

Probing Helical states with Mesoscopic Superconductivity

Mesoscopic Physics group, Laboratoire de Physique des Solides, Orsay, U. Paris-Saclay, France

A. Bernard, A. Murani, B. Dassonneville, X. Ballu, L. Bugaud, M. Bard, C. Li, J. Lefeuvre, R. Delagrangé, A. Kasumov, R. Deblock, M. Ferrier, H. Bouchiat and S. Guéron

Theory collaborators:

Y. Peng, Y. Oreg, F. von Oppen, E. Gerber, B. Wieder, A. Bernevig, T. Neupert, G. Vigniale, C. Gorini
(Caltech USA, Weizmann Institute, Israel, Freie Universität Berlin, Zurich, Princeton, CEA Saclay)

Bard et al, in preparation (2026)

Gorini et al, in preparation (2026)

Ballu et al Phys Rev Research (2026)

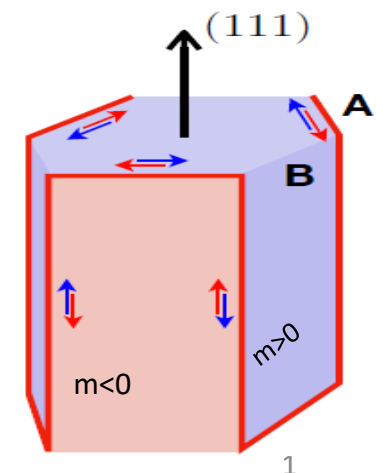
Lefeuvre et al, Phys. Rev. X (2026)

Bernard, Peng et al, Nat Phys (2023)

Murani et al, Phys. Rev. Lett (2019); Nature Comm. (2017); Phys.Rev. B (2017)

Schindler et al, Nature Physics (2018)

Li et al., Phys. Rev. B (2014)



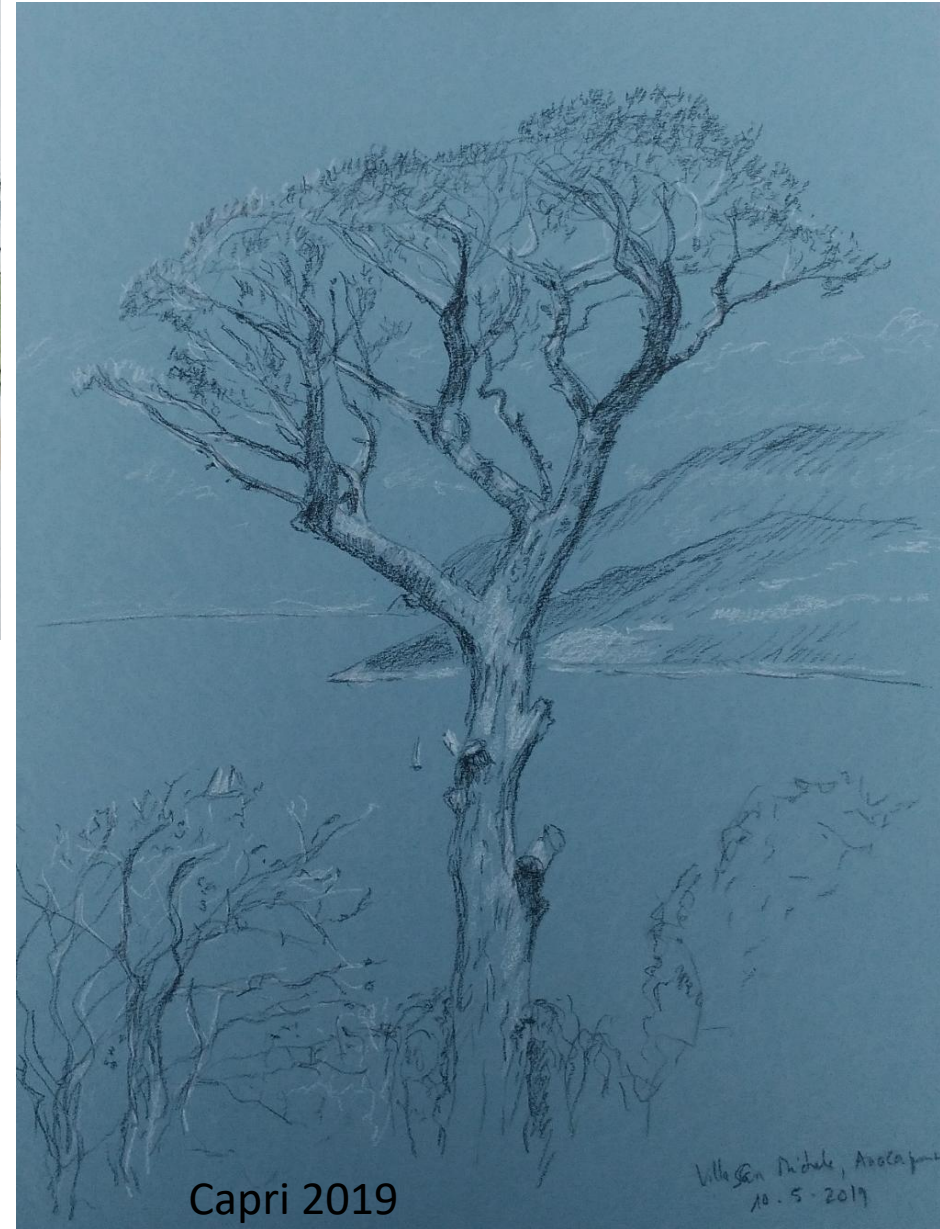
Capri 2008, Capri 2019, 2026: Thank you so much dear organizers!!!



Capri 2026



Capri 2019

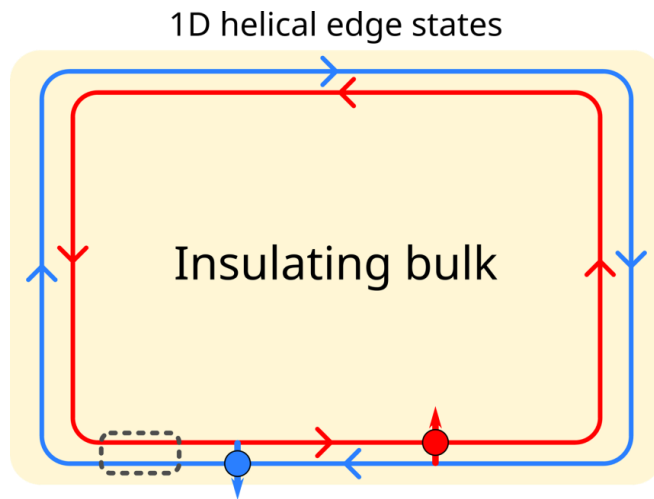


Capri 2019

Outline:

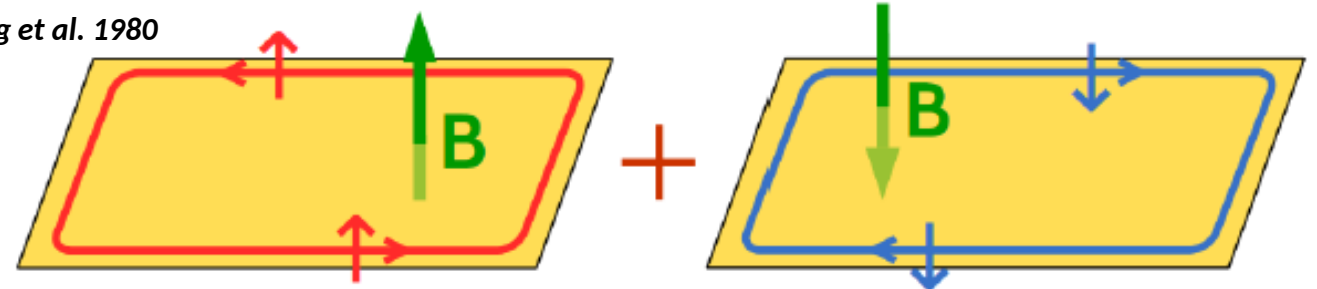
- What are helical states and why are they interesting?
- Helical states in 2D Topological Insulators, first discovered and probed with Non-superconducting contacts
- New type of helical states, in 3D materials: « Second Order » Topological Insulator:
 - **Need to go to S contacts!** (I'll explain why)
 - Experiments in bismuth:
 - Simple Josephson junctions
 - Asymmetric SQUID to determine supercurrent versus phase relation; relaxation
 - RF susceptibility (complex impedance) to test topological protection
 - Bilinear magnetoresistance/Josephson diode to probe spin-momentum locking and spin-axis
 - Helical states also seen in WTe₂
- A better SOTI: Bi₄Br₄, back to N contacts (for now)
- Projects: -no contacts! dc and ac: noise of supercurrent through protected helical states

Introduction: Probing 1D « helical states »

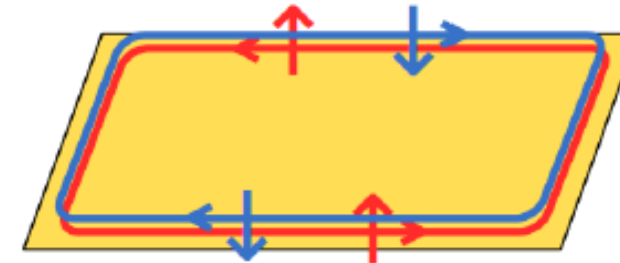


$$\text{helical} = \text{chiral}(B, \uparrow) + \text{chiral}(-B, \downarrow)$$

Klitzing et al. 1980



=



« Helical »
« Quantum spin Hall »

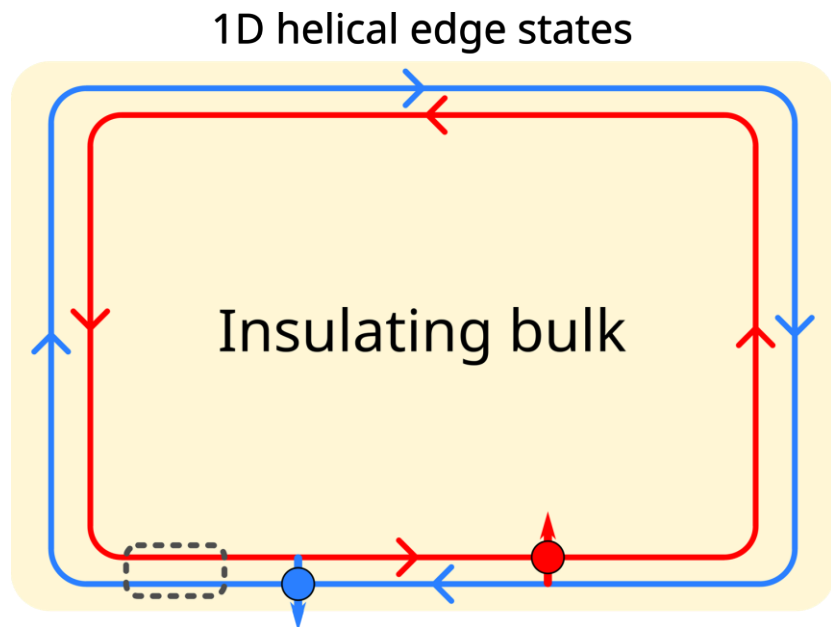
Kane & Mele 2005

Why are they interesting:

- Time-reversal symmetric, Spin-momentum locked -> Protection

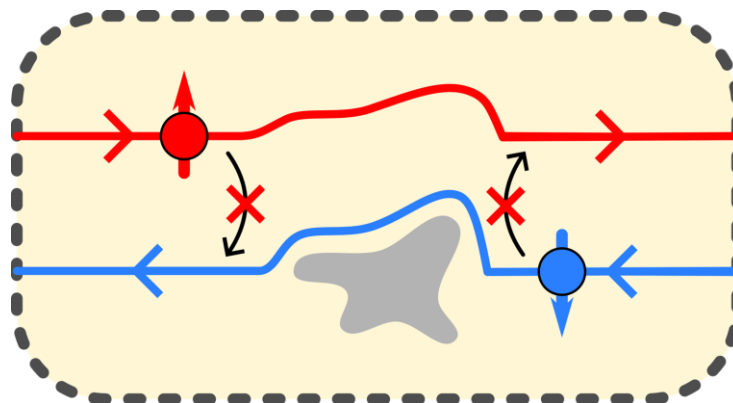
Rare opportunity to find a 1d state that is not localized by disorder!

More specifically: 1D helical states properties



Spin-momentum locking :

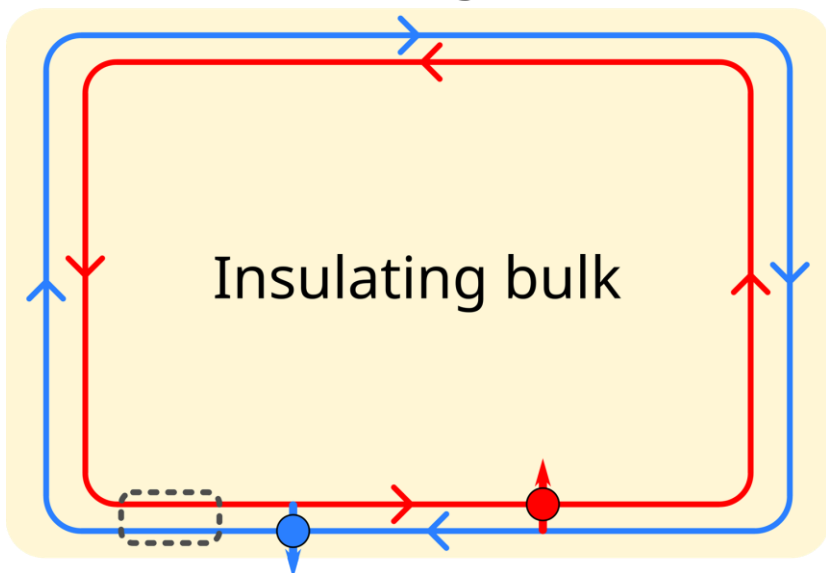
Opposite directions of propagation have opposite spins



Topological protection
(non-magnetic, elastic disorder)
→ **Ballistic transport**
→ **Edge transport**
→ **Spin-momentum locking**

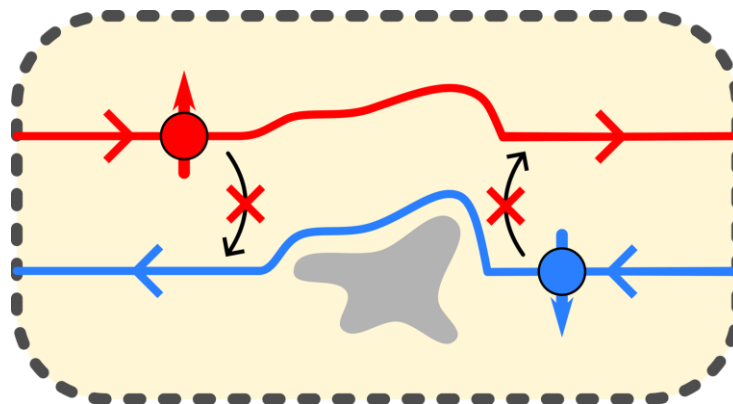
How to probe these special properties of QSH states?

1D helical edge states



Spin-momentum locking :

Opposite directions of propagation have opposite spins



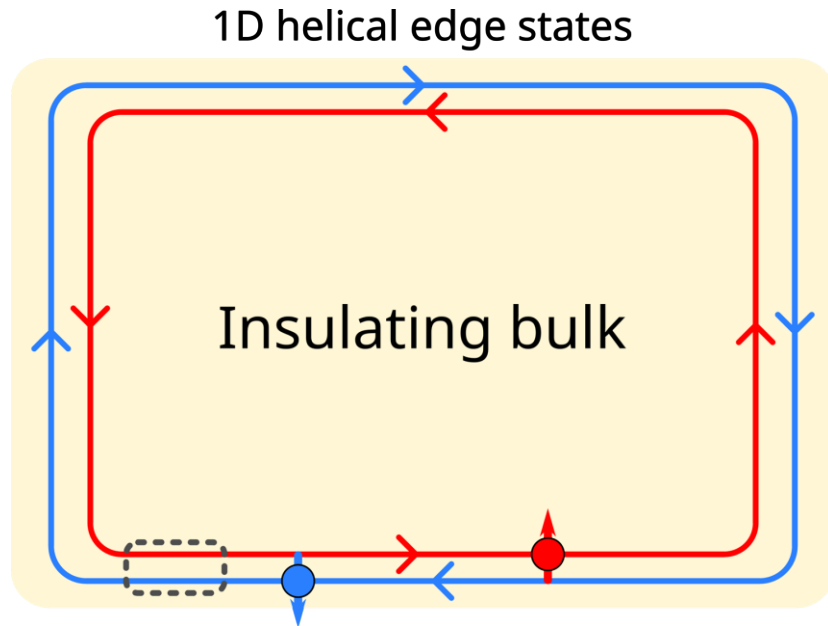
Topological protection

(non-magnetic, elastic disorder)

→ **Ballistic transport**

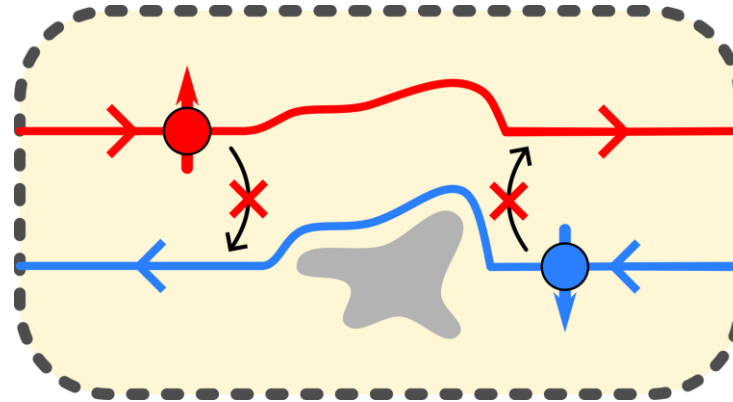
How to probe this?

Probe 1D ballistic states?



Spin-momentum locking :

Opposite directions of propagation have opposite spins



Topological protection
(non-magnetic, elastic disorder)

→ **Ballistic transport**

How to probe this?

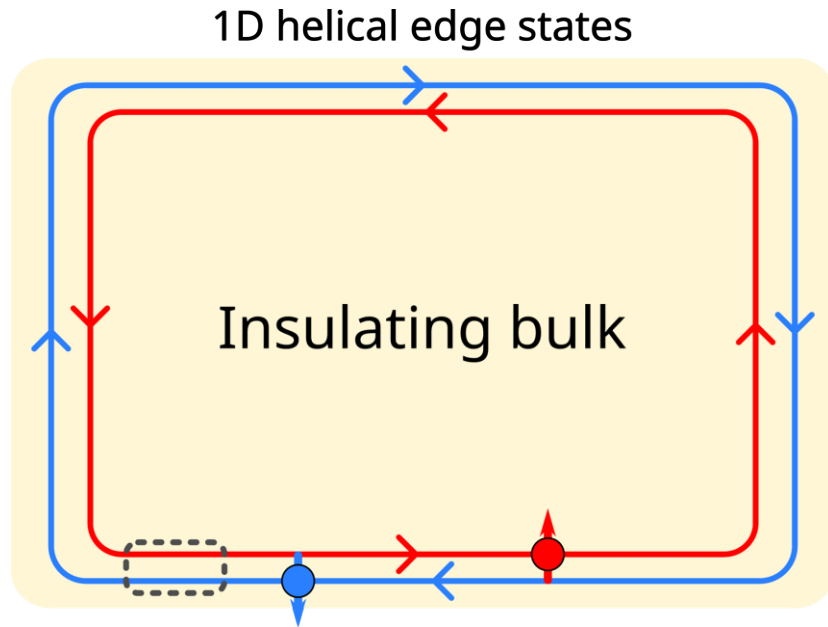
With normal electrodes:
Quantized 2 wire conductance

$$G_Q = 2 \frac{e^2}{h}$$

With superconducting electrodes:

Supercurrent versus phase relation

Probe transport at edges?

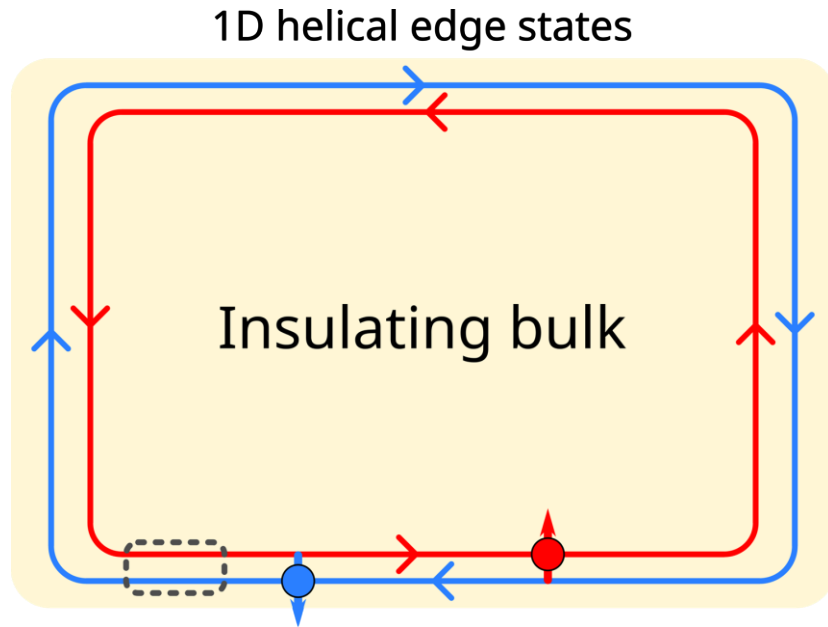


→ Edge transport
How to probe this?

Spin-momentum locking :

Opposite directions of
propagation have opposite
spins

Probe transport at edges?



→ Edge transport
How to probe this?

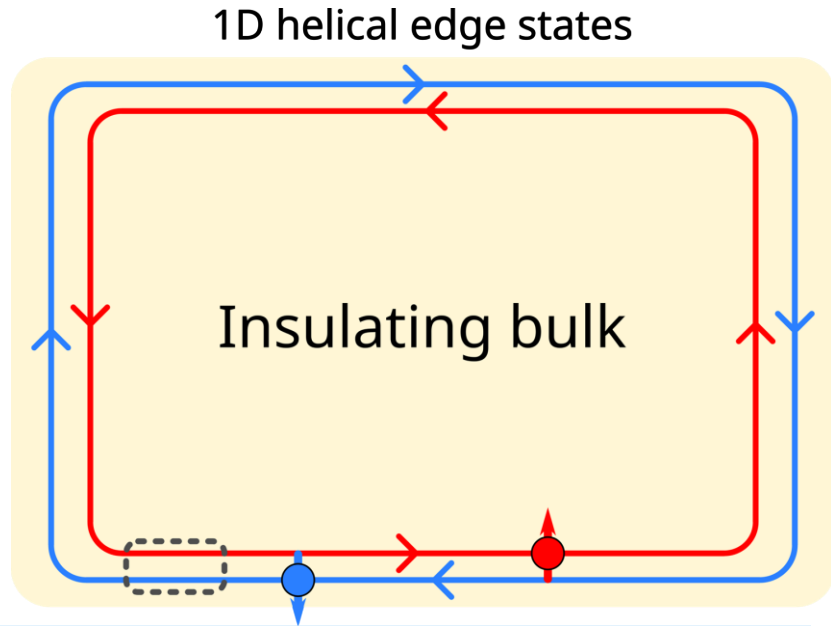
Spin-momentum locking :

Opposite directions of
propagation have opposite
spins

With normal electrodes:
Non local conductance

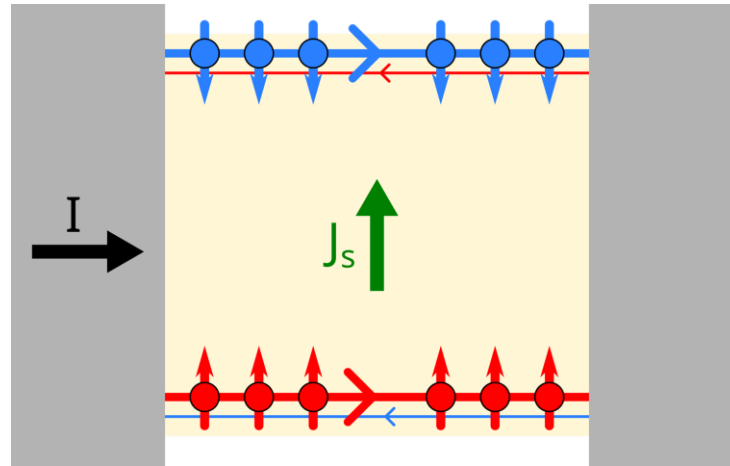
With superconducting
electodes:
Josephson interferometry

Probe spin-momentum locking?



Spin-momentum locking :

Opposite directions of propagation have opposite spins



How to probe spin-momentum locking

Quantum spin Hall effect

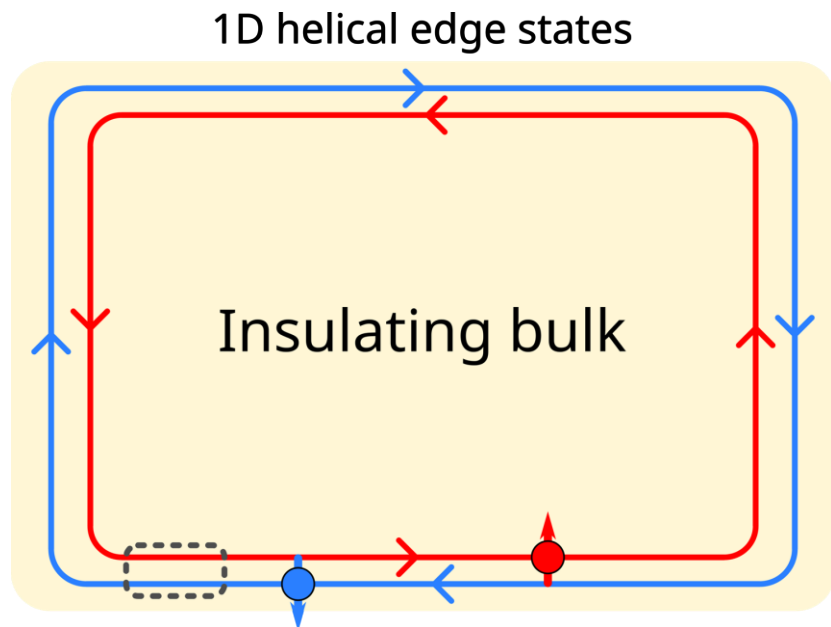
Charge current



Transverse spin current

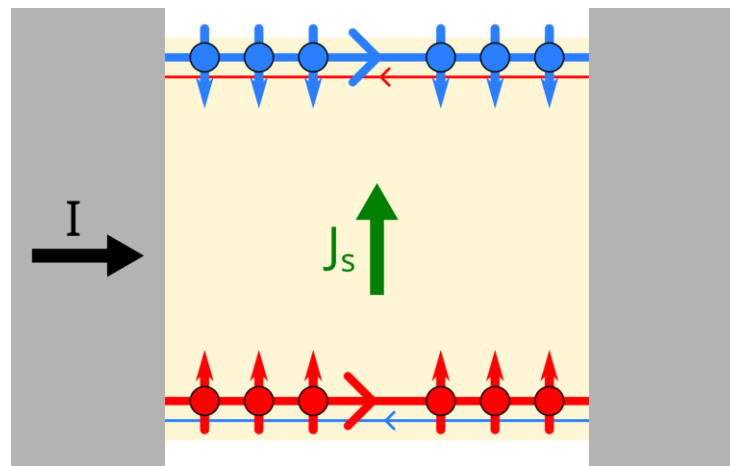
→ Non-quantized

Probe spin-momentum locking?



Spin-momentum locking :

Opposite directions of propagation have opposite spins



Quantum spin Hall effect

Charge current



Transverse spin current

→ Non-quantized

How to probe spin-momentum locking

Edelstein effect in sample with broken inversion symmetry and broken TRS:

Current-induced spin-polarisation?

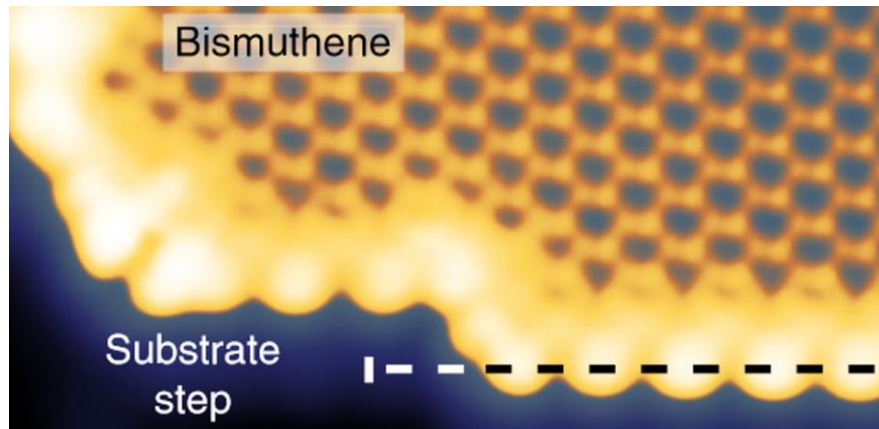
Compare normal and superconducting electrodes

Initially QSH states were predicted/discovered in 2D topological insulators

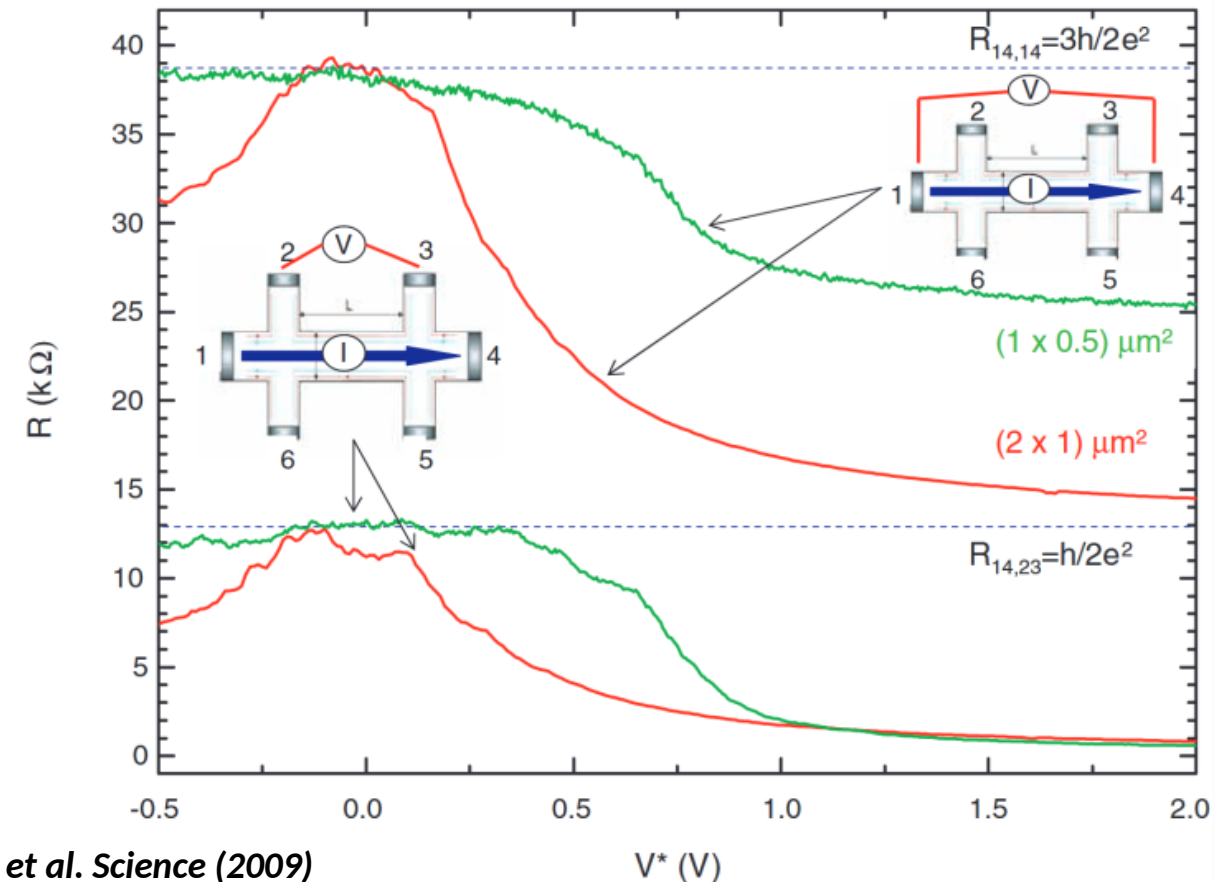
- HgTe/CdTe quantum wells (2007, Molenkamp group, Würzburg)
- Bismuthene (Claeson group, Würzburg, 2017)
- Monolayer WTe₂ (Wu, Herrero, Cobden, Xu, 2017)
- also InAs/GaSb quantum wells (Du, 2011, Rice)

Probed by normal transport

Probed by STM



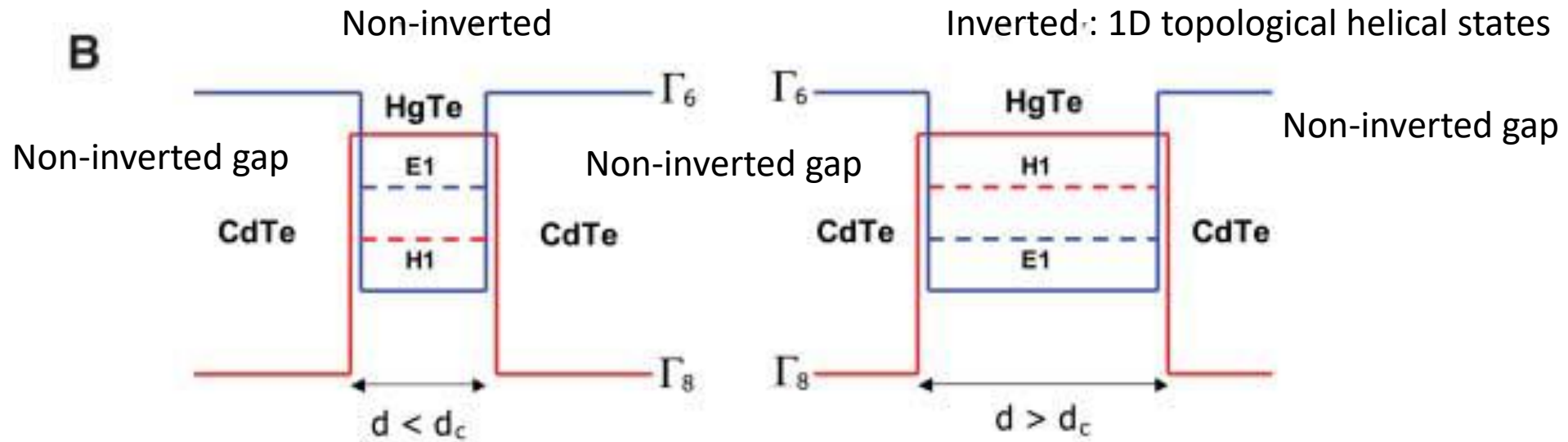
Stühler et al. Nat. Phys. (2020).



Roth et al. Science (2009)

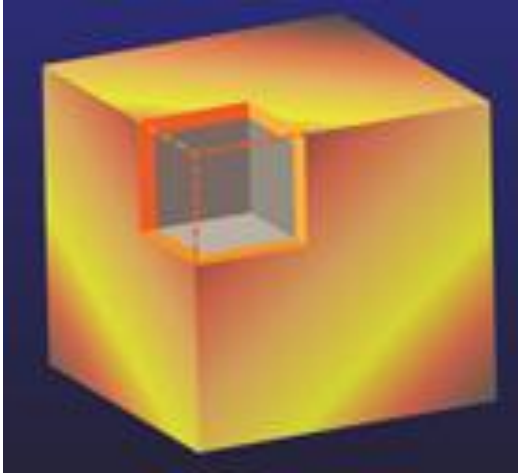
Ballisticity : via quantized conductance (in zero field!),
Edges : via non-local conductance

Mechanism: Band inversion in 2D materials-> 1D edge states.

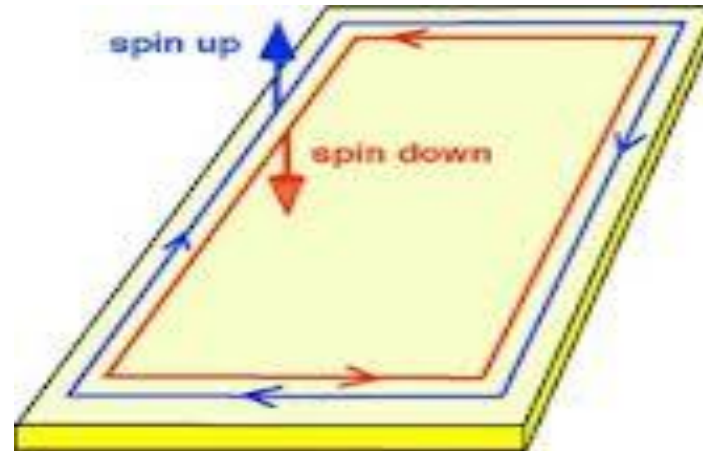


From First Order TI to Second order Topological Insulators

First order Topological Insulators Single band inversion

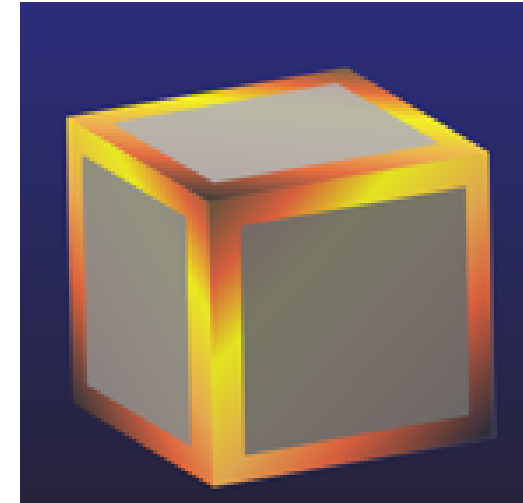


3D topological insulator
Insulating bulk
2D Conducting surfaces



2D topological insulator
insulating bulk
1D conducting helical edges

Extension of the classification Second order Topological Insulator, Double band Inversion



3D Second Order Topological Insulator
insulating bulk
insulating surfaces
1D conducting helical « hinges »

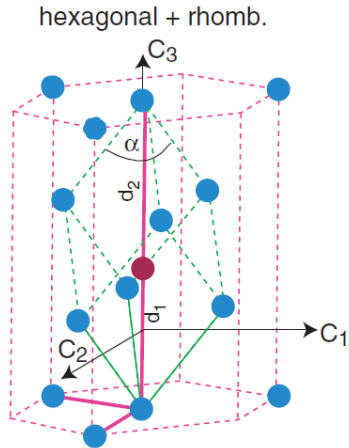
Helical edge states in SOTIs too, but harder to see
(in the first discovered SOTIs. In fact, there is a new promising SOTI...)

Schindler *et al*, Sci. Adv. (2018)
Geier *et al*, Phys. Rev. B (2018)
Xie *et al*, Nat. Rev. Phys. (2021)

Bismuth, a Second Order Topological Insulator ! Theory

Schindler et al, Nat. Phys. 2018

Crystal structure



Bi has inversion (I) and
3-fold rotation symmetries (C_3)
+TRS

Fu-Kane Topological index

$$\nu = \nu_T \nu_\Gamma \nu_X \nu_L = +1$$

Even number of total band inversions:

$\Rightarrow$ not a topological insulator... to first order

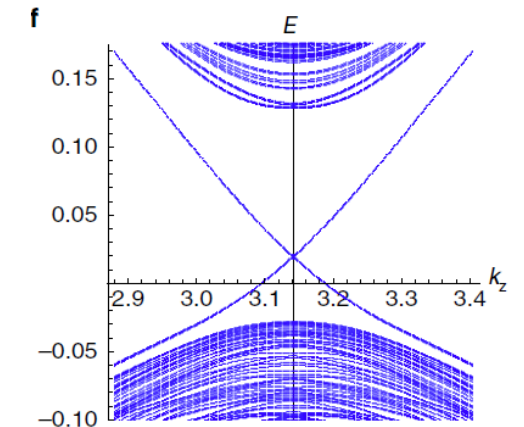
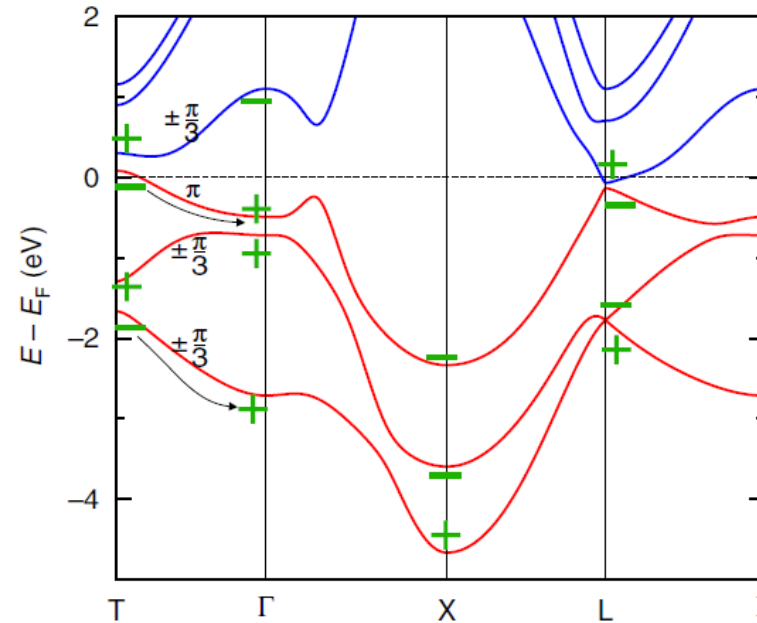
But: odd number of band inversions in each C_3 -subspace

$$\nu^{(\pi)} = \nu_T^{(\pi)} \nu_\Gamma^{(\pi)} \nu_X \nu_L = -1$$

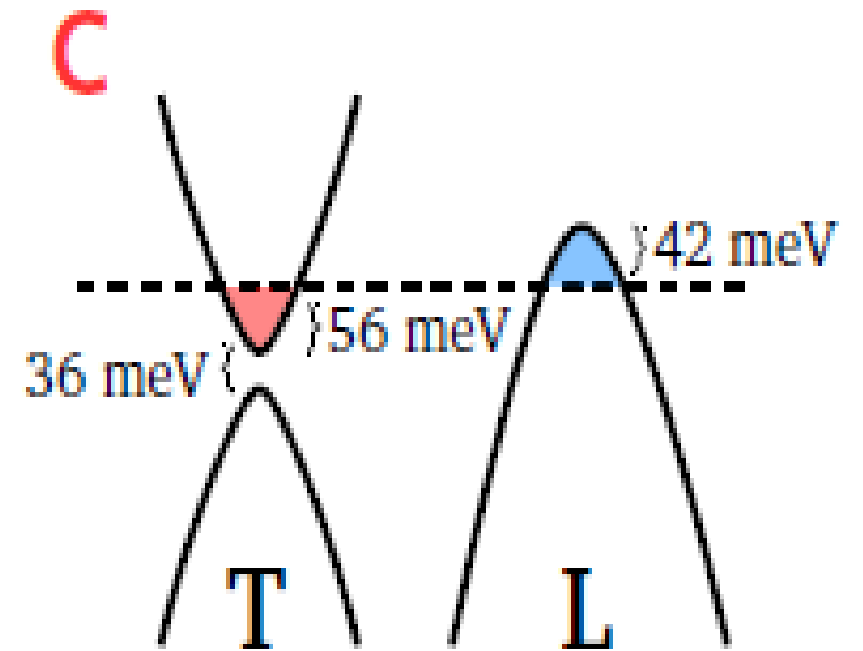
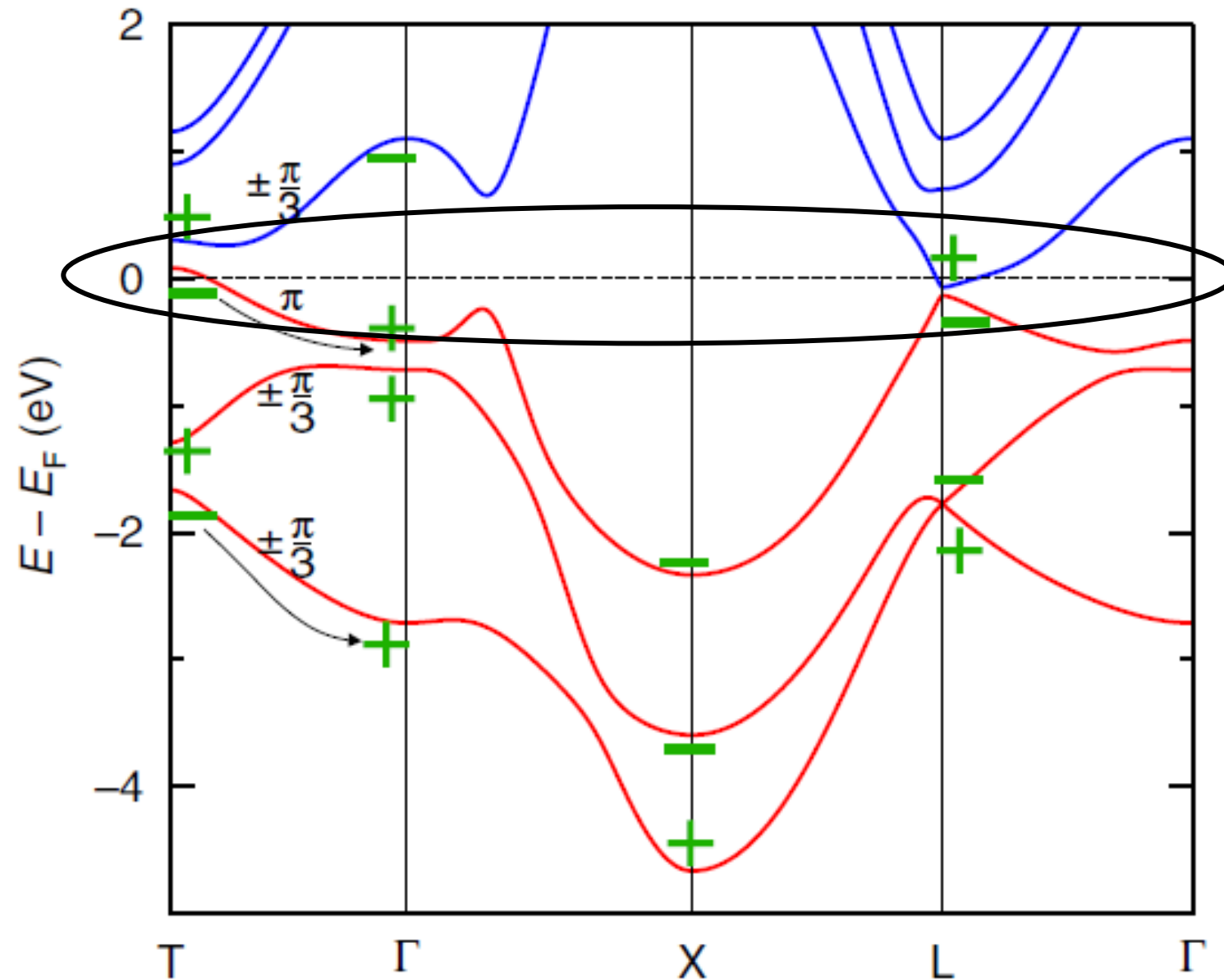
$$\nu^{(\pm\pi/3)} = \nu_T^{(\pm\pi/3)} \nu_\Gamma^{(\pm\pi/3)} = -1$$

$\Rightarrow$ SOTI! With helical states at hinges between surfaces with opposite mass

Band structure: double band inversion



But... Bismuth is a semimetal, not an insulator! Many trivial states



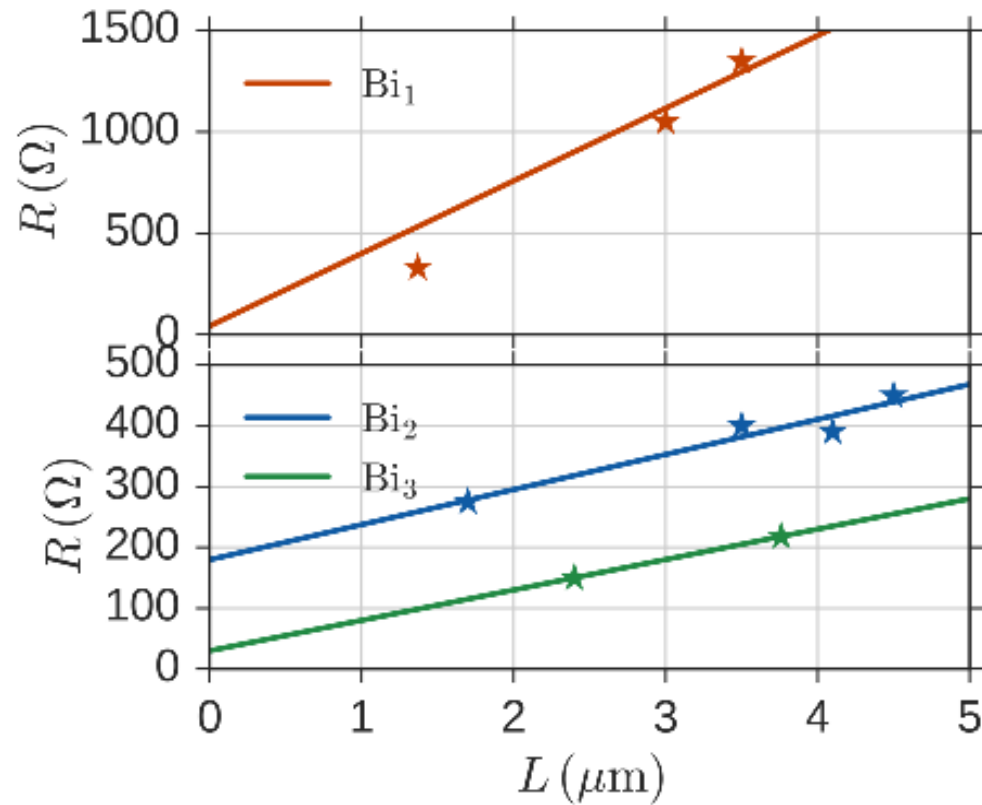
Normal transport in Bismuth doesn't show traces of ballistic states

Bulk $\lambda_F \simeq 50$ nm

Surface $\lambda_F \simeq 5$ nm

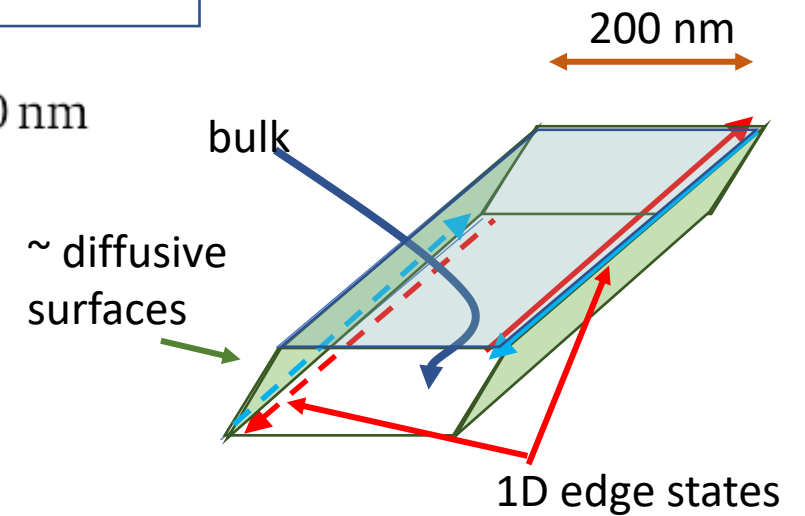
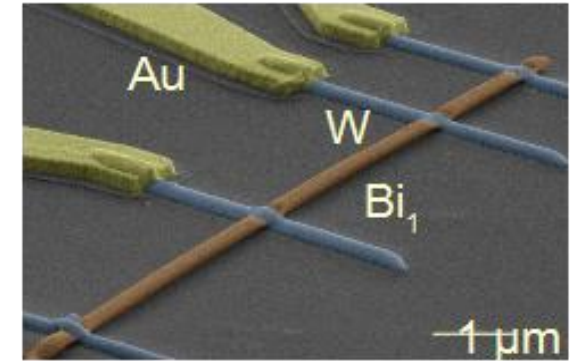
Roughly 50 times more surface states than bulk states

Normal state resistance of nanowire versus segment length



$$R(L) = R_c + \frac{R_Q}{M} \frac{L}{l_e}$$

Thus $l_e \lesssim 200$ nm



Trivial diffusive surface states carry almost all the normal current

$\Rightarrow$ Turn to supercurrent to enhance visibility of ballistic/topological states

Induced superconductivity enhances contribution of helical states

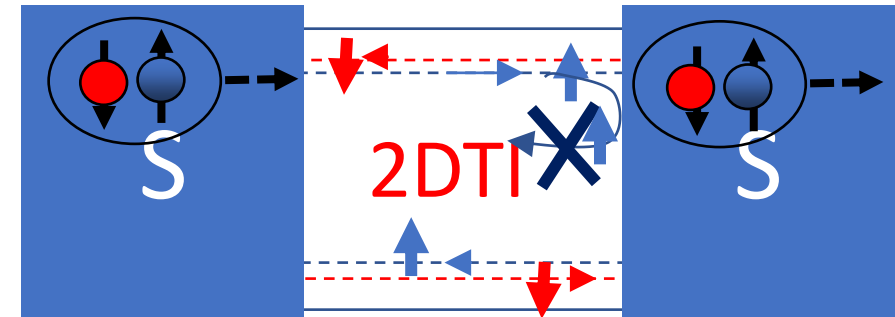
- Critical current carried by diffusive states is much smaller than critical current carried by ballistic/helical states

$$I_{\text{c per channel, ballistic}} \sim \min \left(\Delta, \frac{\hbar v_F}{L} \right) \frac{h}{e^2}$$

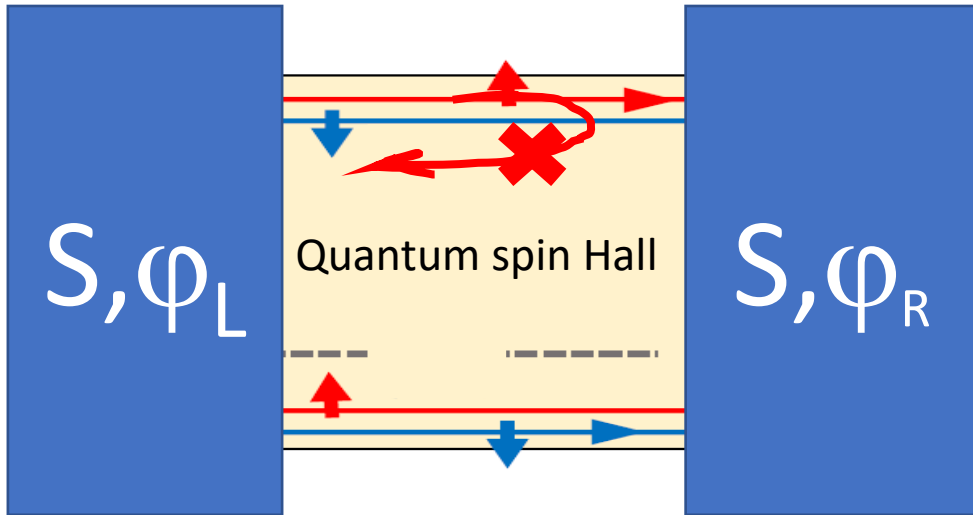
$$I_{\text{c per channel, diffusive}} \sim \min \left(\Delta, \frac{\hbar v_F}{L} \right) \frac{h}{e^2} \left(\frac{l_e^2}{L^2} \right)$$

100 to 1000 times
smaller than
ballistic

- In addition, helical channels should have perfect transmission into S (not true of diffusive channels)



S contacts make it possible to:

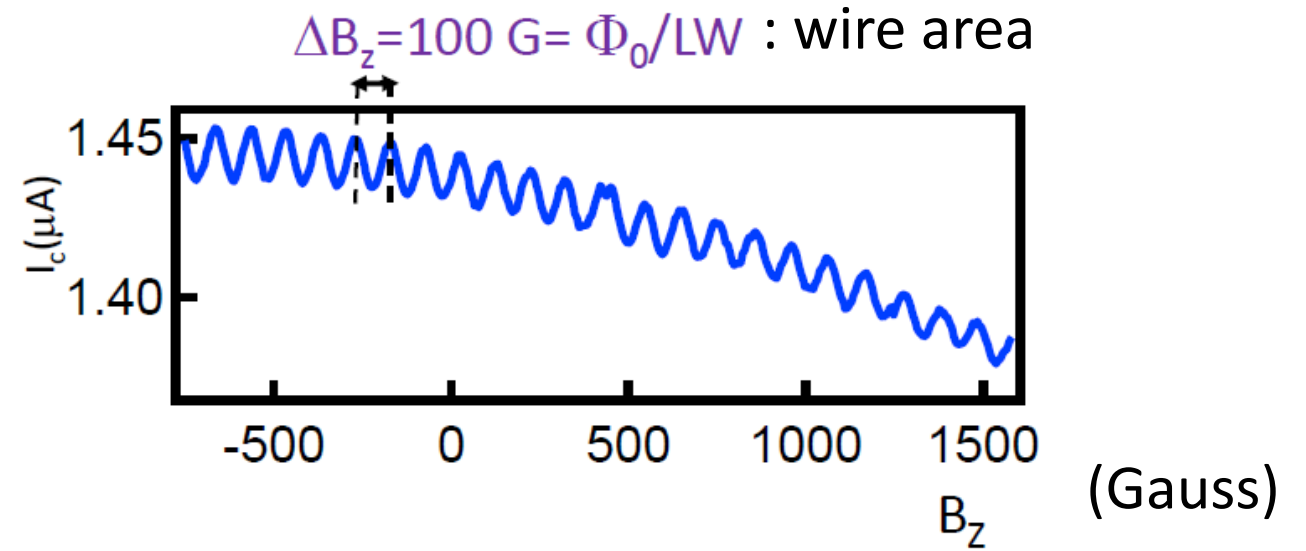
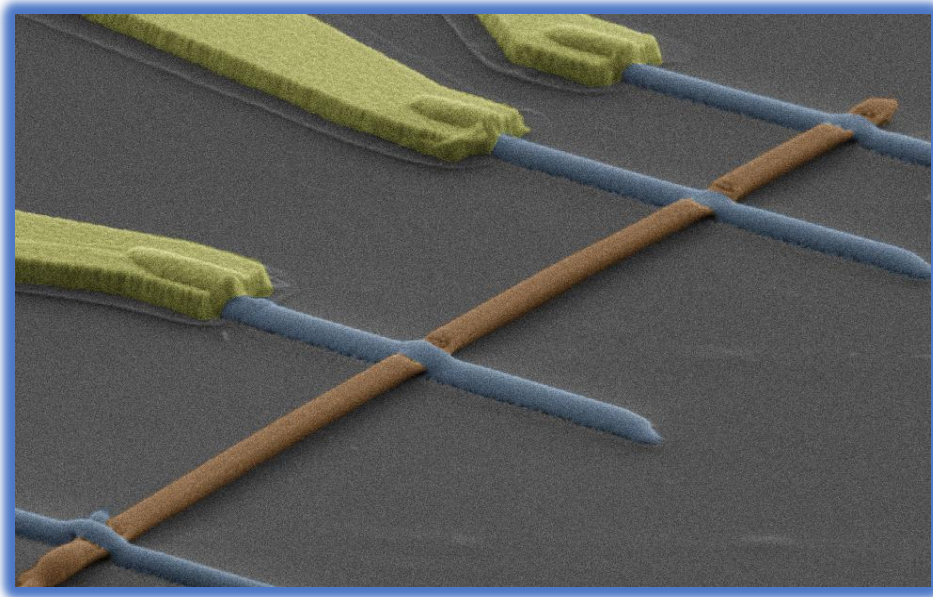


1- Uncover edge channels because of interference (macroscopic wavefunction!), also because much more supercurrent carried by ballistic states

2- Demonstrate topological protection via demonstration of ballistic conduction (perfect transmission?)

3- Probe spin-momentum locking and spin direction via the Josephson diode effect (and compare it to normal non-reciprocal transport)

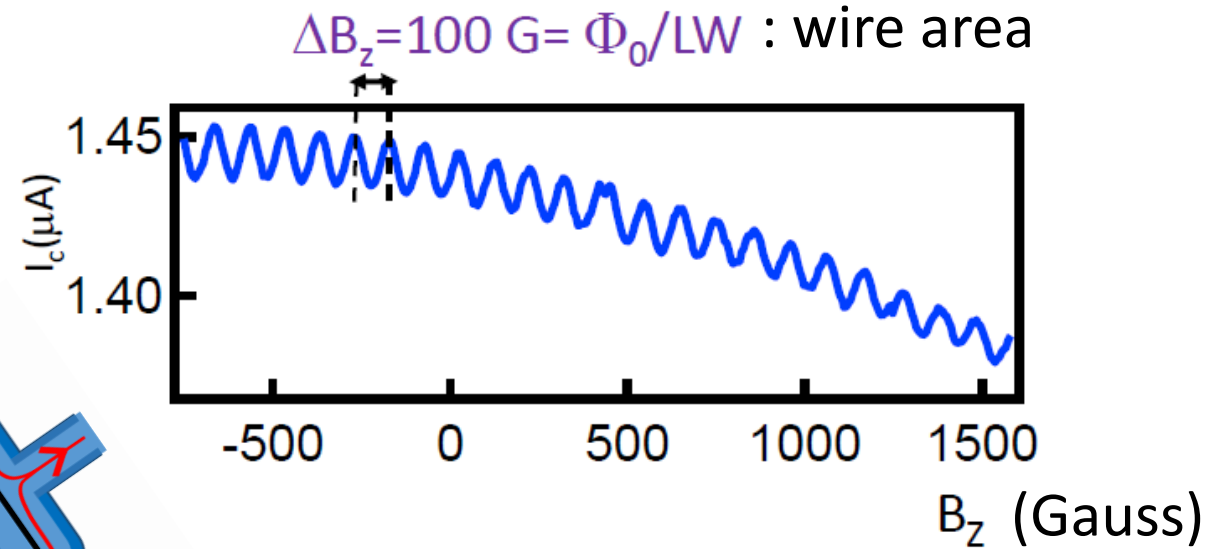
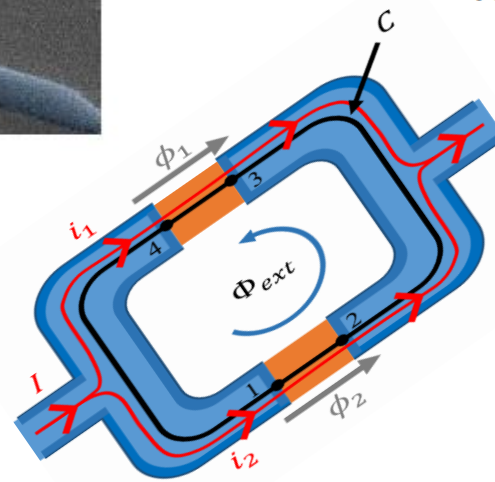
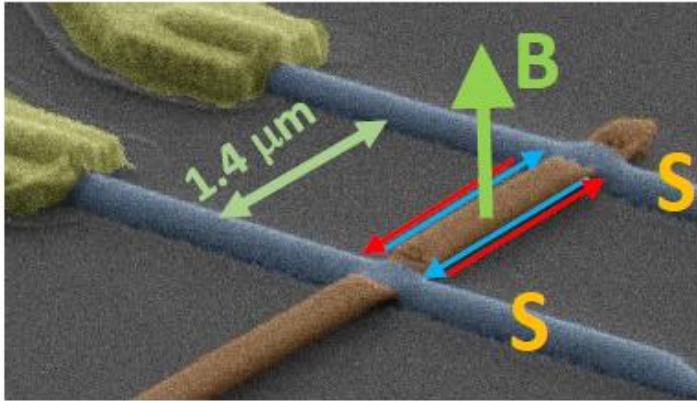
Surprises in first experiments of bismuth nanowires with S contacts



Li 2014, Murani 2017

Critical current oscillates in magnetic field!
What does this mean

Surprises in first experiments of bismuth nanowires with S contacts



- SQUID-like oscillations: there must be two paths (with a hole in the middle)! -> edge states!! discovery that bismuth is a SOTI
- High field (Tesla) decay scale : narrow channels (nm!)
- High critical current : well transmitted channels

Li 2014, Murani 2017

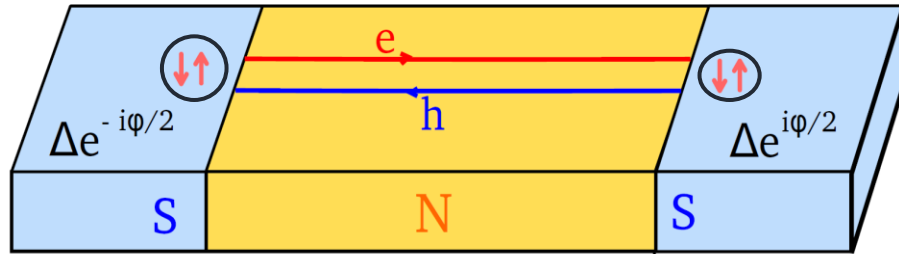
cf Hart et al, 2014 for Squid-like interference in HgTe/HgCdTe quantum wells

But can we say more, beyond the fact that are at edge?

2- Use S contacts to demonstrate ballisticity/topological protection

Andreev spectrum and supercurrent versus phase depend on transport regime

Long SNS junction:



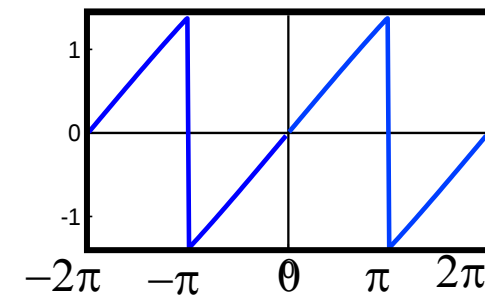
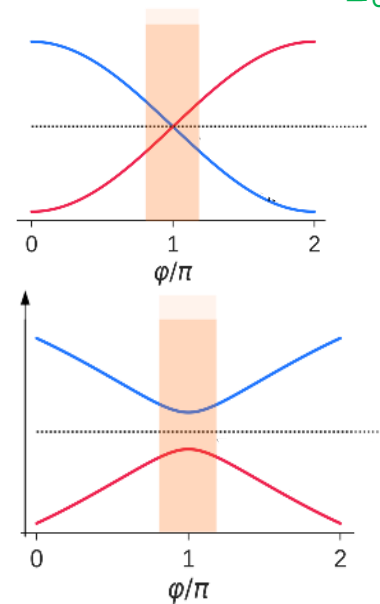
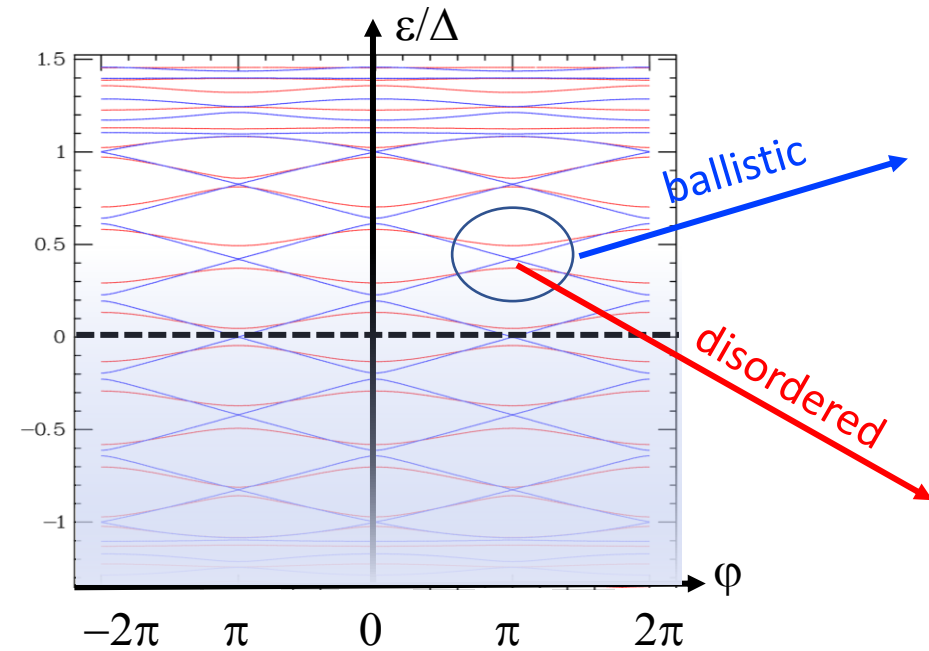
Andreev spectra

$$\frac{2\epsilon L_N}{\hbar v_F} - 2 \arccos \frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

$\sim \pi$

$$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n)$$

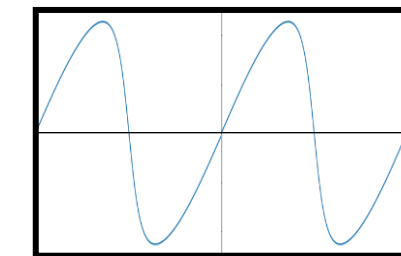
supercurrent versus phase



ev_F/L

ballistic : « Sawtooth »

ϕ



Disordered: almost $\sin\phi$

ϕ

Disorder lifts Andreev level degeneracy at π and rounds $I(\phi)$
 $\Rightarrow$ Measure CPR to distinguish between ballistic and diffusive conduction

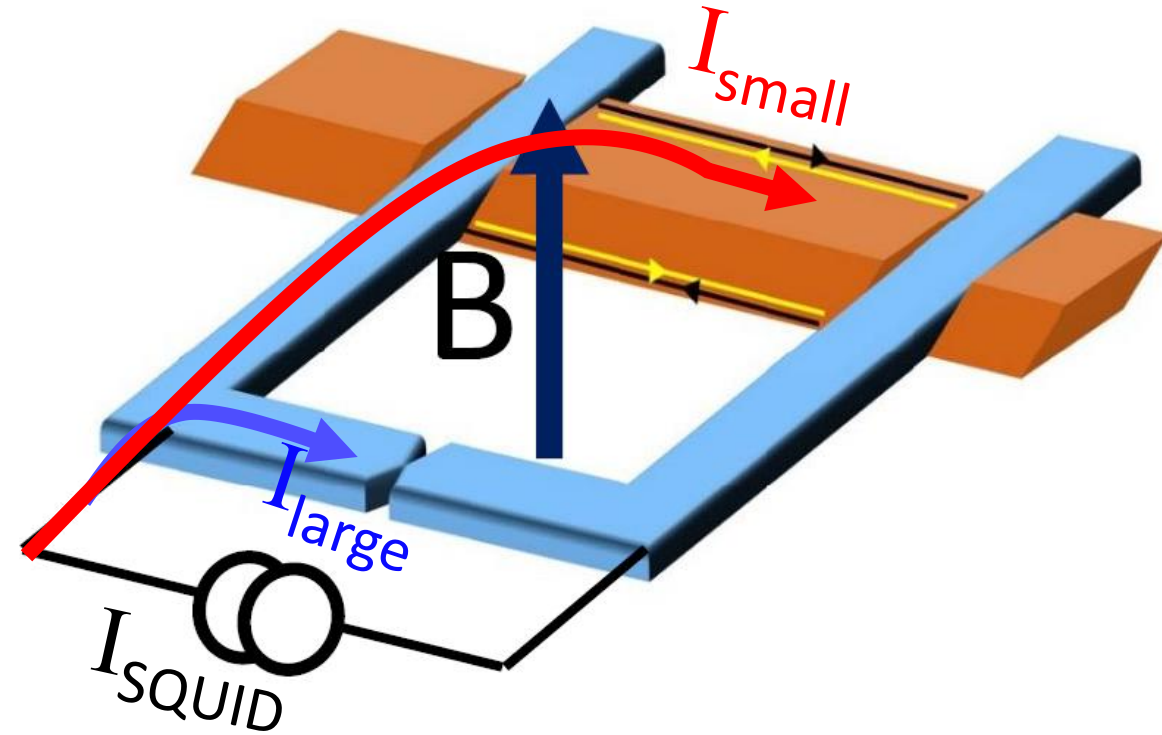
Principle of Supercurrent versus Phase Relation (CPR) measurement:

Phase-biased **asymmetric SQUID** loop

$$\Phi = B \times \text{loop area}$$

$$\varphi = -2\pi \Phi / \Phi_0$$

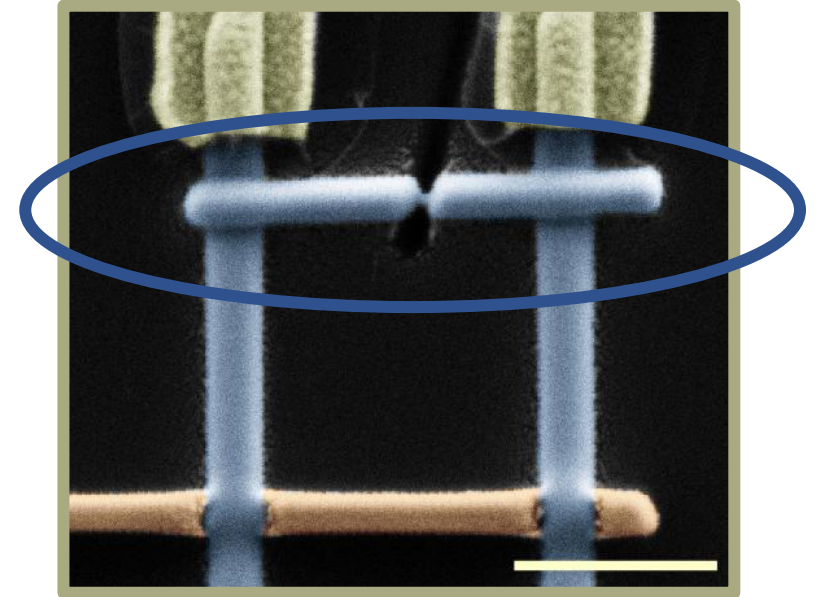
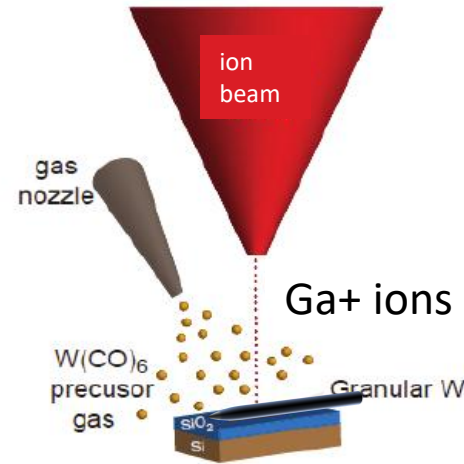
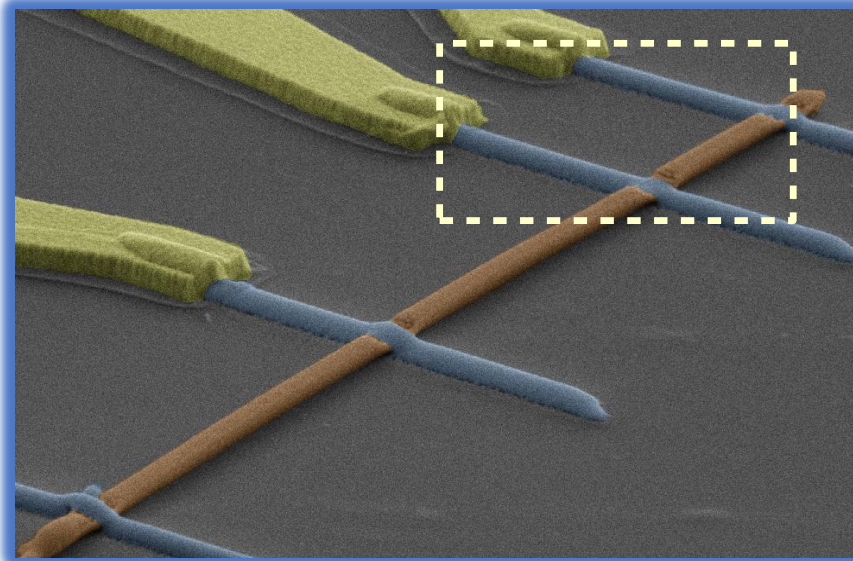
$$I_{c, \text{SQUID}} \approx \underbrace{I_{\text{large}}(\varphi_{\text{max}}) + I_{\text{small}}(\varphi_{\text{max}} + 2\pi \frac{\Phi}{\Phi_0})}$$



$I_c(B)$ measures the phase dependence of the Josephson current of the smallest junction

Measurement of current-phase relation of S/bismuth nanowire/S to probe channels that carry the supercurrent

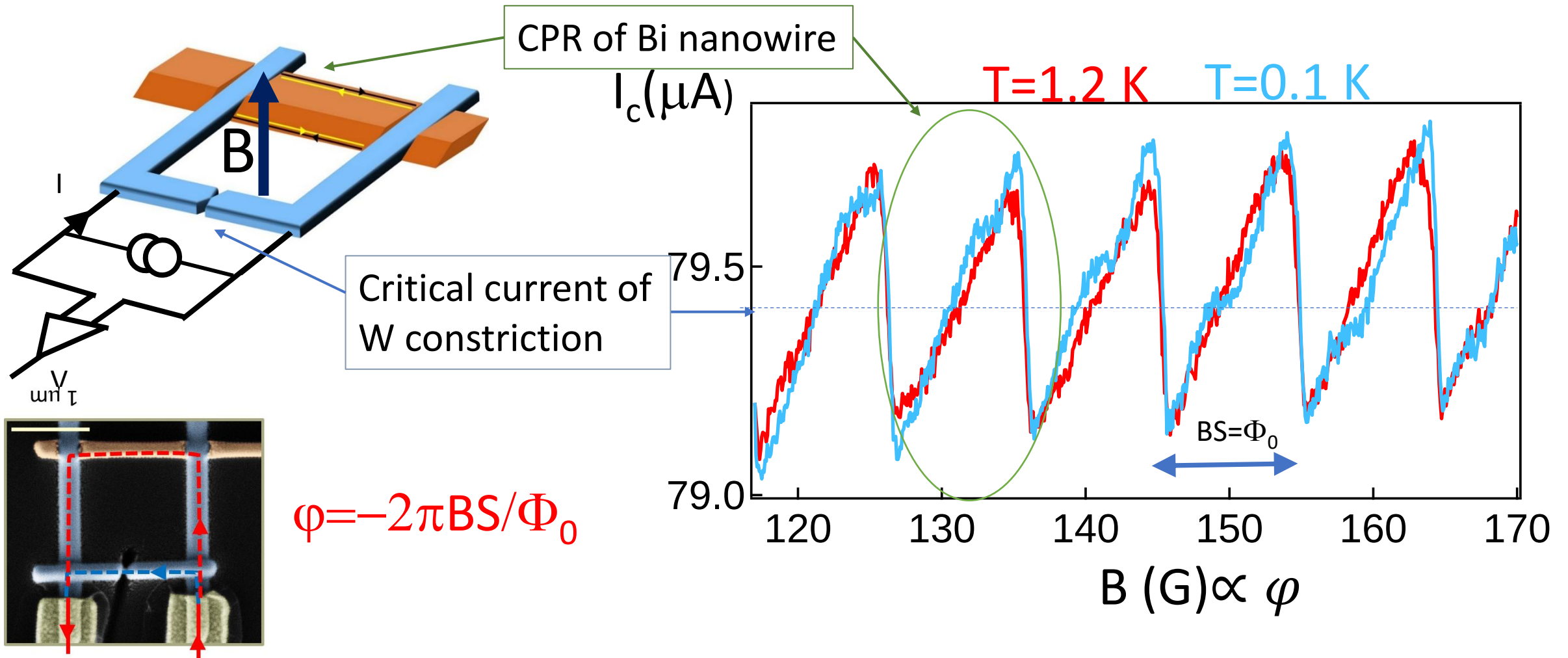
Add superconducting constriction in parallel



1 μm

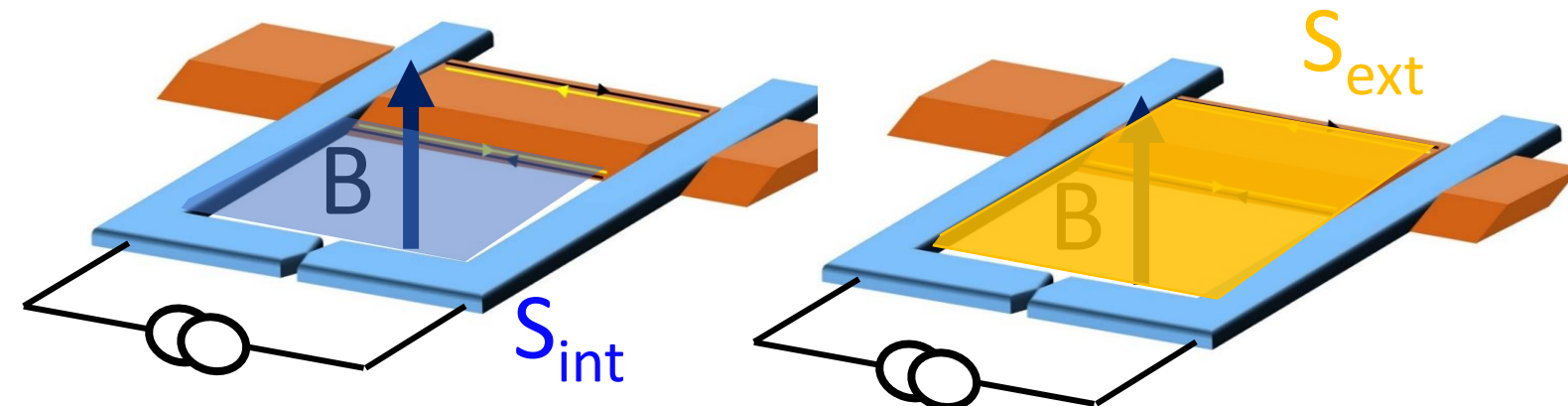
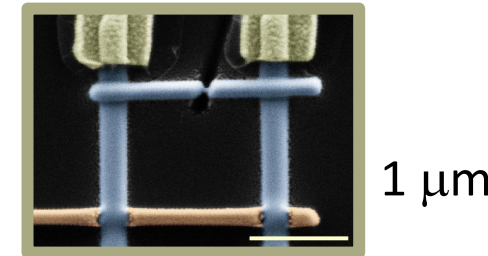
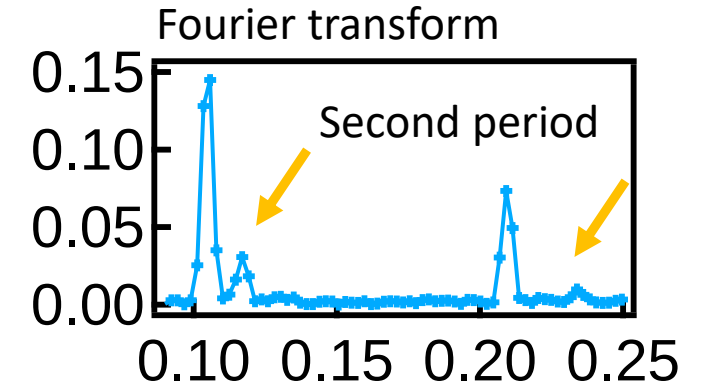
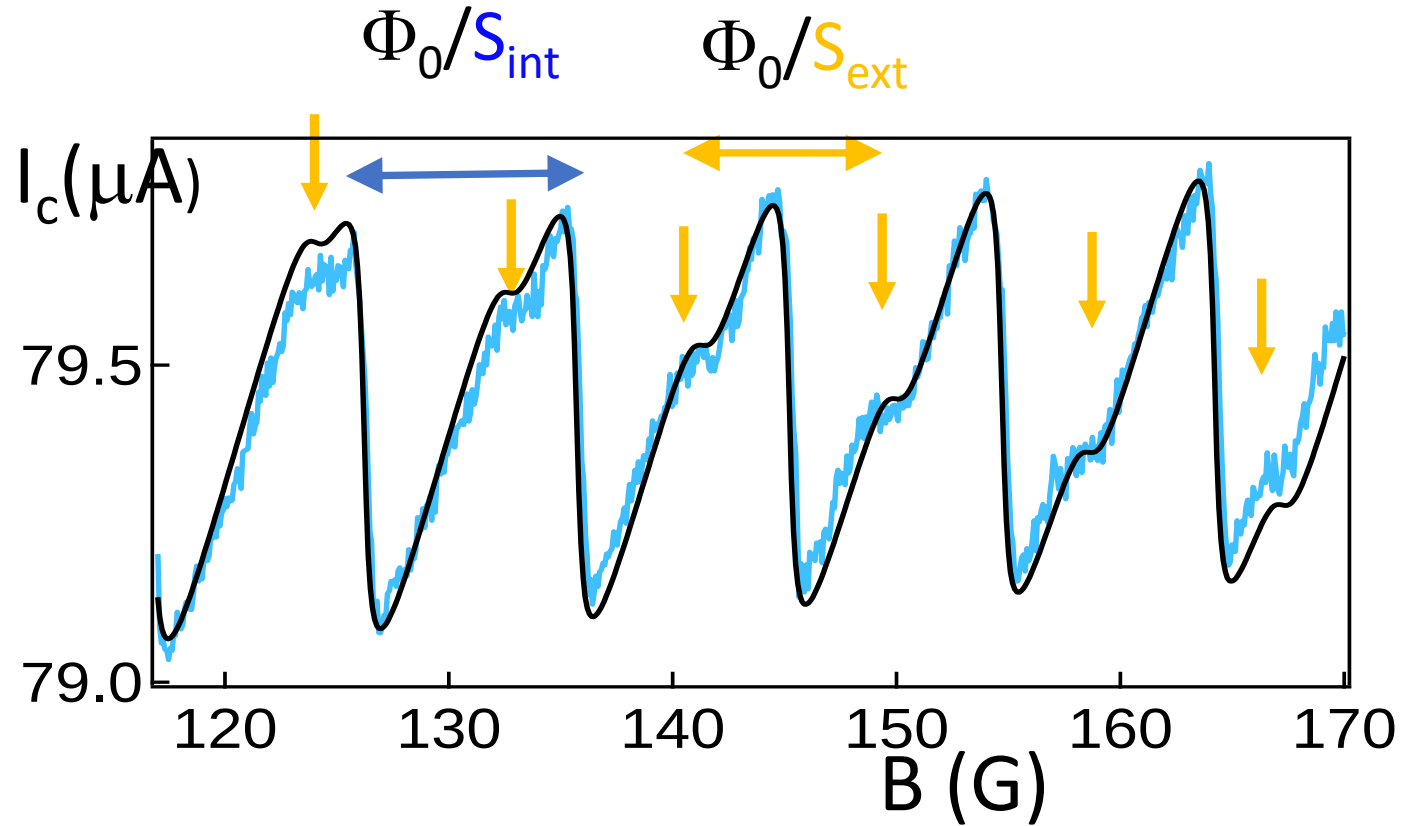
Build an asymmetric SQUID to measure the $I(\varphi)$ relation

Supercurrent-versus Phase relations of S/Bi/S: deduced from $I_c(B)$



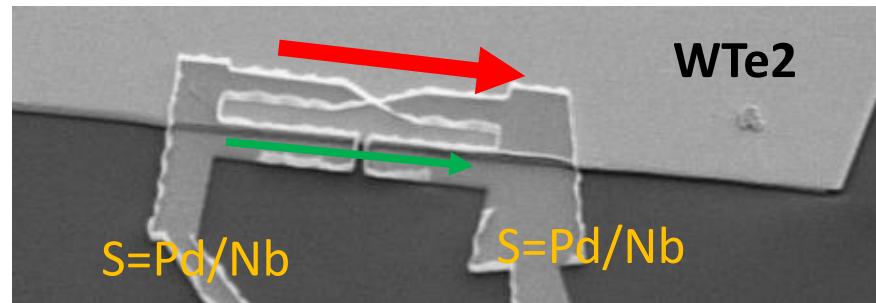
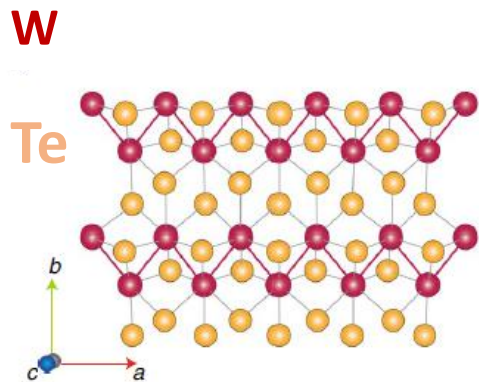
Sawtooth-shaped current phase relation: long ballistic (unusual)!

Two sawtooths? Because two edges!

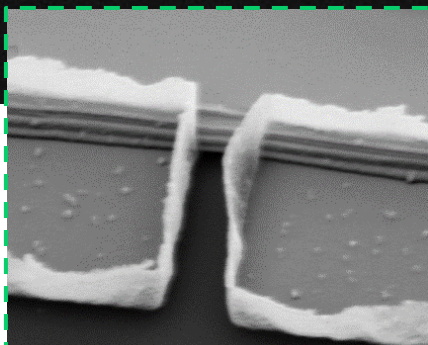
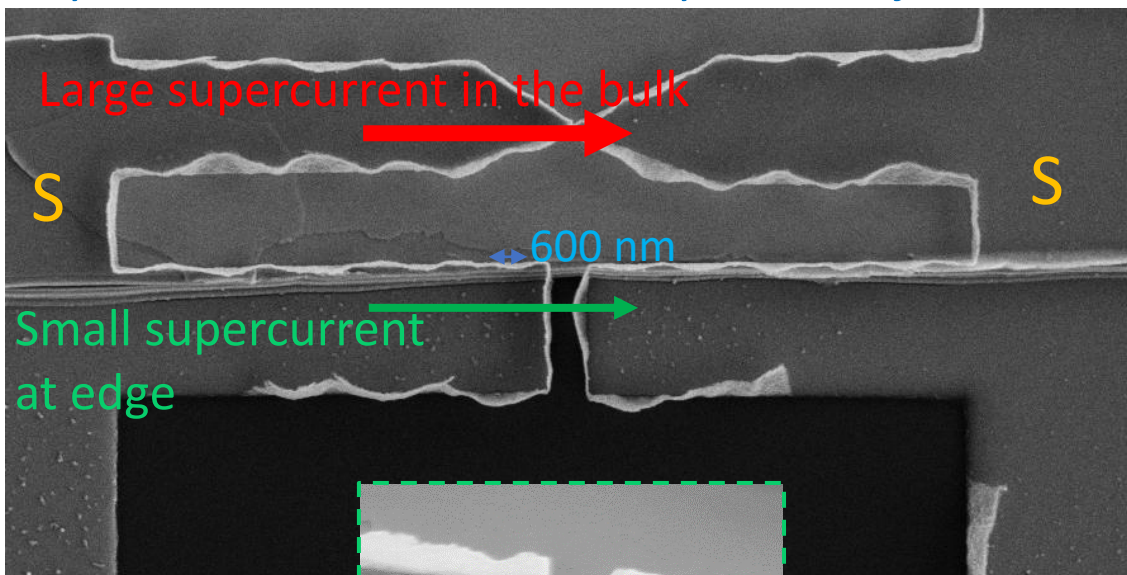


In bismuth:
Ballistic channels (with $T \sim 0.8$)
over $1.4\text{ }\mu\text{m}$ at two edges:
topological protection and hinge
conduction

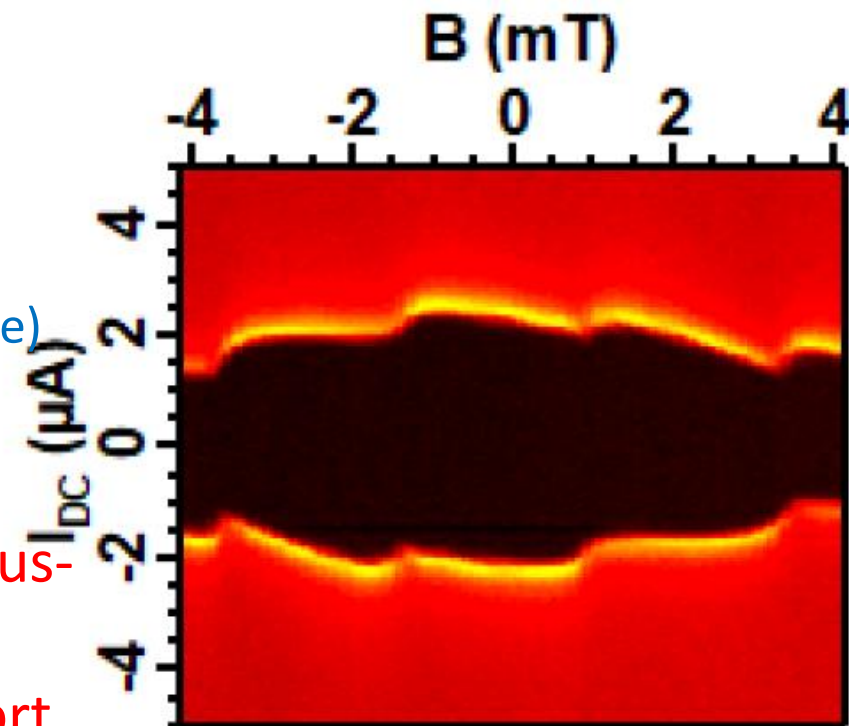
Ballistic edge states also seen in WTe₂ multilayers (Weyl metal? Second Order Topological Insulator?)



Asymmetric SQUID on monocrystal: one junction on bulk, the other on edge



Sawtooth
supercurrent-versus-
phase relation :
-> ballistic transport
over 500 nm ($T \sim 0.8$)!

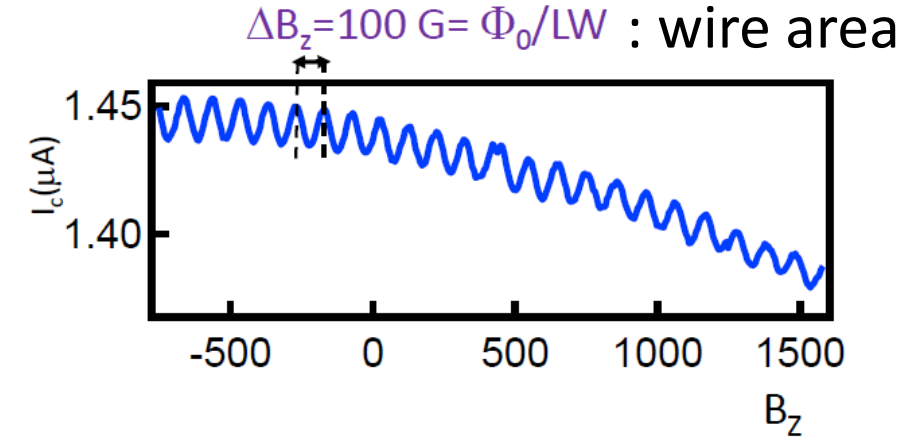
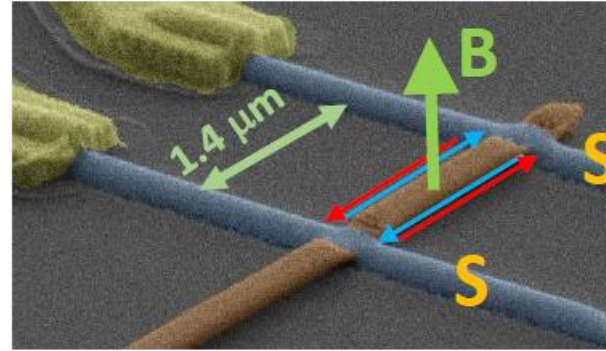


See also Schönenberger group, G-H Lee group
X. Ballu et al., Phys. Rev. Research (last Friday, April 10th 2026)
Also group of Chuan Li, Twente (2026)

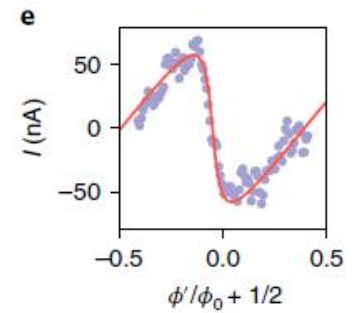
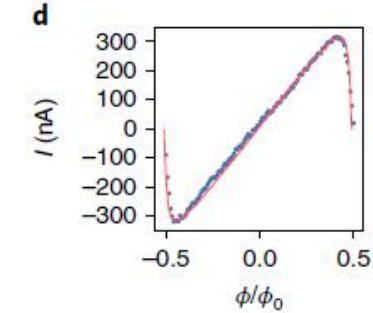
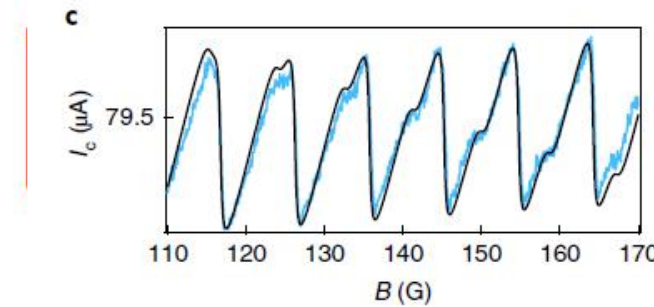
But: not all samples show the sawtooth CPR!

So far, properties of Quantum Spin Hall (helical) states that we tested:

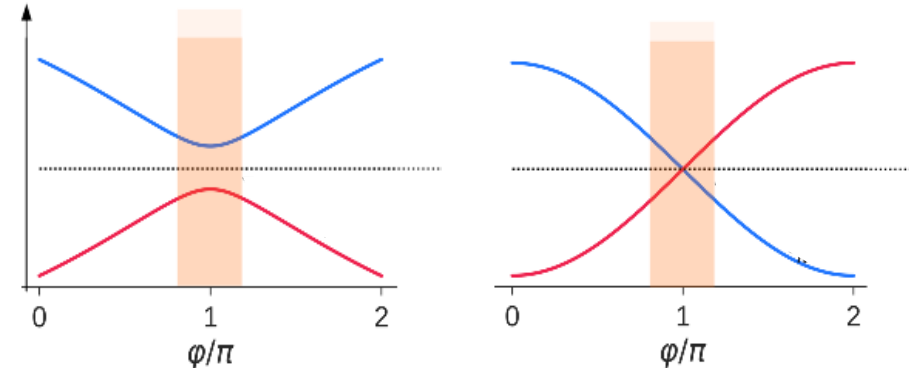
Edge transport



Ballistic transport



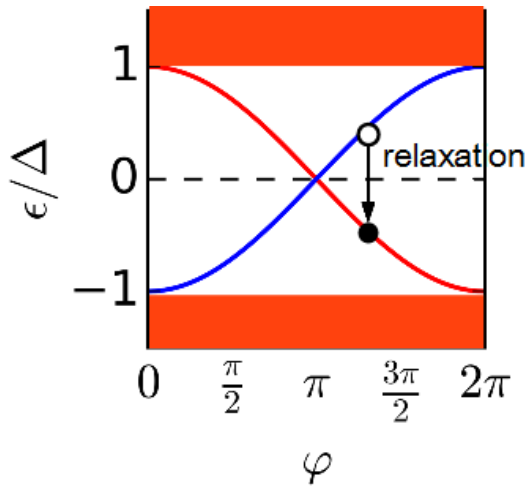
Test crossing of Andreev spectrum $\varepsilon(\varphi)$ at $\varphi = \pi$



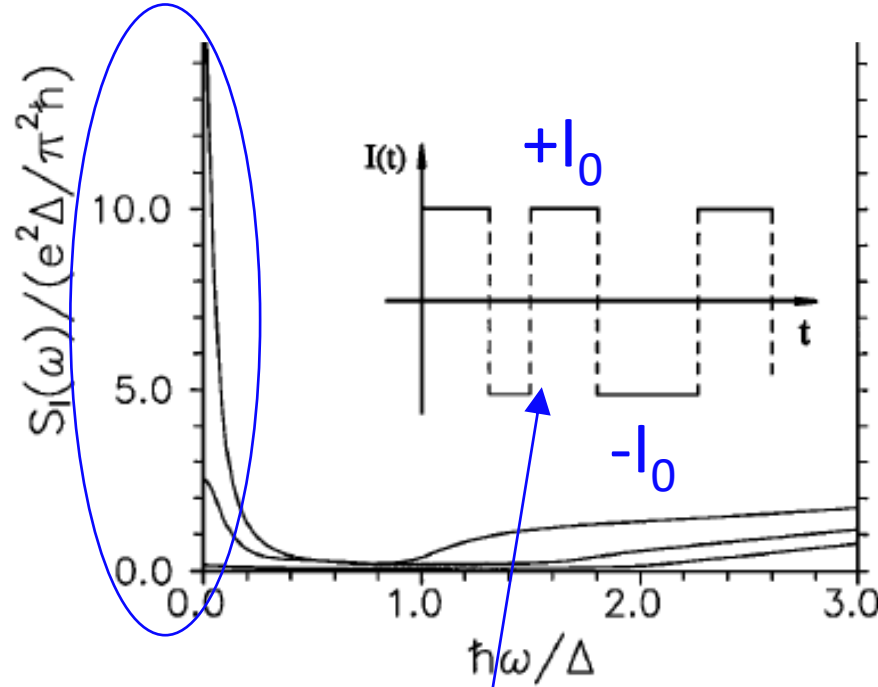
Next: Topological protection

Probe topological crossing with thermal noise ?

Fu Kane 2009 for QSH states-based Josephson junction
Averin Imam 1996 for point contacts



$$i_n = \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n)$$



Noise power $S(\omega)$:
FT of current-current correlation

$$S(\omega) = 2 \int_{-\infty}^{\infty} e^{i\omega t} \langle I(t) I(0) \rangle$$

$$I(t) I(0) = I_0^2 f(1 - f) e^{-t/\tau}$$

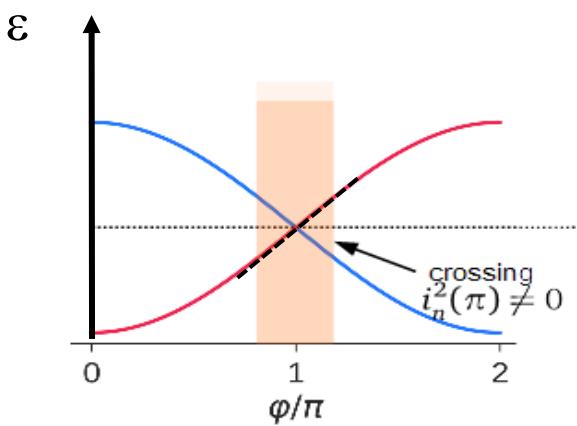
$$S(\omega) = \frac{4I_0^2}{\cosh^2 \epsilon_0(\phi)/2T} \frac{\tau}{1 + \omega^2 \tau^2}$$

At π , occupation $f_{FD} = 1/2$

Huge telegraphic noise of the supercurrent if perfect crossing, and low T

Either thermal noise or RF response should probe topological crossing

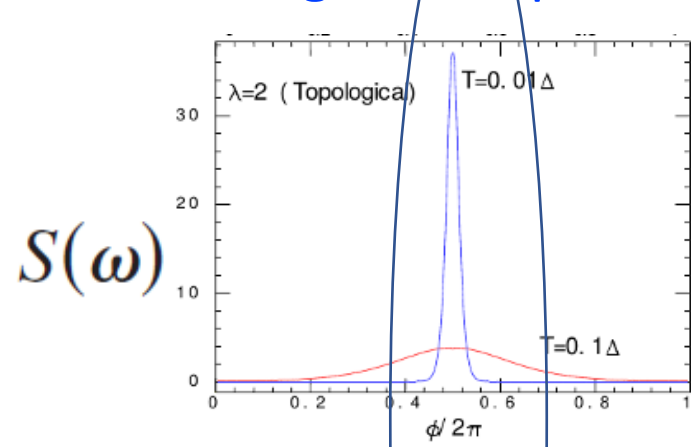
ballistic: crossing at π



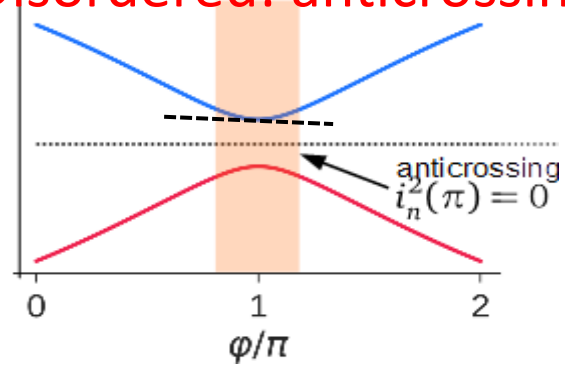
$$i_n = \frac{\partial \epsilon_n}{\partial \phi} f(\epsilon_n)$$

Current is either 0 or max at π , noise reflects that

ballistic: huge noise peak at π

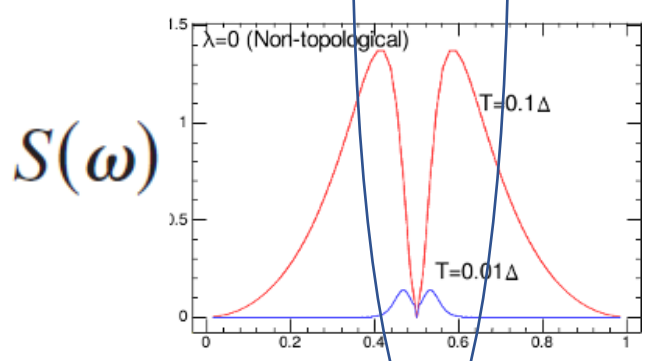


Disordered: anticrossing



$$S(\omega) = \frac{4I_0^2}{\cosh^2 \epsilon_0(\phi)/2T} \frac{\tau}{1 + \omega^2 \tau^2}$$

disordered



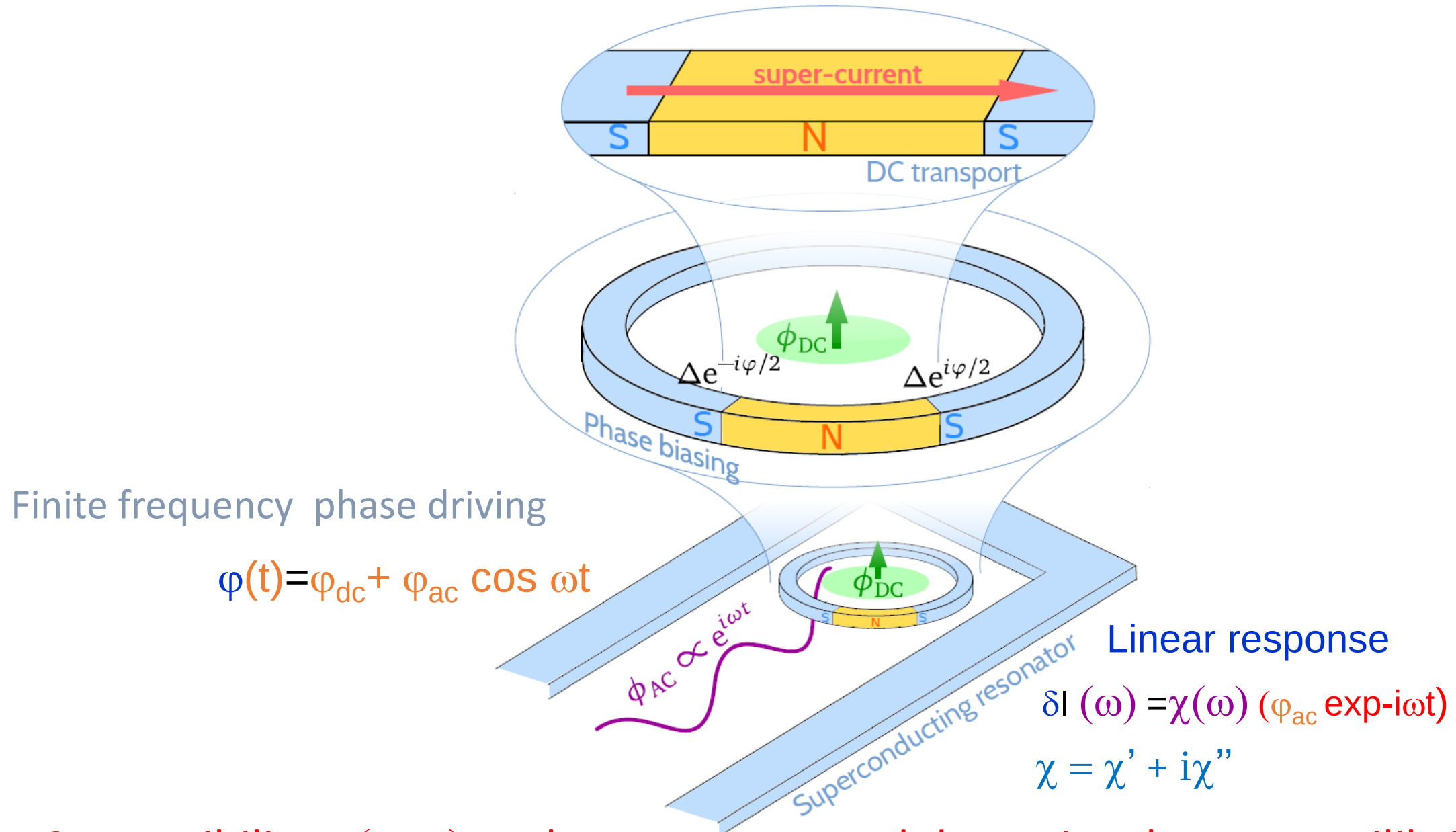
Noise=fluctuations

$$S_I(\omega) = \frac{2}{\pi} \frac{k_B T \chi''(\omega)}{\omega}$$

χ'' =dissipation

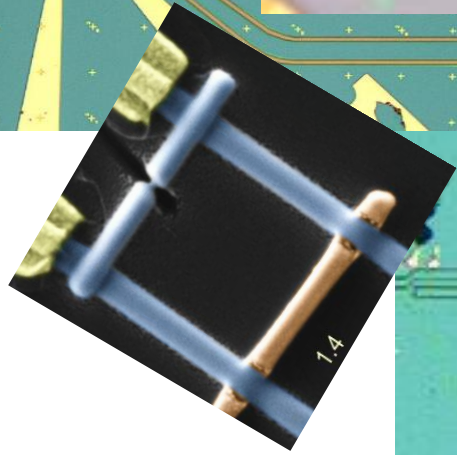
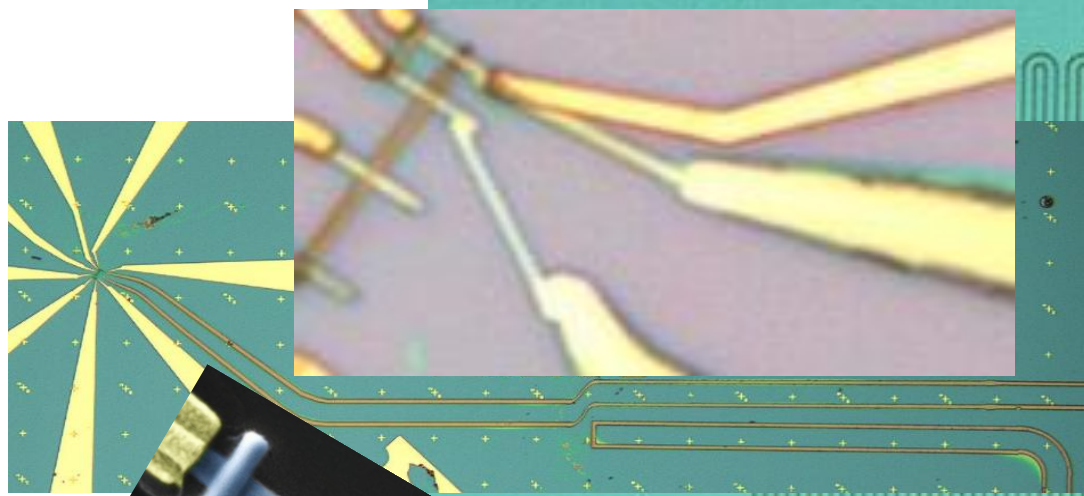
Huge noise at π means susceptibility peak in $\chi''(\pi)$
(via Fluctuation-Dissipation Theorem)

RF probing of QSH states to test topological protection at phase π



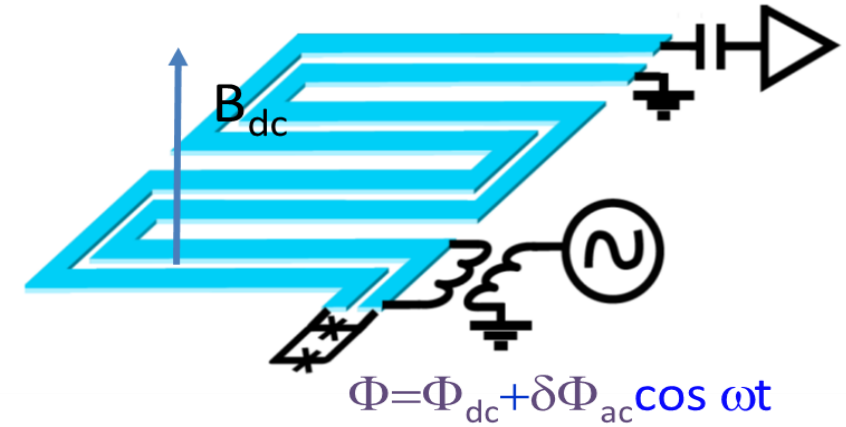
Susceptibility $\chi(\varphi, \omega)$ probes spectrum and dynamics close to equilibrium

In practice : multimode resonator coupled to S/Bi/S asymmetric SQUID

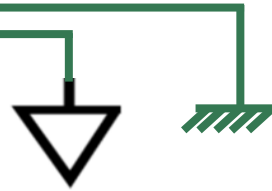


1.5 meter-long Nb
meander:
high Q resonator

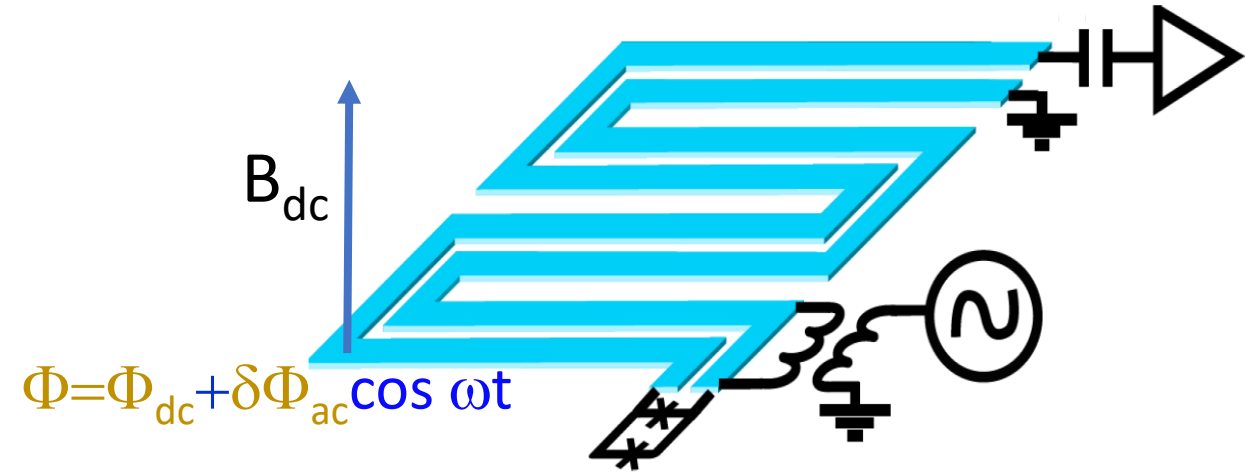
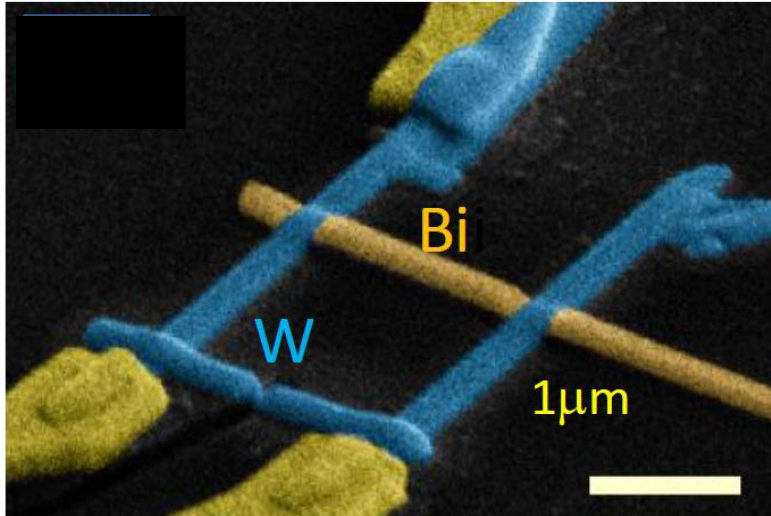
Inductive
coupling to
ac
generator



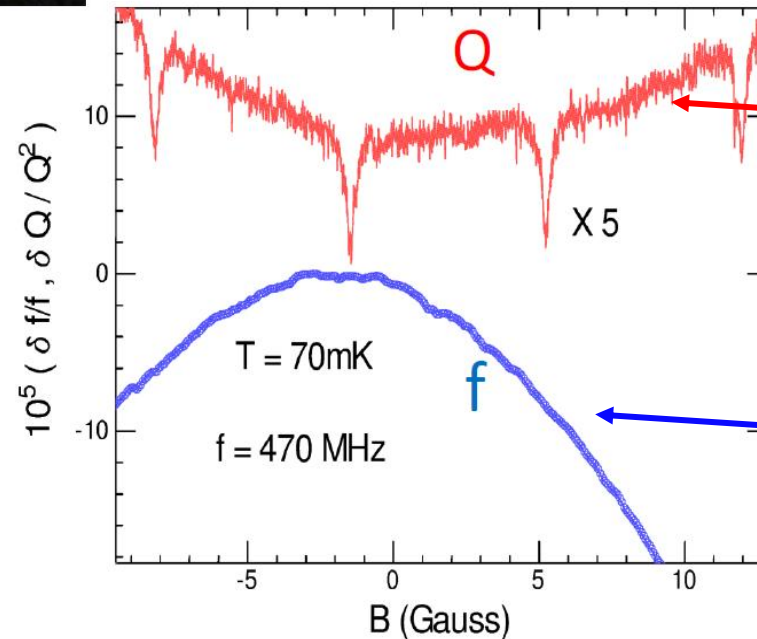
Capacitive
coupling to
ac
cryogenic
amplifier



Extract dissipation from change in quality factor



Measure $\delta Q(B)$ and $\delta f(B)$ at each resonator eigenfrequency



Dissipative response

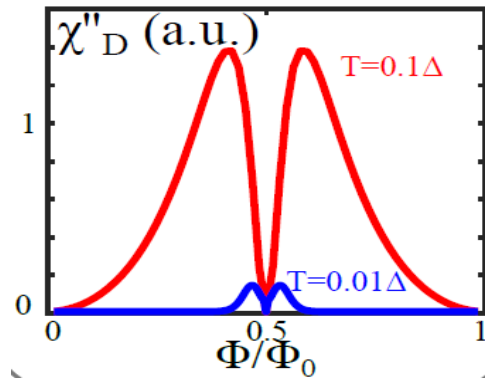
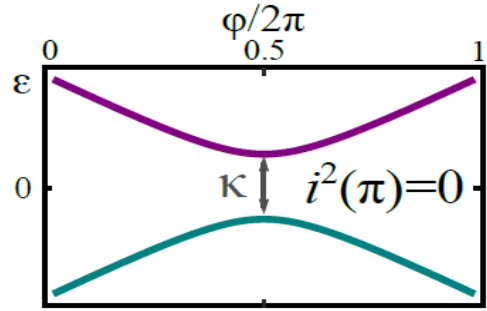
$$\chi''(\varphi) = \frac{L_R}{L_W^2} \delta \left[\frac{1}{Q_n} \right] (\Phi)$$

Non-dissipative response

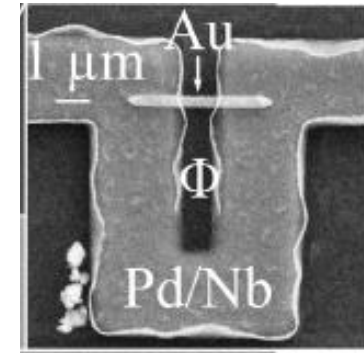
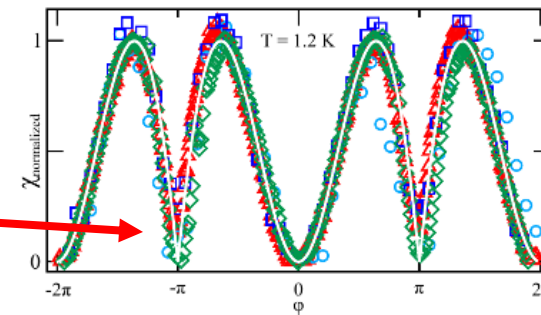
$$\chi'(\varphi) = -\frac{L_R}{L_W^2} \frac{\delta f_n(\Phi)}{2f_n}$$

Absorption peaks at π !

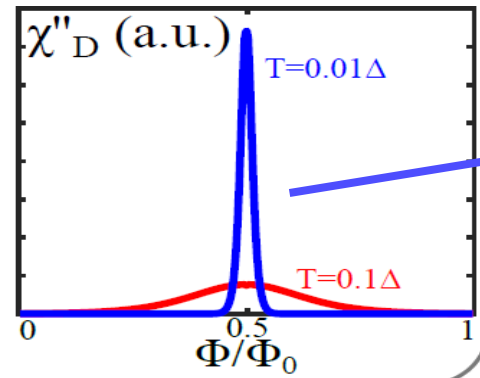
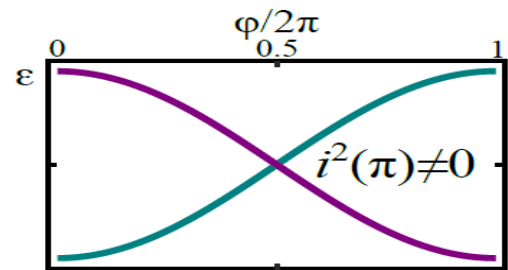
Peaked absorption QSH Josephson junctions, Quite different from trivial SNS (albeit different temperature ranges)



In SNS: zero absorption at π ! S/diffusive Au/S

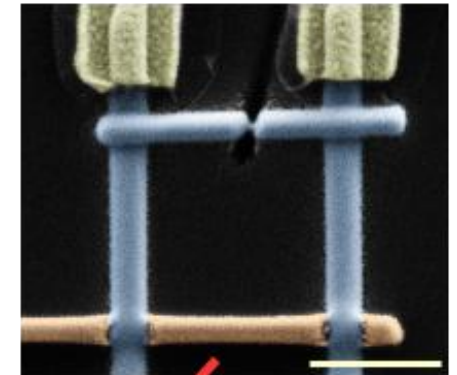
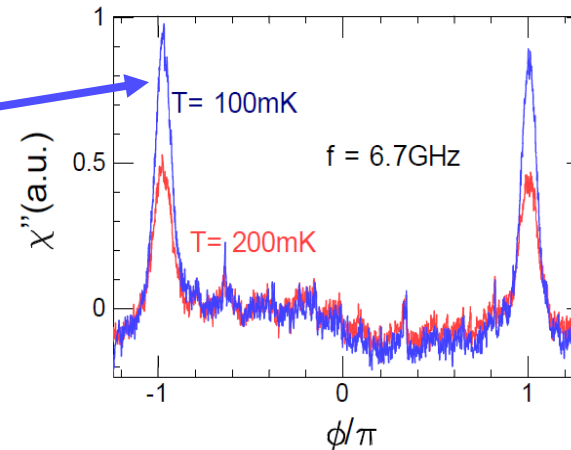


Dassonneville2013



In S/Bi/S: max absorption at π !

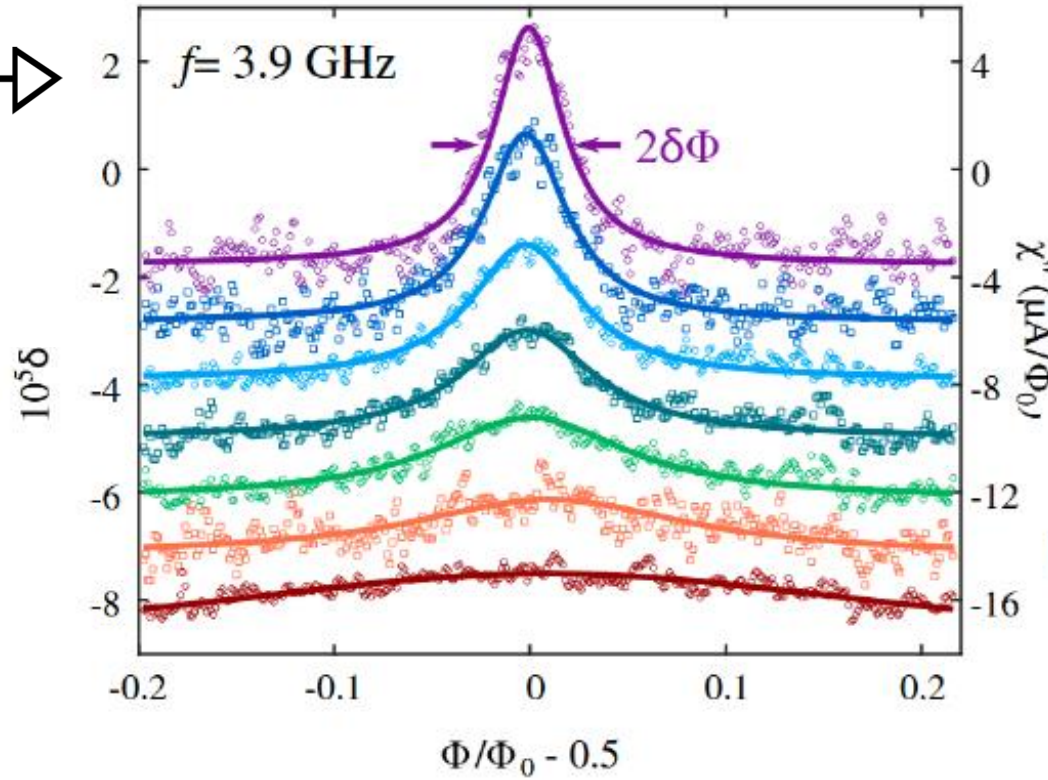
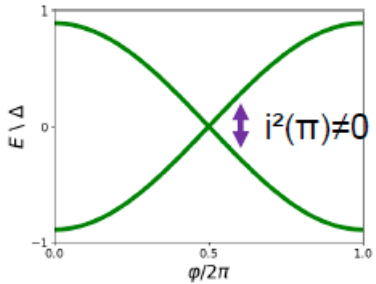
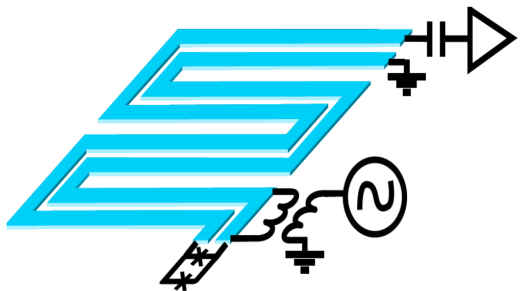
S/Bi/S



Murani2019

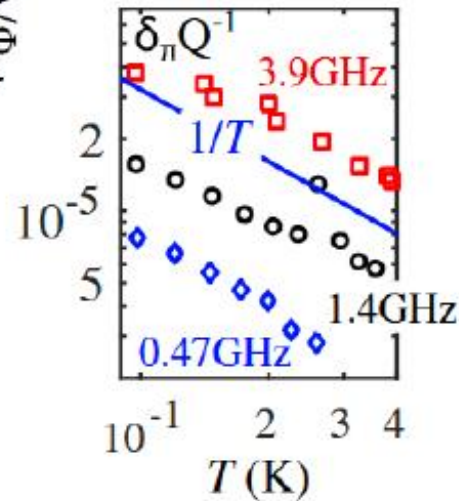
This is the thermal noise of a QSH insulator (Fu Kane)!

Dissipation peak at π is expected signature of protected Andreev level crossing !

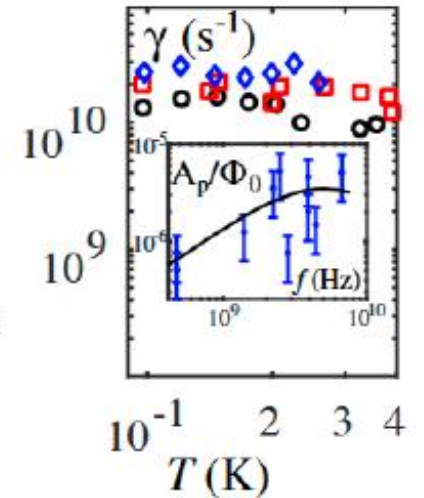


$$\chi''_D = \text{Im}(\chi_D) \propto \frac{\omega\gamma}{\omega^2 + \gamma^2}$$

Peak width



Relaxation rate

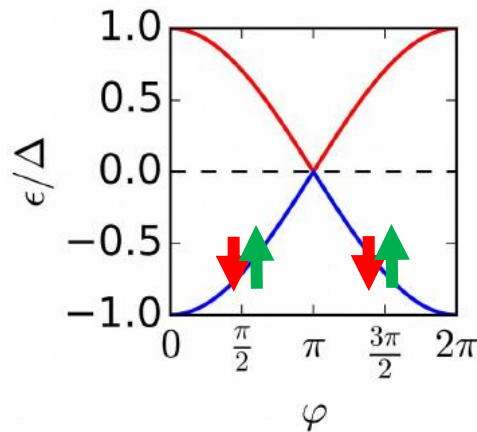
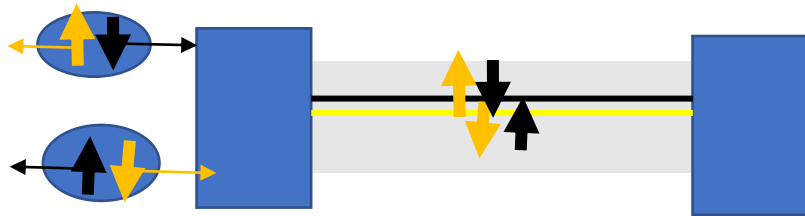


Murani PRL 2019

But frequency dependence indicates a very short (ns) relaxation time
(maybe because multimode resonator too coupled to environment)

What is specific to helical (topological, QSH) junction?

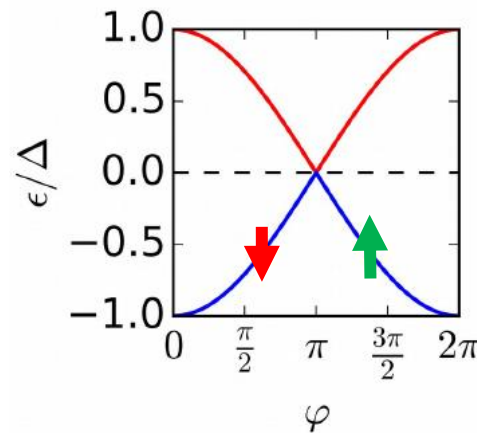
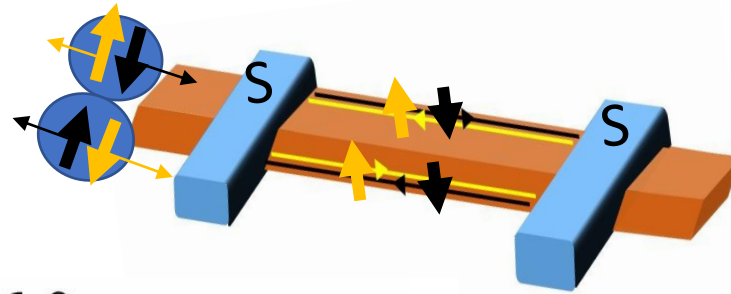
Usual Andreev Bound State
Spin degenerate



Andreev Bound State of one hinge :

« half » usual ABS, Not spin degenerate

Helical: one direction, one spin (other at other edge)

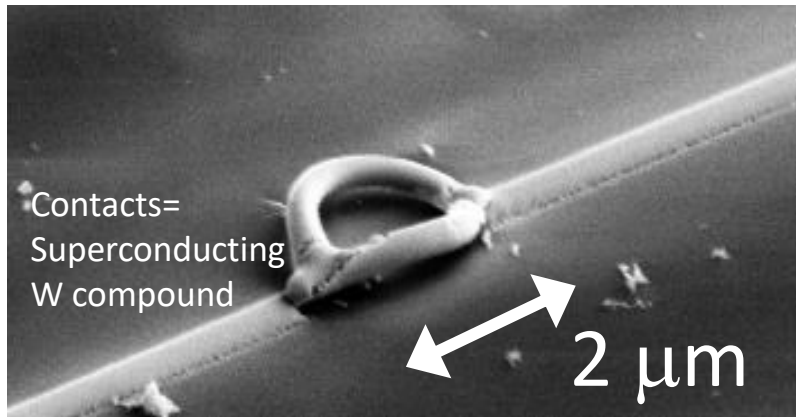


Beenakker 2013,
Trauzettel 2014,
Peng 2016,
Frombach 2020, ...

Fundamental consequences on :

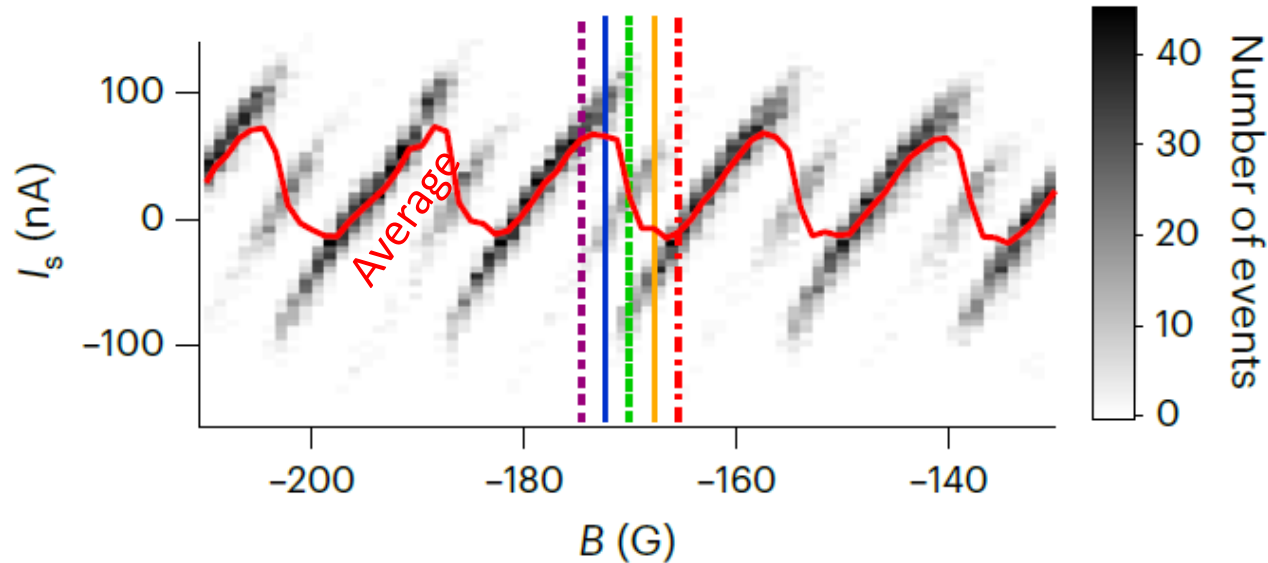
- possible ground and excited states
- relaxation mechanisms and times

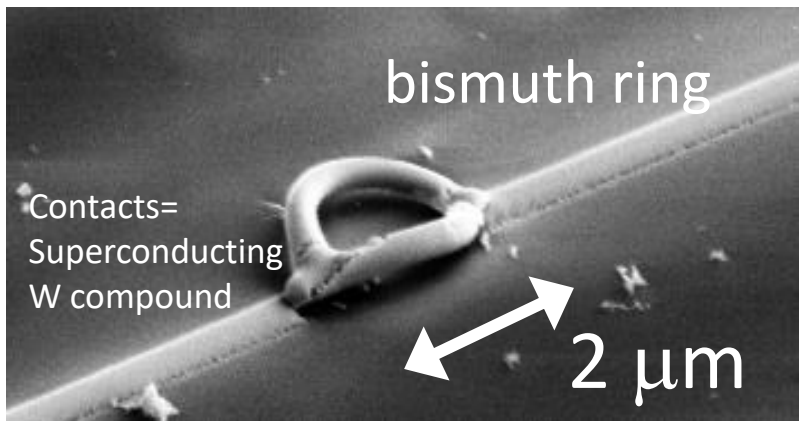
Could helicity explain the millisecond pair relaxation times we found in another experiment?



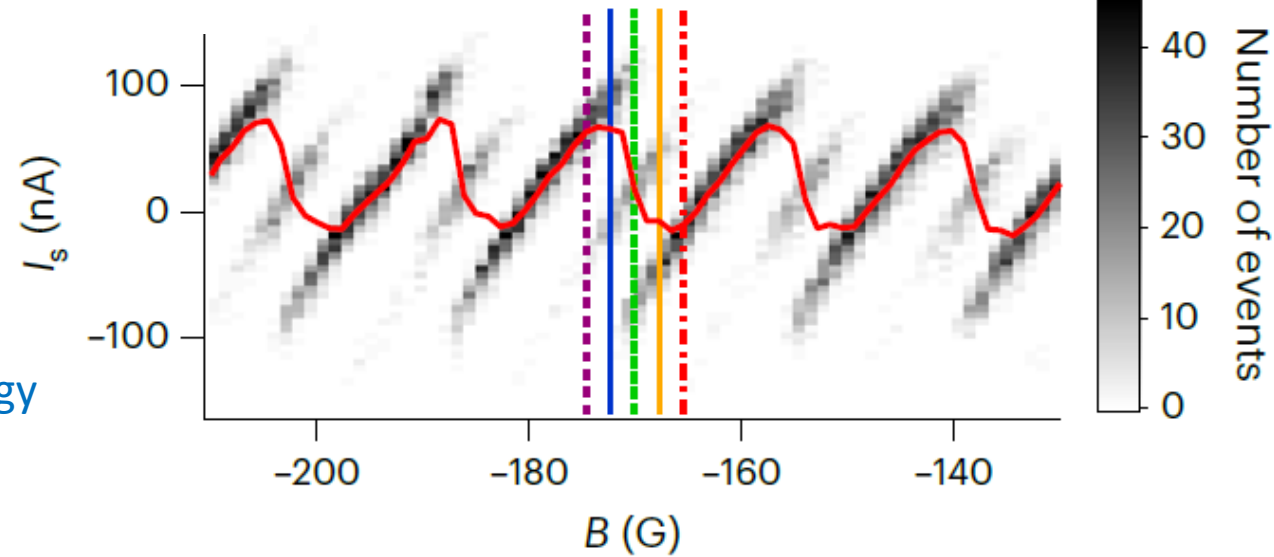
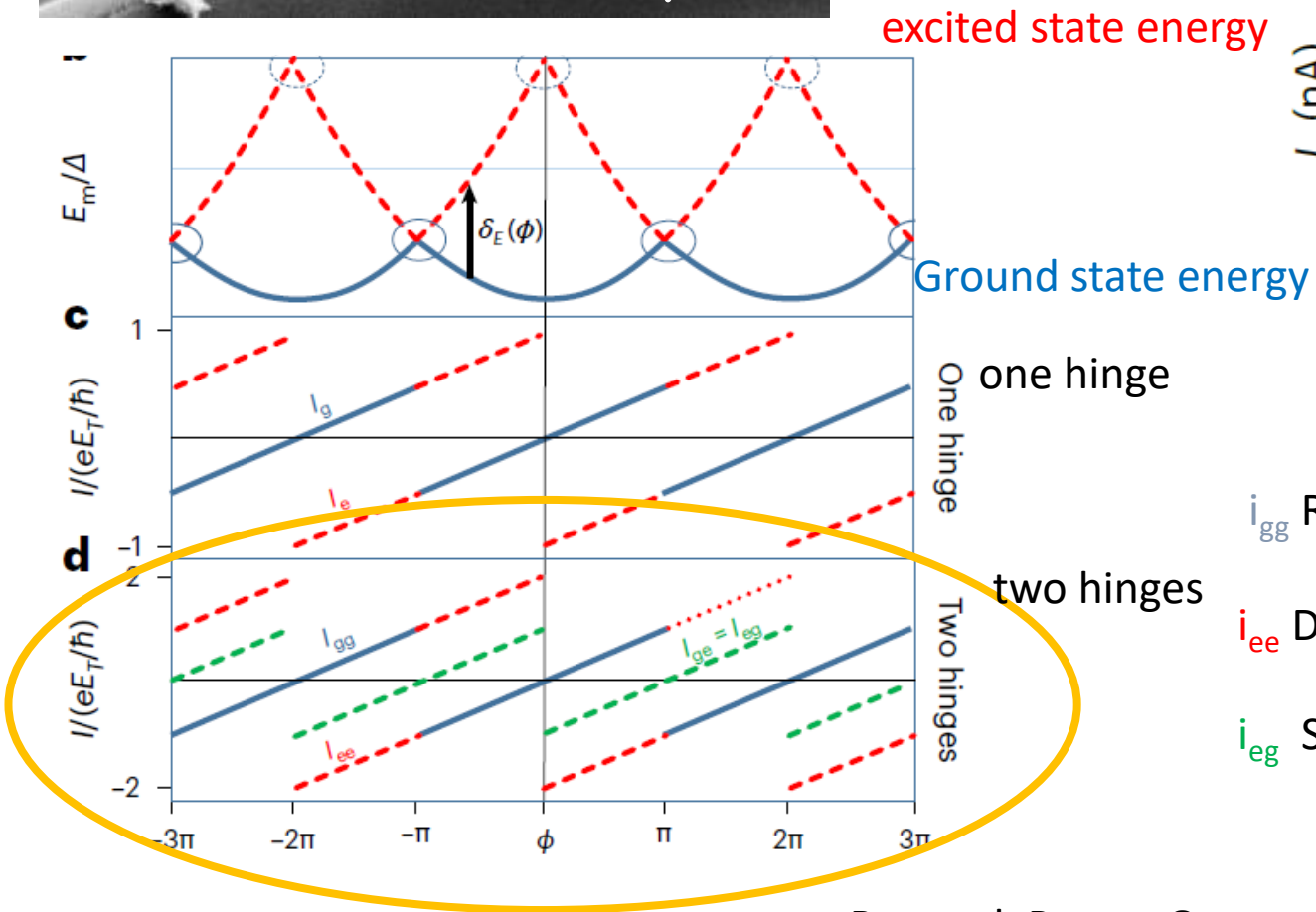
bismuth ring !

- **Average** switching current in intrinsic asymmetric SQUID: sawtooth CPR
- Switching current **distribution** reveals long-lived excited states!





Two hinges in one branch



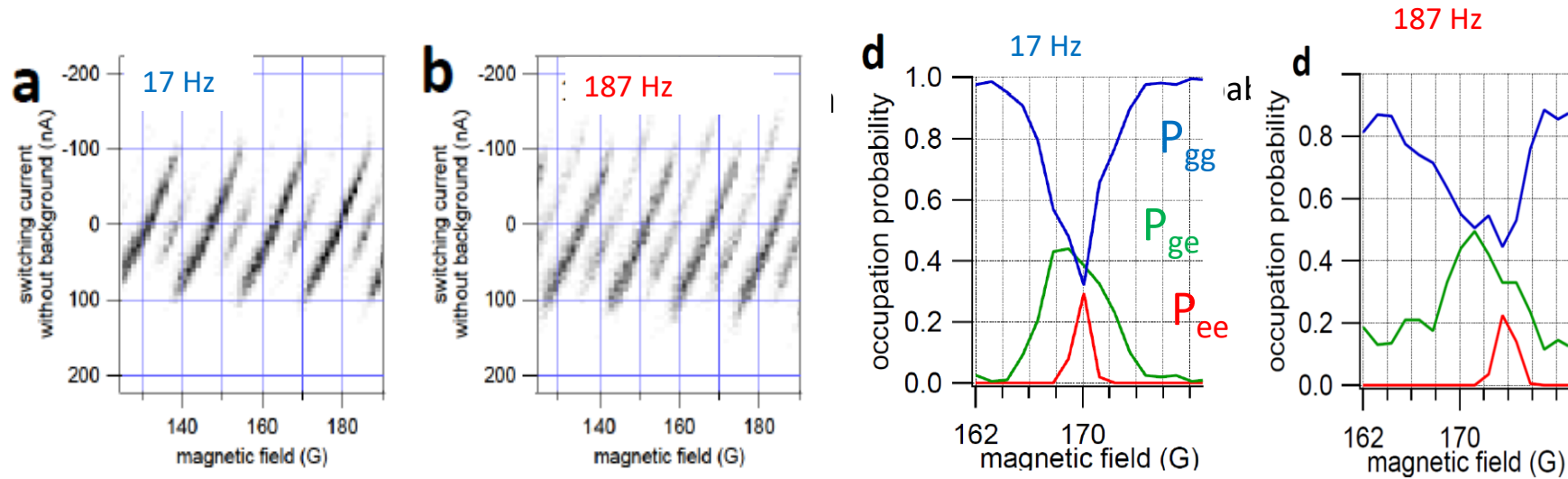
i_{gg} Relaxed current phase relation $2i_g$

i_{ee} Doubly excited Josephson junction $=2i_e$

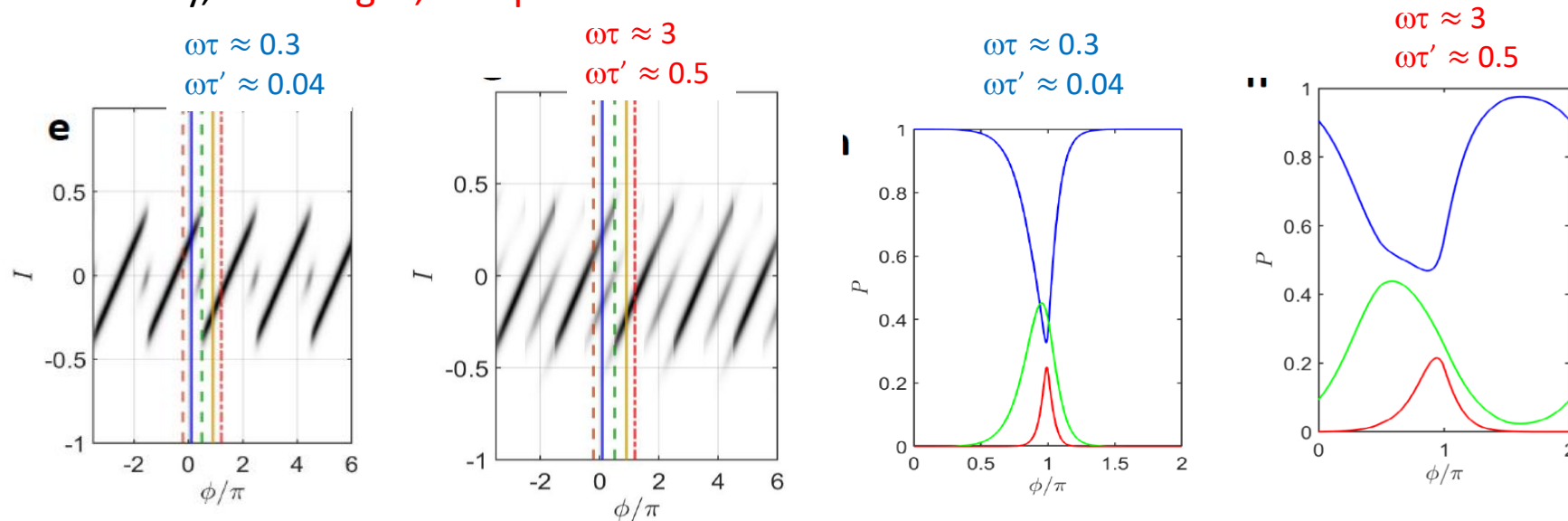
i_{eg} Singly excited junction $=i_e + i_g$

Comparison of experiment /Theory in the case of two hinges

coll. with Yang Peng, F. Von Oppen, Y. Oreg

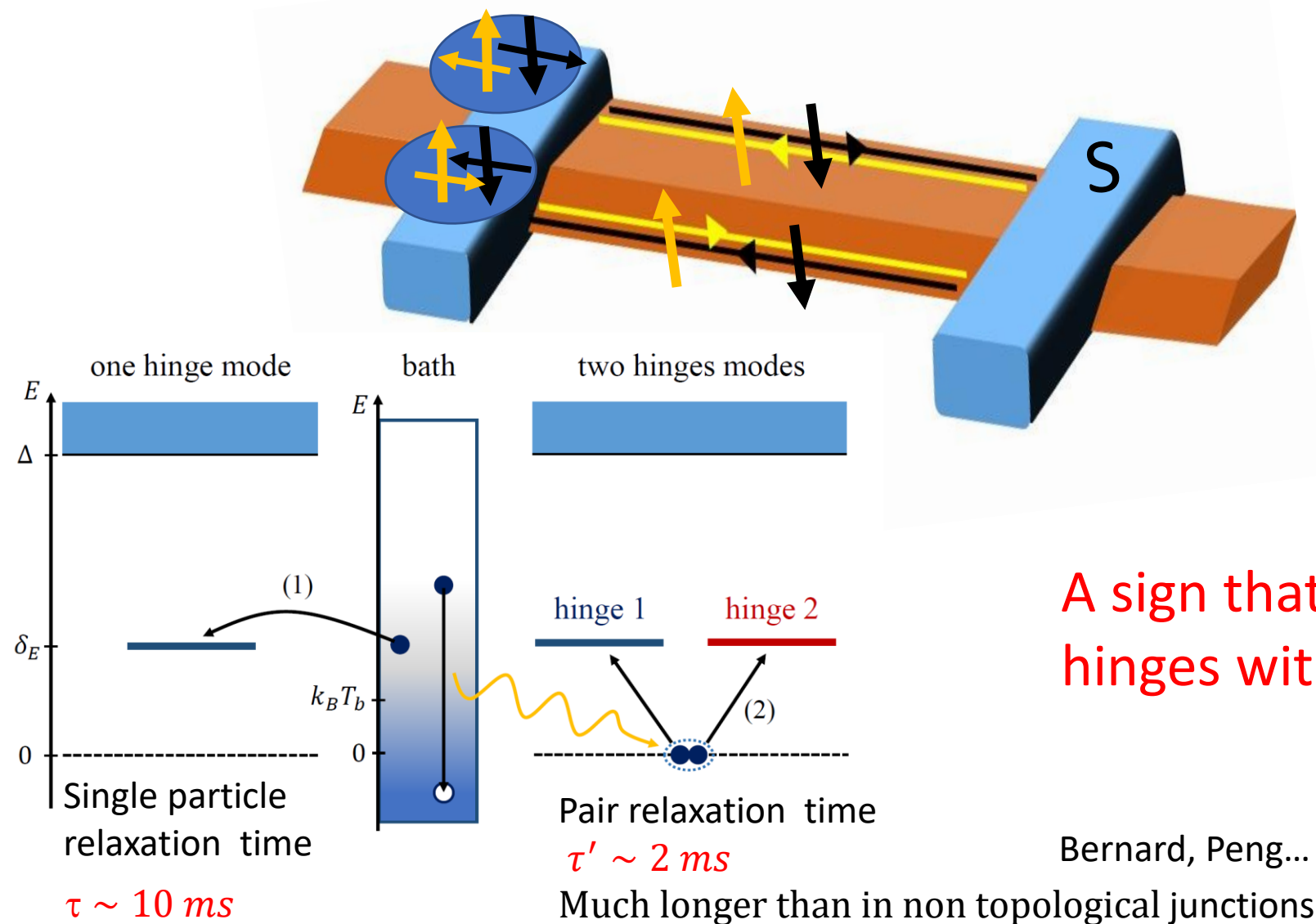


Theory, two hinges, two phase variation rates



$\Rightarrow$ Two (long!) relaxation times $\tau \sim 10$ ms and $\tau' \sim 2$ ms

Long relative reaxation time of pairs with respect to quasiparticle
a sign of spin-filtered transport (helicity?)



A sign that pair is « split » on two hinges with different helicity ?

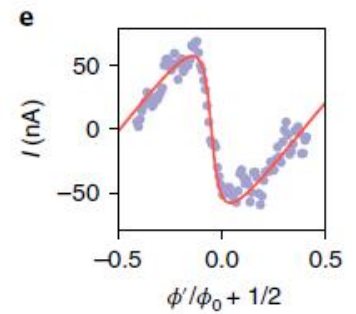
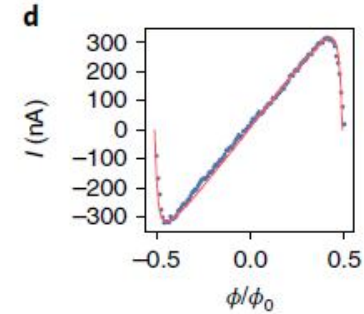
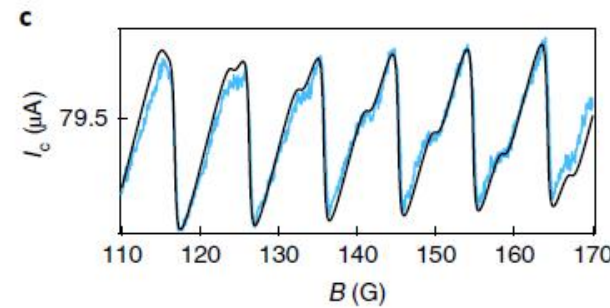
Bernard, Peng... Oreg and von Oppen Nat Phys 2022

So far, properties of Quantum Spin Hall states that we tested:

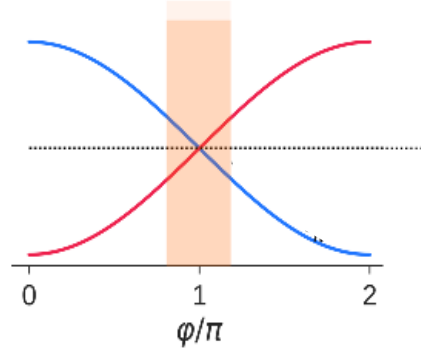
Edge and ballistic transport



- Sawtooth-shaped supercurrent-versus phase relation
- Interference between two edges



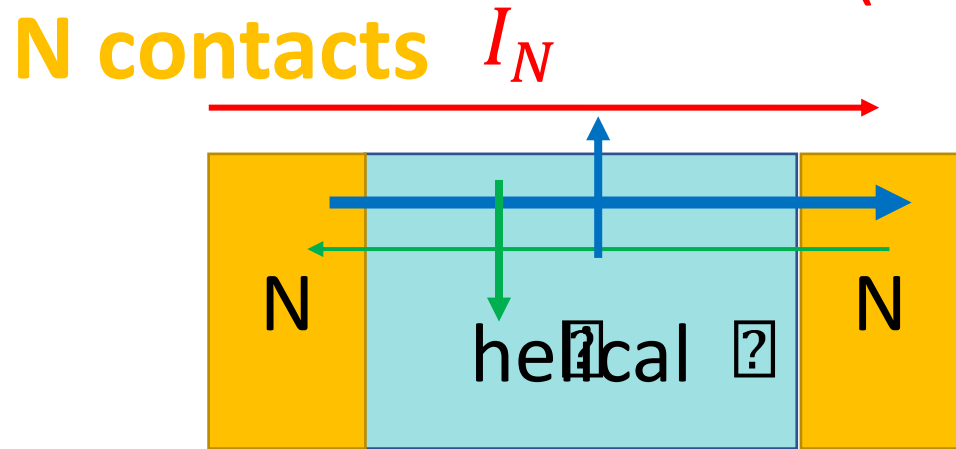
Topological protection



- Dissipation peak at π
- Relatively long pair relaxation time ($\cong$ quasiparticle relaxation)

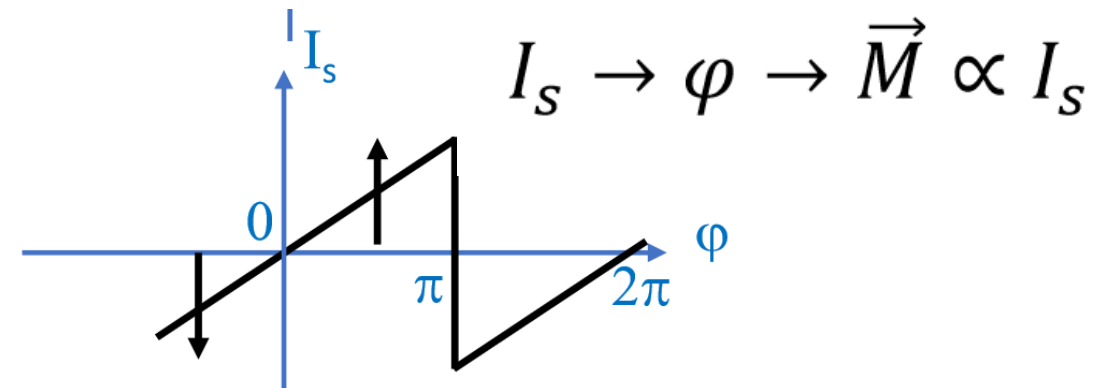
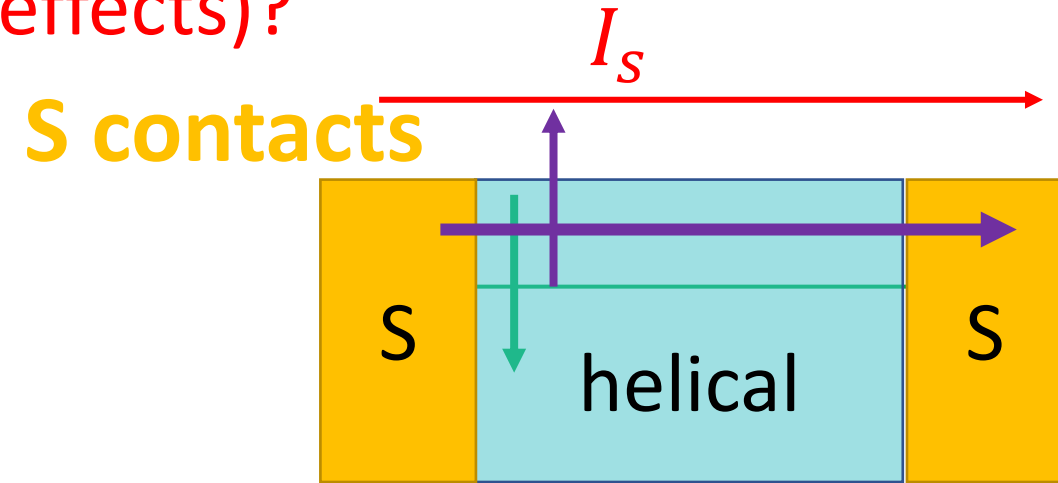
What about spin-momentum locking (helicity)?

Detect helicity via (super)current-induced spin polarisation (Edelstein effects)?



$$I_N \rightarrow \delta S \quad M \quad IN$$

Bilinear magnetoresistance
 $R_{BMR} \sim \gamma IB$



Josephson Diode effect

$$I_{c+}(B) \neq I_{c-}(B)$$

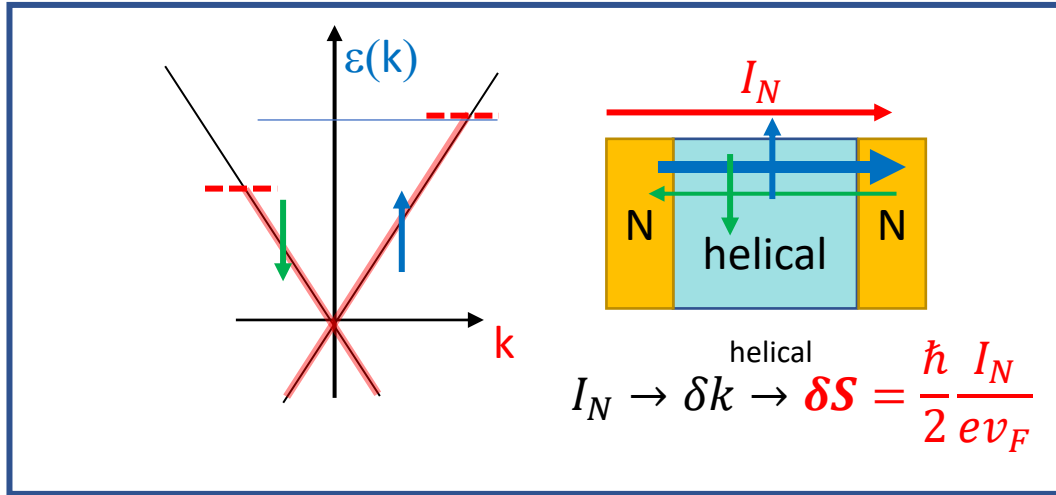
Compare two magnetochiral effects

Max effect as a function of B direction yields orientation of current-induced spin!

Non reciprocal magnetotransport with normal electrodes

Helical state (full spin-momentum locking) with **N** contacts

Huge **spin-polarisation induced by a current** (Edelstein)



$$\delta \mathbf{S} \rightarrow \mathbf{M} \propto I_N, \text{ then } B_{\text{eff}} = B + M$$

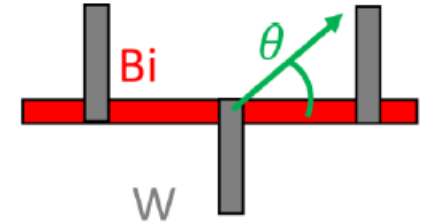
$$R \text{ must depends on } B_{\text{eff}} \text{ as } B_{\text{eff}}^2 \rightarrow R = R_0(1 + \gamma \mathbf{B} \cdot \mathbf{I}_N)$$

→ bilinear magnetoresistance

Max when $\mathbf{B} \parallel$ to spin axis

Inspired by bilinear magnetoresistance in Rashba 2D materials: He Vignale 2019, Vaz Bibes 2020, Dyrda Barnas Fert 2020, Zhao Cobden 2020, Gorini, Vignale, et al, in preparation

Bilinear Magnetoresistance (5K) Normal state



$$I = I_{ac} \cos(\omega t)$$

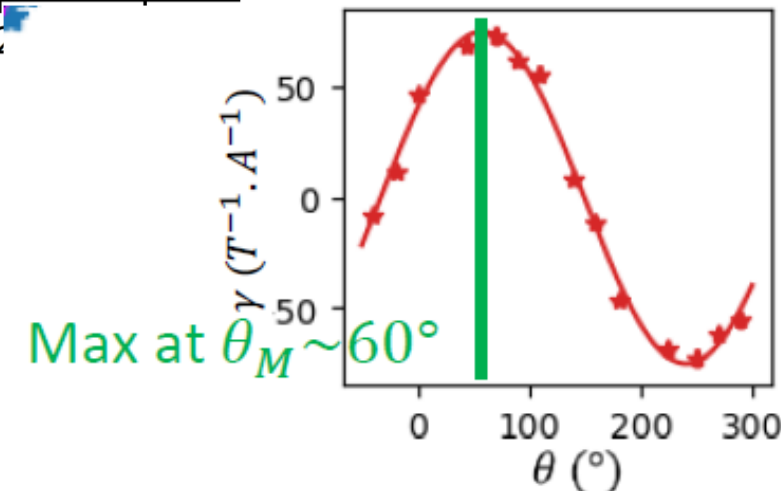
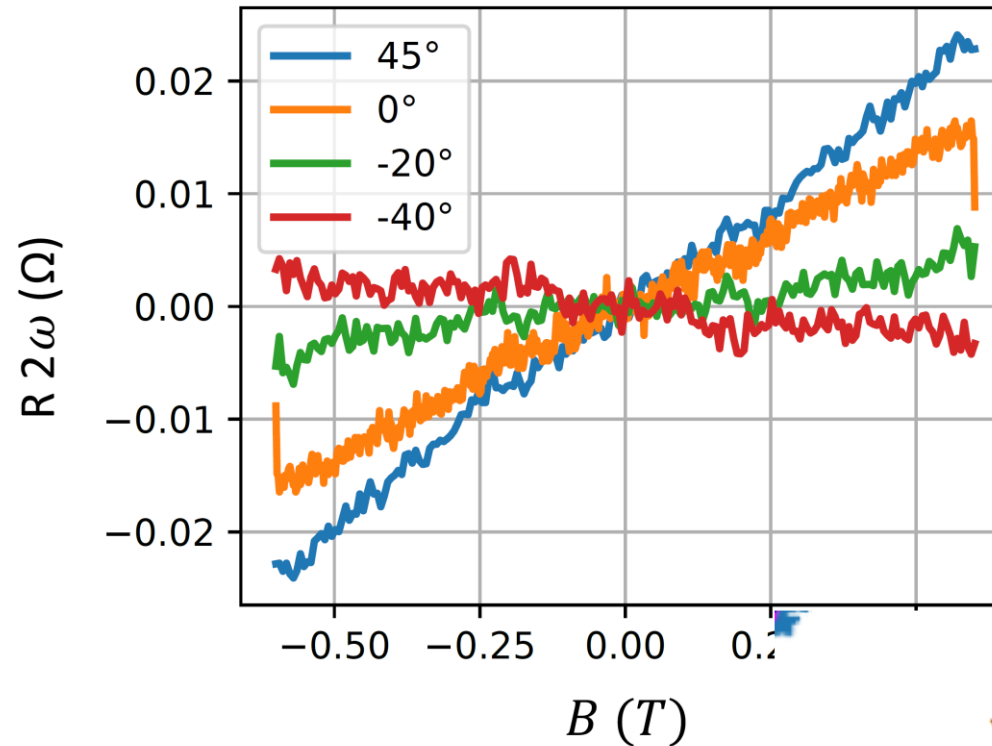
$$V = R_0(1 + \gamma \mathbf{B} \cdot \mathbf{I}) I$$

$$\text{So } V_{BMR} = \gamma R_0 I_{ac}^2 \cos^2(\omega t) B$$

$$R_{2\omega} = \frac{1}{2} R_0 \gamma I_{ac} B$$

→ Linear dependance in **B**

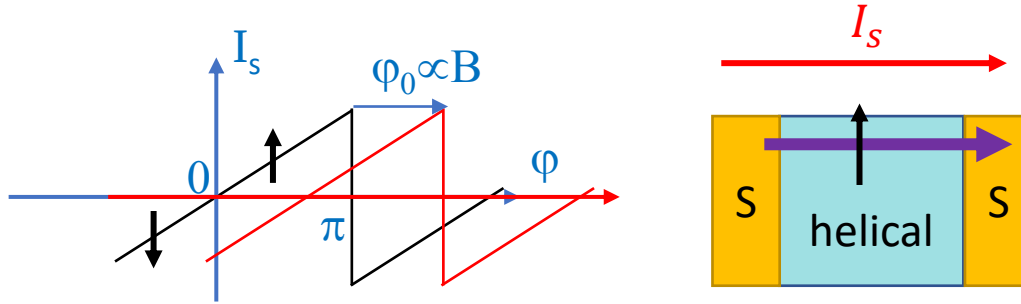
γ is thus obtained at different angles
in plane of the field



S contacts to probe spin-momentum locking

Helical state with **S contacts**:

Spin-polarisation induced by a supercurrent



Anharmonic CPR shifted by Zeeman $B \rightarrow "$ φ_0 $"$ shift
+ spatial asymmetry (several edge states, unbalanced helicity)

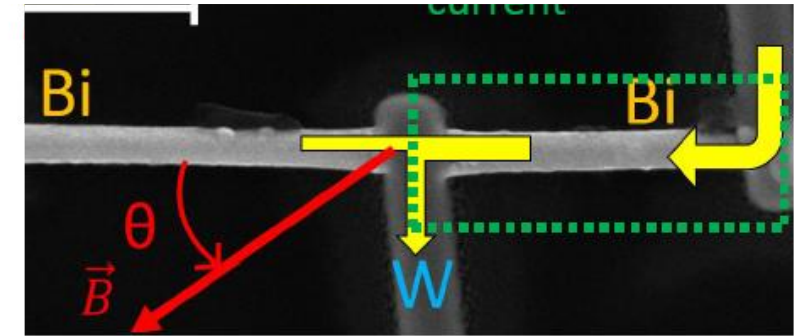
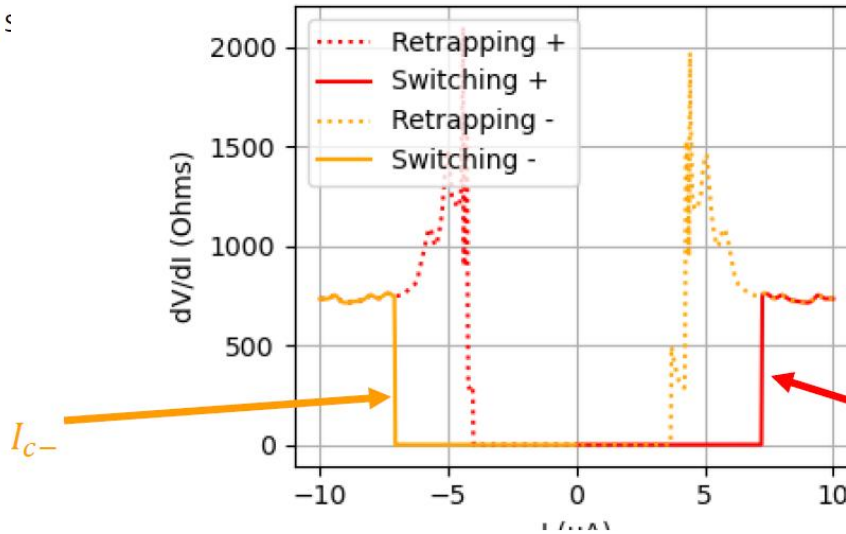
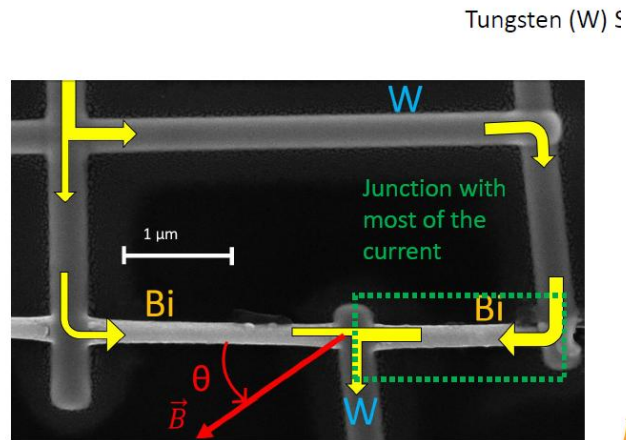
→ field-dependent Superconducting diode $I_c^+(B) \neq I_c^-(B)$
+ distortion by spin polarisation induced by supercurrent

*Yokoyama, Eto, Nazarov 2014,
Dolcini Houzet Meyer, 2015,
Meyer Houzet 2024,
Fu 2022*

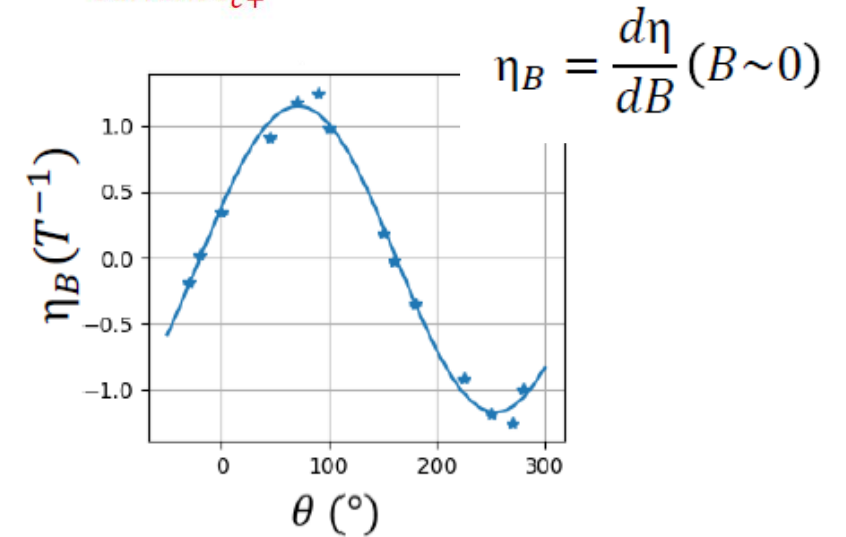
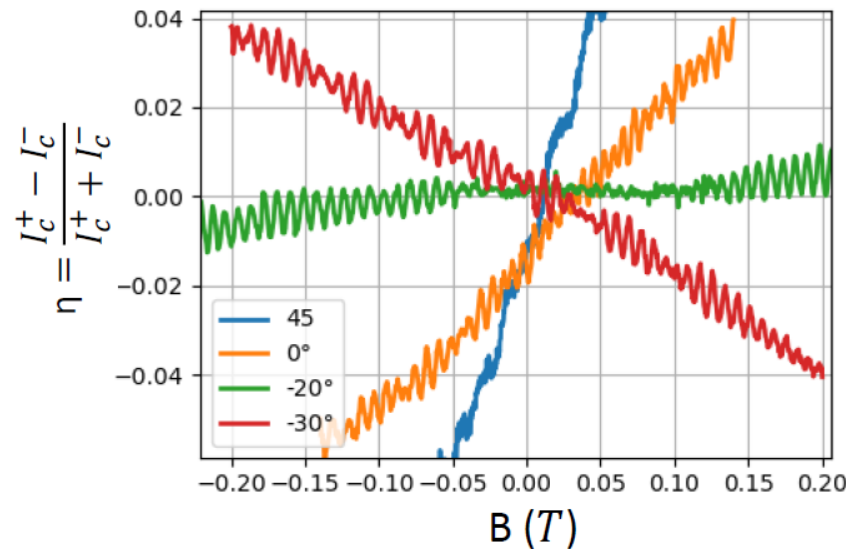
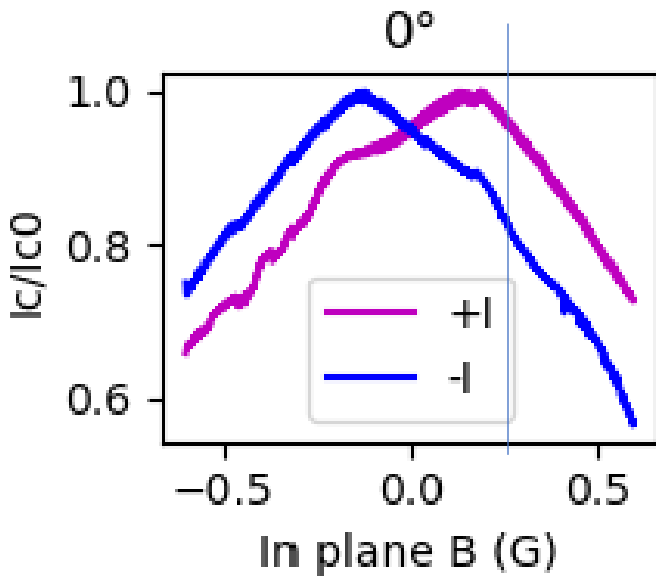
Non-reciprocity = Josephson diode
Max when $B \parallel$ spin axis

Superconducting diode effect and its angle dependence

Bismuth SQUID



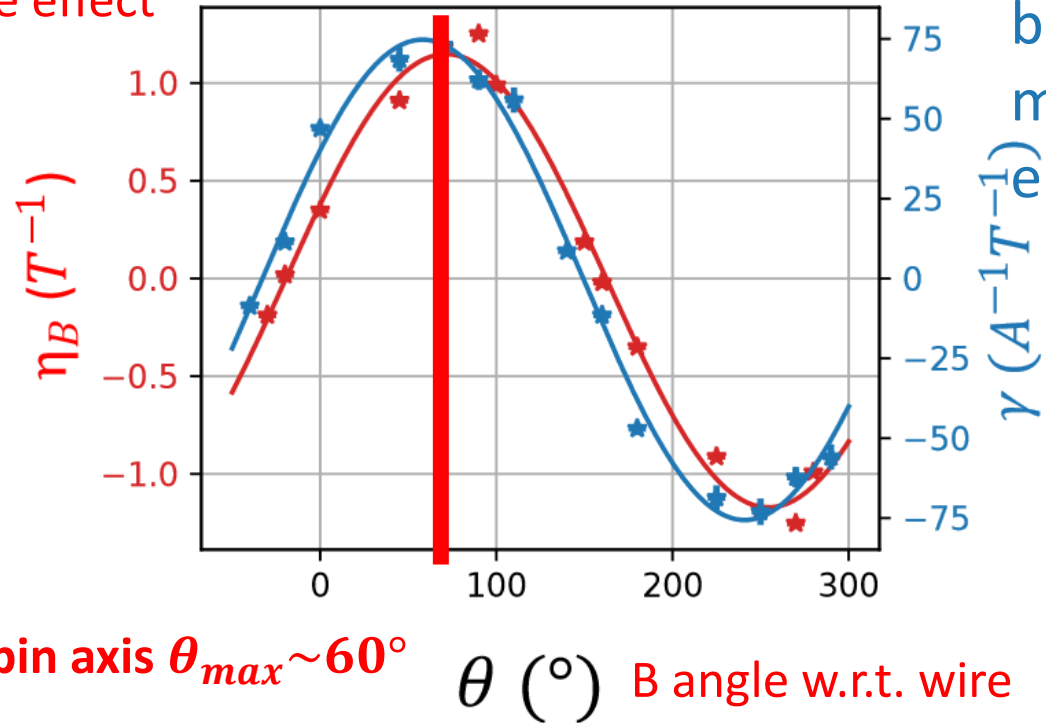
Diode effect : $I_+ \neq I_-$



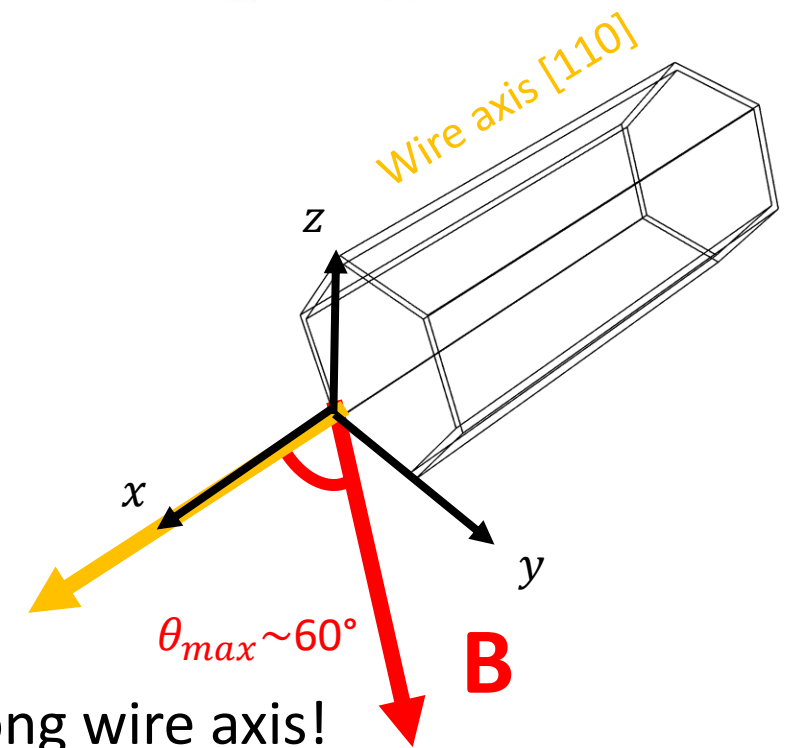
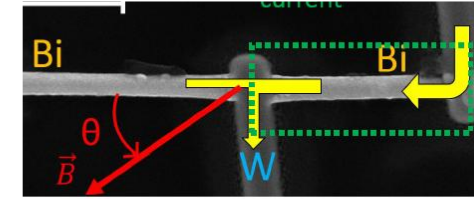
(Note also: max is not at B=0)

Same angular dependence of non-reciprocal transport in N and S regime

Superconducting
diode effect



Normal state
bilinear
magnetoresistance



- Not exclusively perpendicular to current, also component along wire axis!
→ Exclude purely orbital origin; → Not entirely caused by the Rashba surfaces

Common origin for the two effects:

current-induced spin polarization along an axis, with supercurrent carried by several ballistic one-dimensional channels, with both helicities in unequal number.

Three theoretical analyses of these results

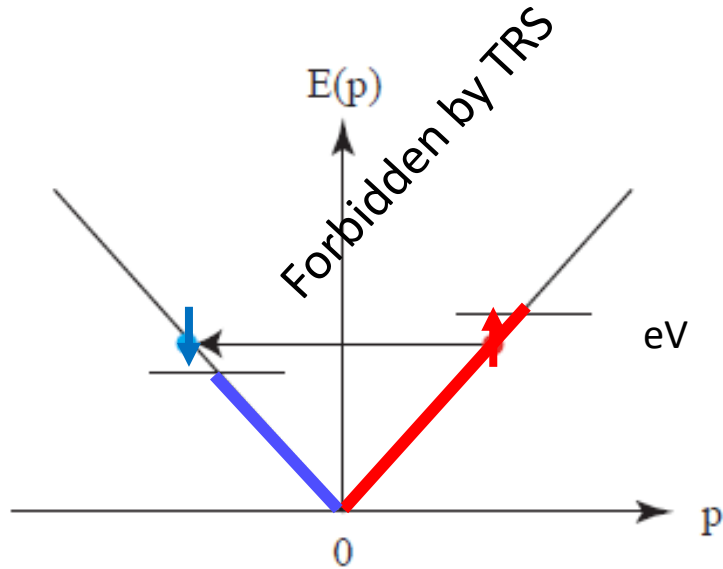
1- non reciprocity in N state:

How can B induce backscattering in a QSH state? Gorini et al, in preparation.

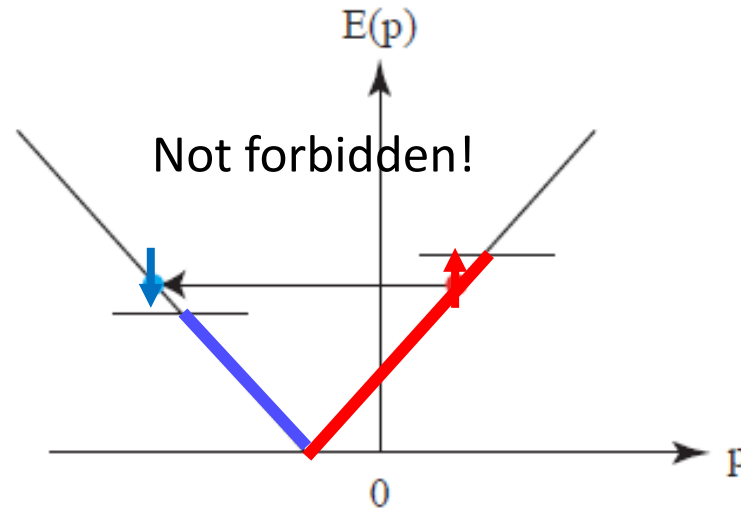
2- E. Gerber and B. Wieder: ad-initio calculation of the current-induced spin direction, given the orientation of wire surfaces (Bard et al, in preparation), and comparison to experiments (two wires)

3- H. Bouchiat, how current-induced spin-polarization distorts the diode effect (Self-consistent : negative inductance!)

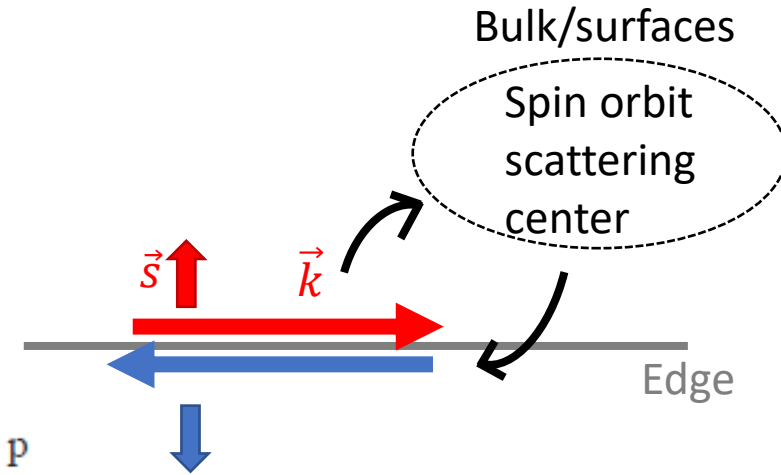
1-Gorini& Vignale: how B induces a huge bilinear magnetoresistance in helical states



$B=0$



$B \neq 0$



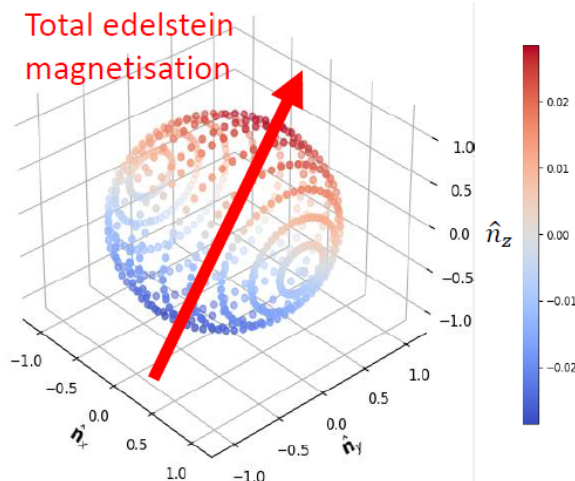
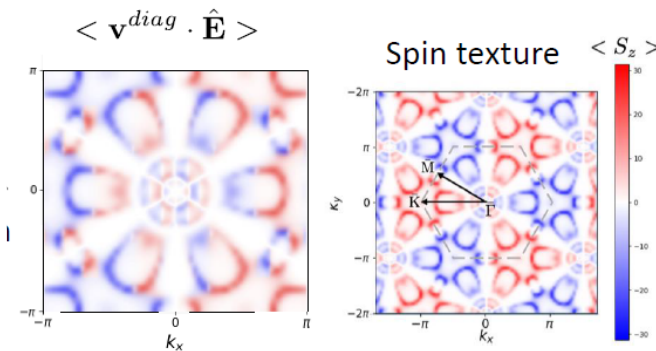
Current-induced spin polarization (Edelstein),
& extrinsic spin-orbit interactions needed to flip spin

2-Ab initio simulations to determine direction of induced spin

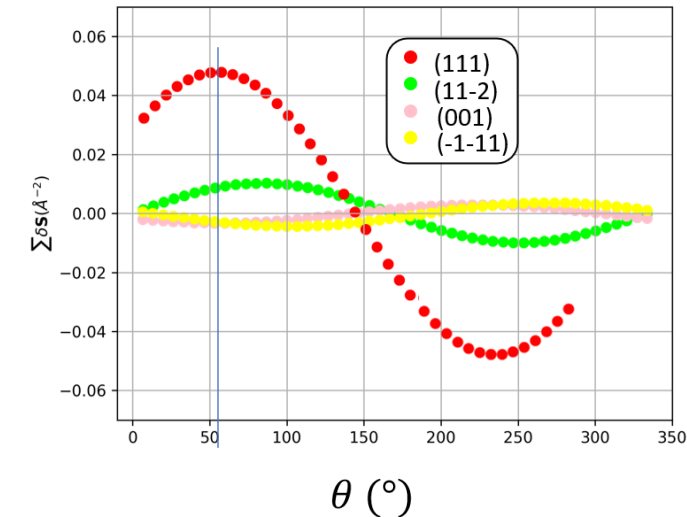
1-Assumption: Edge states inherit spin behavior of parent surfaces

2- With transmission microscopy, identify wire facets (or closest computable orientation, i.e. low miller indices), and current direction

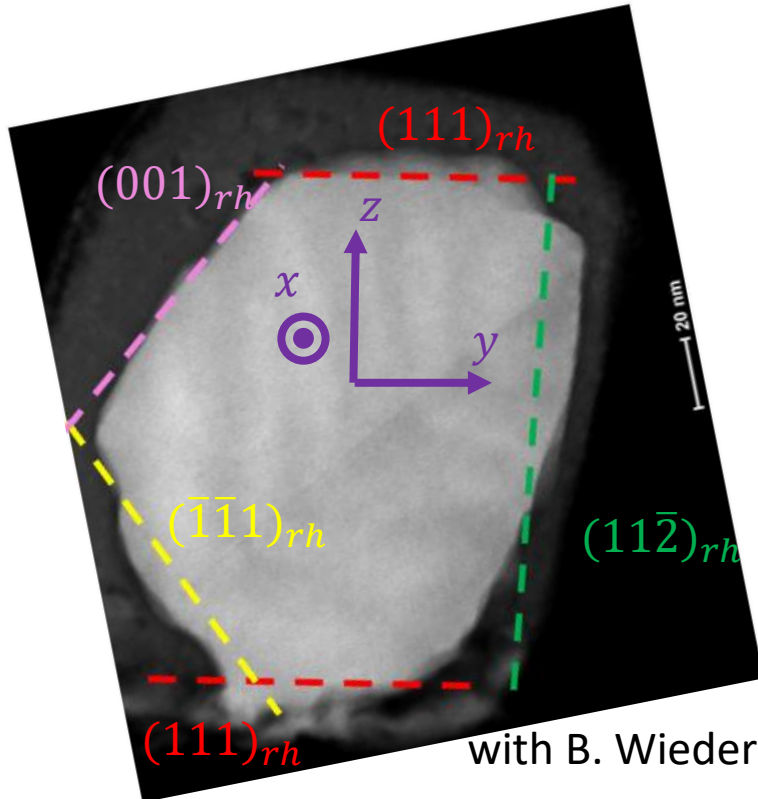
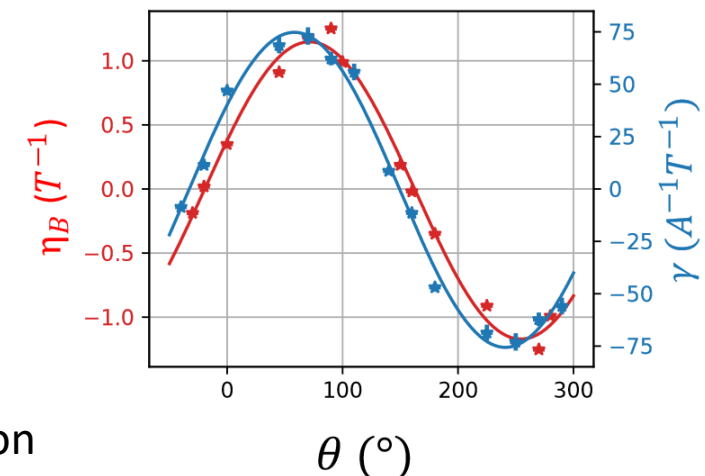
3- For each facet, compute Edelstein response (current-induced magnetisation): combination of velocity direction and spin texture



On the chosen surfaces (in plane of the wire)

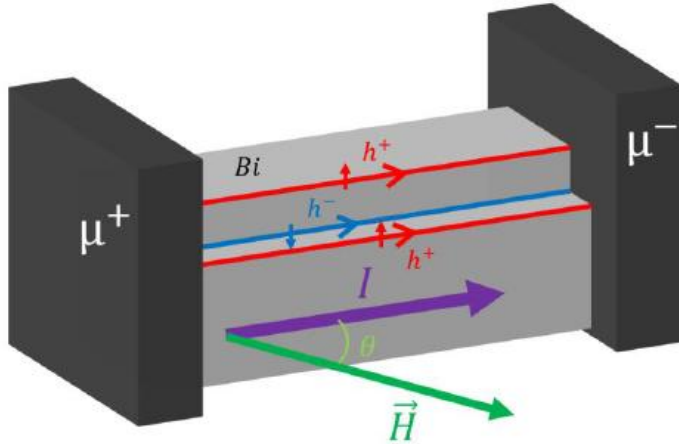


Main spin axis not so different from experiment



with B. Wieder and E. Gerber, CEA Saclay, Bard et al, in preparation

3-Simulating the shape of the diode effect (H. Bouchiat)



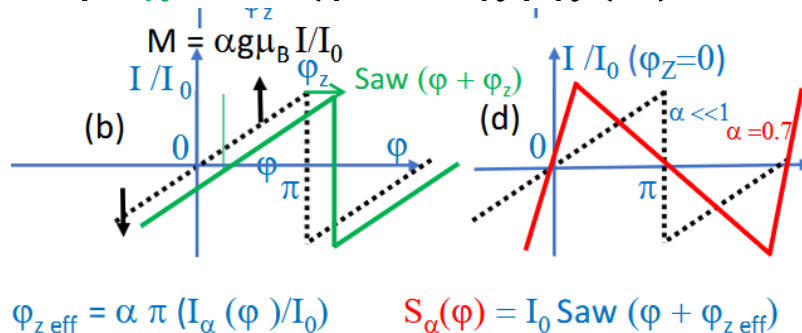
Hypotheses:

- Supercurrent carried by all edge states, **with both helicities (in unequal number)**
- **Extra phase differences:** due to Zeeman term and induced phase, due to spin-momentum locking / inverse Edelstein effect

$$I_J(\varphi) = \sum_n i_n * \text{Saw}(\varphi + h_n \varphi_n(B))$$

$$\varphi_n = \cos(\theta) g_{eff} \mu_B B_{eff} / E_{th}(n) \quad \theta \text{ Angle between spin axis and B}$$

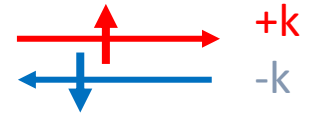
$$B_{eff} = B_z + \beta i_n \text{Saw}(\varphi + h_n \varphi_n(B)) \quad (\text{Self consistent})$$



→ Then compute critical current $I_c = \max(I_J)$

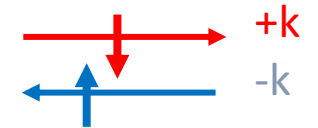
Helicity $h_n = \pm 1, N_+ \neq N_-$

Helicity $h=+1$:



Helicity $h=-1$

∴

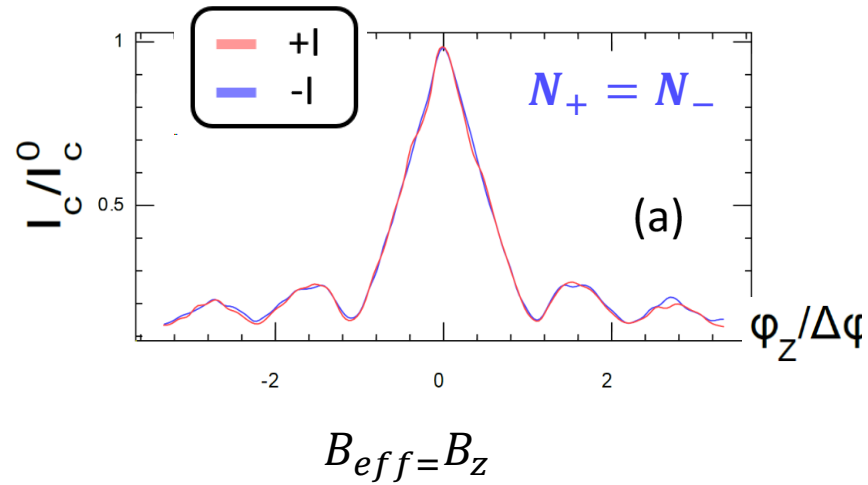


Same current I induces opposite magnetic moments for opposite helicities

$$M(I, h=+1) = -M(I, h=-1)$$

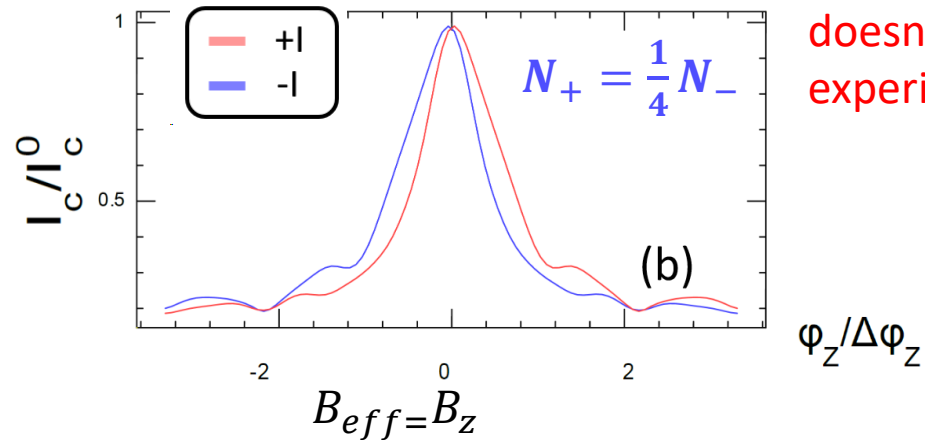
Simulated $I_c(B)$ summing 100 sawtooth CPR (H. Bouchiat)

(a) No spatial asymmetry: no diode effect!

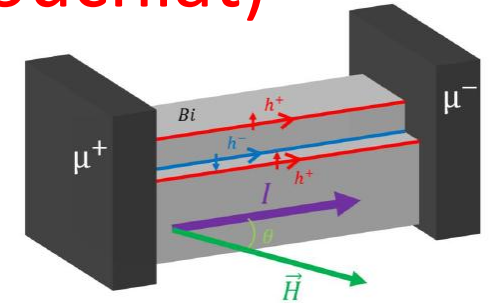


(b) Spatial asymmetry: unequal number of channels with + and - helicity

- No additional change of phase due to current-induced spin



Get diode, but doesn't look like experiment

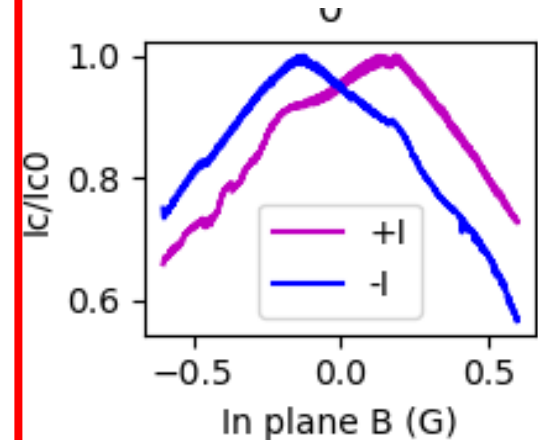


$$I_J(\varphi) = \sum_n \mathbf{i}_n * \text{Saw}(\varphi + h_n \varphi_n(B))$$

$$\varphi_n = \cos(\theta) g_{eff} \mu_B B / E_{th}(n)$$

Diode but not Edelstein

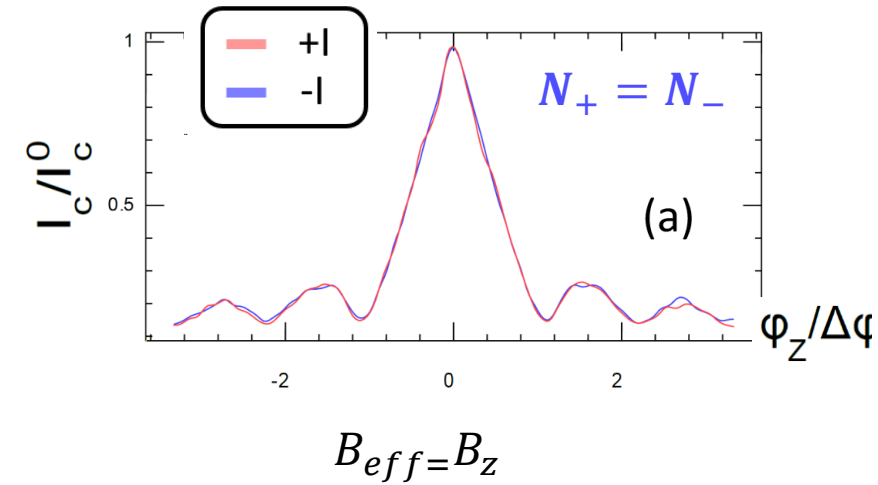
Experiment



Now looks like experiment!

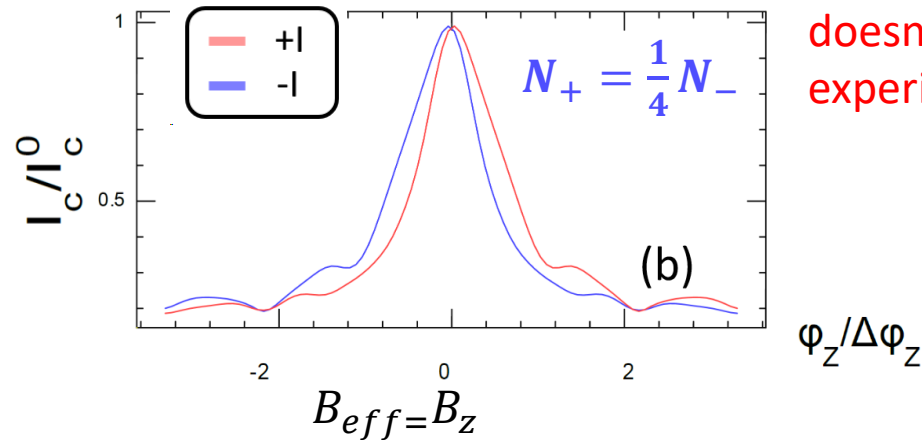
Simulated $I_c(B)$ summing 100 sawtooth CPR (H. Bouchiat)

(a) No spatial asymmetry: no diode effect!

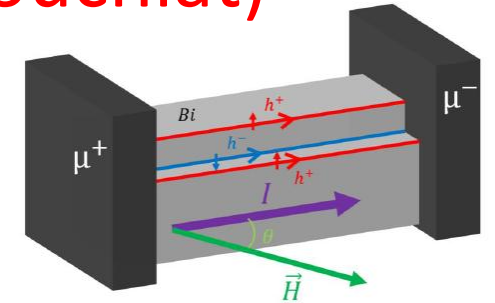


(b) Spatial asymmetry: unequal number of channels with + and - helicity

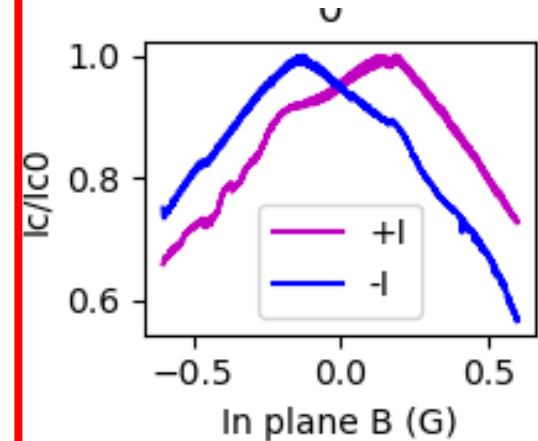
- No additional change of phase due to current-induced spin



Get diode, but doesn't look like experiment



Experiment



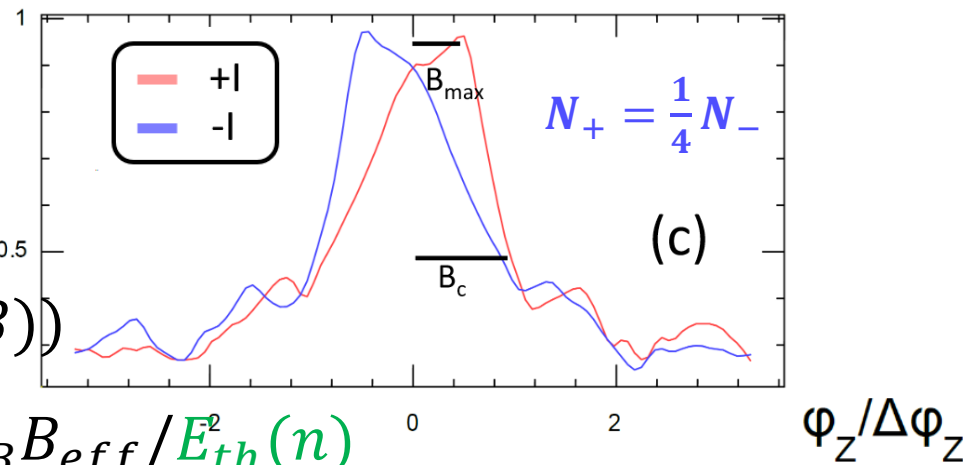
Now looks like experiment!

(c) Spatial asymmetry ($N_+ = 1/4 N_-$)
AND: Additional phase due to current-induced spin polarisation

$$B_{eff} = B_z + \beta \mathbf{i}_n \text{Saw}(\varphi + h_n \varphi_n(B))$$

$$I_J(\varphi) = \sum_n \mathbf{i}_n * \text{Saw}(\varphi + h_n \varphi_n(B))$$

$$\varphi_n = \cos(\theta) g_{eff} \mu_B B_{eff} / E_{th}^2(n)$$



Finally: A new remarkable SOTI: Bi₄Br₄, for now Quantum transport with N contacts (No S yet)

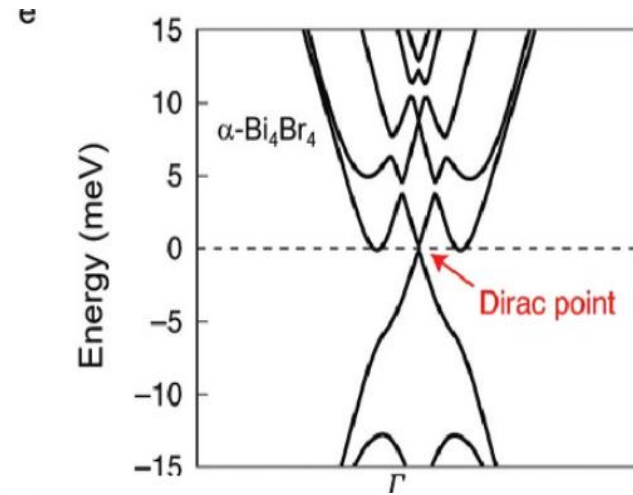
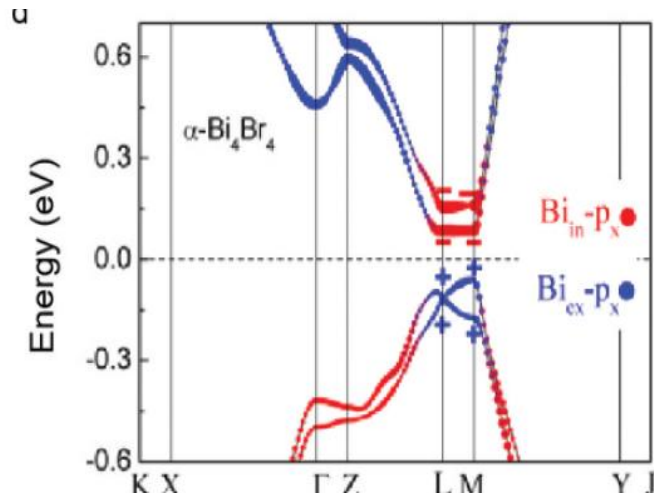
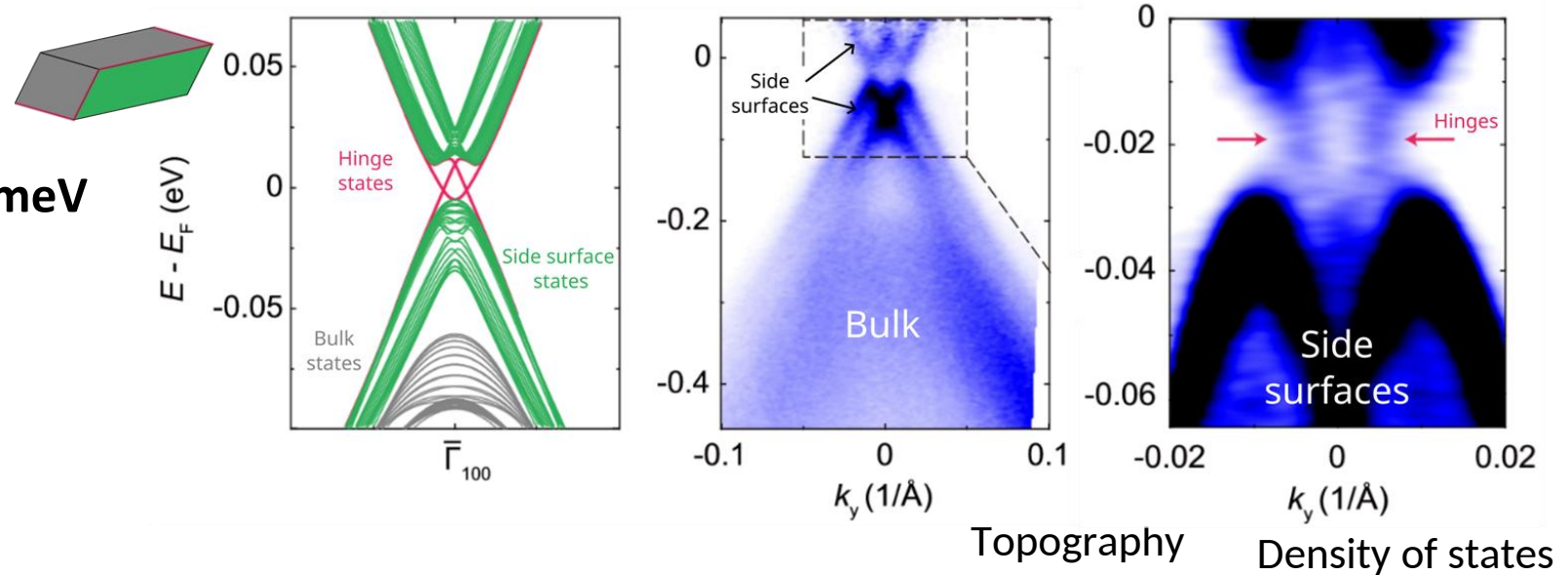
Jules Lefeuvre, Richard Deblock, et al PRX 2026

A better SOTI:

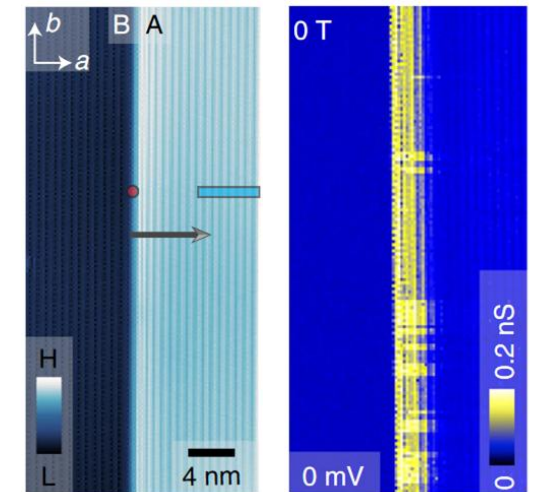
Large (double inverted) bulk gap : **230 meV**

Side surface gap : **~ 25 meV**

No other in-gap states !

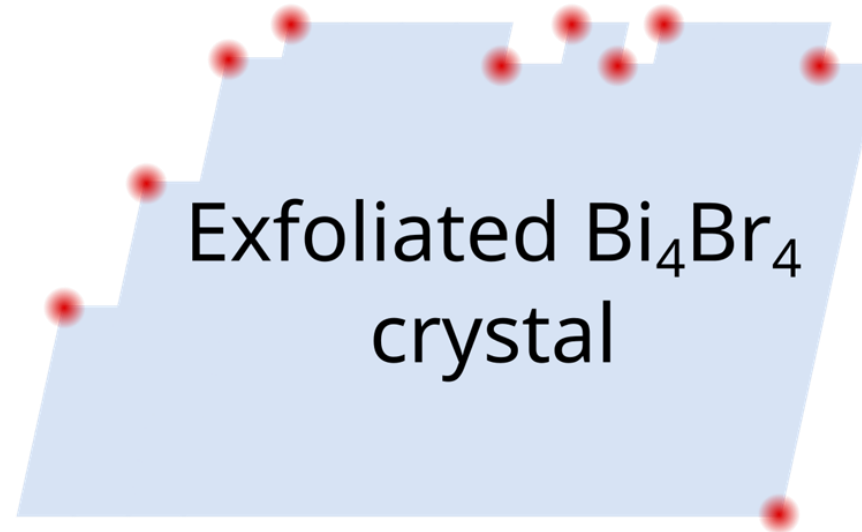


Confirmed by STM:
Edge states up to room temperature!

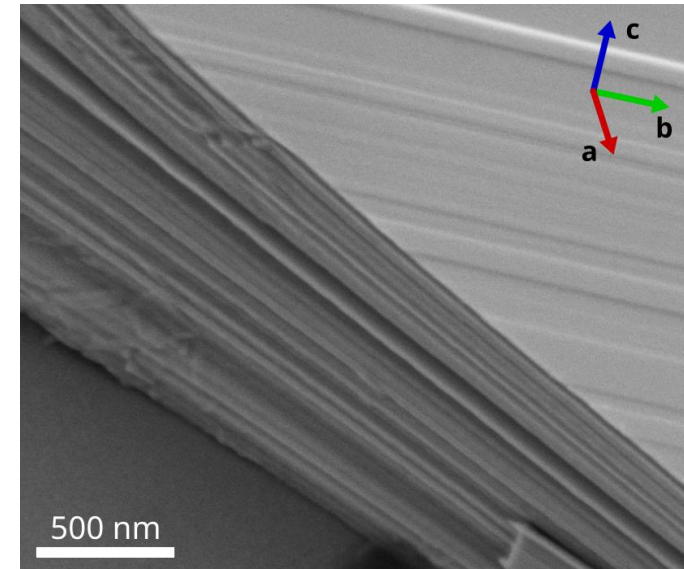


Shumiya, N. et al. Nat. Mater. (2022)

II) Conducting hinges

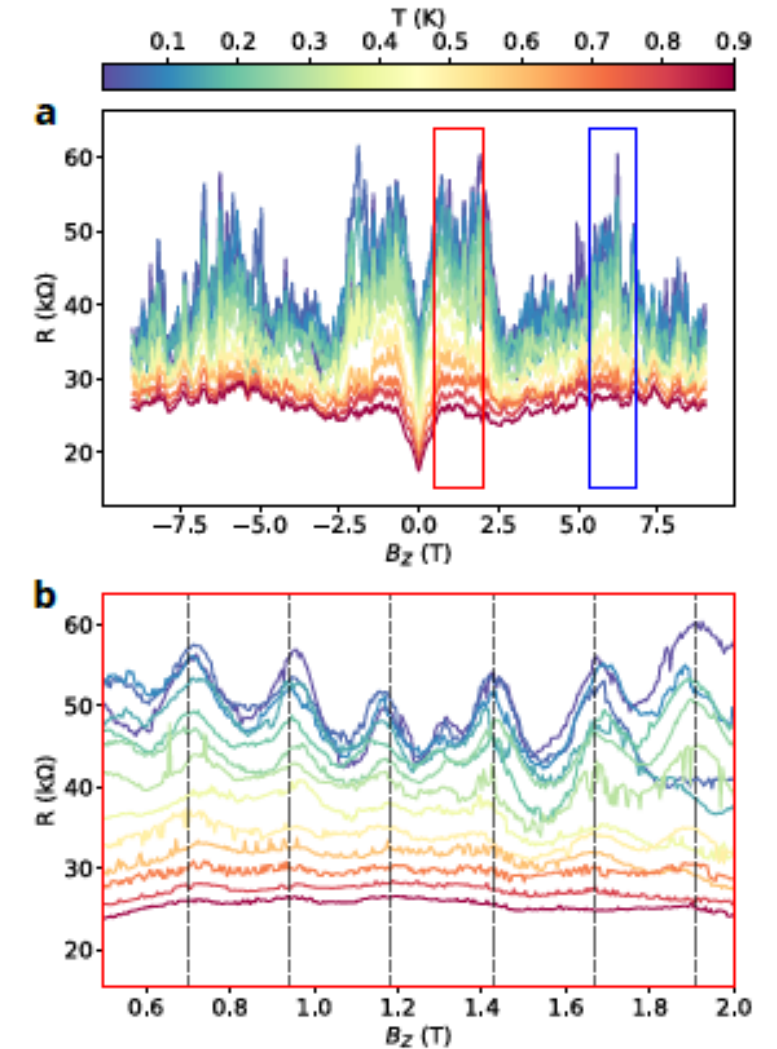
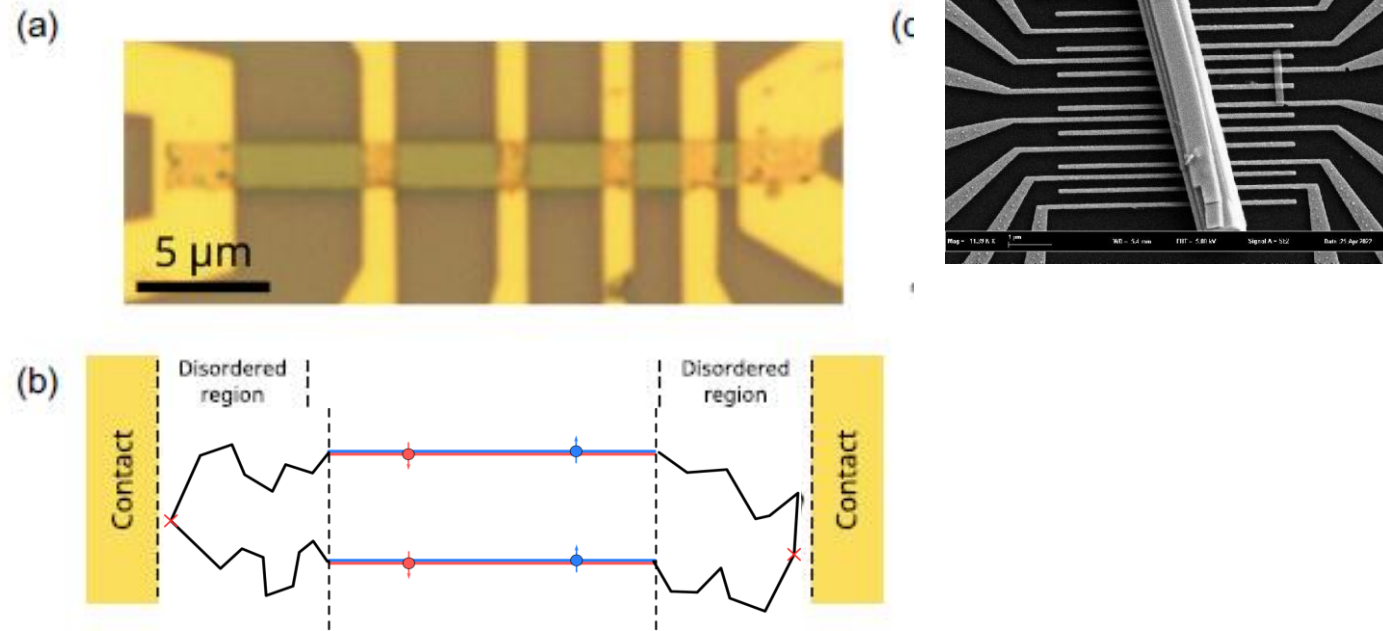


Exfoliation may create a lot of step edges
→ **Many hinge states**



A better SOTI: Bi_4Br_4 , Quantum transport with N contacts (No S yet)

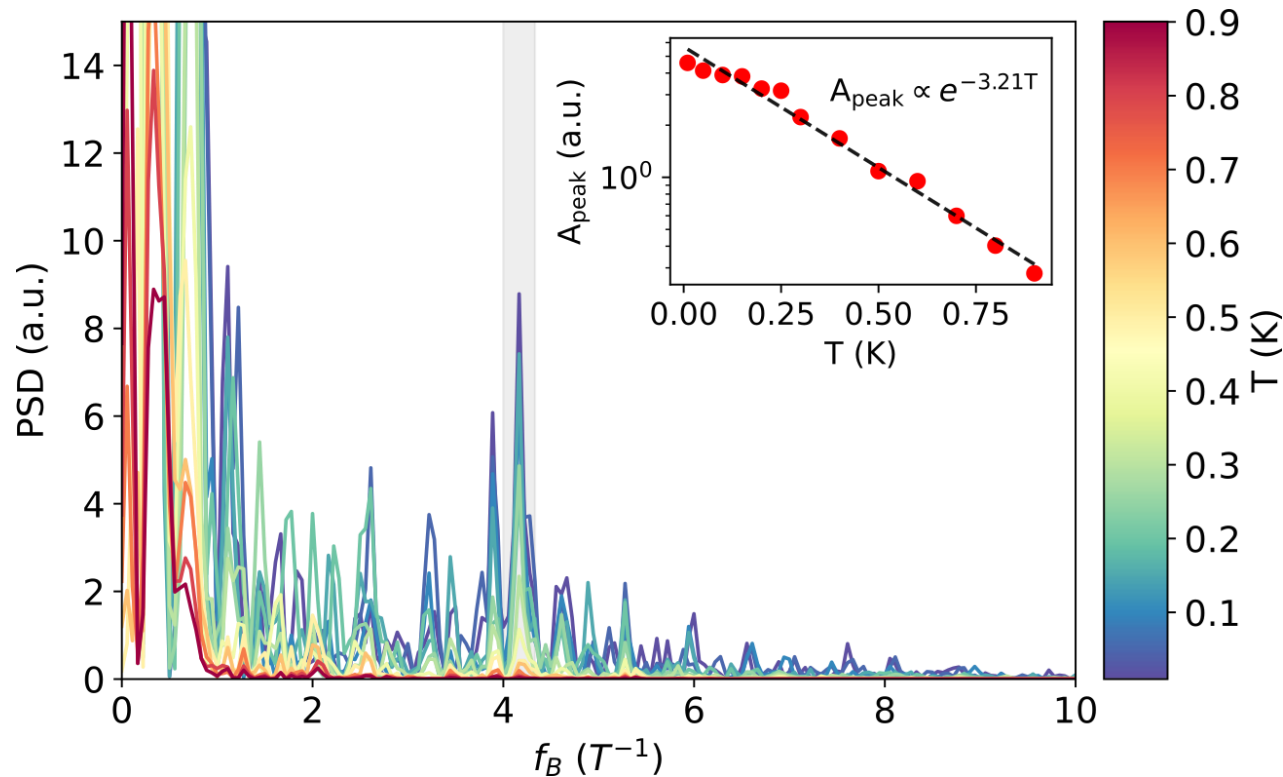
Jules Lefeuvre, Richard Deblock, et al PRX 2026



Aharonov-Bohm oscillations

Fourier transform, T dependance: transport regime and interfering paths

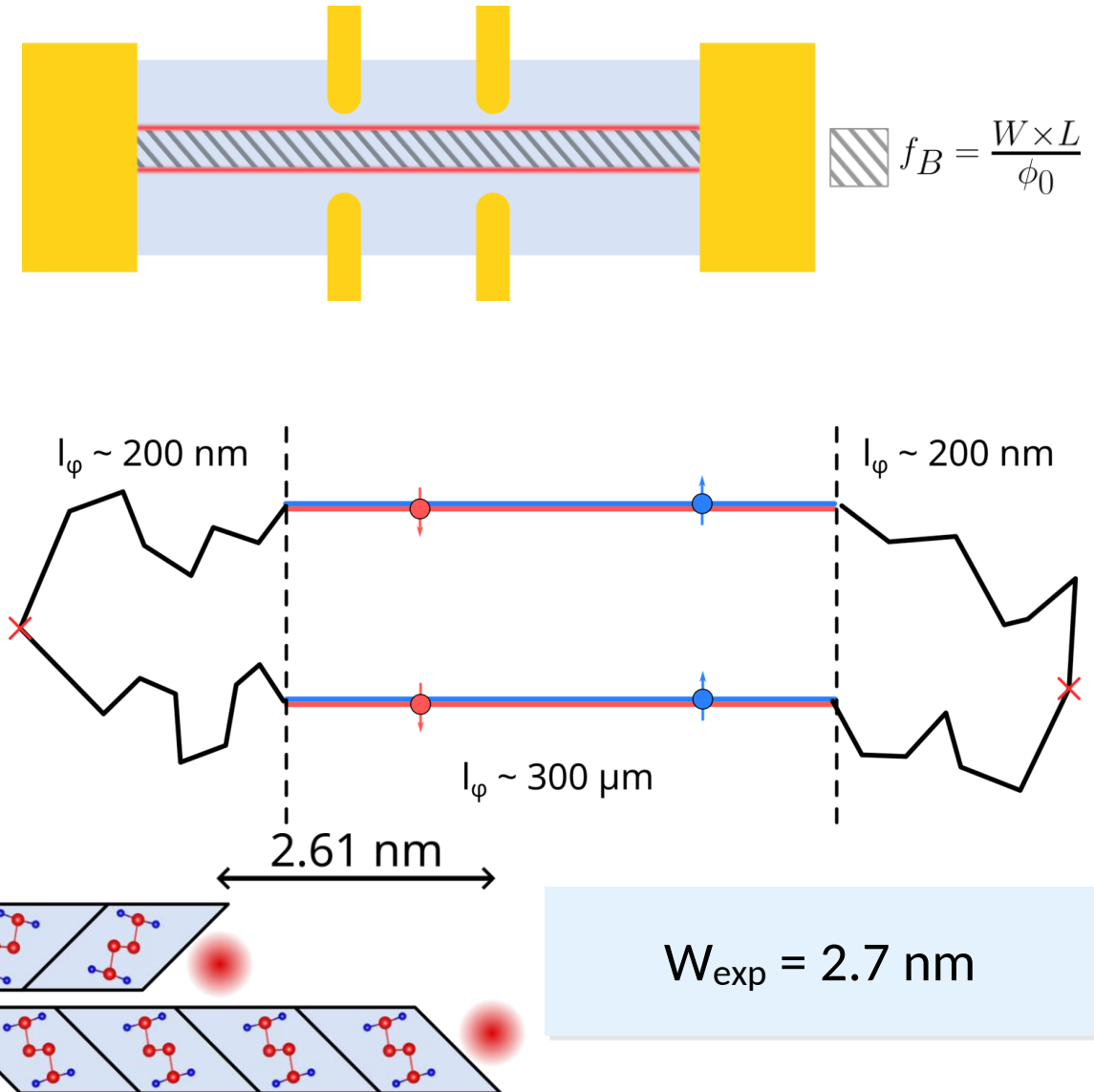
Fourier Transform



Cf ballistic rings: Hansen 2001, Kobayashi 2002, Dufouleur 2013

Aharonov Bohm oscillations due to one pair of hinge states

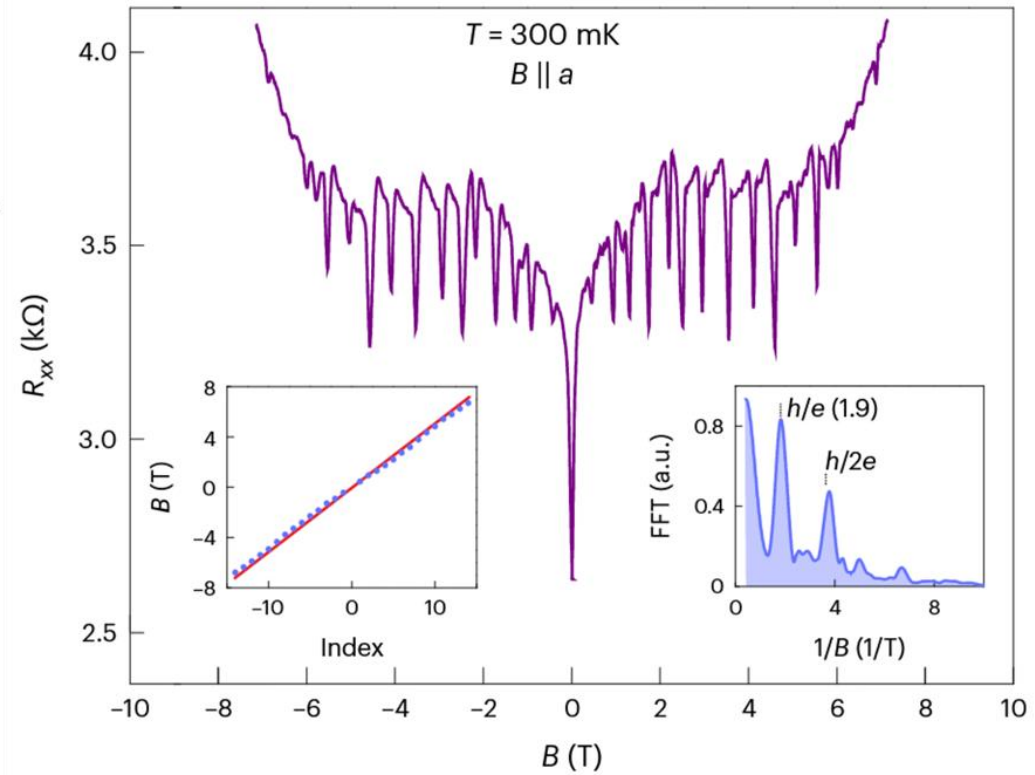
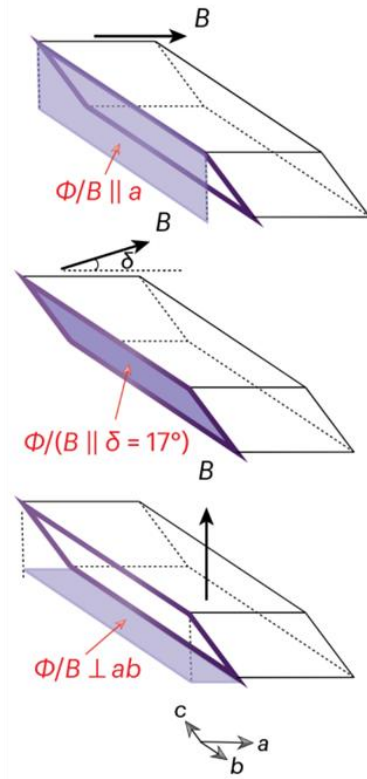
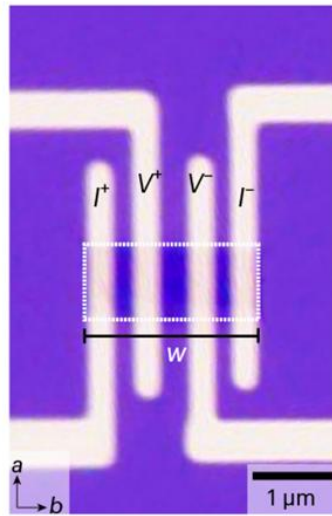
Crosses the flake coherently over $L = 6 \mu\text{m}$!



Aharonov-Bohm oscillations also seen by Hossain et al

4- and 6-layer flakes

Clear Aharonov-Bohm oscillations

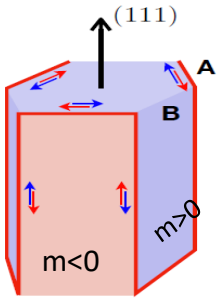


Hossain, MS. et al. Nature Physics, 20(5) (2024)

Hinge state loops

→ Large I_ϕ !

Further explore helical edge states



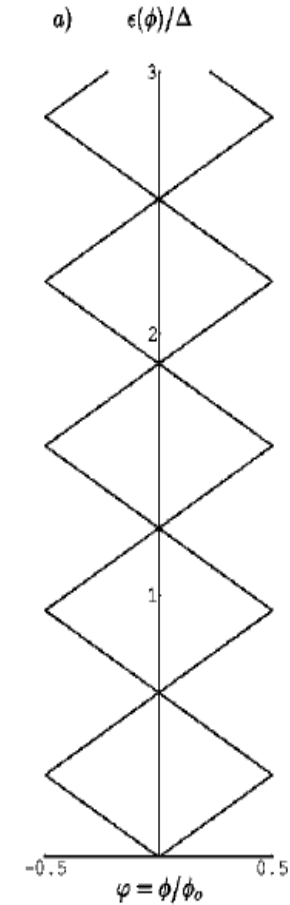
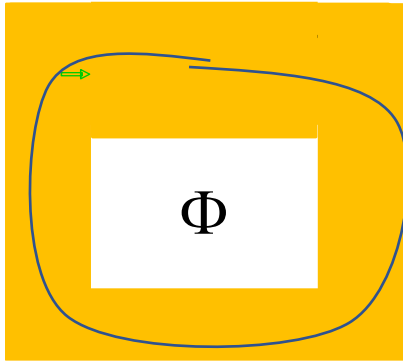
Planned experiments: no contacts!

- Detect **periodic** orbital moments in topological insulators with 1D edge states?
- Measure thermal equilibrium noise at π phase difference (Meydi Ferrier)

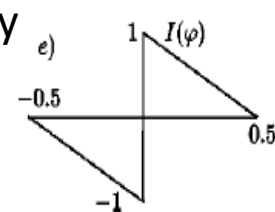
Outlook: Periodic persistent current in material with helical edge states ? (another mesoscopic test of topology)

Preamble:

A mesoscopic ring has a **periodic persistent** current
(no need for superconducting loop, since phase coherence)



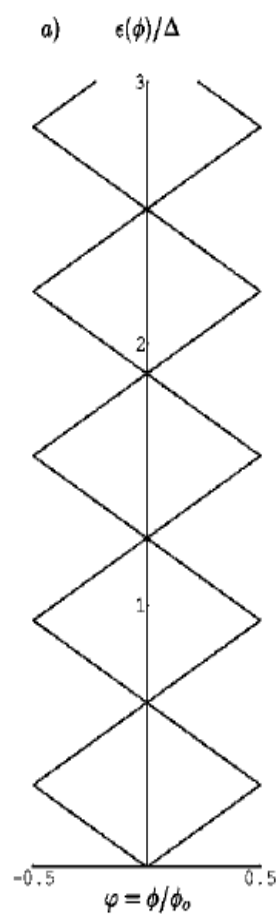
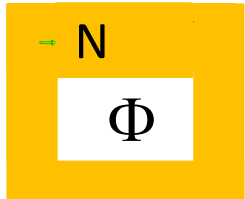
h/e periodicity



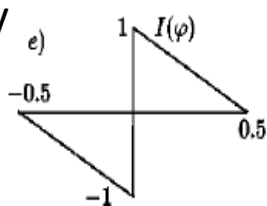
For fun: Compare Normal spectrum to Andreev Bound State ballistic spectra

Cayssol, Kontos Montambaux 2003.

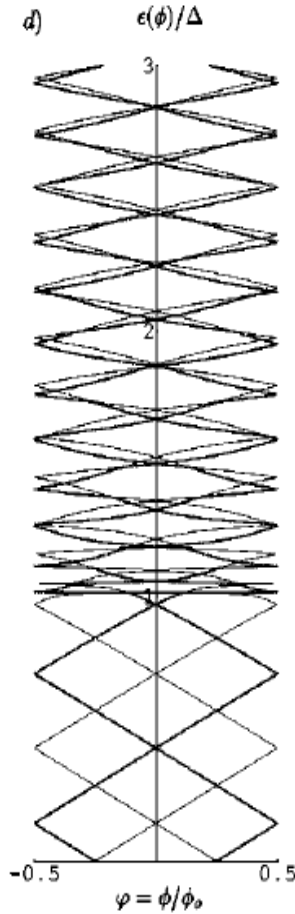
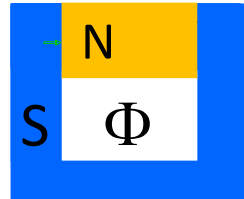
Normal ring



h/e periodicity



NS ring (long junction)

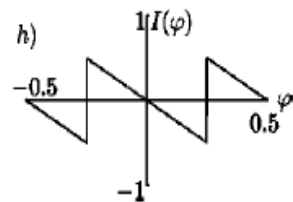


h/e periodicity above gap (like N)

$E = \Delta$

$h/2e$ periodicity in subgap region

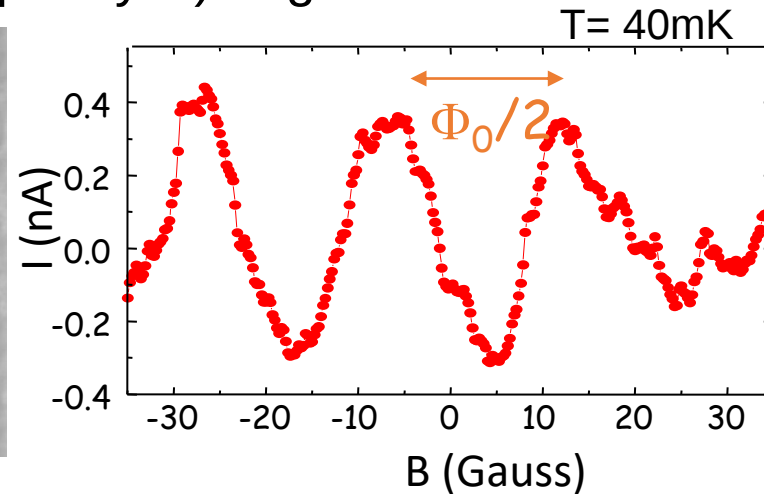
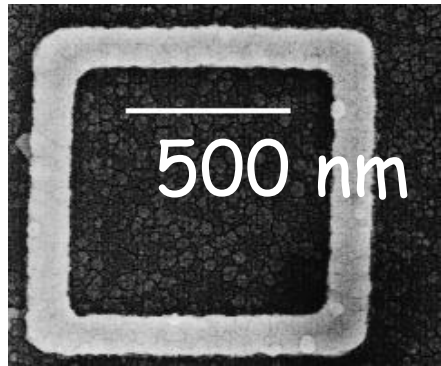
$h/2e$ periodicity



Outlook: Periodic persistent current in material with helical edge states ? (another mesoscopic test of topology)

Preamble: A mesoscopic ring has a **periodic persistent** current (no need for superconducting loop, since phase coherence)

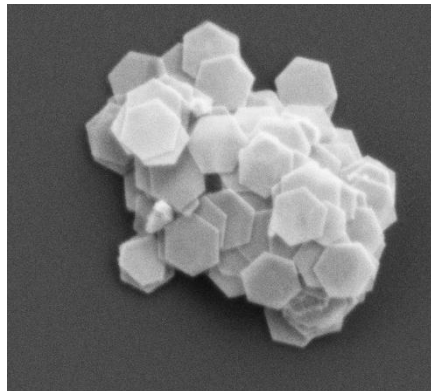
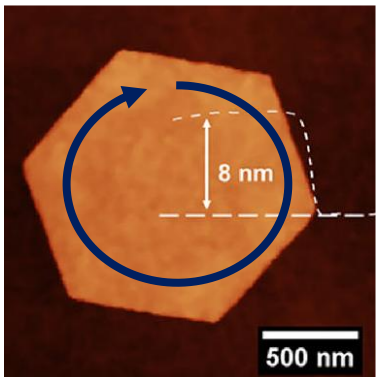
Detection of persistent current of 100 000 Ag (purely N) rings



- Ensemble average: $\Phi_0/2$ periodicity
- Persistent current in each ring $= e/\tau_D = 0.3$ nA

Deblock, Bouchiat et al. 2002

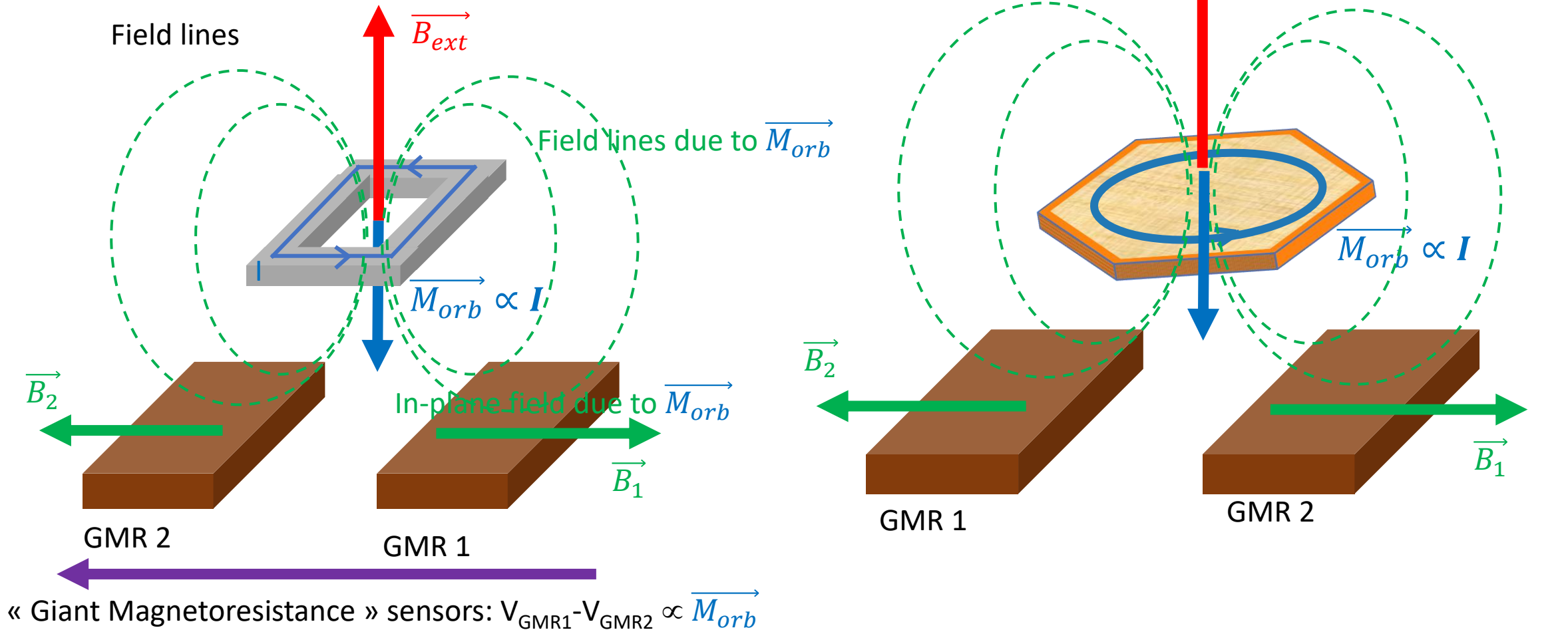
If helical edge states, should see the same in a flake (no need for ring!)



Bismuth hexagons, almost identical,
G. Abellan, Valencia, JACS 2023

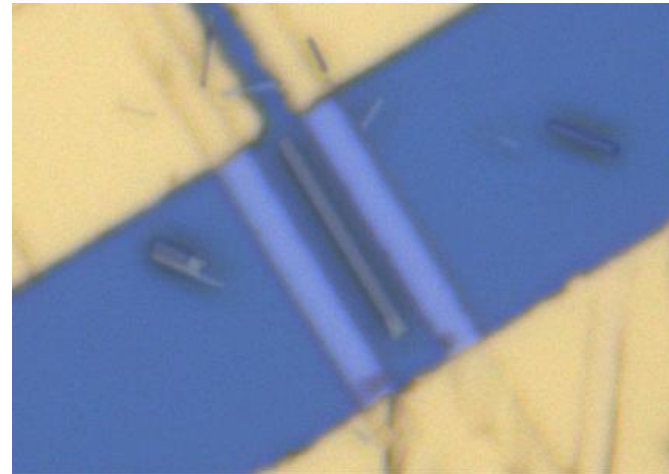
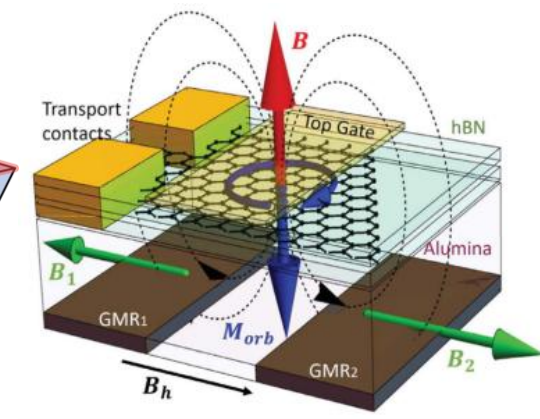
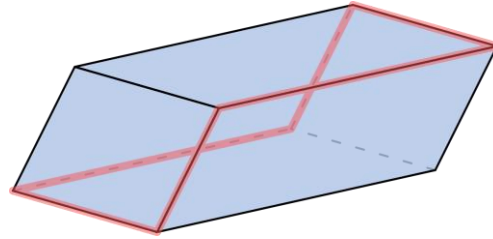
Goal: **Periodic** persistent current in 2DTIs and HOTIS?

Detect with Magnetoresistive Sensors, only sensitive to in-plane field

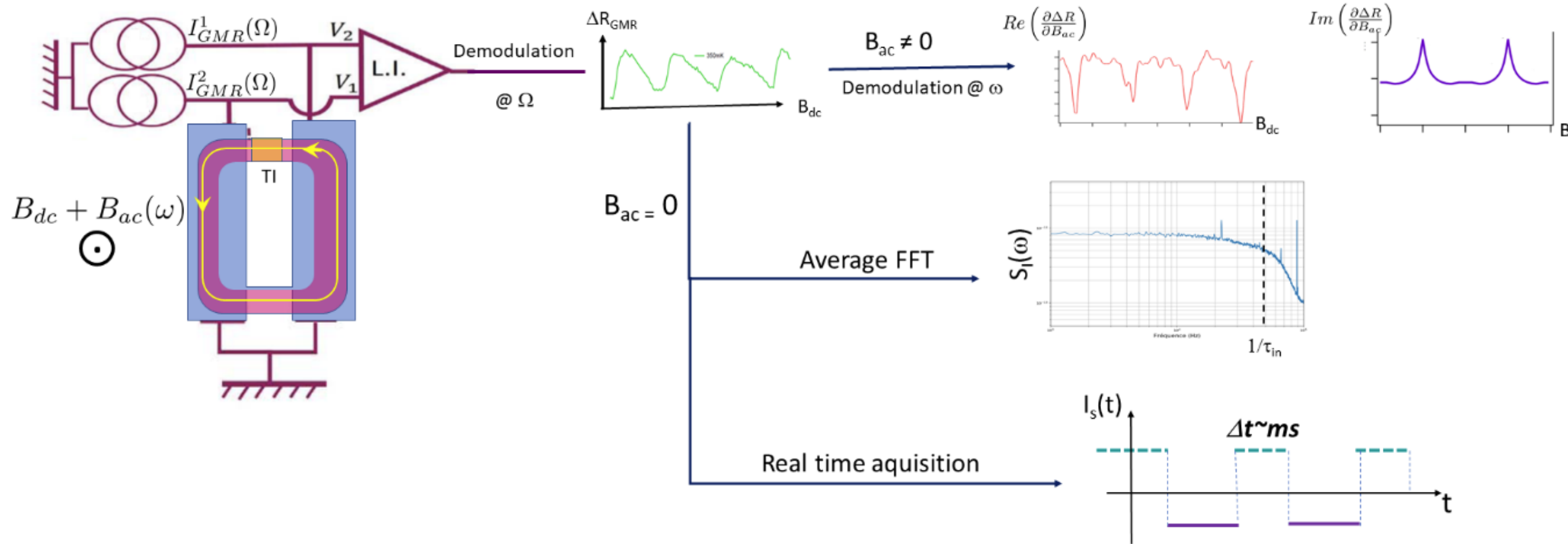


Detector has exquisite sensitivity (nT in 0.1 Tesla vertical field),
demonstrated in experiments on graphene and moiré (Vallejo, Science 2021, PRL 2023)

Persistent currents in 2DTIs and HOTIs ?



Time-resolved noise of a S/ topological hinge state/S junction (Meydi Ferrier and rest of Meso group)



Mesoscopic physics group in LPS Orsay, 2026

