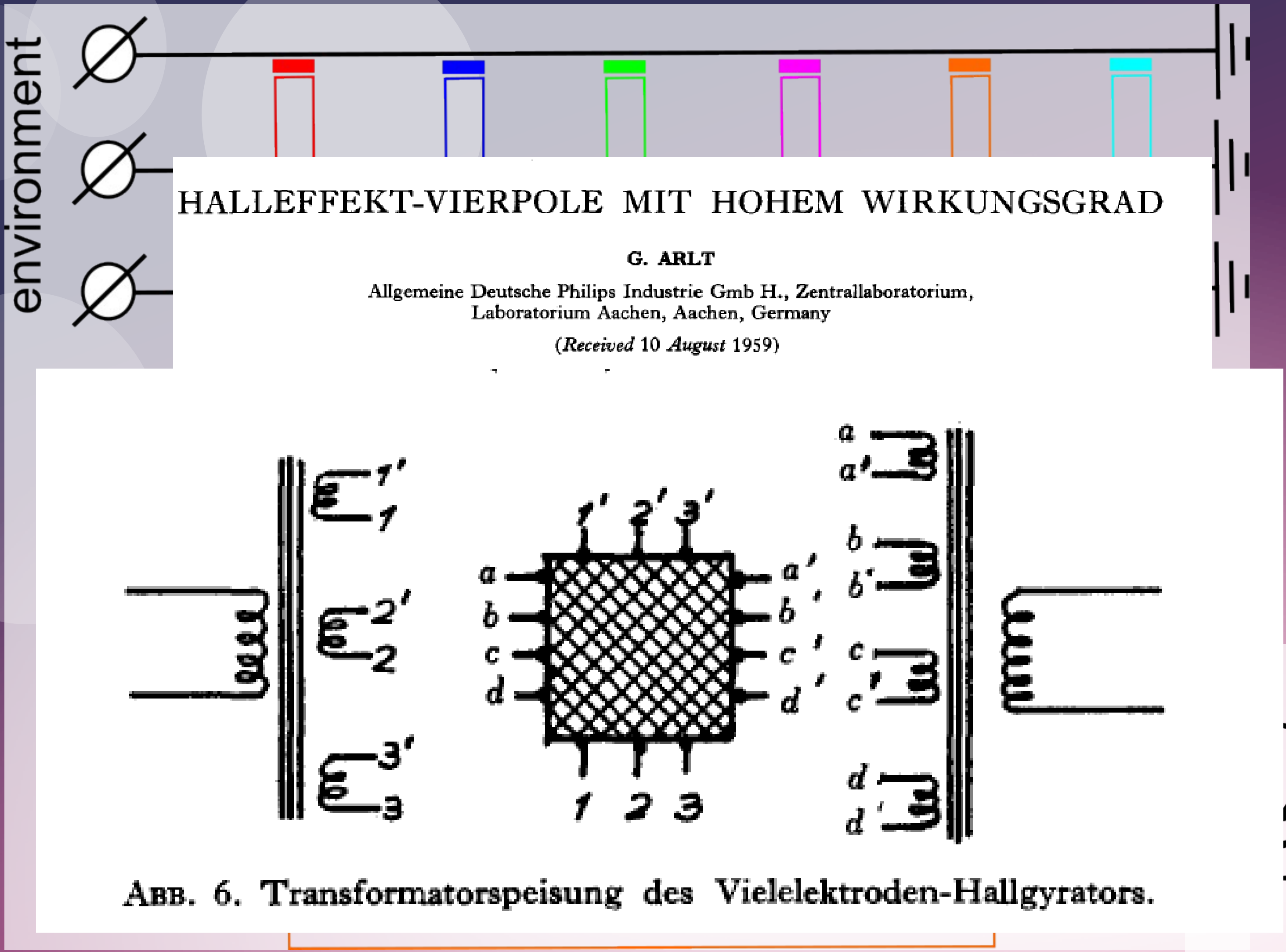
Implementation



- Hall elements with a plenty of contacts
- To achieve high asymmetry

$$\frac{\check{Z}^A}{Z^S} \simeq \frac{2n}{\pi}$$

Implementation oder Durchführung?

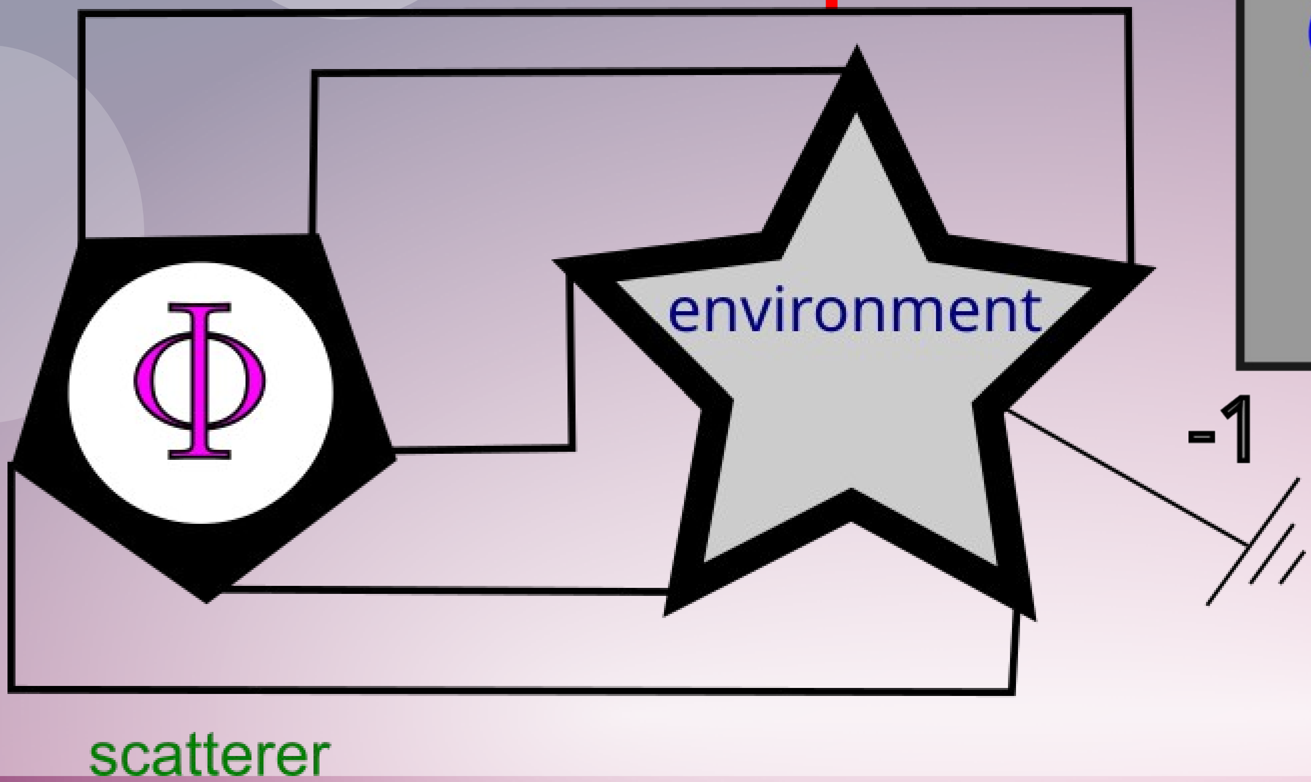


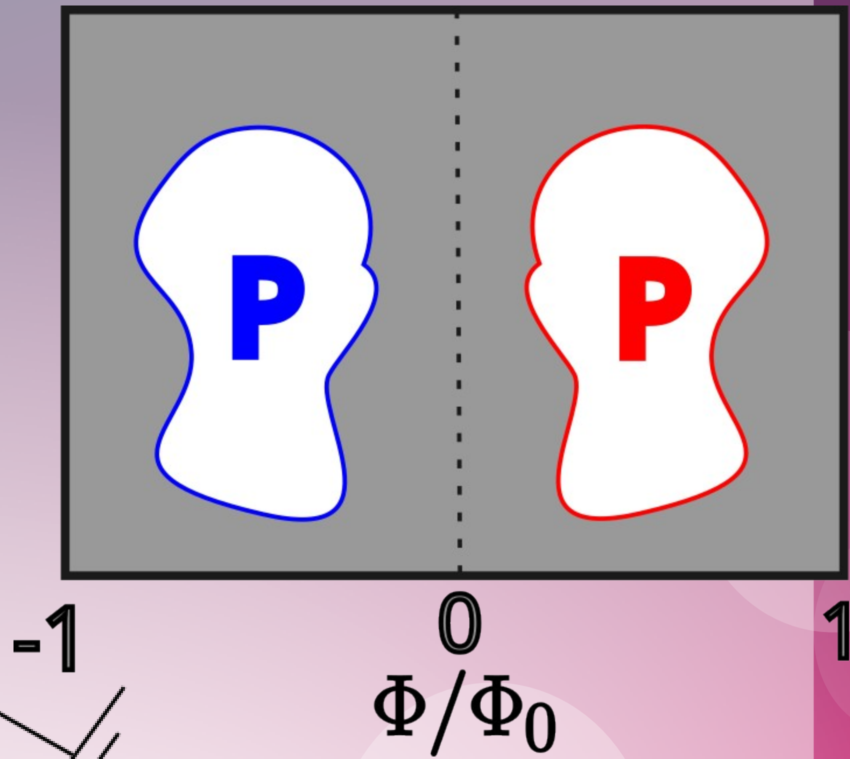
- Hall elements with a plenty of contacts
- To achieve high asymmetry

$$\frac{\check{Z}^A}{\check{Z}^S} \simeq \frac{2n}{\pi}$$

New research: **Penta**

- 5 terminal time-symmetric environment
- 5 terminal circuit with flux
- **Permutation fixed points!**





Conclusions

- Quantum Transport is not yet dead
- Many interesting and profound things still wait to be explored
- One can have QHE without topology
- Quantum fluctuations can lead to complexity
- Challenge of realization