Quantum Hall Superconductor interfaces

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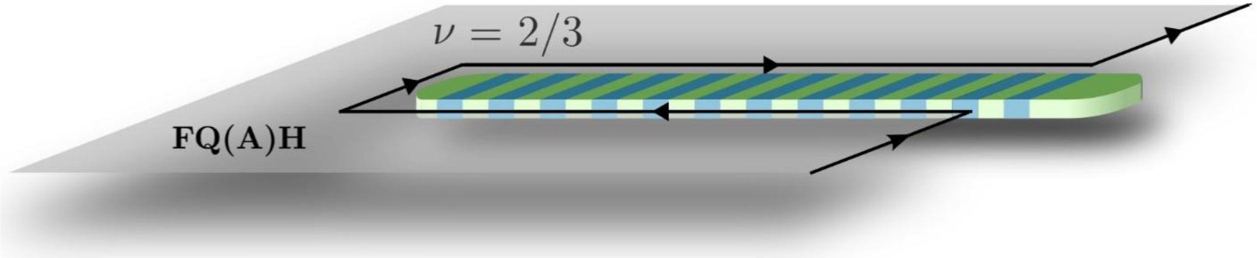
arXiv:2510.26686, arxiv:2604.09463

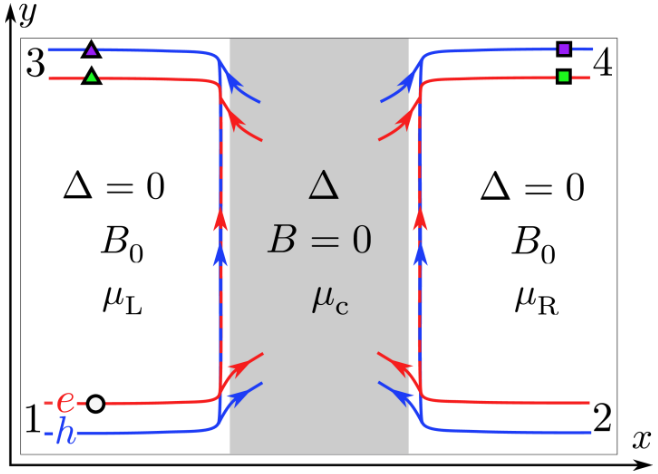
Outline

Phases of QH/SC interfaces with order parameter fluctuations

arXiv:2510.26686



Superconducting Hong-Ou-Mandel interferometry



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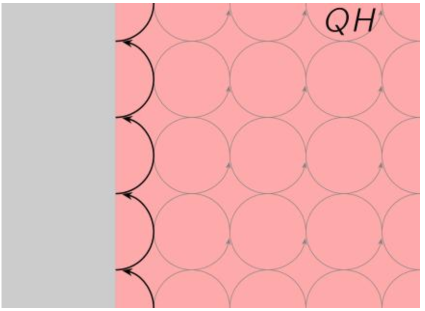
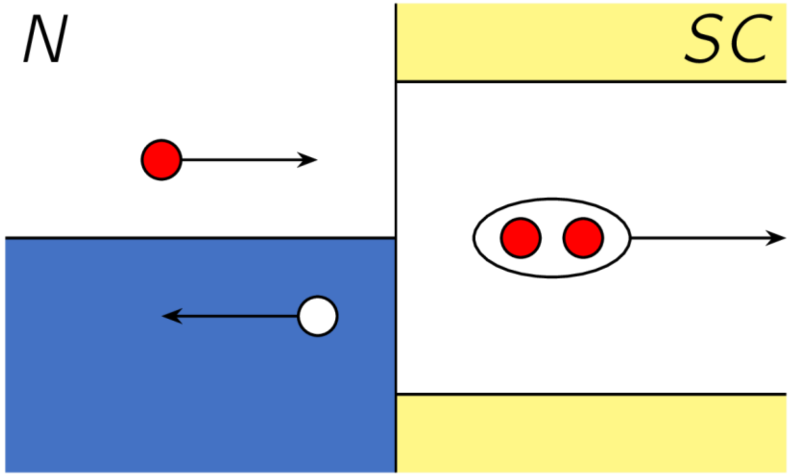
Semiclassical picture

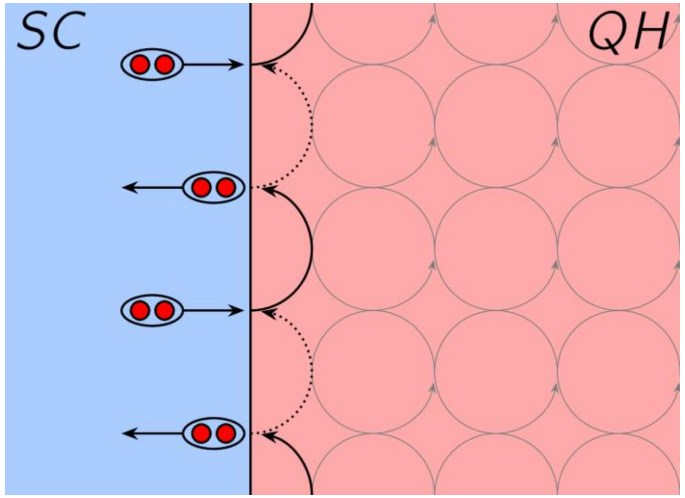
Skipping orbits in quantum Hall systems:



Andreev reflection at the superconductor:



Repeated Andreev reflection gives rise to **chiral Andreev edge states**:



Zyuzin, Phys. Rev. B 50, 323 (1994)
Takagaki, Phys. Rev. B 57, 4009 (1998)
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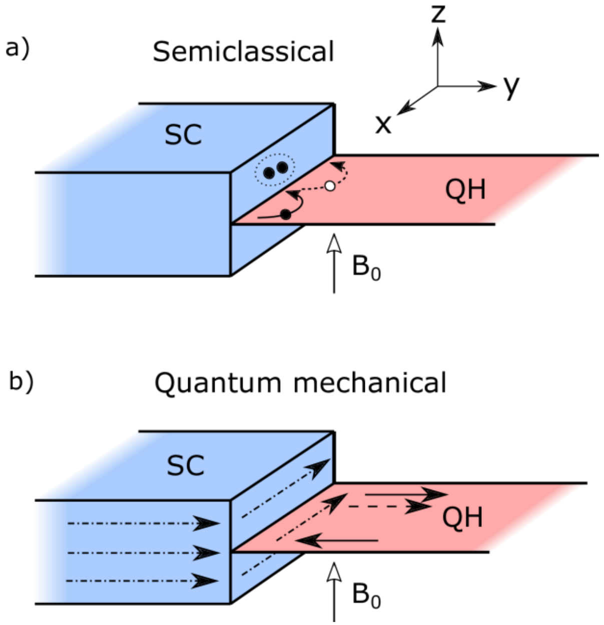
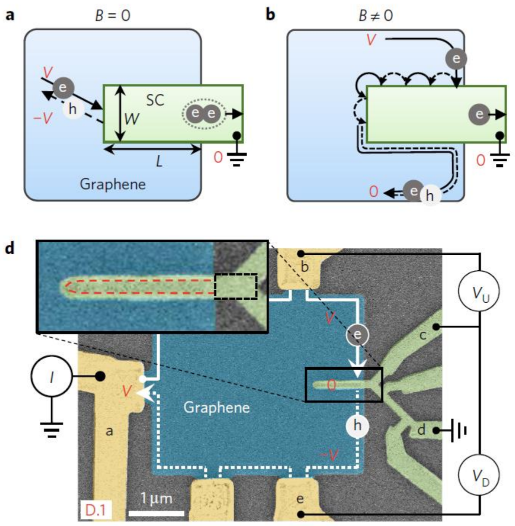
Semiclassical skipping orbits versus quantum mechanical edge states:



Experimental realizations



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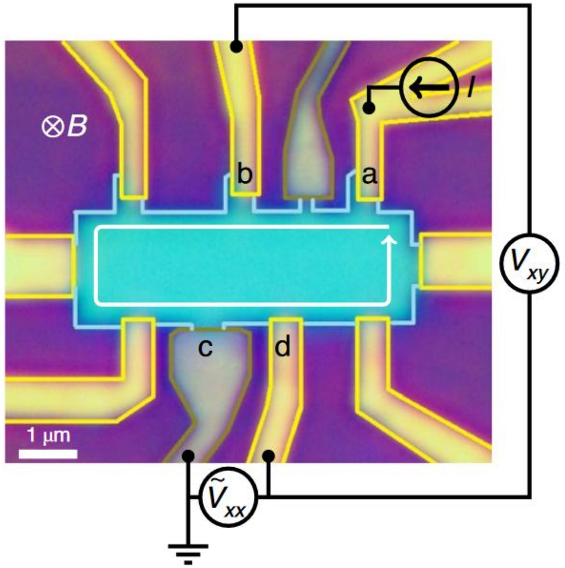
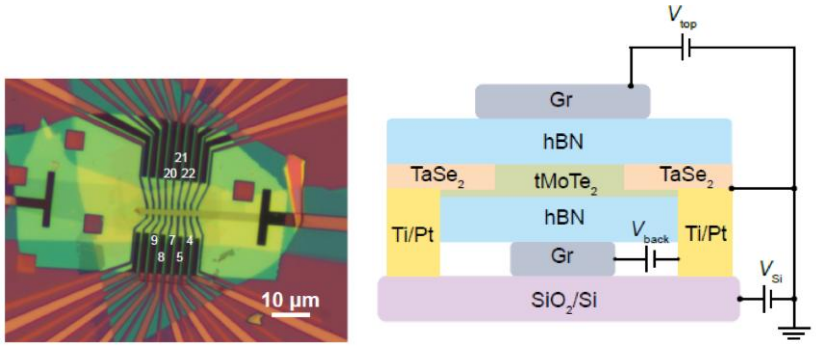


Zhao *et al.* (Finkelstein group),
Nat. Phys. 16, 862 (2020)

Material platforms:
Graphene, InAs as quantum Hall systems
NbN, MoRe as superconductors
Twisted bilayer MoTe2



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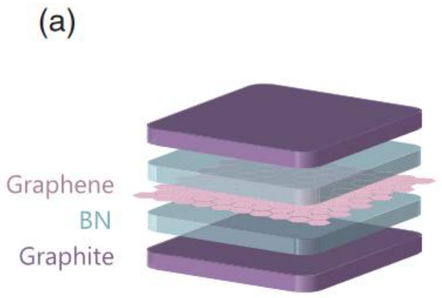

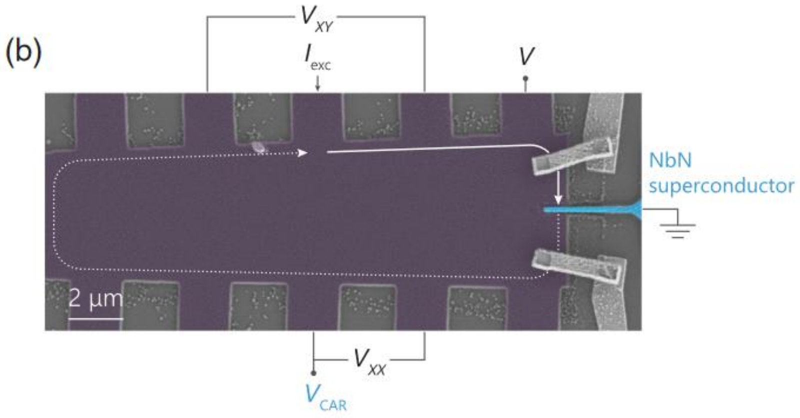

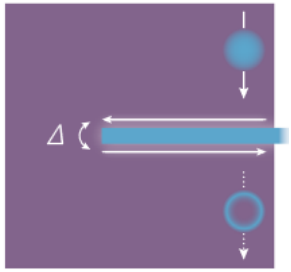

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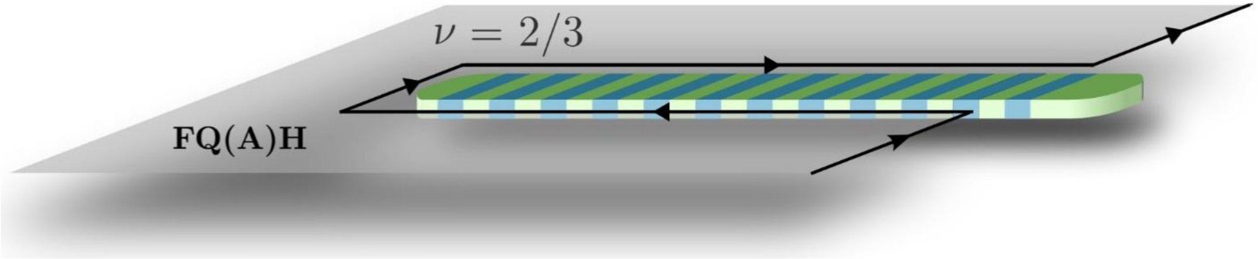
Rickhaus *et al.* (Schönenberger group), Nano Lett. 12, 4, 1942 (2012)

Quantum Hall/Superconductor interfaces



Action for general chiral fractional QH edge states:

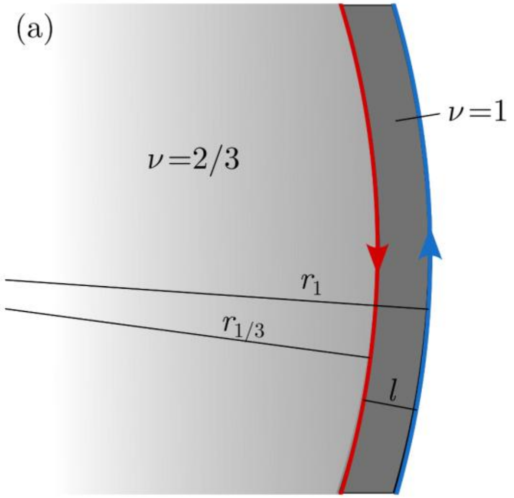
$$S = \frac{1}{4\pi} \int dx d\tau \left(i\partial_\tau \boldsymbol{\phi}^\top \mathcal{K} \partial_x \boldsymbol{\phi} + \partial_x \boldsymbol{\phi}^\top \mathcal{V} \partial_x \boldsymbol{\phi} \right)$$

We are interested in spin-polarized $\nu = 2/3$ states:

- Downstream charge mode: ϕ_ρ
- ~~Upstream neutral mode: ϕ_σ~~

  Gapped out by disorder

Relevant for $\mathbb{Z}_3$ parafermions and FCI phase in twisted bilayer $MoTe_2$
Nature 622, 69–73 (2023), arXiv:2504.06972



Two edges coupled by a superconducting nanowire:

- Pairing: $\Delta e^{i\varphi} \psi_t \psi_b + \text{H.c.}$
- Normal tunneling: $-t\psi_t^\dagger \psi_b + \text{H.c.}$

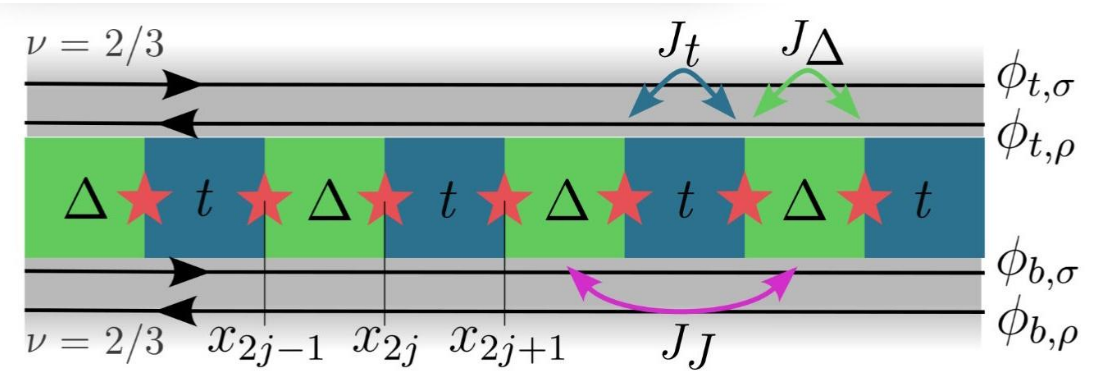
What is the effect of SC phase fluctuations $\varphi \to \varphi(x)$?

Parafermion lattice model

Alternating tunneling and pairing regions give rise to Z_3 parafermions at the interfaces:

$$\alpha_i \alpha_j = e^{2\pi i \,\mathrm{sgn}(j-i)/3} \alpha_j \alpha_i$$
$$\alpha_i^3 = 1$$

Charge $q = 2e/3$



Neighboring parafermions are coupled by tunneling:

Fractional charge across SC:

$$-J_\Delta \alpha_{2j}^\dagger \alpha_{2j-1}$$

$$-2J_\Delta \cos(\pi n_j)$$

Fractional charge across insulator:

$$-J_t e^{i(\varphi_{j+1}-\varphi_j)/3} \alpha_{2j+1}^\dagger \alpha_{2j}$$

$$-J_t b_{j+1}^\dagger b_j$$

Josephson current through insulator:

$$-J_J \cos(\varphi_{j+1} - \varphi_j)$$

$$-J_J B_{j+1}^\dagger B_j$$

Charging energy of SC region: $E_c \left(n_j - Q_g\right)^2$

For the DMRG simulation, we translate the parafermion to a **rotor model**:

b_i: Charge-2e/3 bosons

$$n_j = b_j^\dagger b_j$$
$$B_j = b_j^3$$

Phase diagram

Simulation parameters:

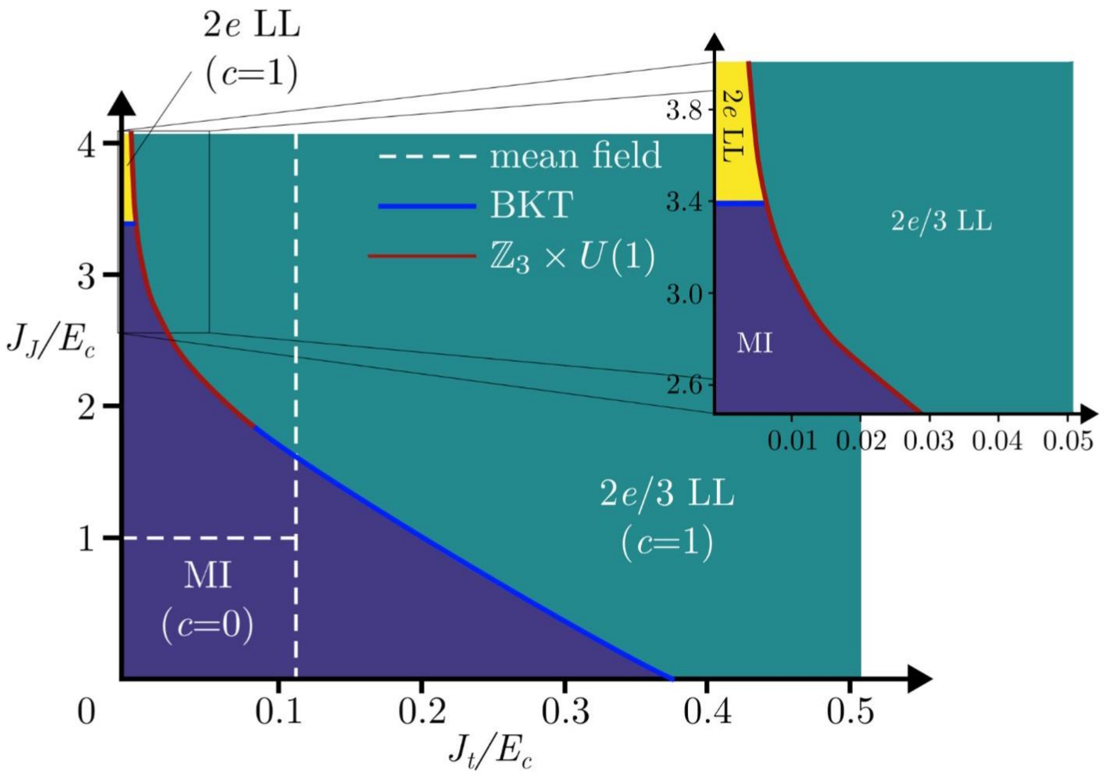
- Typical system sizes $L = 128$
- Local Hilbert space dimension $d = 19$ (charges $Q = -6$ to $Q = 6$ in steps of $\delta Q = 2/3$)
- Bond dimension from $M = 32$ to $M = 1024$

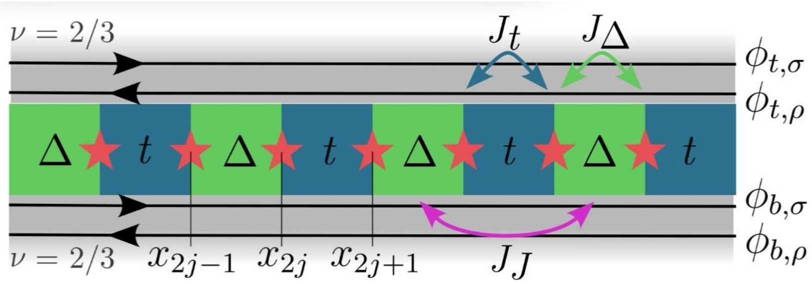
Main phases of the system for $J_\Delta = 0$:

Mott insulator (MI) phase:
Large charging energy

2e Luttinger liquid (2e LL) phase:
At large J_J Cooper pair tunneling dominates

2e/3 Luttinger liquid (2e/3 LL) phase:
At large J_t tunneling of fractional charges dominates



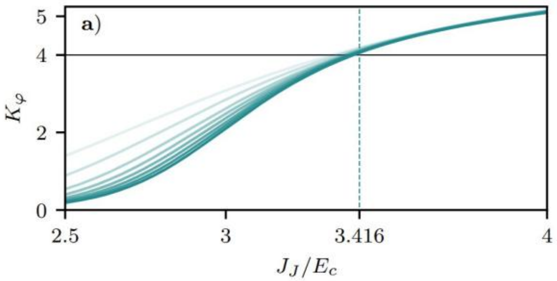


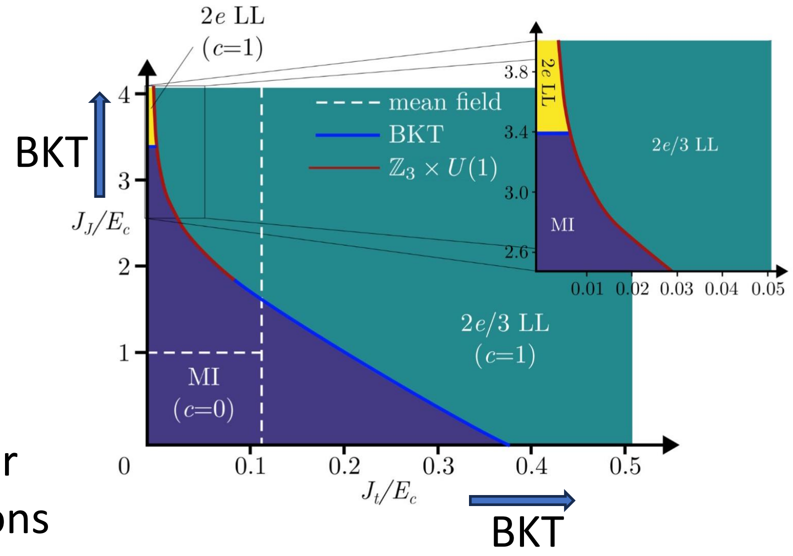
Characterization of the phases

Order parameters in terms of rotor operators: $B_j = b_j^3$

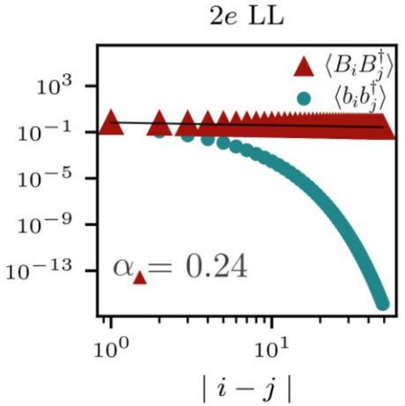
$\langle b_i^\dagger b_j \rangle$ charge-2e/3 order

$\langle B_i^\dagger B_j \rangle$ charge-2e order



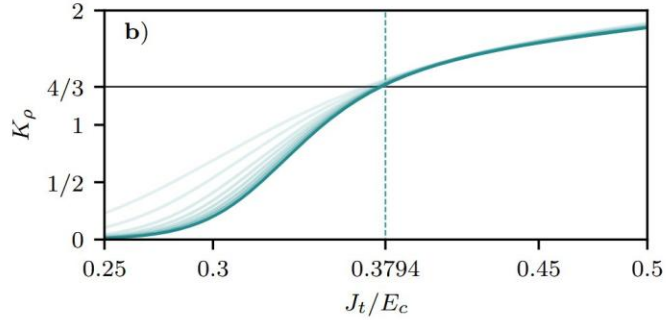


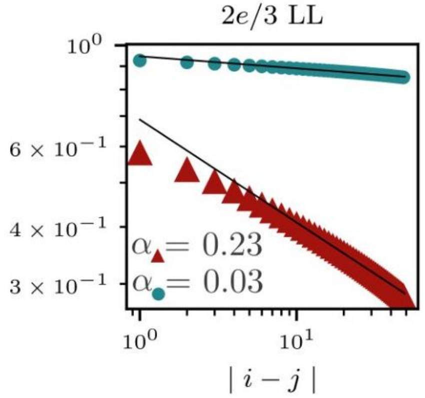
Mott insulator (MI) phase:
No long-range order



Luttinger parameter across BKT transitions

2e Luttinger liquid (2e LL) phase:
Quasi long-range order of $\left\langle B_i^\dagger B_j \right\rangle$





2e/3 Luttinger liquid (2e/3 LL) phase:
Quasi long-range order of $\left\langle b_i^\dagger b_j \right\rangle$ and $\left\langle B_i^\dagger B_j \right\rangle$

Luttinger parameter K from charge fluctuations:

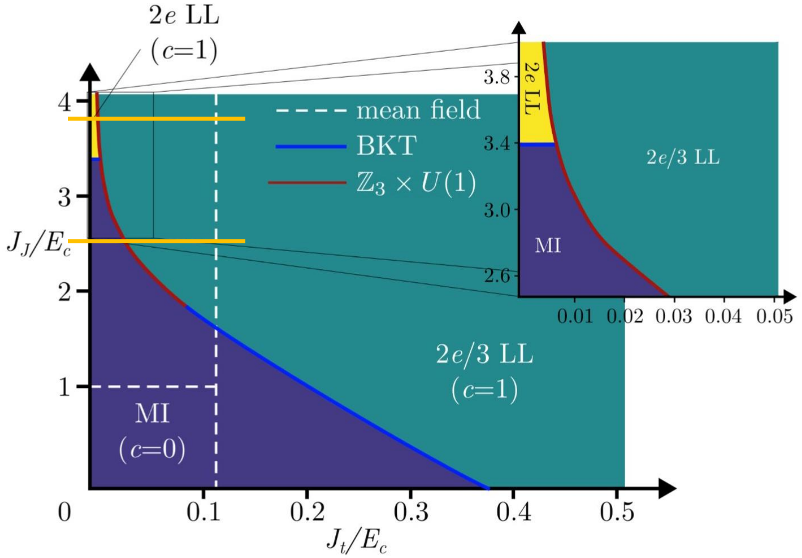
$$\langle N^2(\ell) \rangle - \langle N(\ell) \rangle^2 \propto K \log\left[\frac{2L}{\pi} \sin\left(\frac{\pi \ell}{L}\right)\right]$$

Conformal field theory

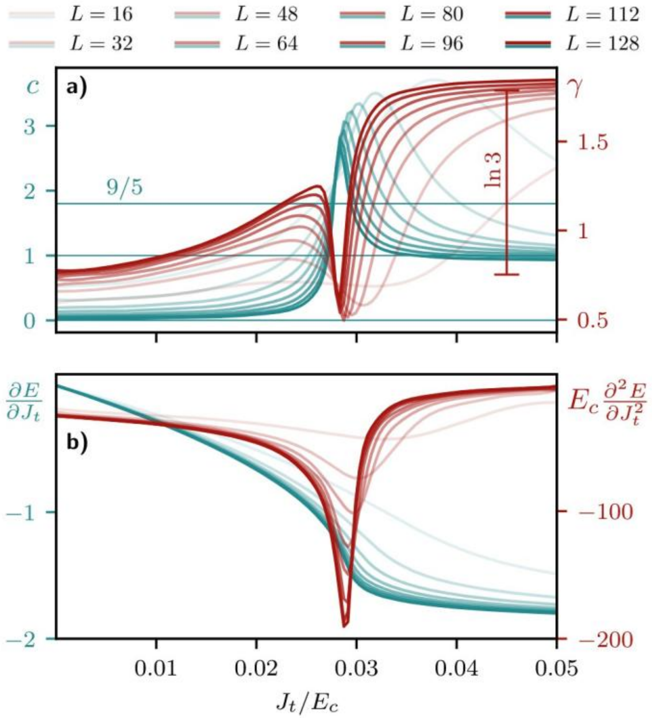
Entanglement entropy from Cardy-Calabrese formula:

Central charge　　　　Ground state degeneracy

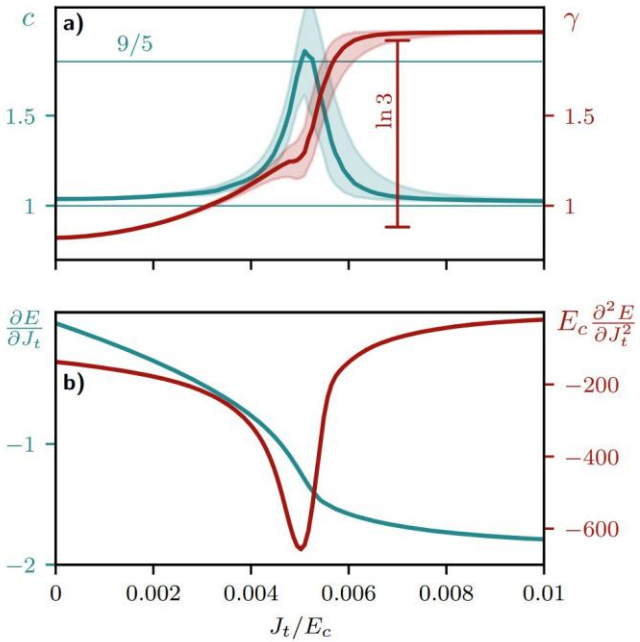
$$S_A = \frac{c}{6} \ln\left[\frac{2L}{\pi a}\sin\left(\frac{\pi \ell}{L}\right)\right] + \gamma$$



$J_J = 2.5\ E_C$



$J_J = 3.8\ E_C$



Phenomenological field theory

Reduced effective theory:

$$H_{\text{FQH}} = \frac{3u_\rho}{4\pi}\int dx \left(K_\rho(\partial_x\Phi_\rho)^2 + \frac{(\partial_x\Theta_\rho)^2}{K_\rho}\right)$$

$$H_{\text{SC}} = \frac{u_\varphi}{4\pi}\int dx \left(K_\varphi(\partial_x\varphi)^2 + \frac{(\partial_x\theta)^2}{K_\varphi}\right)$$

Luttinger parameters:

$$K_\rho = \sqrt{J_t/E_c}$$

$$K_\varphi = \sqrt{J_J/E_c}$$

Superconducting phase

$\boldsymbol{\phi} = (\Theta_\rho, \Phi_\rho, \theta, \varphi)$

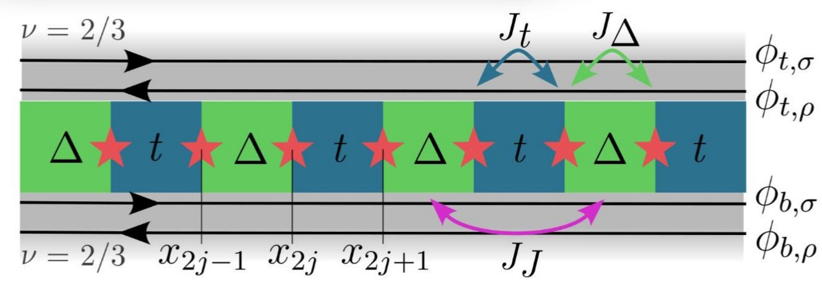
Charged mode

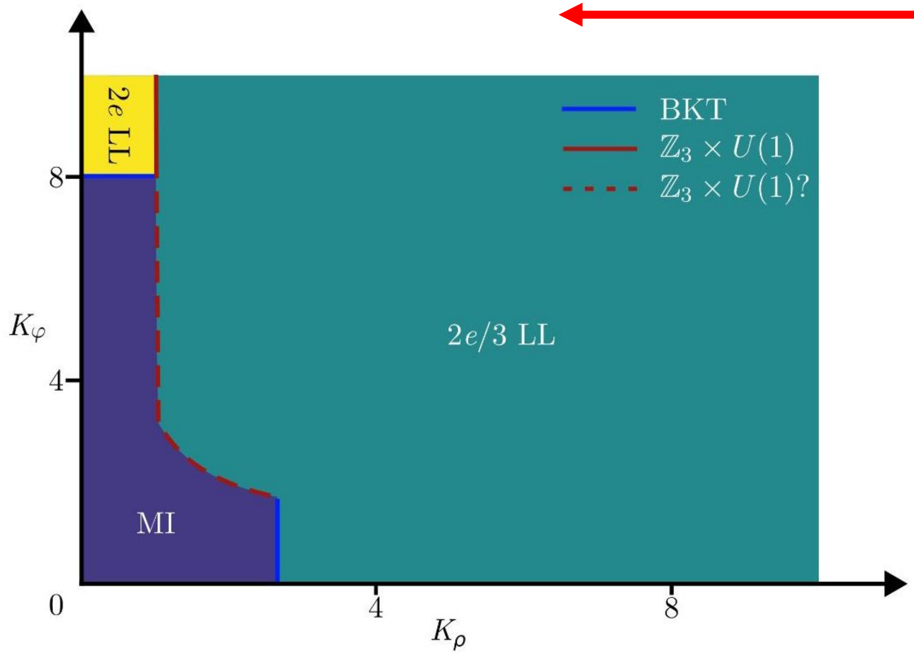
Coupling terms allowed by locality and charge conservation:

Inter-edge pairing: $g_1 \cos\left(3\Phi_\rho + \varphi\right)$

Electron backscattering: $g_2 \cos\left(3\Theta_\rho\right)$

SC phase slips: $g_3 \cos\left(\theta\right)$

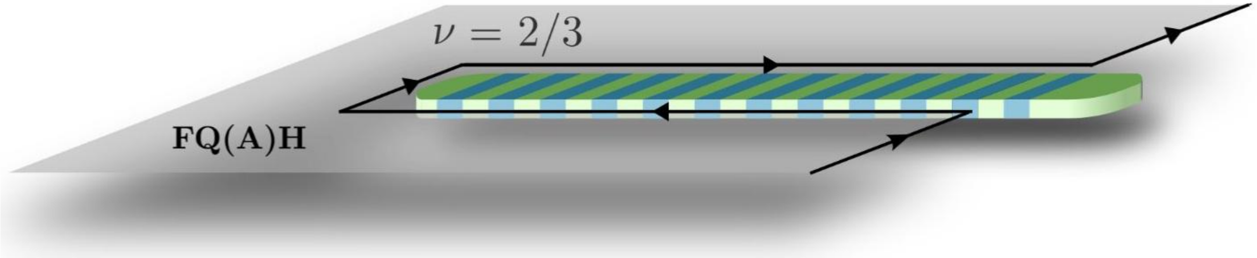




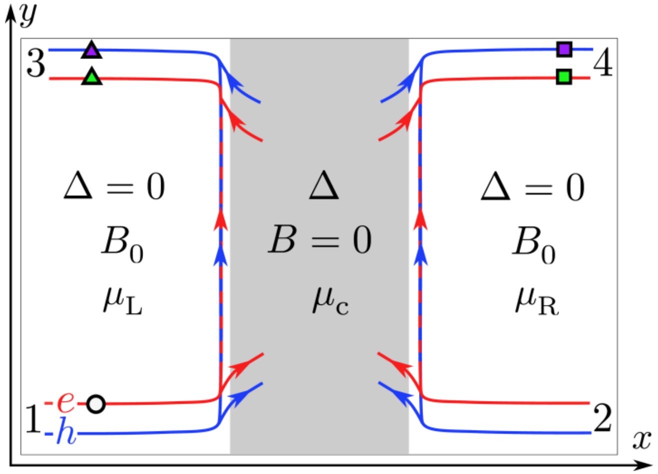
Outline

Phases of QH/SC interfaces with order parameter fluctuations

arXiv:2510.26686

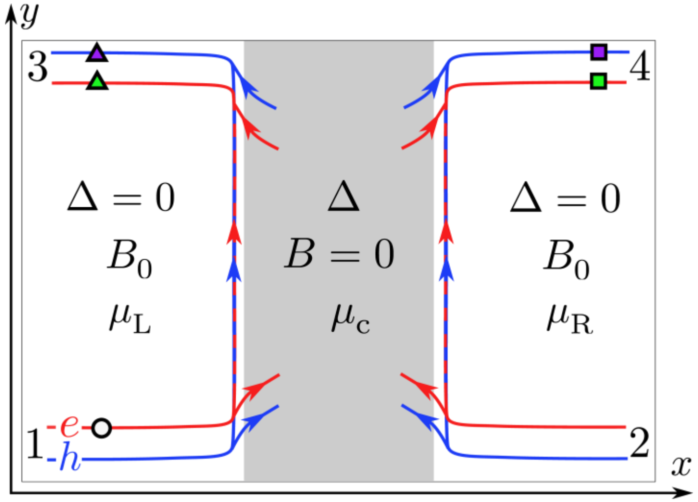


Superconducting Hong-Ou-Mandel interferometry



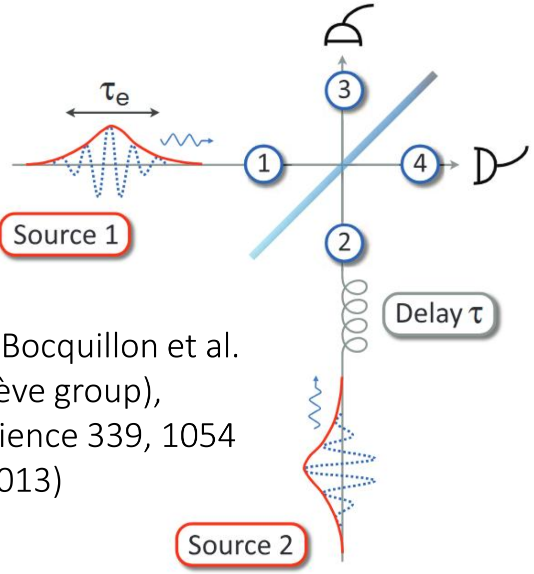
arXiv:2604.09463

Hong-Ou-Mandel interferometry

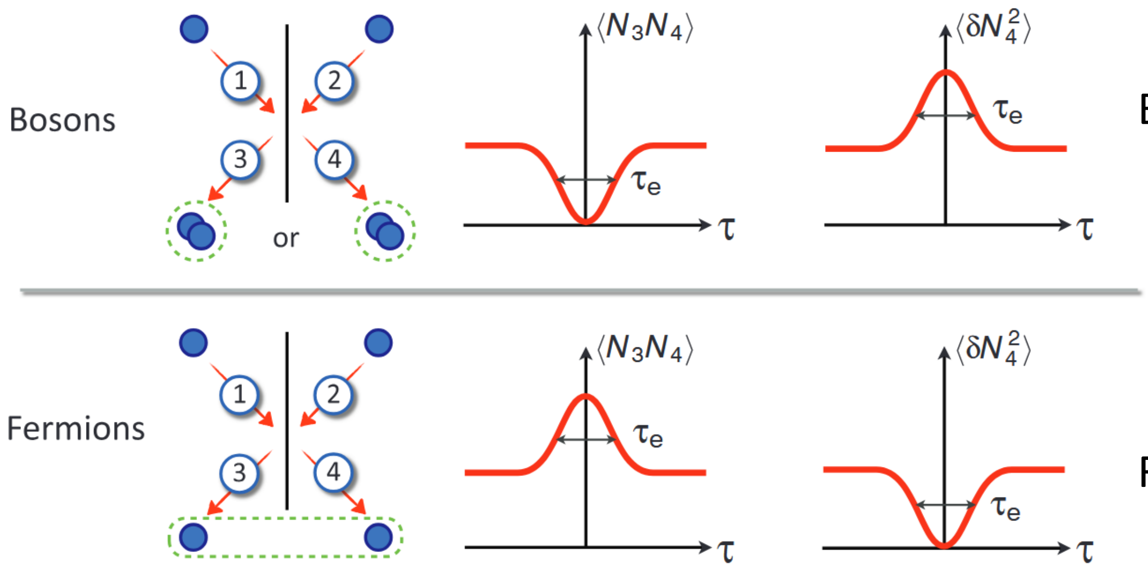


Model:
- ➢ Graphene-based quantum Hall bar at lowest Landau level
- ➢ Scattering region in the center (normal insulator or superconductor)

Hong-Ou-Mandel interferometry:
- Injection of two electron pulses at the source contacts
- Time delay τ
- Current correlations at the drain contacts



E. Bocquillon et al. (Fève group), Science 339, 1054 (2013)



Boson bunching

How does this change for a superconducting beam splitter?

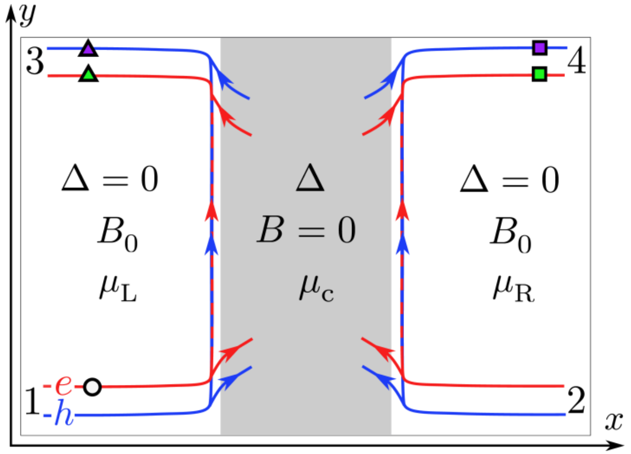
Fermion anti-bunching

Scattering theory

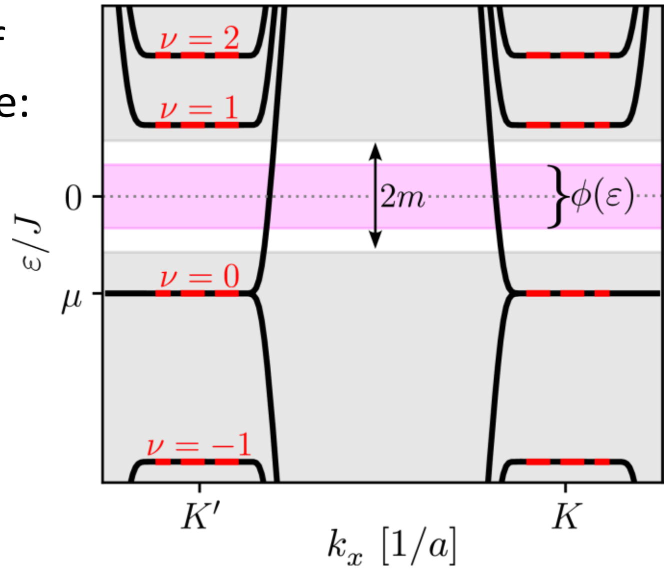
Method:

- Tight-binding simulation to determine the scattering matrix in Nambu space
- Landauer-Büttiker theory to determine current correlations
- Gaussian wave packets in source contacts

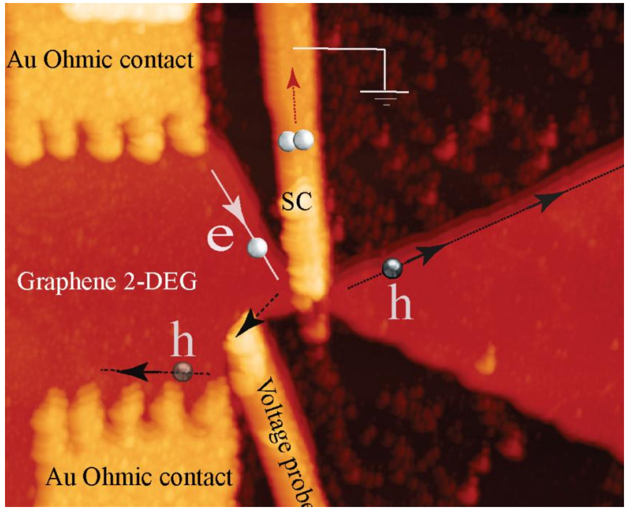
$$|\Psi(\tau)\rangle = \int d\varepsilon_1 d\varepsilon_2 \phi(\varepsilon_1)\phi(\varepsilon_2) e^{-i\varepsilon_2\tau} \hat{a}^\dagger_{1e}(\varepsilon_1)\hat{a}^\dagger_{2e}(\varepsilon_2)\,|0\rangle\,,$$

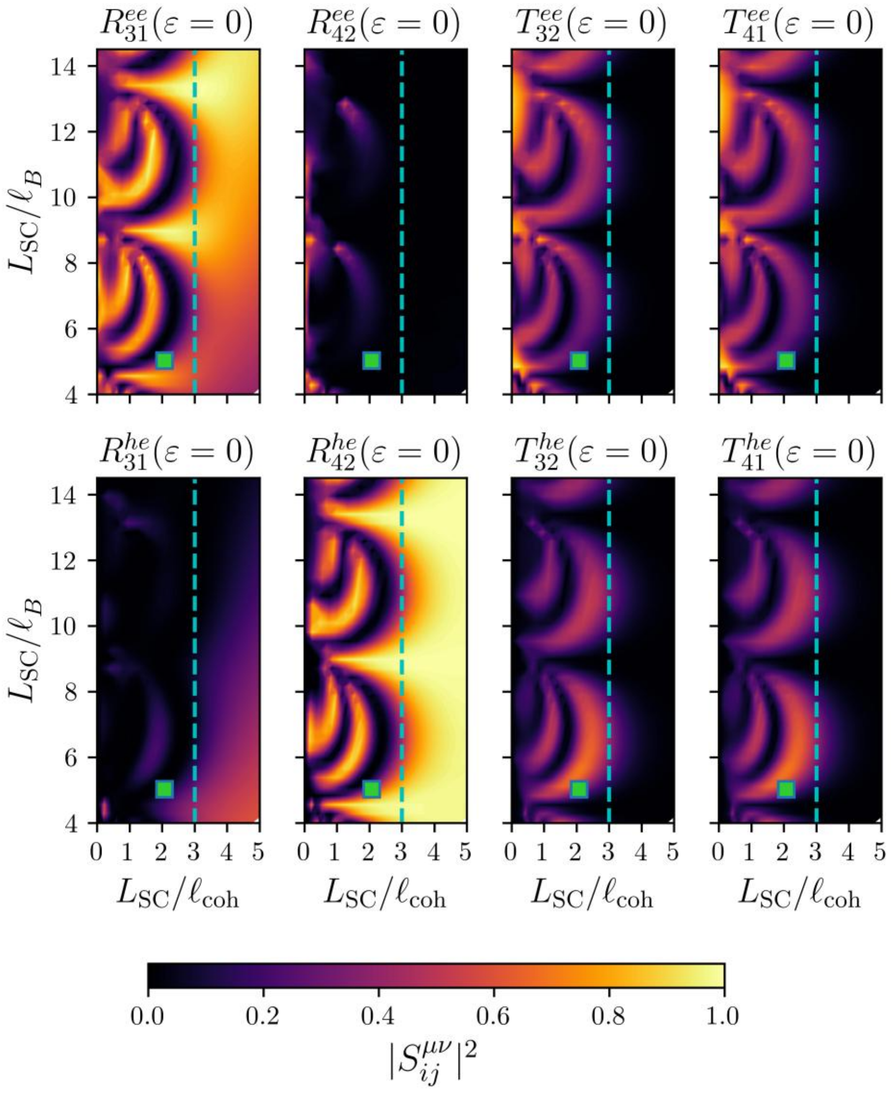


Edge state spectrum of quantum Hall graphene:



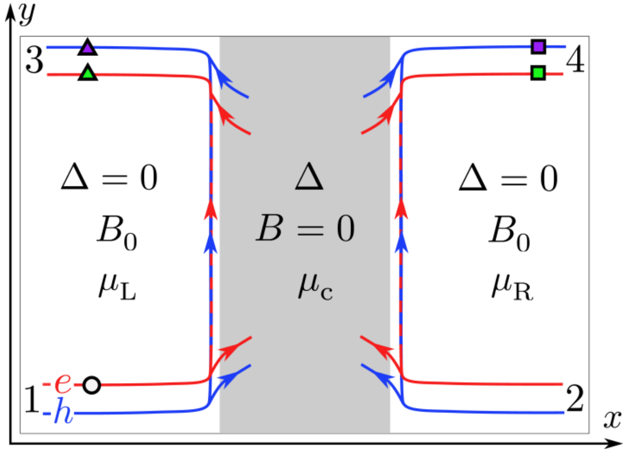
$$\begin{pmatrix} \hat{b}_{3e} \\ \hat{b}_{3h} \\ \hat{b}_{4e} \\ \hat{b}_{4h} \end{pmatrix} = S \begin{pmatrix} \hat{a}_{1e} \\ \hat{a}_{1h} \\ \hat{a}_{2e} \\ \hat{a}_{2h} \end{pmatrix}$$





Scattering processes:

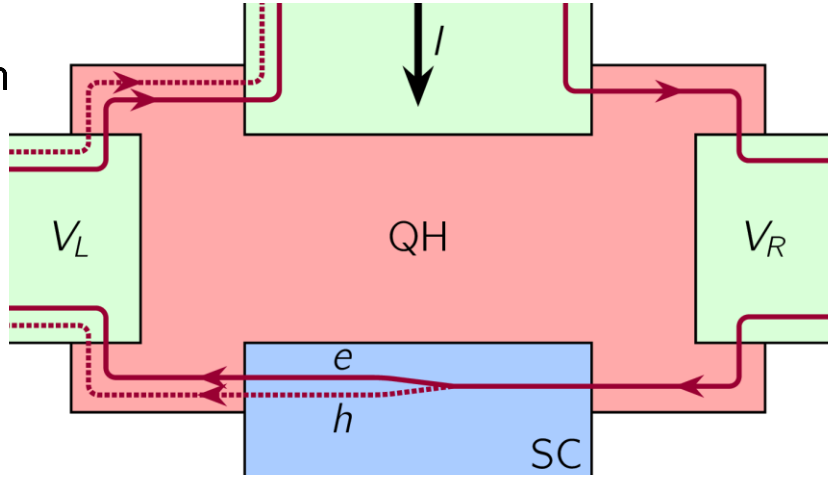
- Normal reflection R^{ee}
- Normal transmission T^{ee}
- Andreev reflection R^{eh}
- Andreev transmission T^{eh}



Qualitative behavior:

- Fabry-Perot oscillation with superconductor size L_{SC}
- Exponential decay if longer than SC coherence length

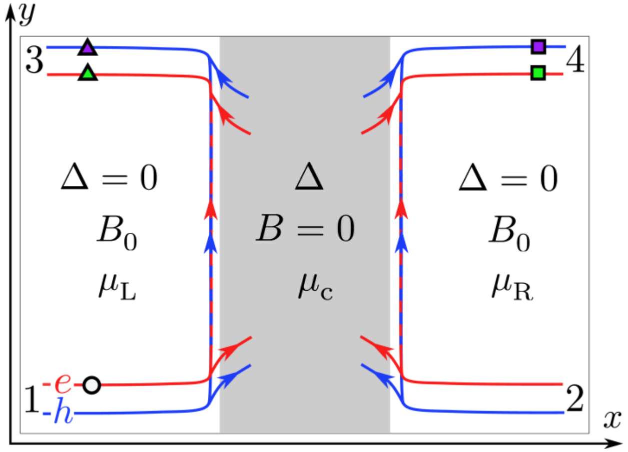
Negative downstream resistance:

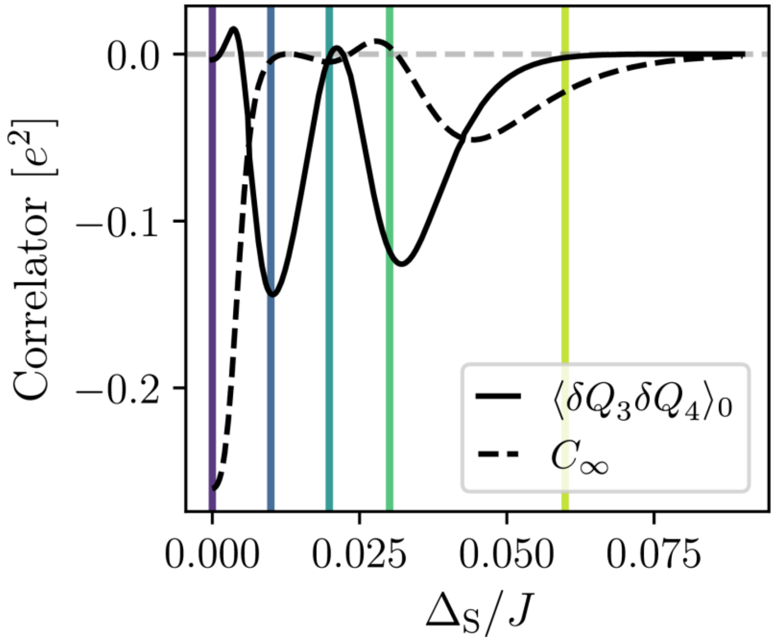


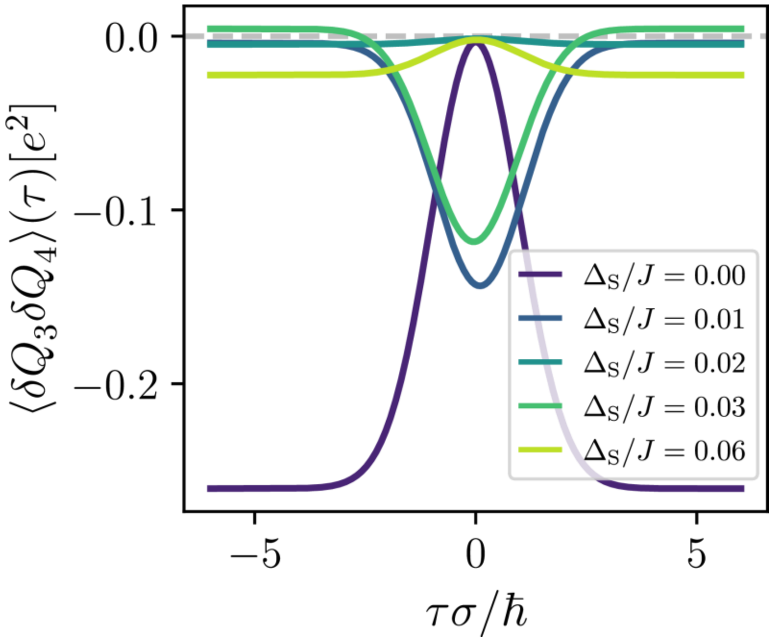
Superconducting HOM signal

Results:

- Cross correlation retains extremum for time delay $\tau = 0$.
- Switching between maximum and minimum as function of SC strength



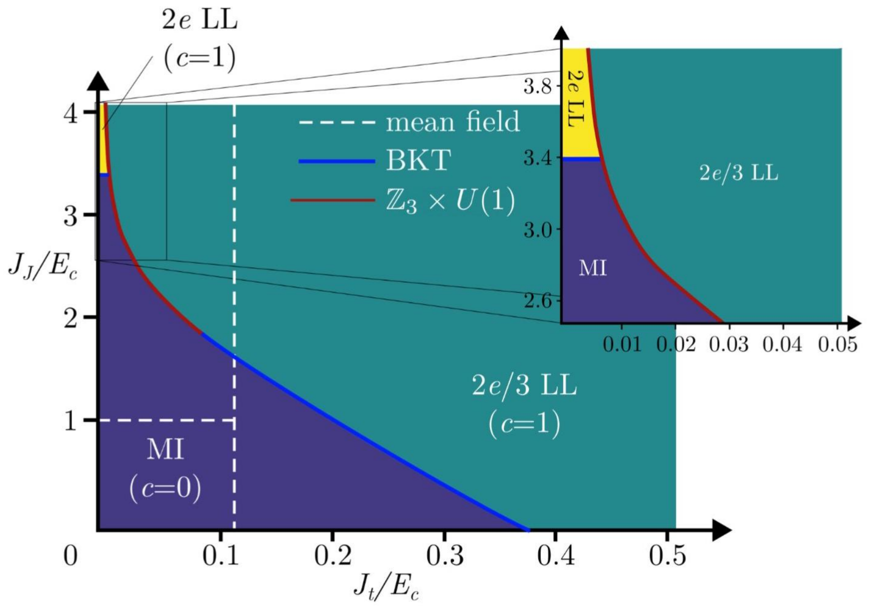




Conclusions

- We studied a pair of $\nu = 2/3$ quantum Hall states coupled via a superconductor with order parameter fluctuations.
- The phase diagram shows gapless charge-2e and charge-2e/3 phases as well as a Mott insulator phase.

arXiv:2510.26686



- We numerically simulated a superconducting Hong-Ou-Mandel interferometer.
- Sign change in anti-bunching correlation reveal Andreev processes.

arXiv:2604.09463