Mixing of edge modes in a quantum Hall system proximitized by a superconductor

Shalini Maji, M.P. Nowak

Academic Centre for Materials and Nanotechnology, AGH University of Krakow, Poland

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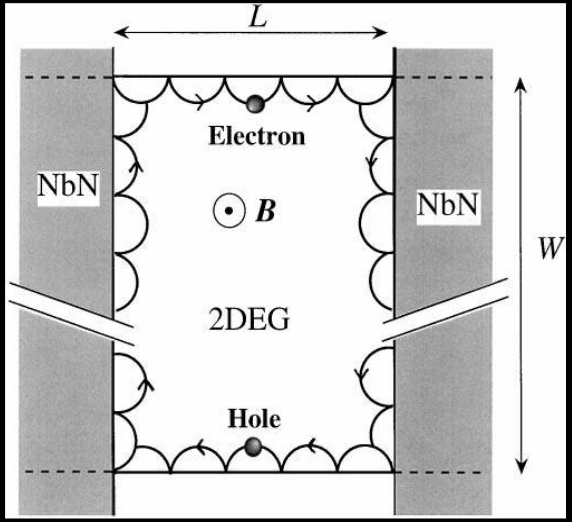
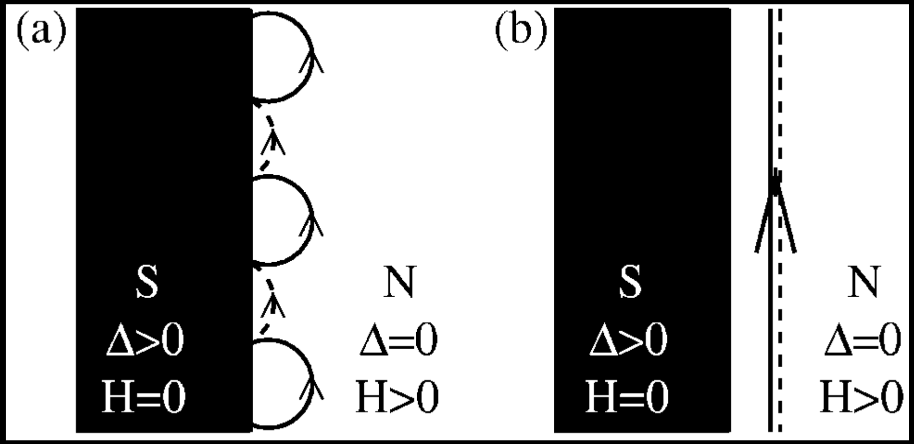
Early study of QH-SC systems:

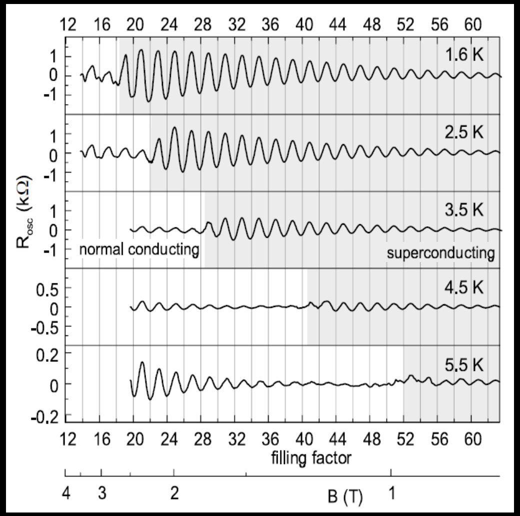


Physica B 249-251 (1998) 462-466



PRL 84, 1804 (2000)

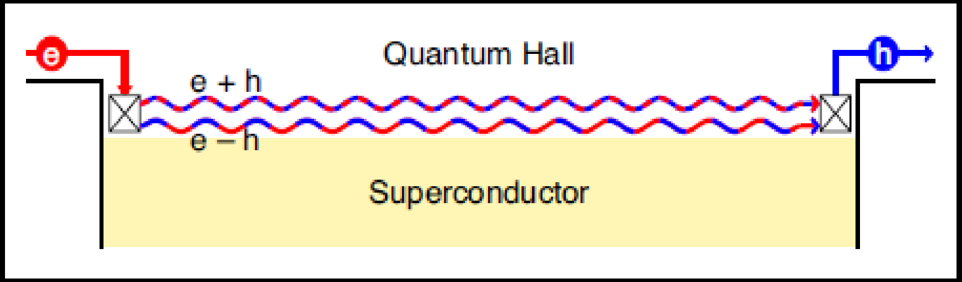
- The Andreev reflection becomes a nonlocal edge-state process in the QH-SC interface, and mixed bound states form.
- Observation of Magneto-resistance oscillations as a signature of Andreev reflection.

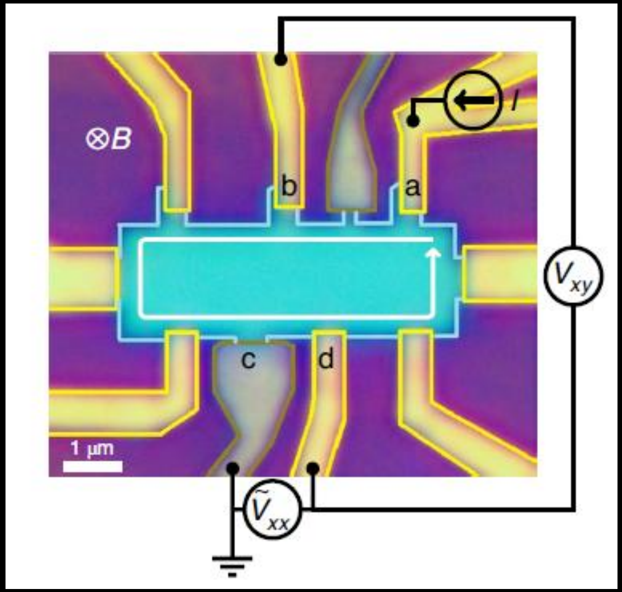


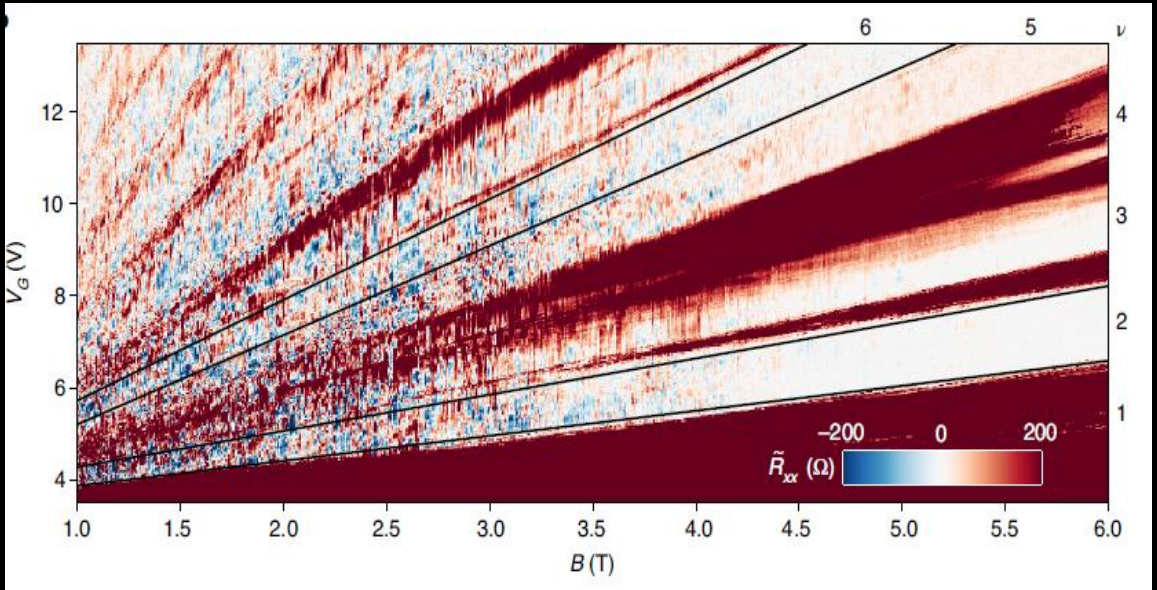
PRL 95, 107001 (2005)

Proposition of Chiral Andreev Edge States (CAESs):

- Andreev reflection becomes a chiral, nonlocal process in the QH regime.
- Successive reflections hybridize the electron and hole into two propagating eigenstates.
- The two eigenstates carry different momenta.
- The interference was directly observed in oscillating signatures, and the states were named CAES







Nat. Phys, 16, 862–867 (2020)

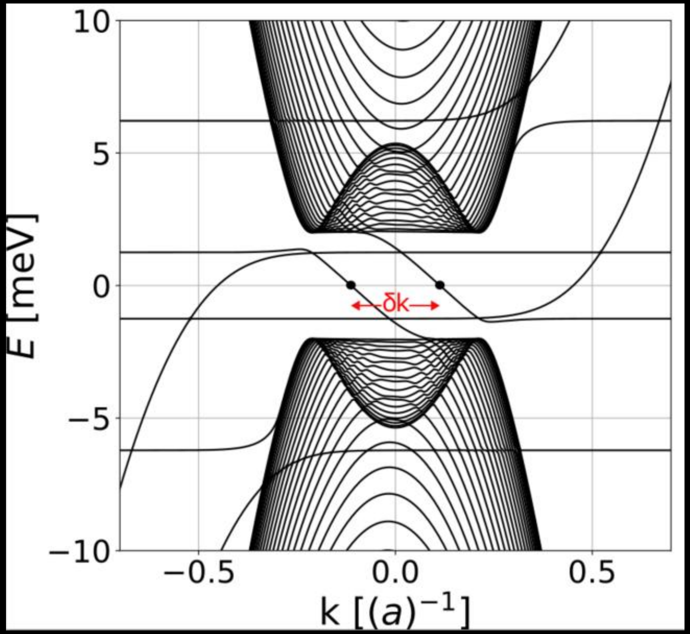
Analytical Formulation of CAESs:

Two orthogonal hybridized eigenstates from BdG particle-hole symmetry can be written as

$$|\psi_1\rangle = \alpha|e\rangle + \beta|h\rangle$$

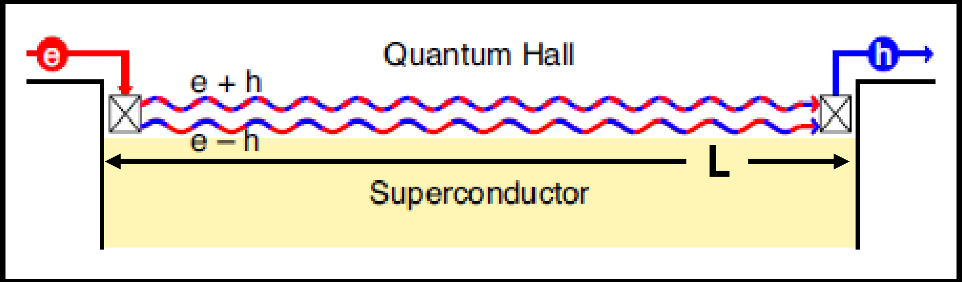
$$|\psi_2\rangle = \beta^*|e\rangle - \alpha^*|h\rangle$$

$$P_e - P_h = 1 - 8|\alpha|^2|\beta|^2 \sin^2(\delta k\ L/2)$$



$P_e - P_h$ = electron-hole conversion probability, directly measured as nonlocal conductance

$\delta k = k_1 - k_2,\ k_1 and\ k_2$ are the two wavevectors corresponding to two CAESs at zero energy

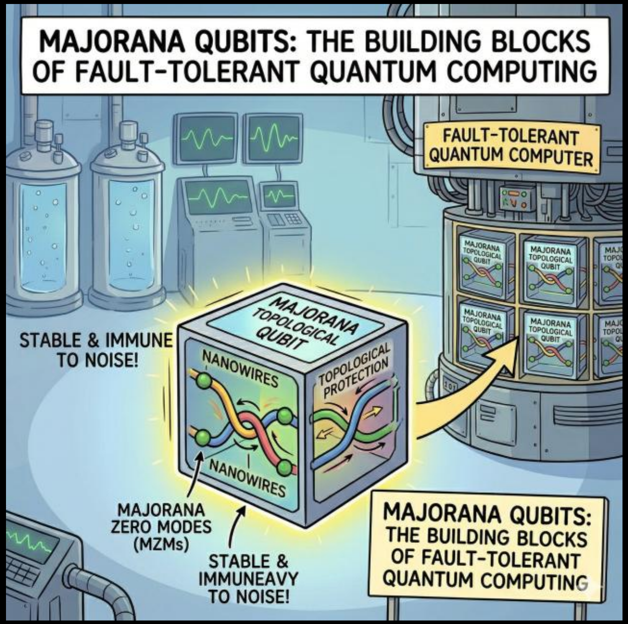


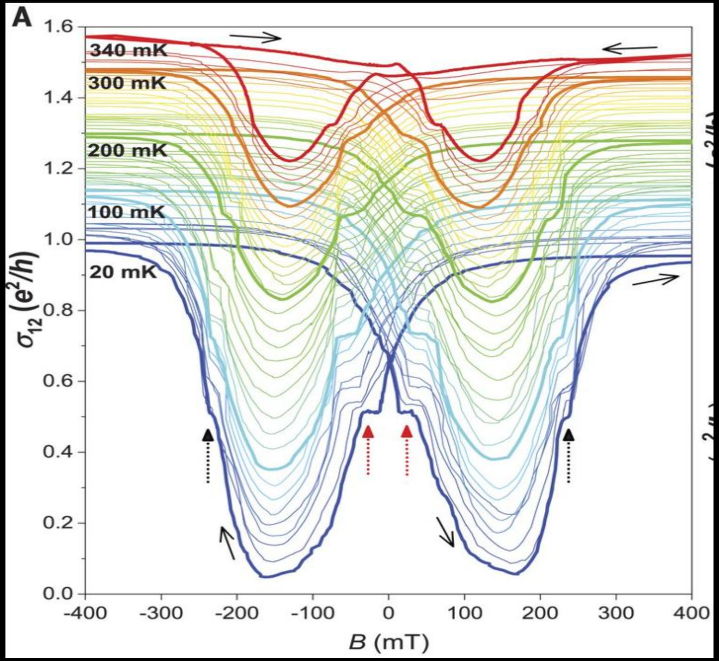
L = interference length

Possible Applications:

- QH-SC platforms have been considered as the possible platform to realize exotic topological states like CAESs.

- Under certain conditions, CAESs are proposed to appears self conjugate Majorana fermions.





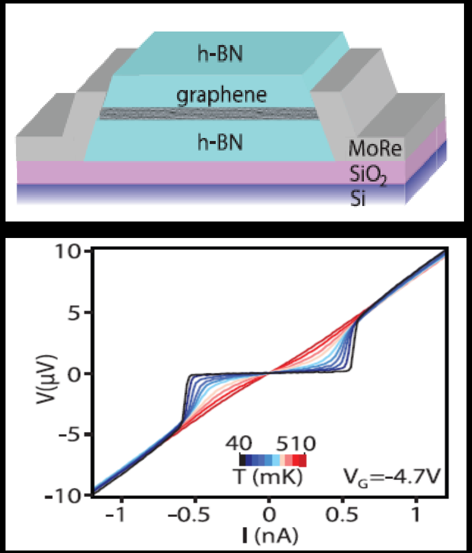
Q. L. He et al., Science 357, 294–299 (2017).

- A single spin-polarized edge channel, a transparent QH-SC interface with finite induced pairing, and an exactly equal electron-hole superposition are the key ingredients for CAESs to become chiral Majorana fermions.

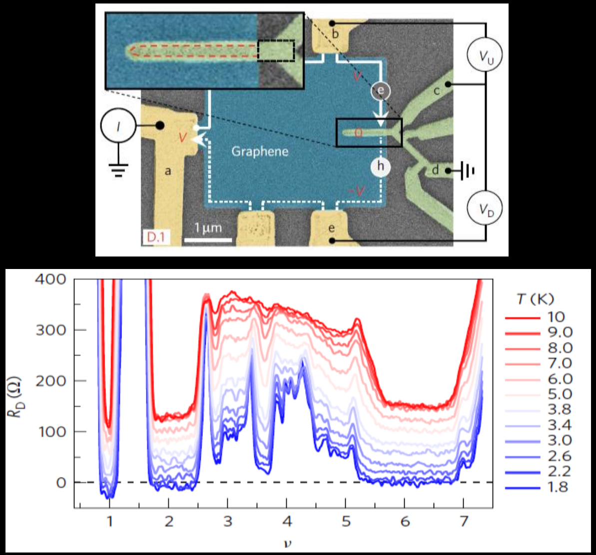
- No experimental evidence yet.

From First Signatures to Spinful Systems:

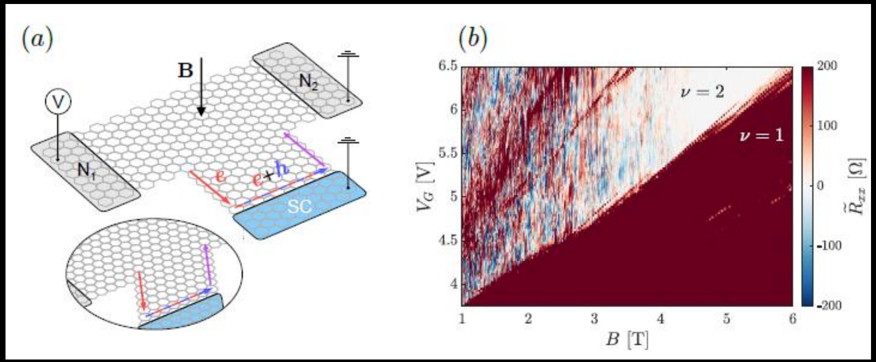
Graphene based results:



Science 352, 6288 (2016)

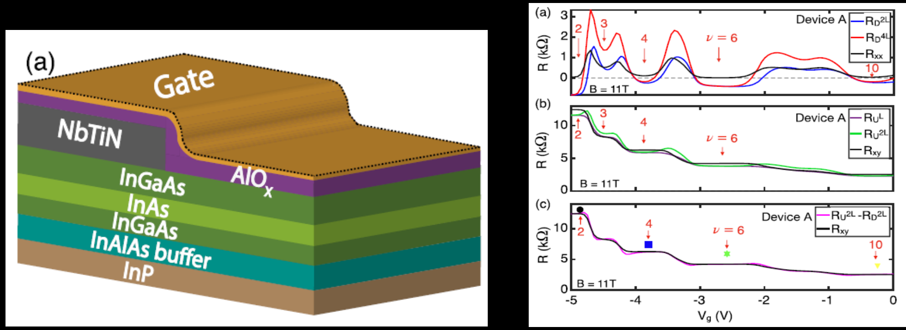


Nat. Phys., 13, 693–698 (2017)

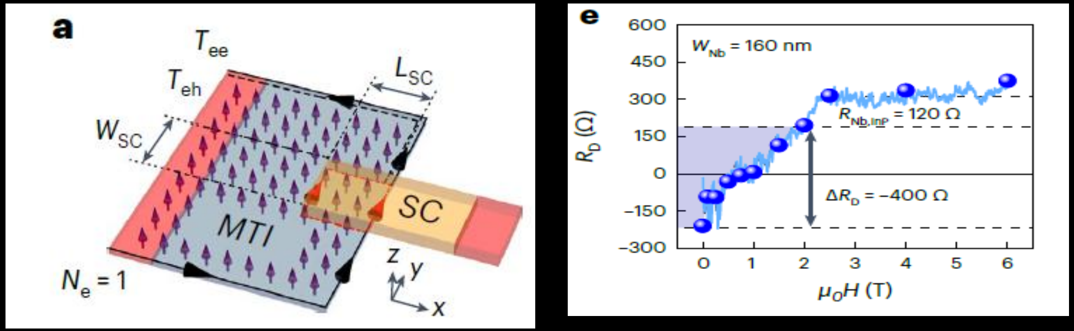


SciPost Phys. Core 5, 045 (2022)

Spinful semiconductor-based results:



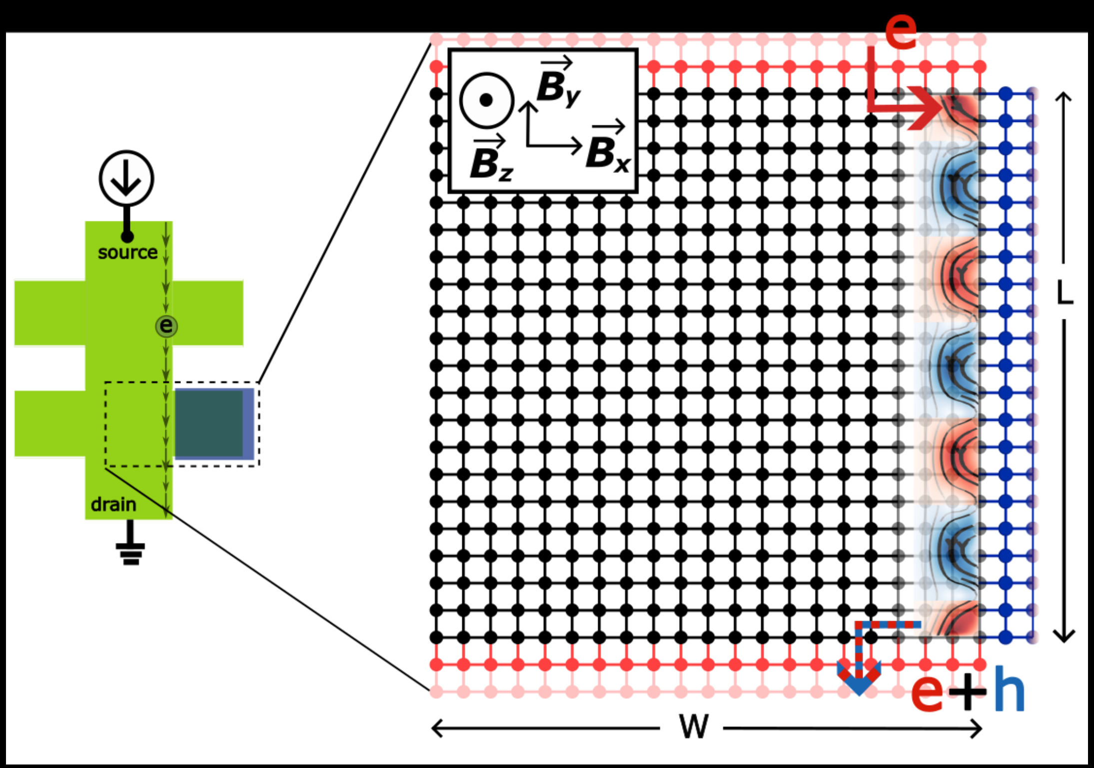
Nano Lett. 2022, 22, 6173−6178



Nat. Phys., 20, 1589-1595 (2024)

System and Hamiltonian:

$$H = \left(\frac{\hbar^2 \boldsymbol{k}^2}{2m^*} - \mu\right) \sigma_0 \otimes \tau_z + \Delta(r)\sigma_0 \otimes \tau_x$$
$$+\alpha(\sigma_x k_y - \sigma_y k_x) \otimes \tau_z + \frac{1}{2} g\mu_B \boldsymbol{B}\, \sigma \otimes \tau_0$$



Perpendicular magnetic field is included by the inclusion of the Peierls phase in the hopping term.

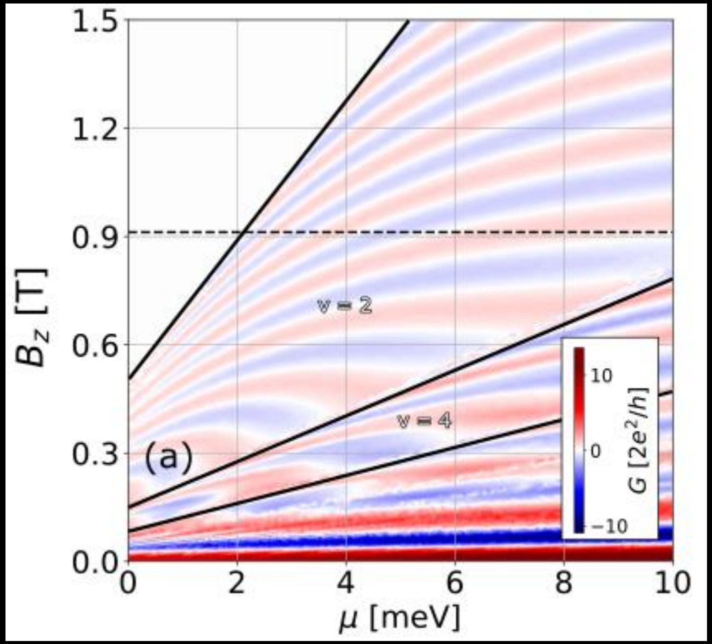
$$t_{ij} \longrightarrow t_{ij} \exp[\mp \frac{ie}{\hbar} \int \vec{A} \cdot \vec{dl}]$$

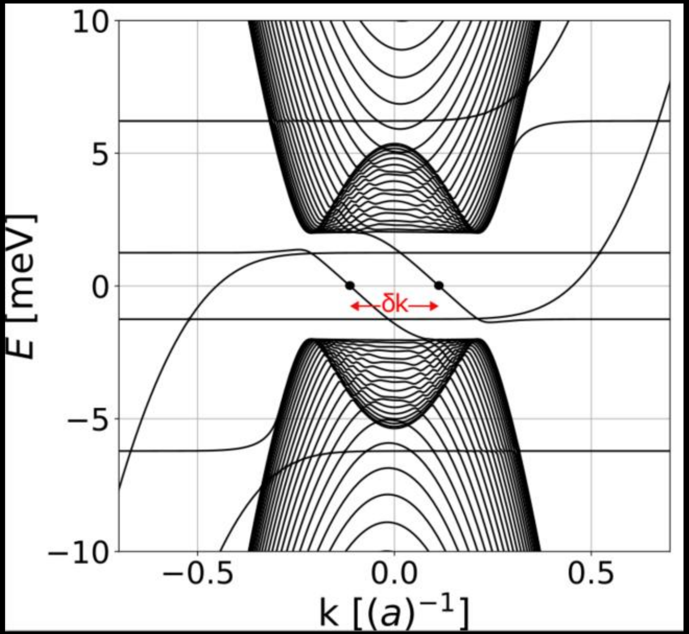
Non-local conductance of the system is measured by the Landauer- Buttiker formalism,

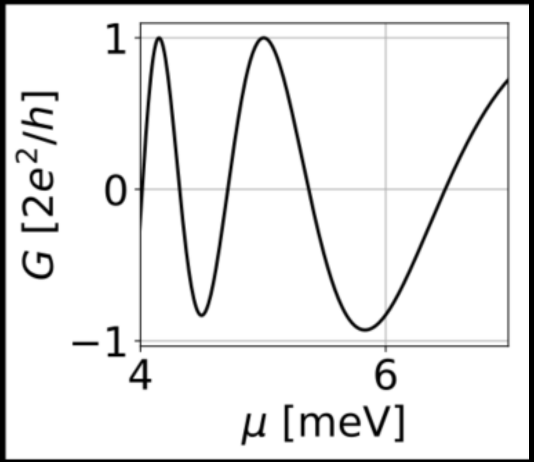
$$G_{ij}(E) = \frac{\partial I_i}{\partial V_j} = \frac{e^2}{h}\left(\delta_{ij} N_i^e(E) - T_{ij}^{ee}(E) + T_{ij}^{he}(E)\right)$$

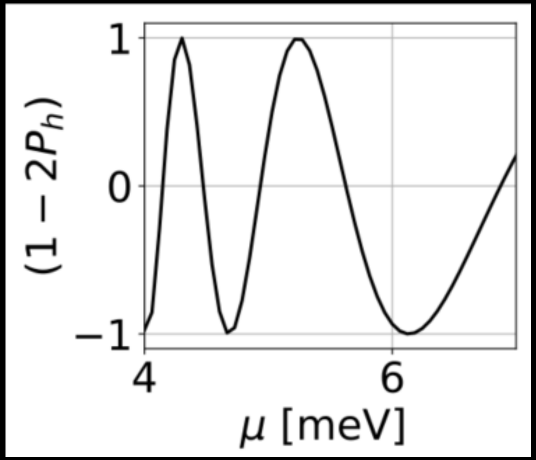
Results:

Spin degenerate case: at $\nu = 2$



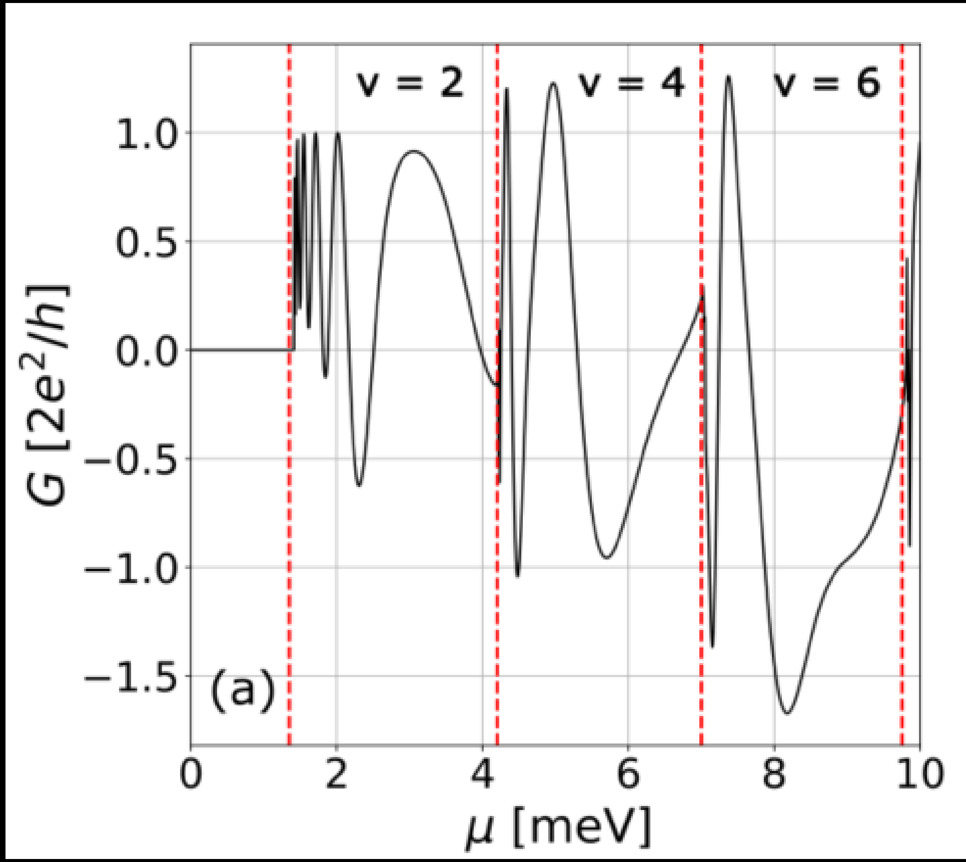


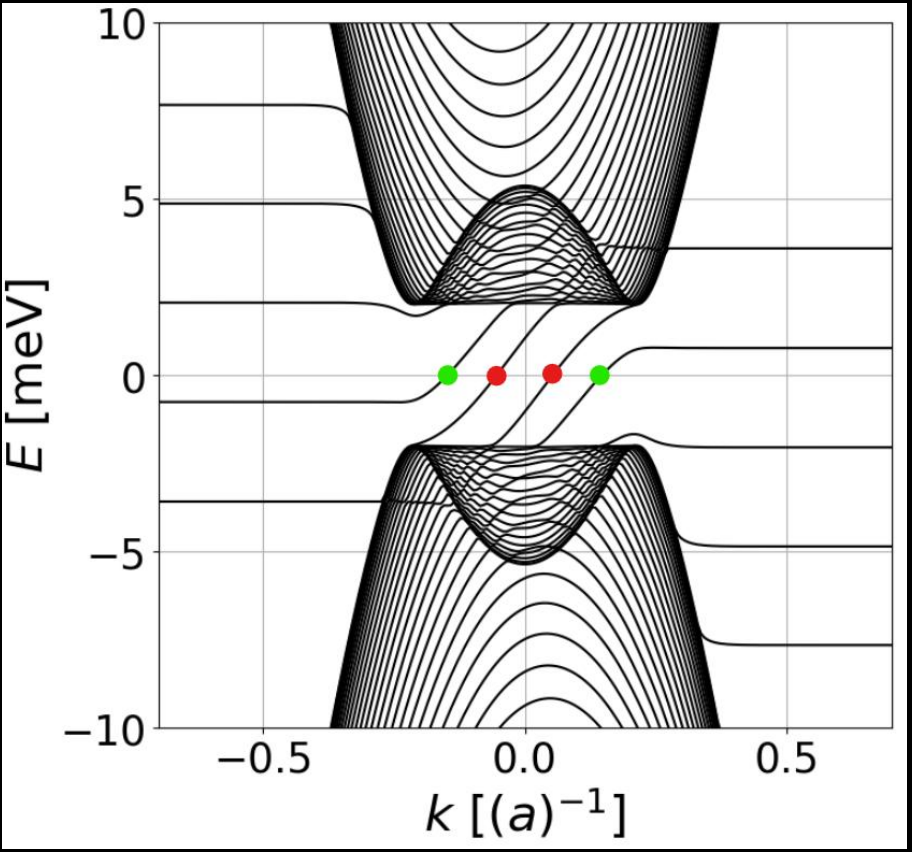




$$P_e - P_h = 1 - 8|\alpha|^2|\beta|^2 \sin^2(\delta k \, L/2)$$

Spin degenerate case: at $\nu > 2$

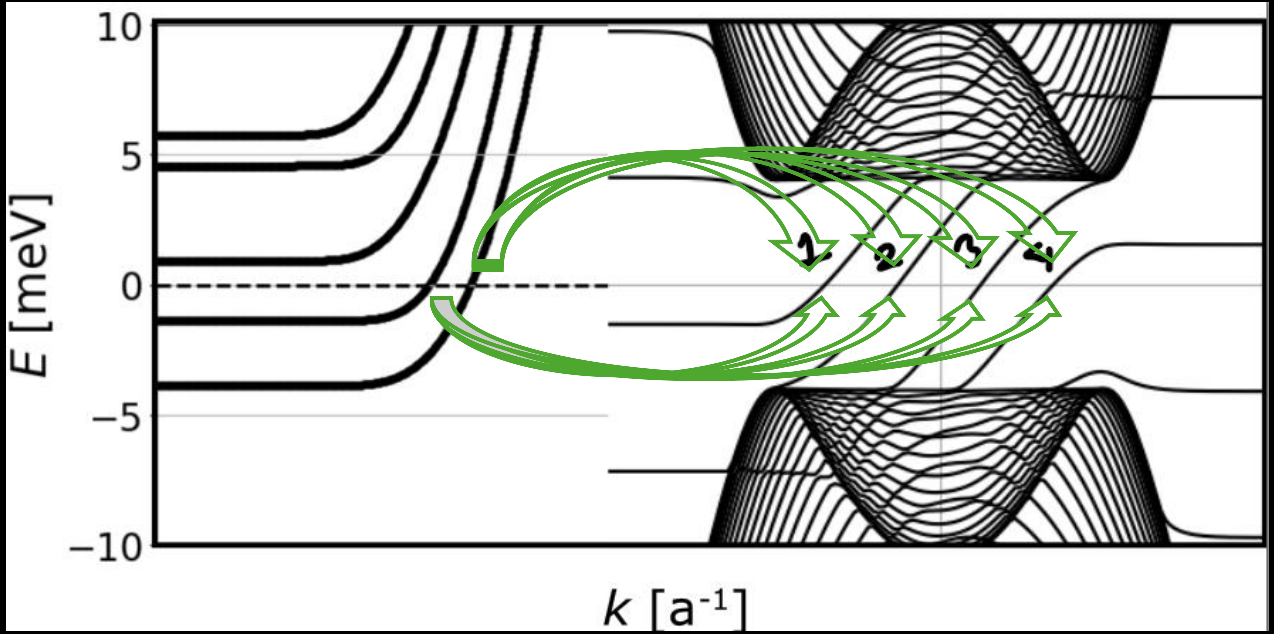


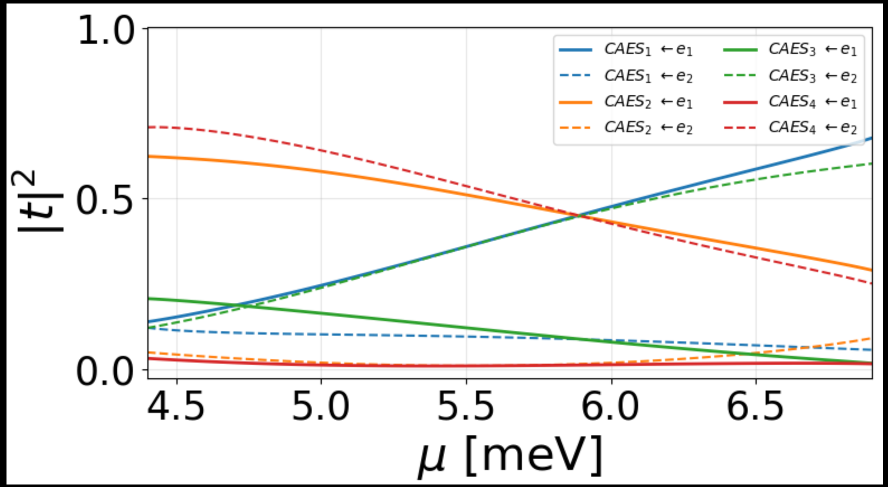


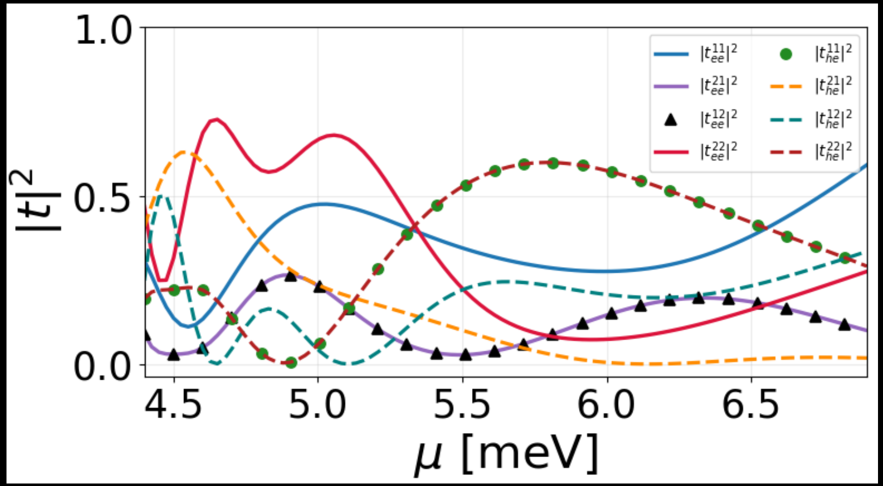
Signatures of mode mixing:

Spin degenerate case: at $\nu > 2$

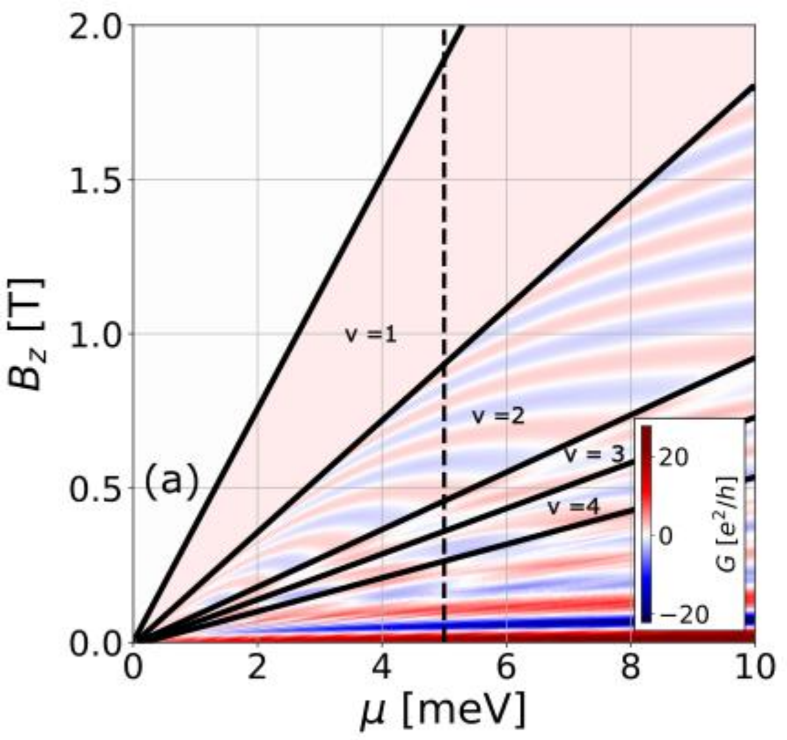
We extract transmission probabilities in different ways:

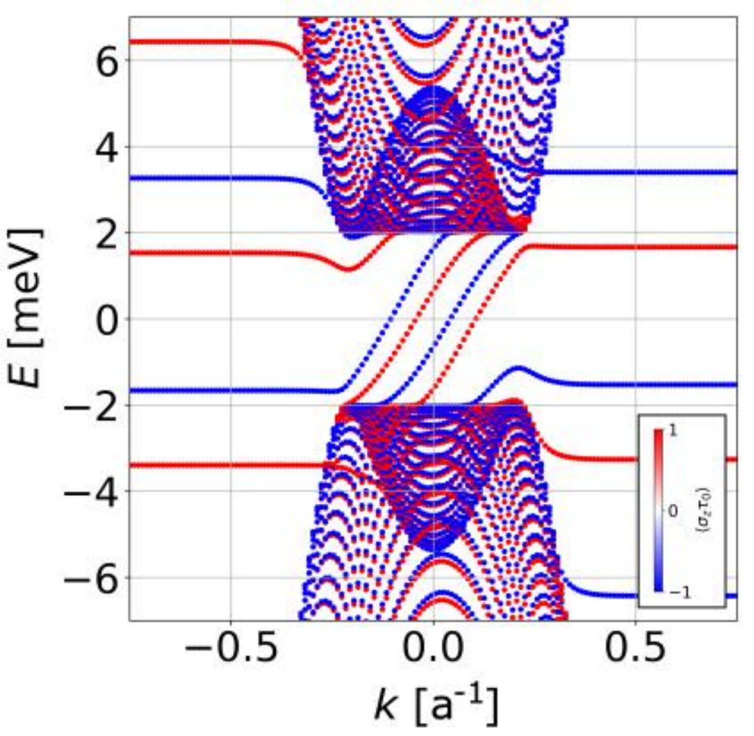


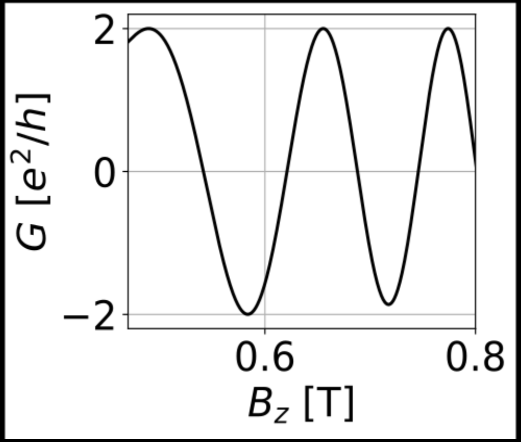


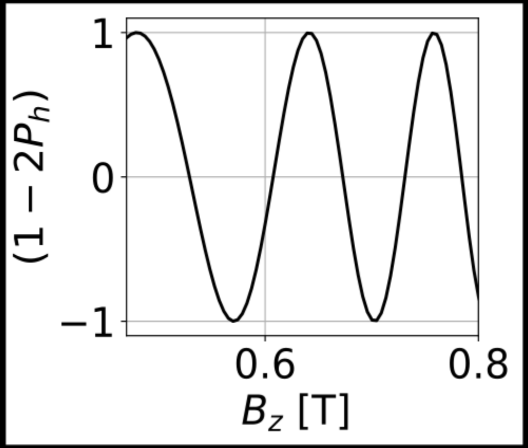


Spinful system with Zeeman field along z axis: at $\nu = 2$

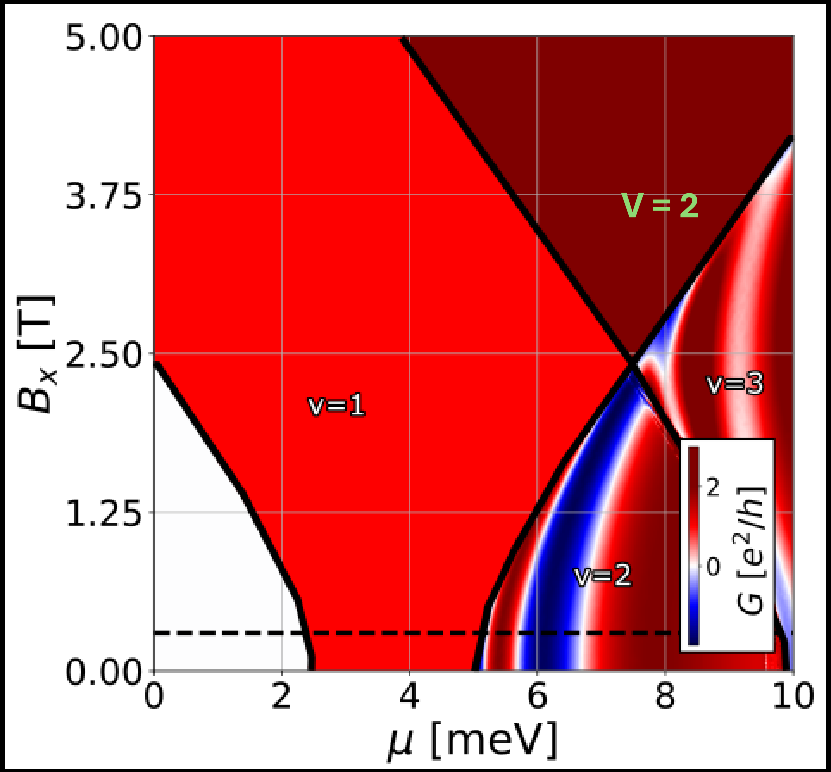


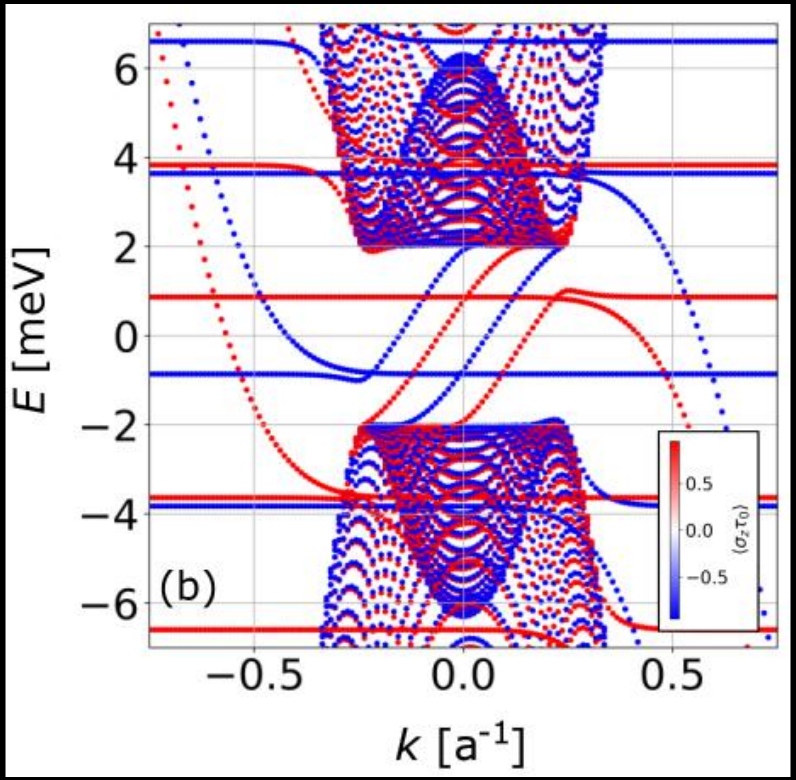


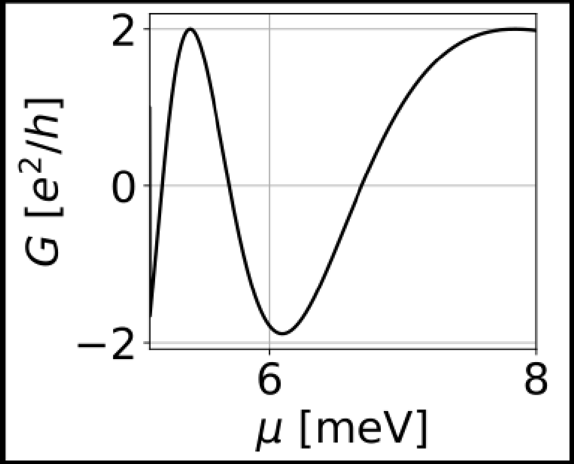


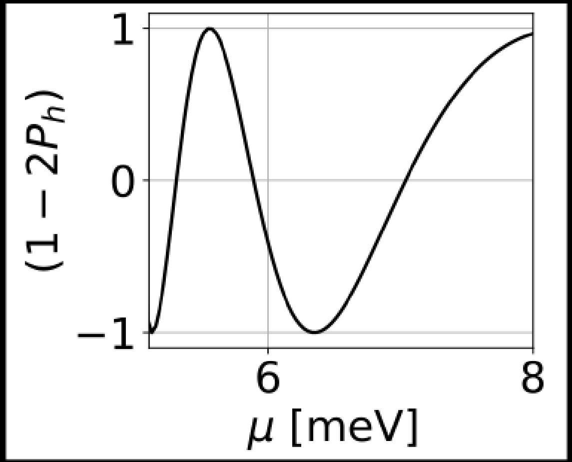


Spinful system with Zeeman field along z axis and external Zeeman field along x/y axis: at $\nu = 2$

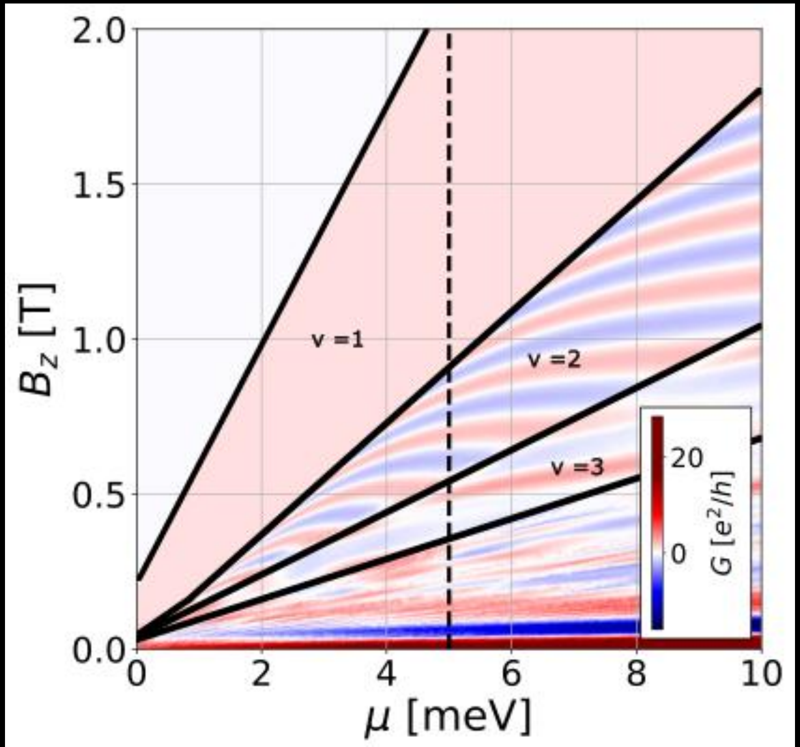


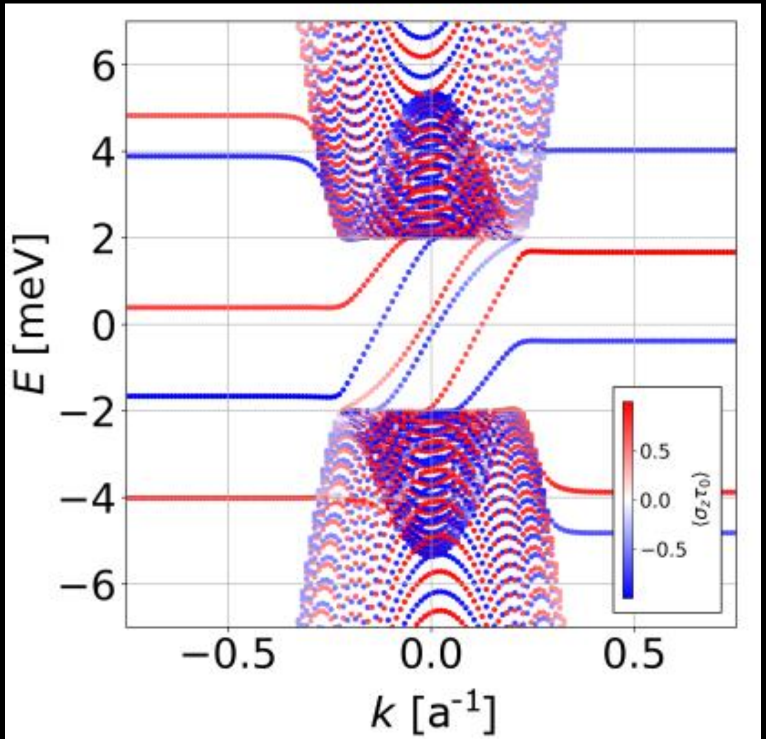


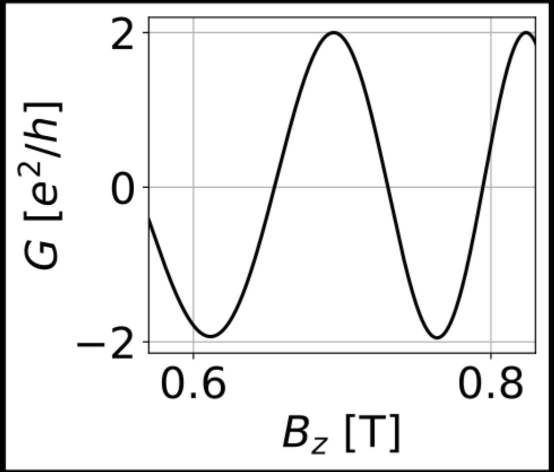


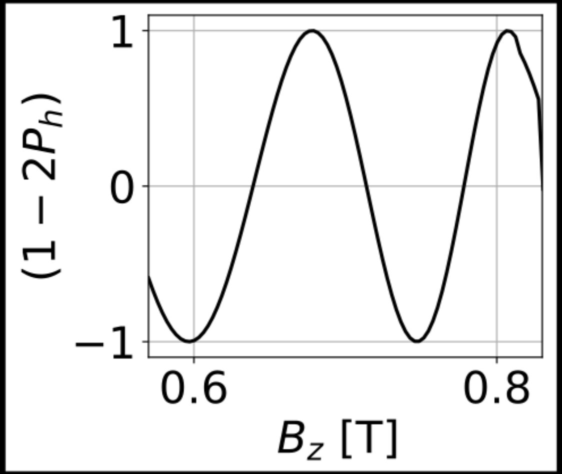


Spinful system with Zeeman field along z axis + spin-orbit coupling: at $\nu = 2$

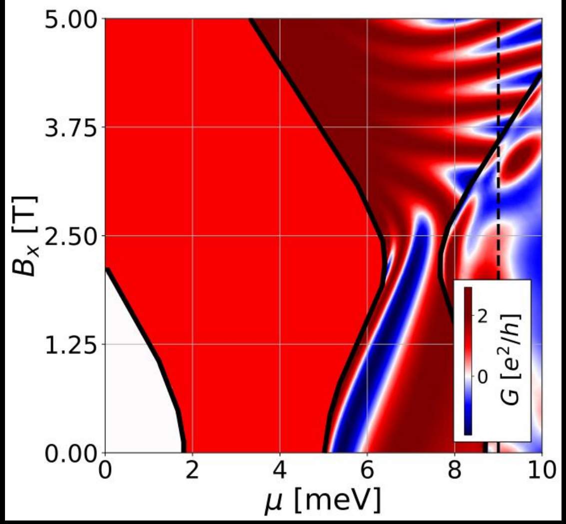


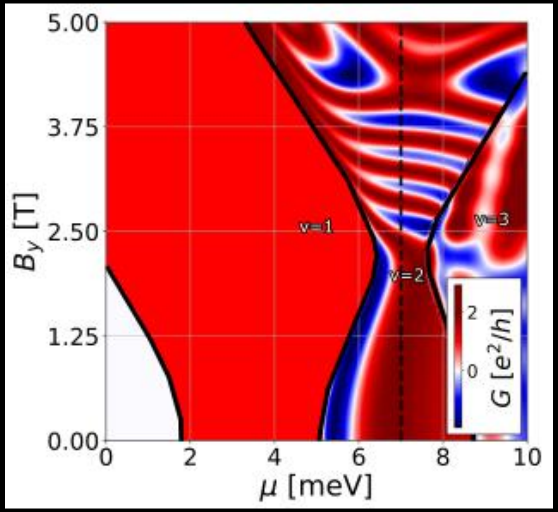


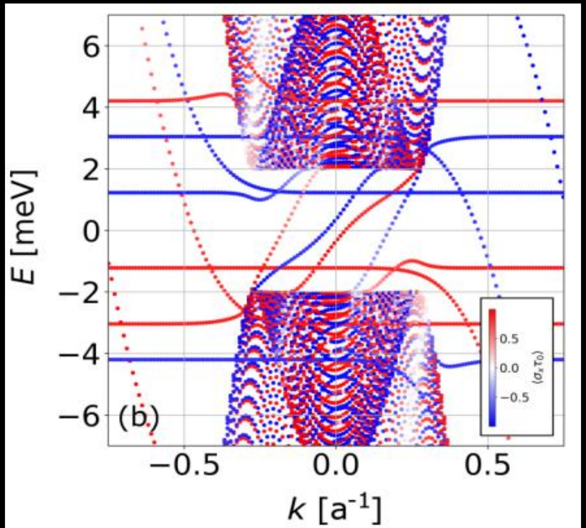


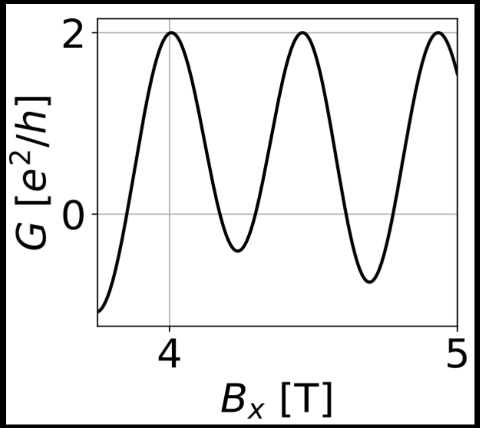


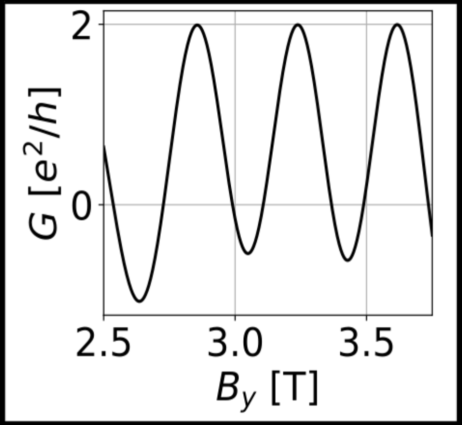
Spinful system with Zeeman field along z axis + external Zeeman field along x/y axis + spin-orbit coupling: at $\nu = 2$

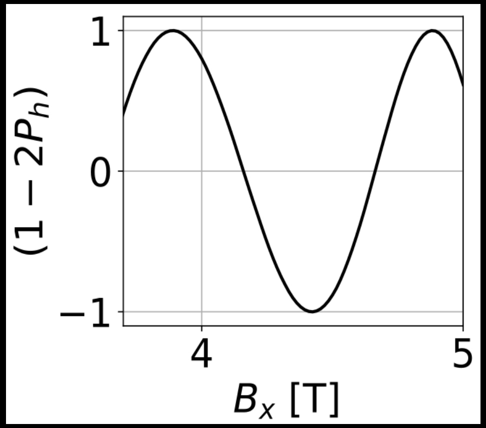


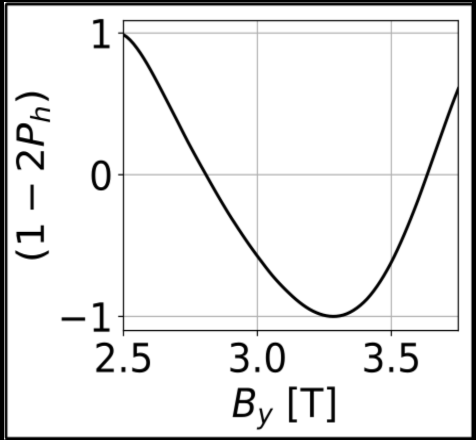




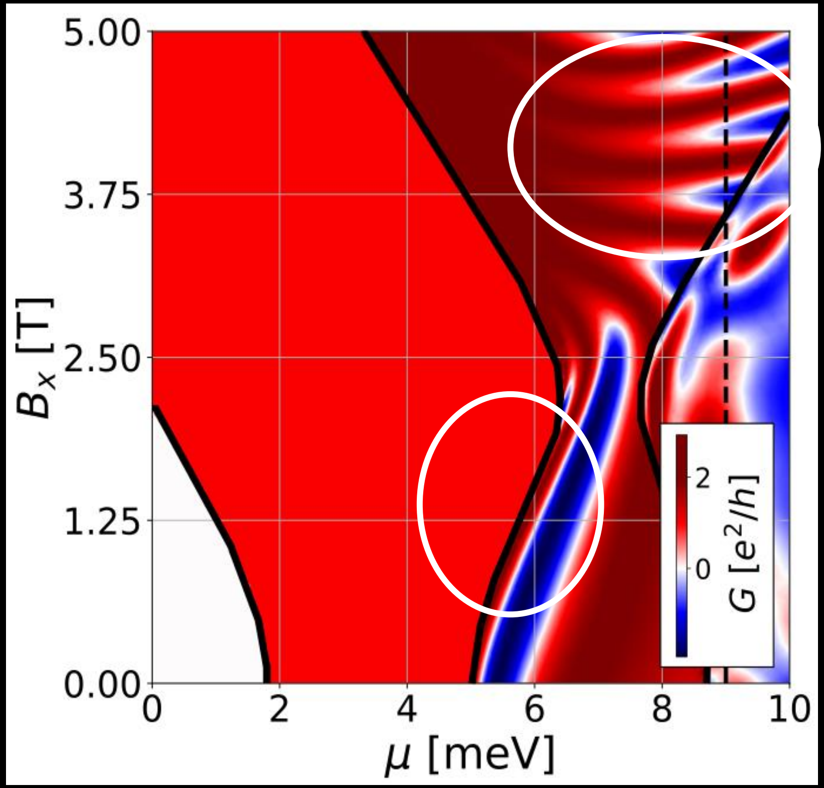


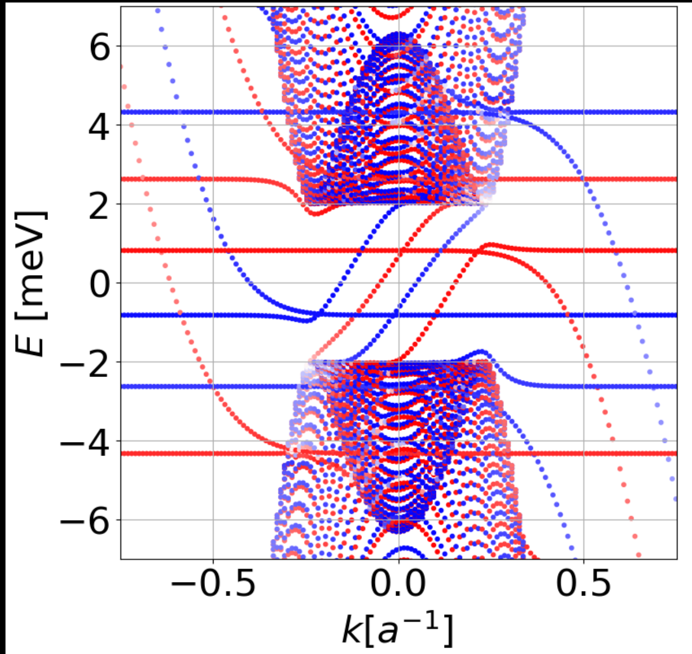


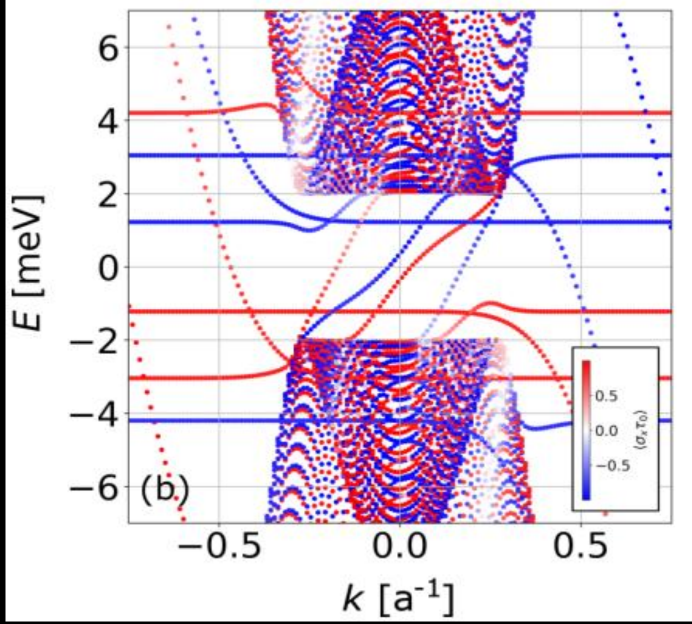


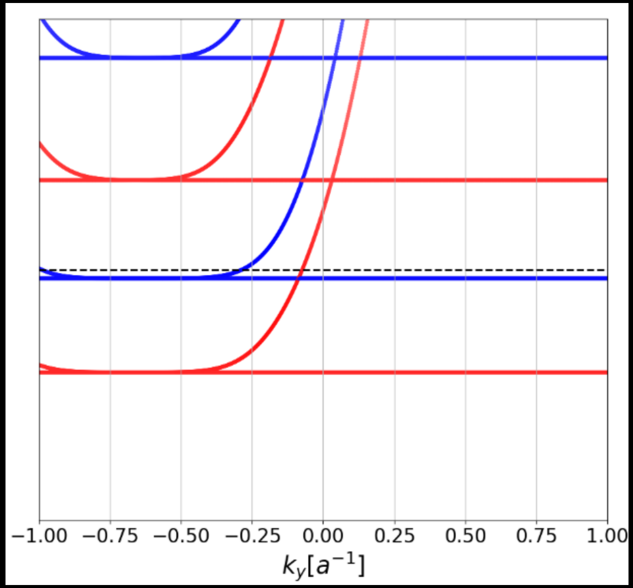


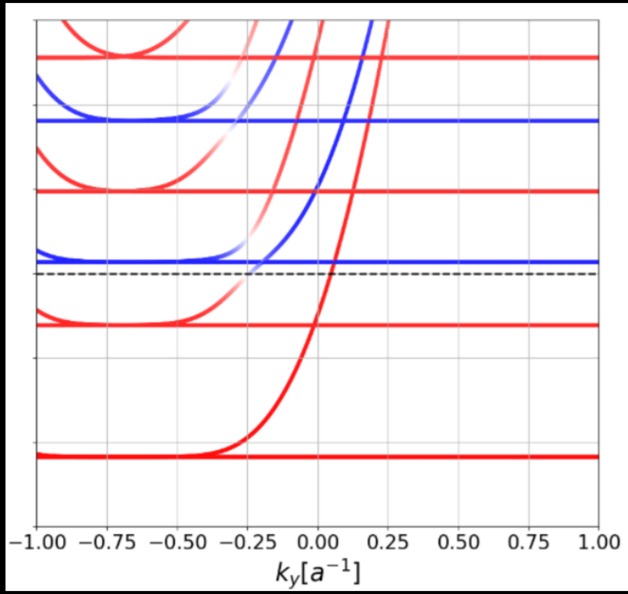
Mixing with SO coupling:



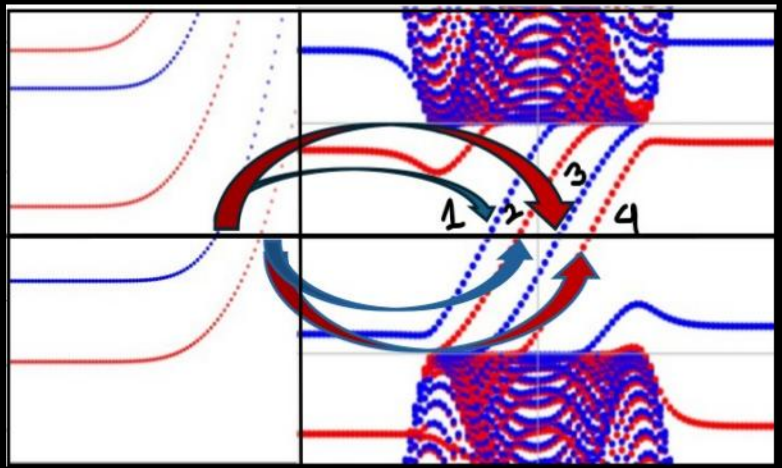


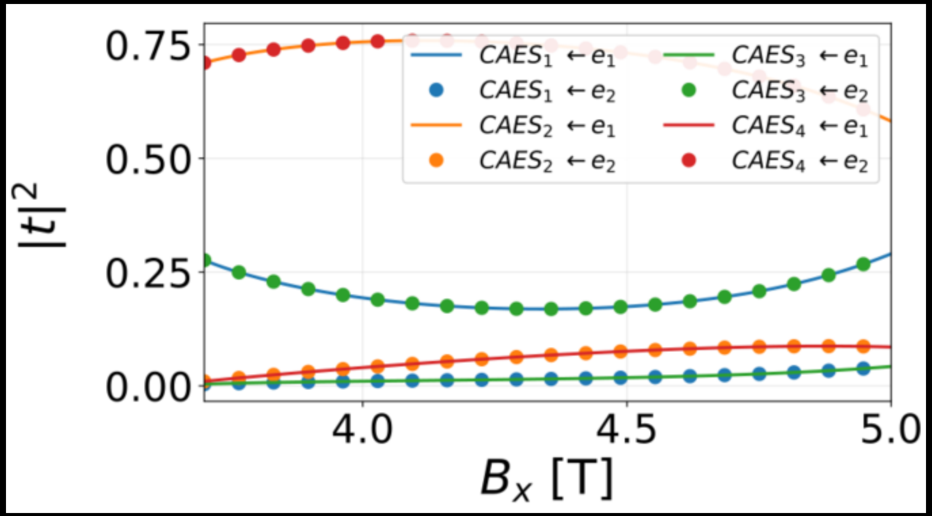


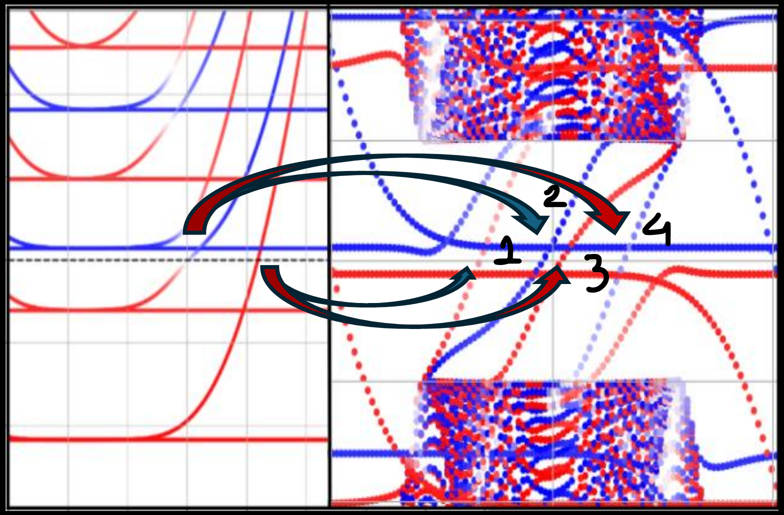


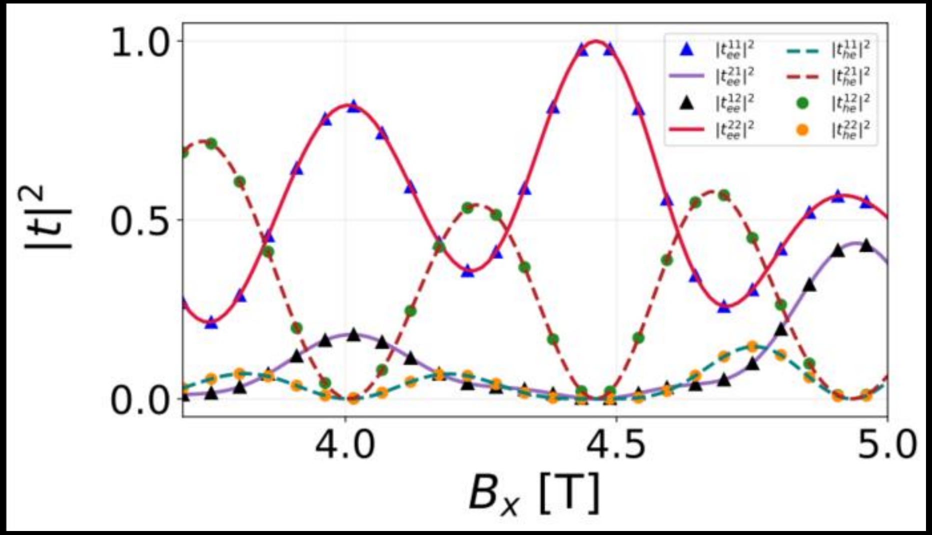


Signatures of mode mixing:









Summary and Conclusion:

- In the spin-degenerate case at v=2, the two-CAES interference formula quantitatively reproduces the conductance oscillations using δk extracted directly from the dispersion relation.
- At v>2, Andreev reflection mixes QH edge modes otherwise independent in a clean system, an electron injected in one channel couples to multiple CAES, breaking the simple two-CAES interference picture entirely.

- Out-of-plane Zeeman interaction splits the spin-degenerate bands into spin-split CAES pairs, spin orthogonality between opposite-spin sectors protects the analytical two-CAES picture.

- With B_z and SOI, spin depolarization is slight, electrons couple only to same-spin CAES and the analytical formula is preserved.

- With in-plane field and SOI the forbidden window disappears, SOI prevents perfect spin polarization, restores Andreev reflection, and the two orbital channels amplify mixing across all four CAES, breaking the analytical formula.

Thank you!