Anyon Braiding in The Time Domain

Anyon Braiding and Tunneling Dynamics in Fractional Quantum Hall States

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Capris Spring School 2026/04/16

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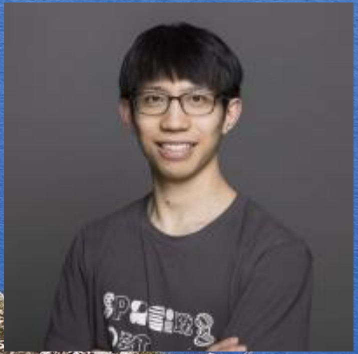

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$G = \nu e^2/h$ GaAs/AlGaAs heterostructure

$T_{el} = 30mK$
$B = 14\,T$

Experiments LPENS

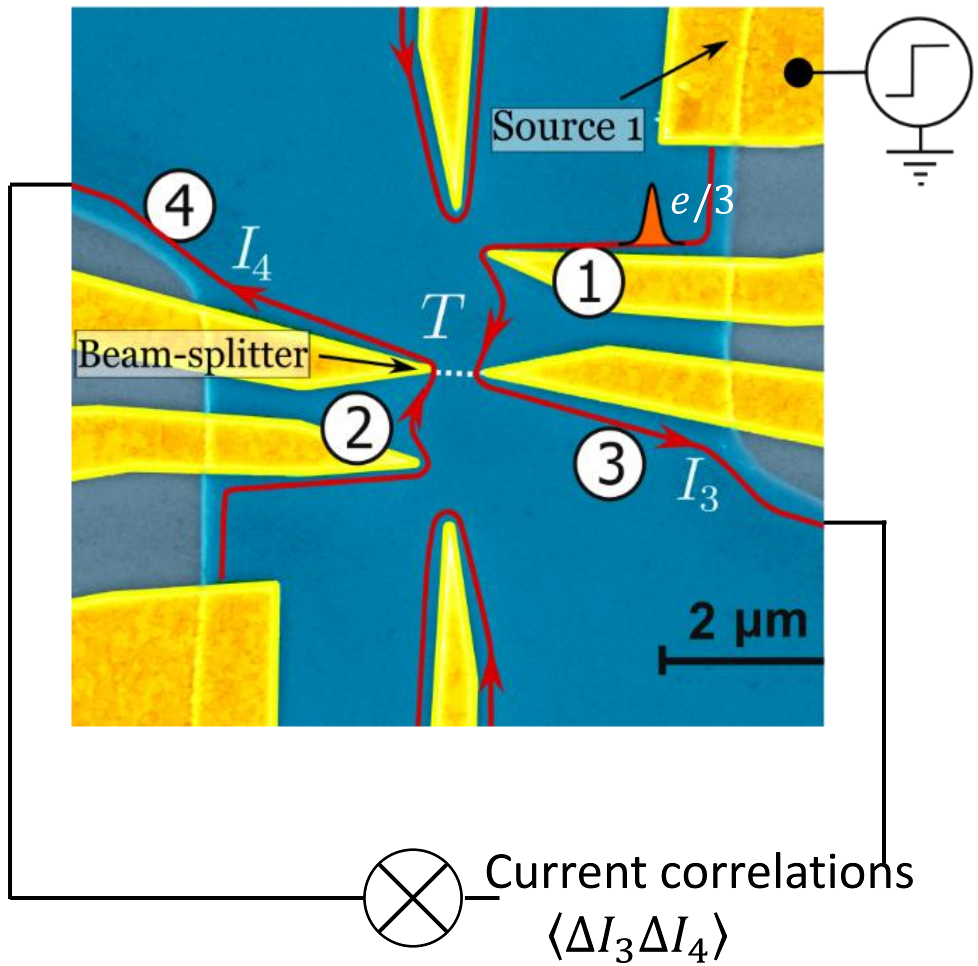
H. Lyu, M. Ruelle, E. Frigerio, H. Bartolomei, M. Kumar, J.M. Berroir, B. Plaçais, F. Parmentier, G. Ménard, G. Fève

Samples Fab,C2N Palaiseau

Y. Jin, A. Cavanna, U. Gennser

Theory, CPT Marseille

K. Iyer, B. Gremaud, T. Jonckheere, T. Martin, L. Raymond, J. Rech

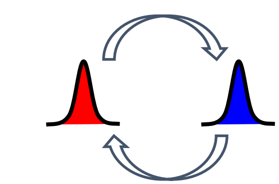


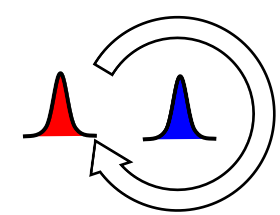
$\nu = 1/3$

$q = e/3$

$\varphi = \pi/3$

$\theta = 2\varphi = 2\pi/3$





T. Jonckheere et al., PRL **130**, 186203 (2023)

M. Ruelle et al., Science389,eadm7695(2025)

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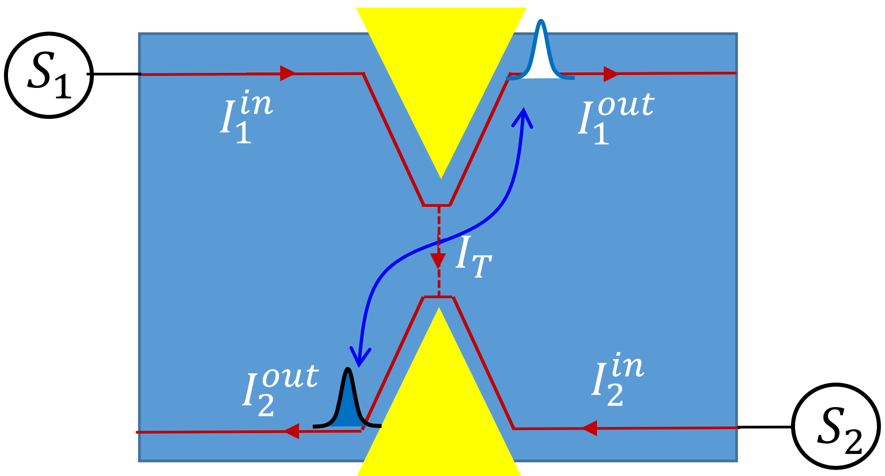
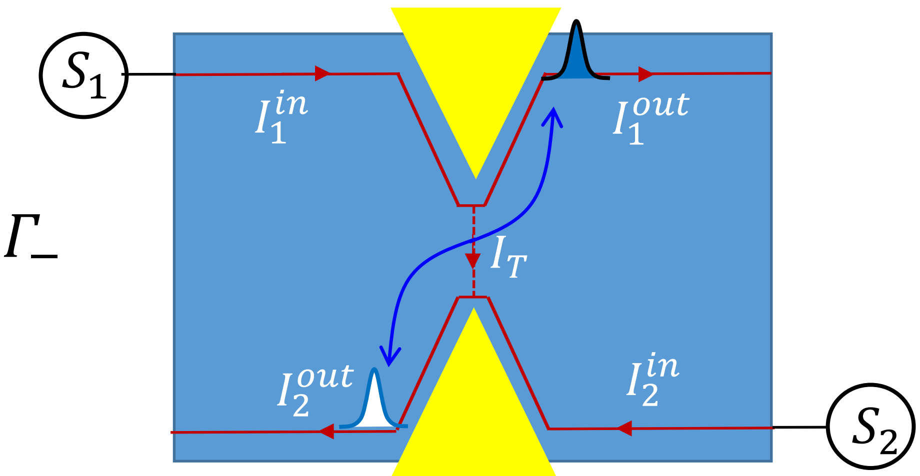
Random anyon backscattering at a quantum point contact

Weak backscattering regime: lowest order in tunneling $H_T = \zeta\psi^{+}_{1,\alpha}\psi_{2,\alpha} + \zeta^{*}\psi^{+}_{2,\alpha}\psi_{1,\alpha}$

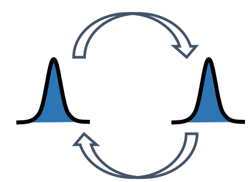
Γ_+



Γ_-



$\nu = 1/3 : \psi^{+}_{1,\alpha} = \psi^{+}_{1,a}$ particles are anyons, $q = e/3$



$\varphi = \pi/3, \quad \theta = 2\pi/3$

$$I_T = q(\Gamma_+ - \Gamma_-)$$ Tunneling current

$$S_{I_T} = 2q^2(\Gamma_+ + \Gamma_-)$$ Noise of tunneling current

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Weak backscattering regime: lowest order in tunneling $H_T = \zeta \psi^{+}_{1,\alpha}\psi_{2,\alpha} + \zeta^{*}\psi^{+}_{2,\alpha}\psi_{1,\alpha}$

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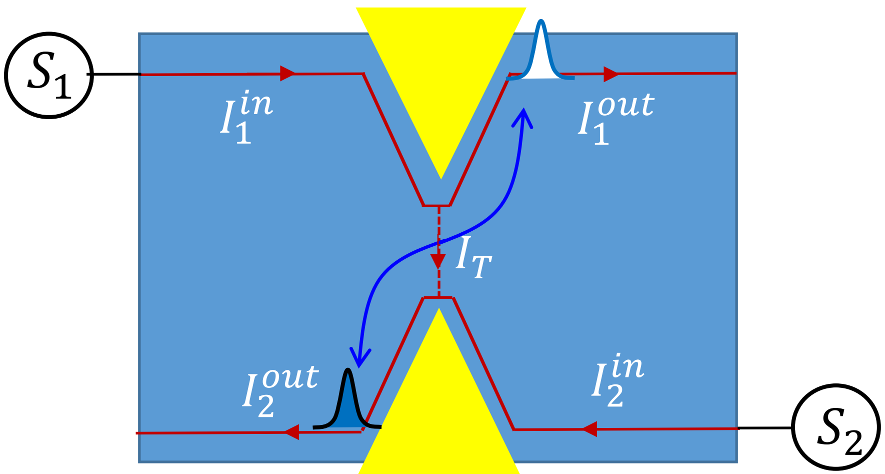
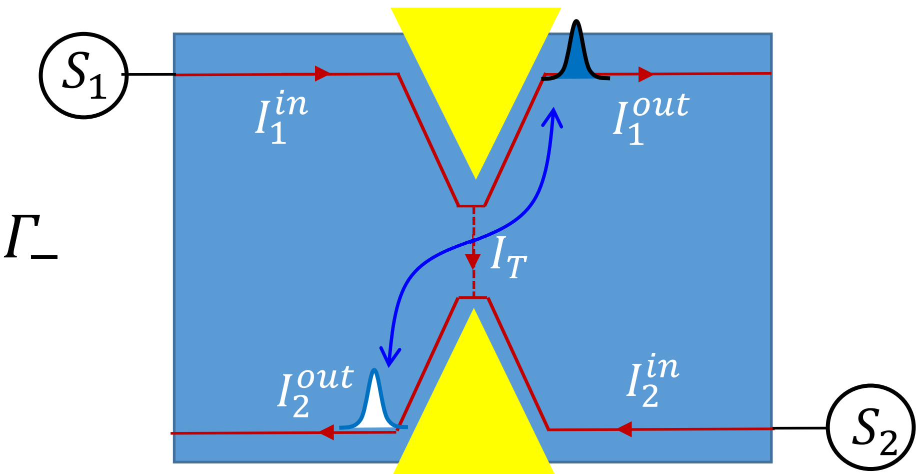
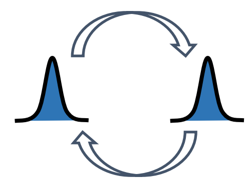


Γ_-



$\nu = 1/3 : \psi^{+}_{1,\alpha} = \psi^{+}_{1,a}$ particles are anyons, $q = e/3$



$\varphi = \pi/3, \quad \theta = 2\pi/3$

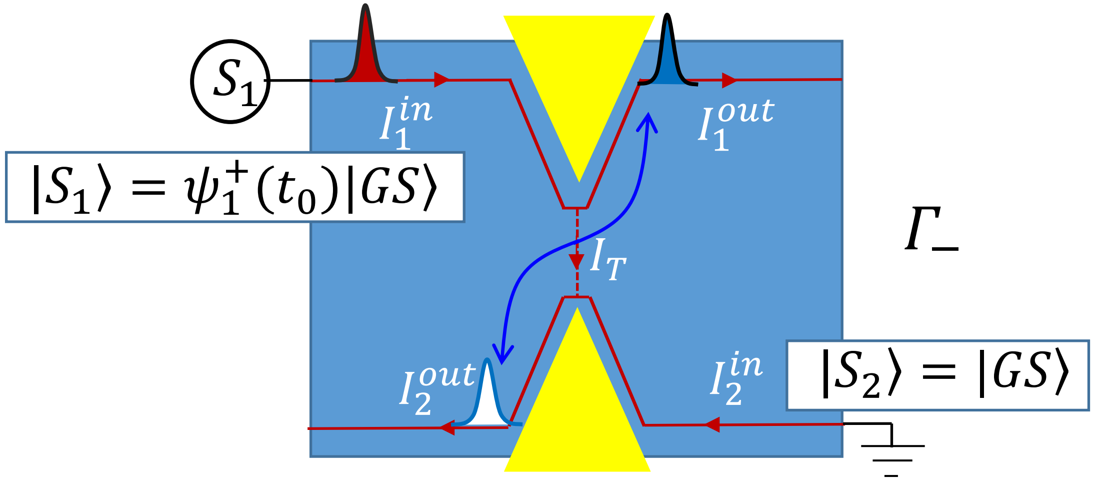
$$I_T = q(\Gamma_+ - \Gamma_-) \qquad S_{I_T} = 2q^2(\Gamma_+ + \Gamma_-)$$

$$\Gamma_-(t) \propto Re\left[\int_{-\infty}^{t} dt' \langle S_1|\psi^{in}_{1,\alpha}(t)\psi^{+,in}_{1,\alpha}(t')|S_1\rangle\langle S_1|\psi^{+,in}_{2,\alpha}(t)\psi^{in}_{2,\alpha}(t')|S_2\rangle\right]$$

Anyon braiding at a QPC

S1: Single anyon incoming on the QPC at time t_0:

$$t' \leq t_0 \leq t$$



Γ_-

Tunneling rate: $\Gamma_- \propto Re \int_{-\infty}^{t} dt' \underbrace{\langle S_1|\psi_1^{in}(t)\psi_1^{+,in}(t')|S_1\rangle}_{\text{Non-equilibrium: 1 anyon emitted}} \underbrace{\langle GS|\psi_2^{+,in}(t)\psi_2^{in}(t')|GS\rangle}_{\text{Equilibrium: } G_{eq,\delta}(t-t')}$

Han et al., Nat Commun **7**, 11131 (2016)
Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)
Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

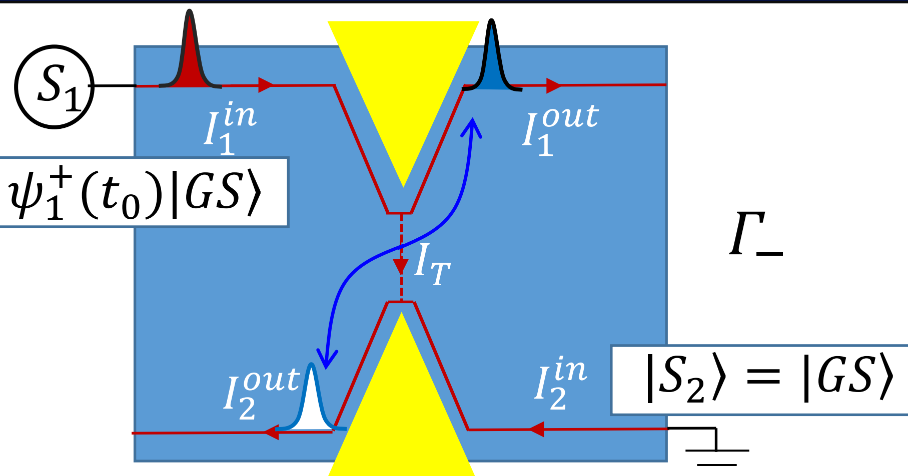
Anyon braiding at a QPC

S1: Single anyon incoming on the QPC at time t_0:

$$t' \leq t_0 \leq t$$

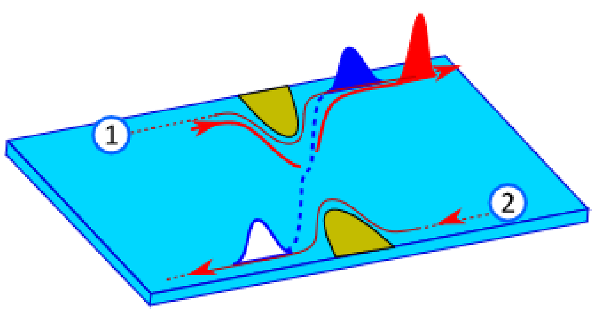
$$|S_1\rangle = \psi_1^+(t_0)|GS\rangle$$

$$|S_2\rangle = |GS\rangle$$



$$\Gamma_- \propto Re \int_{-\infty}^{t} d\tau \underbrace{\langle GS|\psi_1(t_0)\psi_1(t)}\,\underbrace{\psi_1^+(t')\psi_1^+(t_0)|GS\rangle}\langle GS|\psi_2^{+,in}(t)\psi_2^{in}(t')|GS\rangle$$

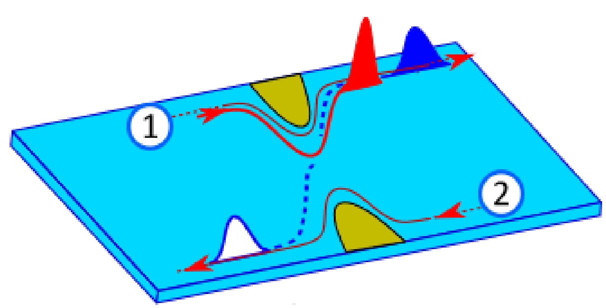
$$\psi_1^+(t)\psi_1^+(t_0)|GS\rangle$$



emitted at t > t_0

Han et al., Nat Commun **7**, 11131 (2016)
Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)

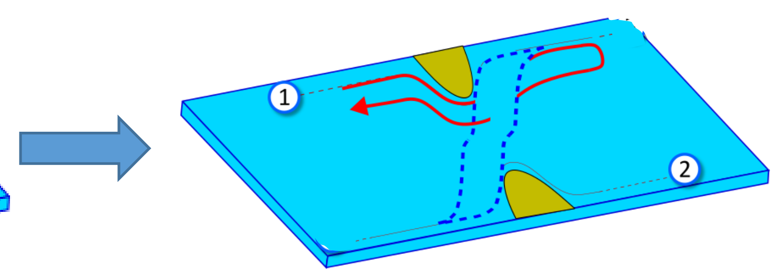
$$\psi_1^+(t')\psi_1^+(t_0)|GS\rangle$$



emitted at $t' < t_0$

Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

$$\langle S_1|\psi_1^{in}(t)\psi_1^{+,in}(t')|S_1\rangle = e^{-i\theta_{rb}} G_{eq,\delta}(t-t')$$



θ_{rb}: braiding phase between red and blue anyons

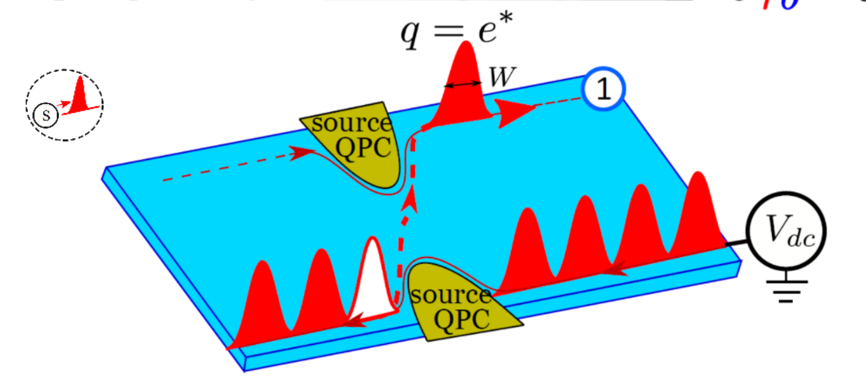
$\nu = 1/3 \qquad \theta = 2\pi/3$

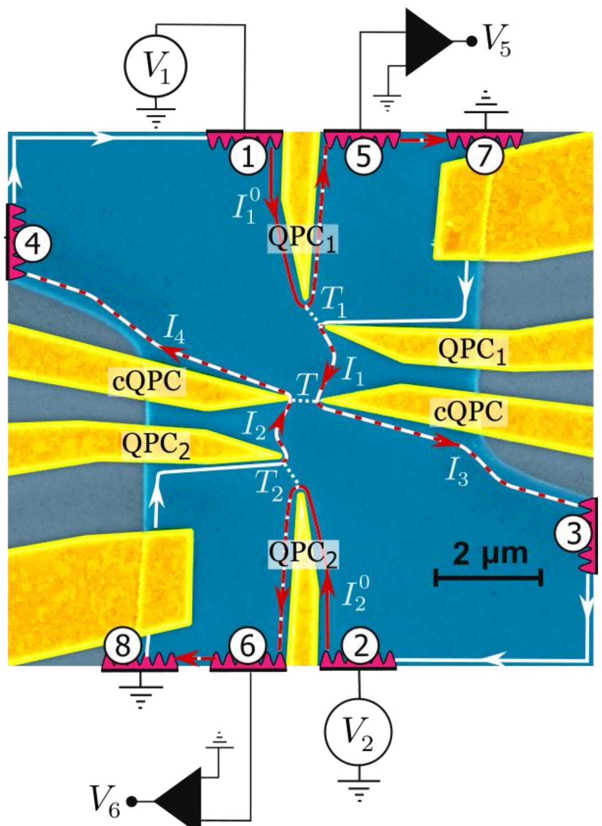
Partitioning of a random anyon source

$N_1(t,t')$ anyons incoming on the QPC between times t' and t

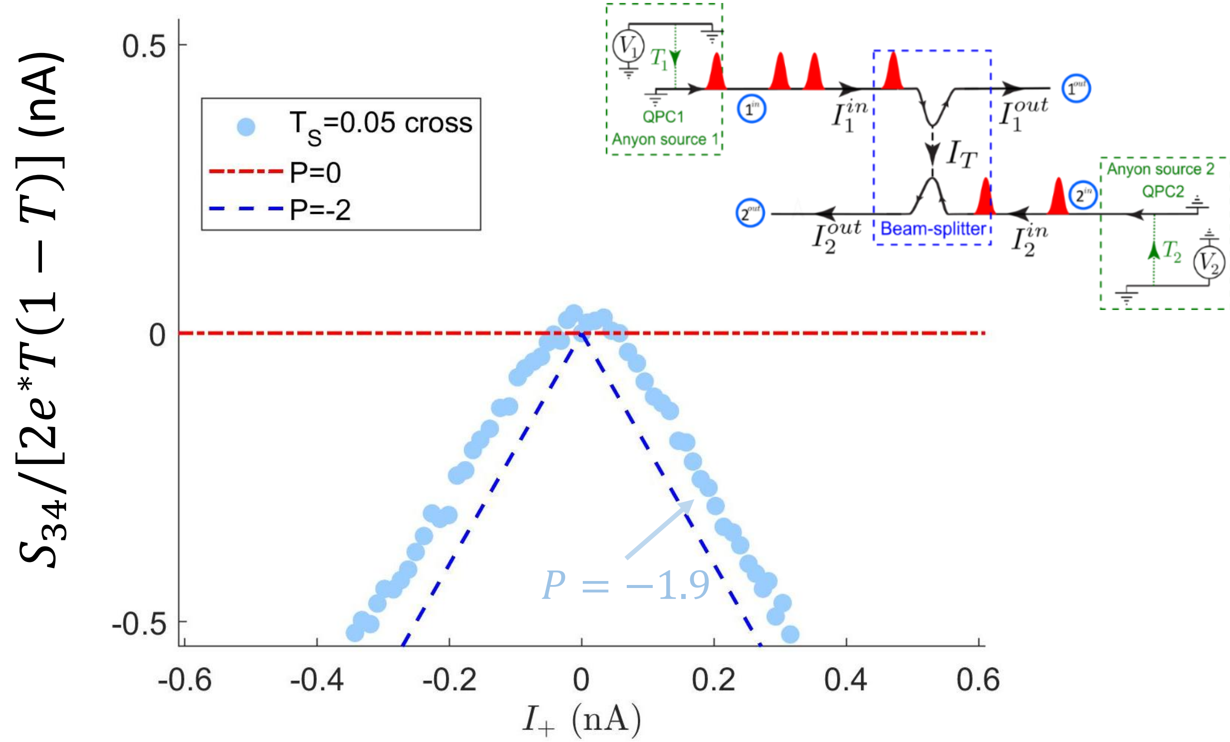
$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

Topological anyon tunneling at input QPC: $\theta_{rb} = \theta$





N_1 is stochastic: random emission

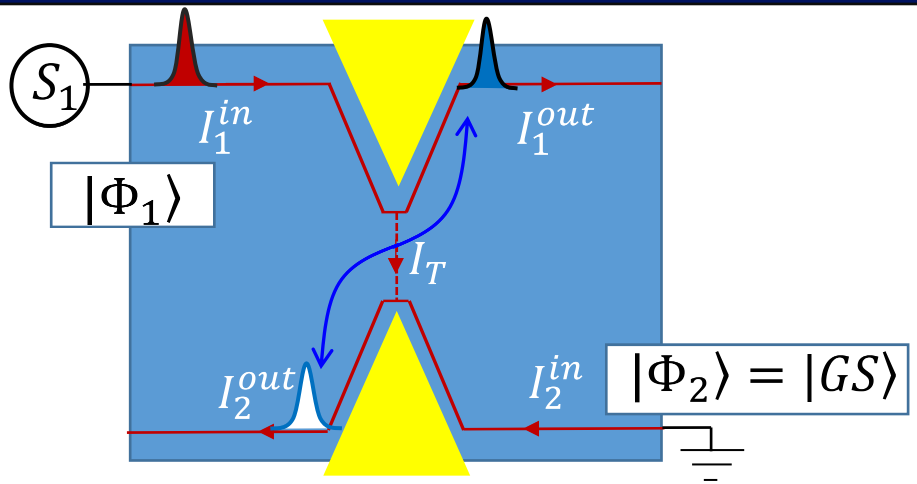


H. Bartolomei et al., Science368,173-177(2020)

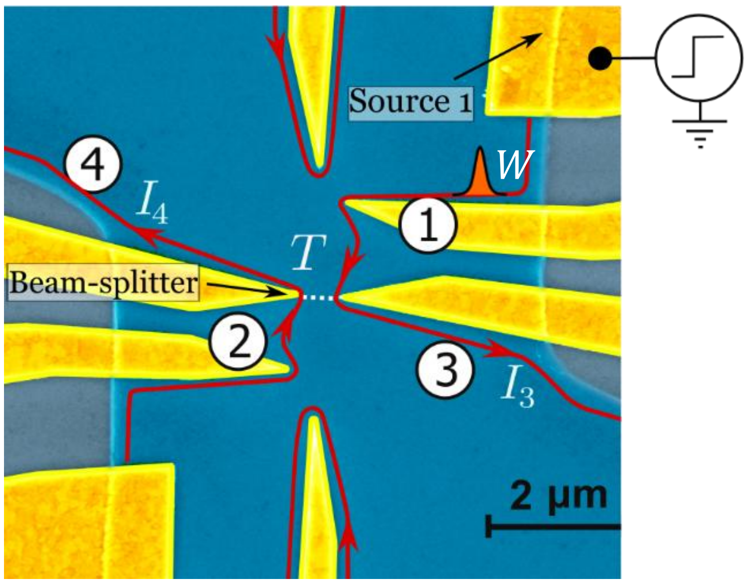
Partitioning of a triggered anyon source

S1: $N_1(t,t')$ anyons incoming on the QPC between times t' and t

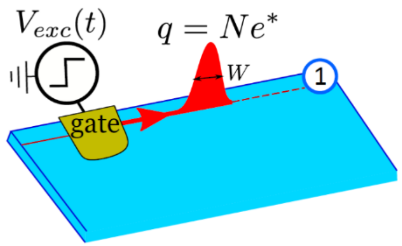
$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

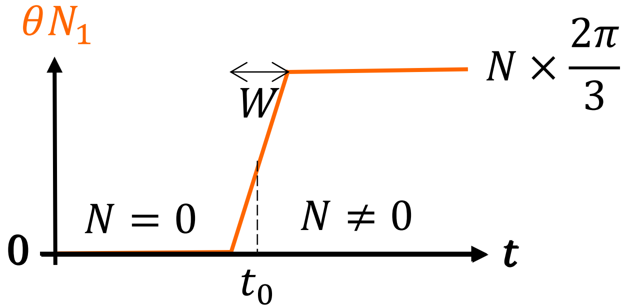


N_1 is deterministic in anyon pulse



anyon pulses: $\theta_{rb} = \theta \times N$





N = 1 reproduces anyon emission at $\nu = 1/3$ $\theta_{rb} = 2\pi/3$

N = 3 reproduces electron emission at $\nu = 1/3$ $\theta_{rb} = 3 \times 2\pi/3$

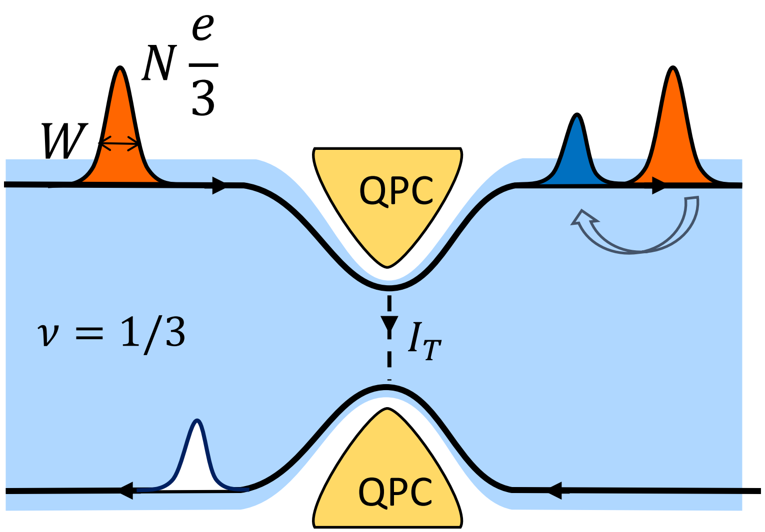
J.Y. Lee, C. Han, H.S. Sim, PRL **125**, 196802 (2020)

Proposed in: T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)

Experiment: M. Ruelle et al., Science389,eadm7695(2025)

Anyon scattering at a QPC: time-domain

T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)



$$\Gamma_{\pm}(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta\, N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

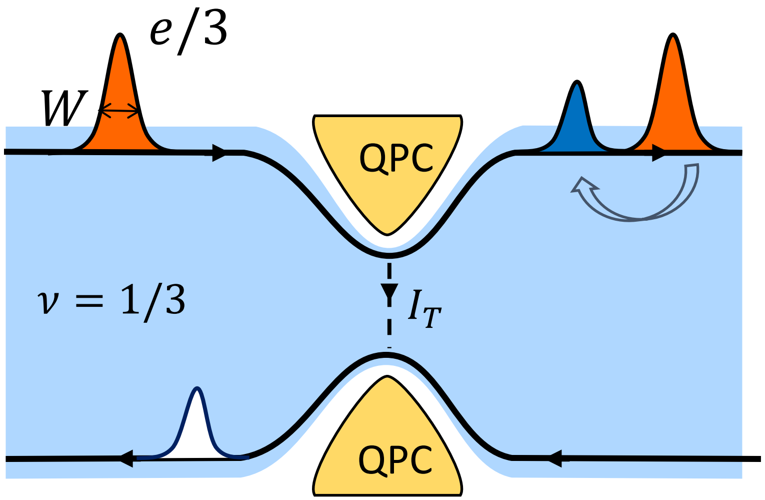
$$I_T = q(\Gamma_+ - \Gamma_-)$$

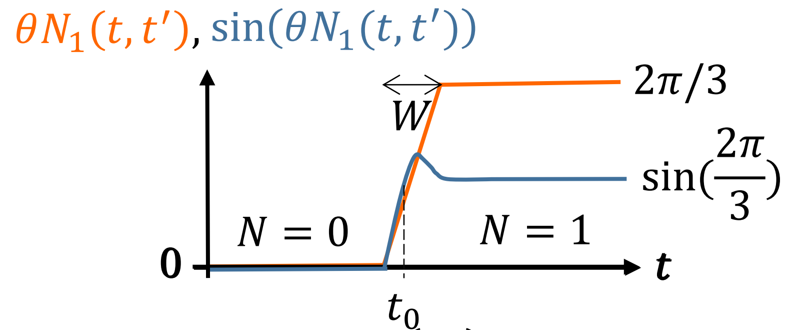
$$I_T(t) \propto \int_{-\infty}^{t} dt' \sin(\theta\, N_1\,(t,t'))\, Im[G_\delta(t-t')^2]$$

time-dependent
braiding phase

Anyon scattering at a QPC: time-domain, $\nu = 1/3, N = 1$

T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)





Long-time memory $\sin(\ \theta\ )$ is erased on time τ_δ

$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta\, N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

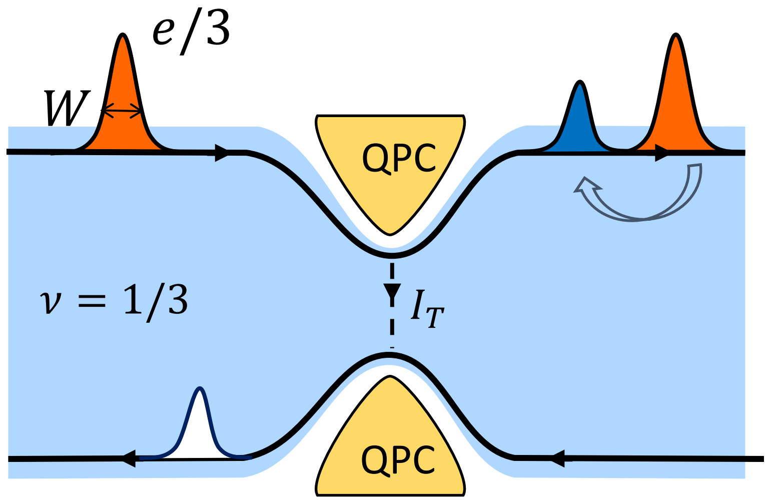
$$I_T = q(\Gamma_+ - \Gamma_-)$$

$$I_T(t) \propto \int_{-\infty}^{t} dt' \sin(\theta\, N_1\,(t,t'))\, Im[G_\delta(t-t')^2]$$

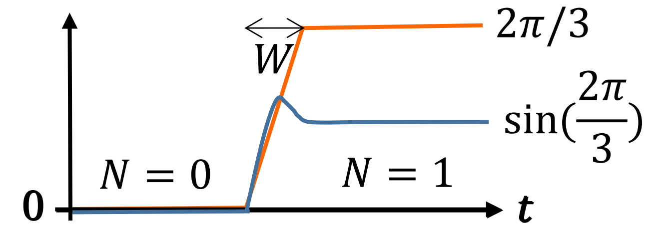
time-dependent
braiding phase

Anyon scattering at a QPC: time-domain, $\nu = 1/3, N = 1$

T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)



$\theta N_1(t,t')$, $\sin(\theta N_1(t,t'))$



Long-time memory $\sin(\theta)$ is erased on time τ_δ

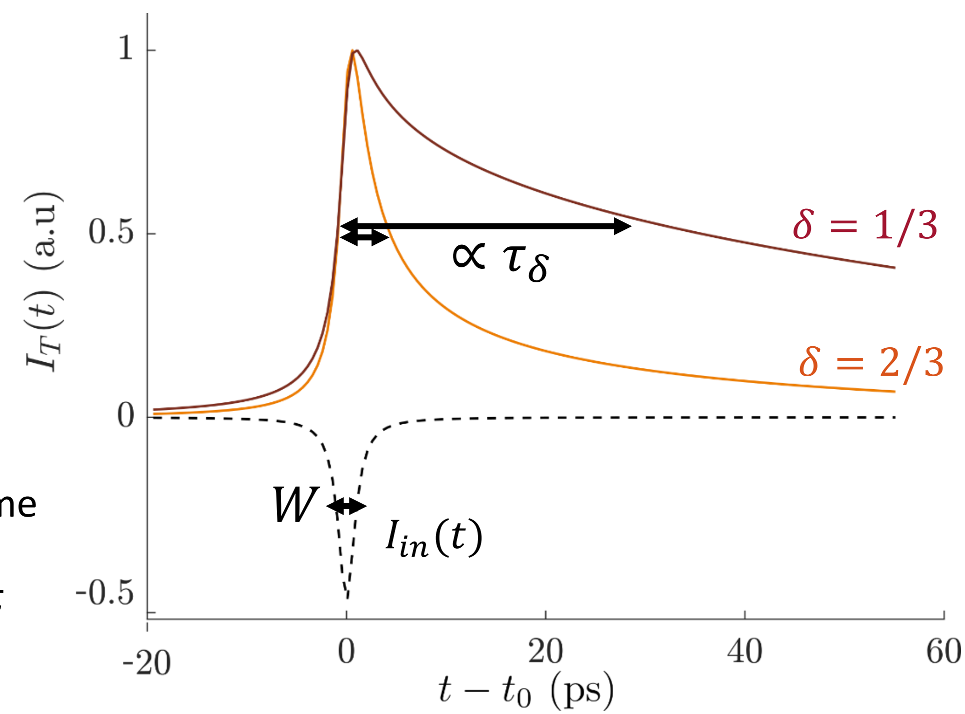
$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta\, N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

$$I_T = q(\Gamma_+ - \Gamma_-)$$

$$I_T(t) \propto \int_{-\infty}^{t} dt' \sin(\theta\, N_1\,(t,t'))\, Im[G_\delta(t-t')^2]$$

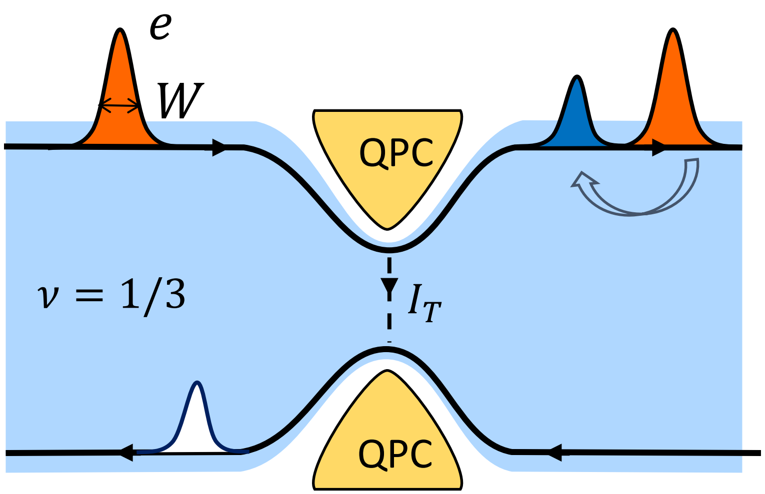
time-dependent braiding phase

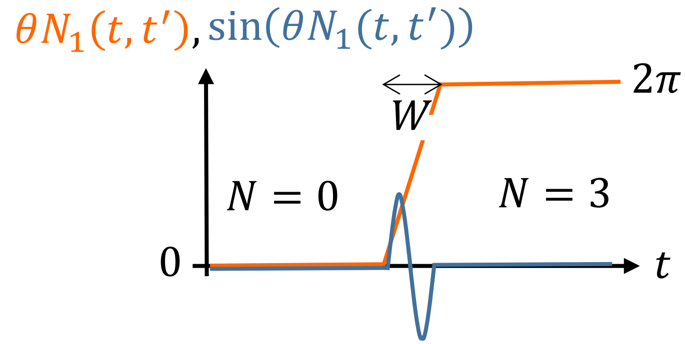
Memory or correlation time decays on $\tau_\delta = \dfrac{\hbar}{k_B T_{el} \pi \delta}$



Anyon scattering at a QPC: time-domain, $\nu = 1/3, N = 3$

T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)





No long-time memory of sin(θN)

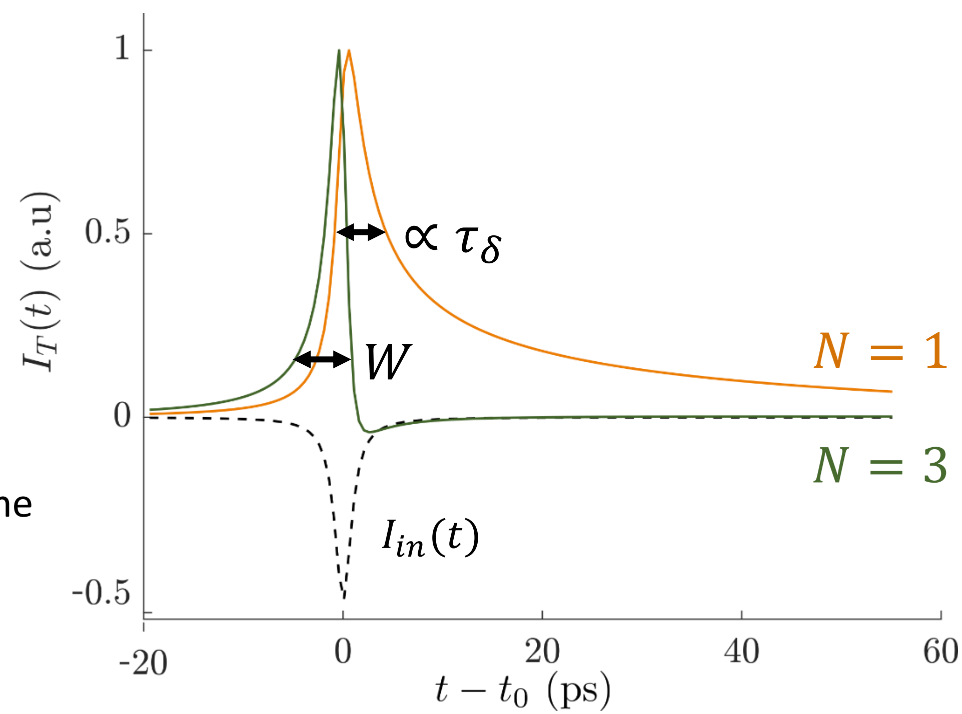
$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta\, N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

$$I_T = q(\Gamma_+ - \Gamma_-)$$

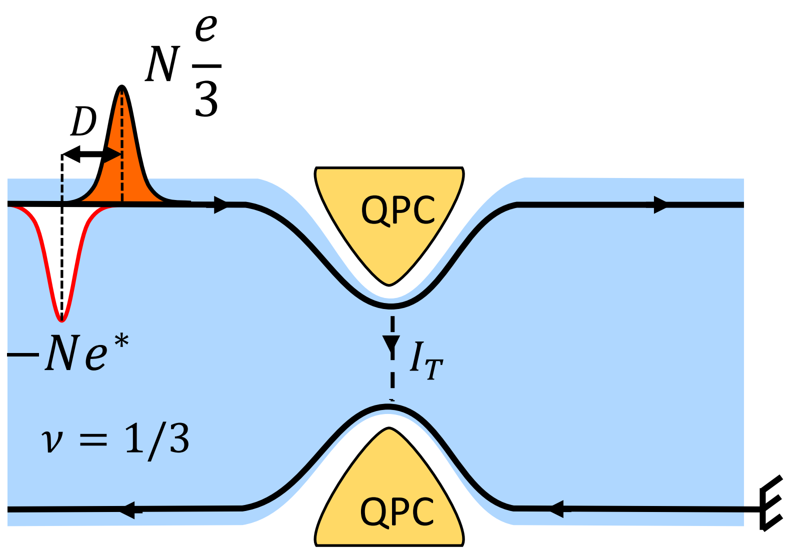
$$I_T(t) \propto \int_{-\infty}^{t} dt' \sin(\theta\, N_1\,(t,t'))\, Im[G_\delta(t-t')^2]$$

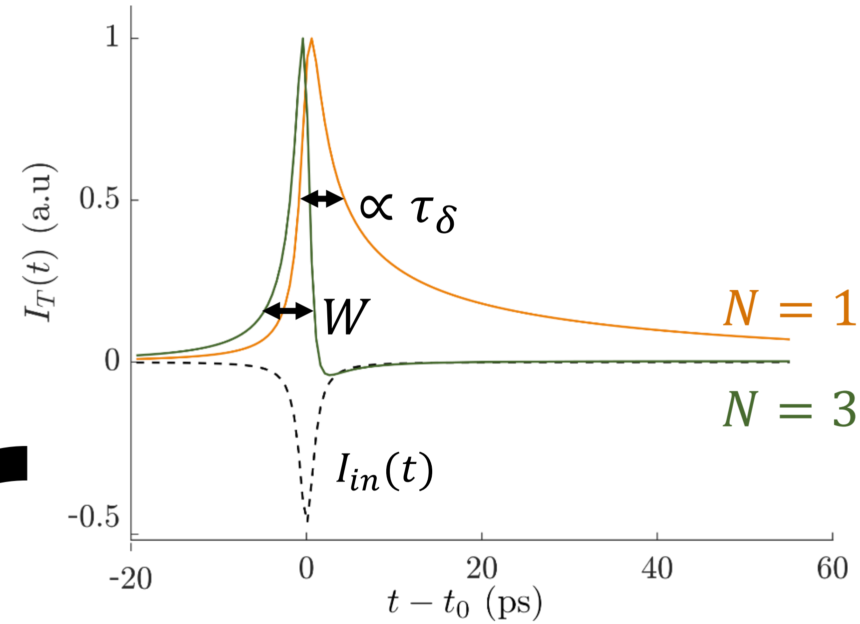
time-dependent braiding phase

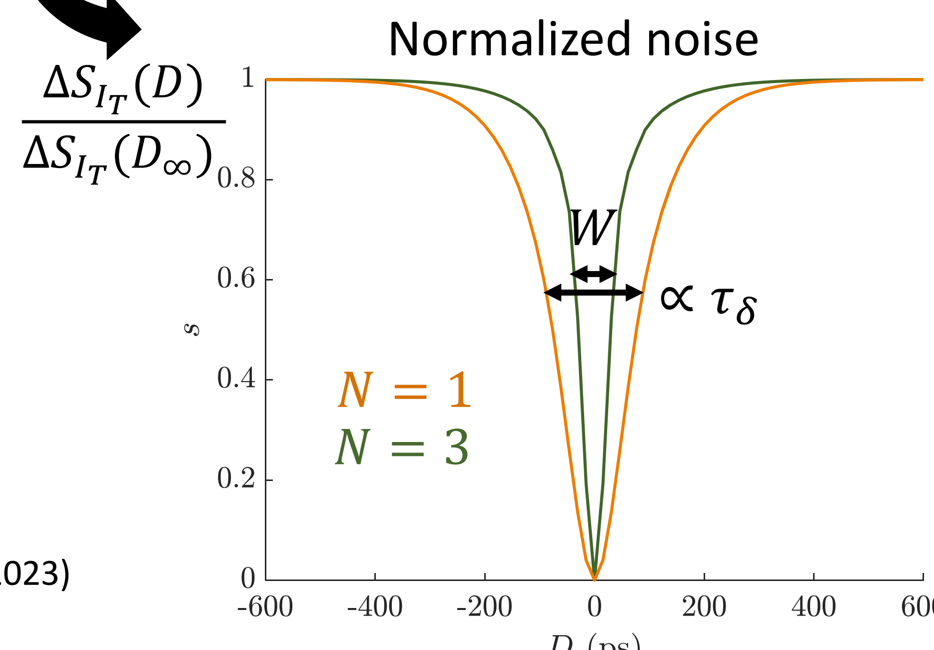
Memory or correlation time decays on $\tau_\delta = \dfrac{\hbar}{k_B T_{el} \pi \delta}$



HOM experiment between anyons pulses at $\nu = 1/3$



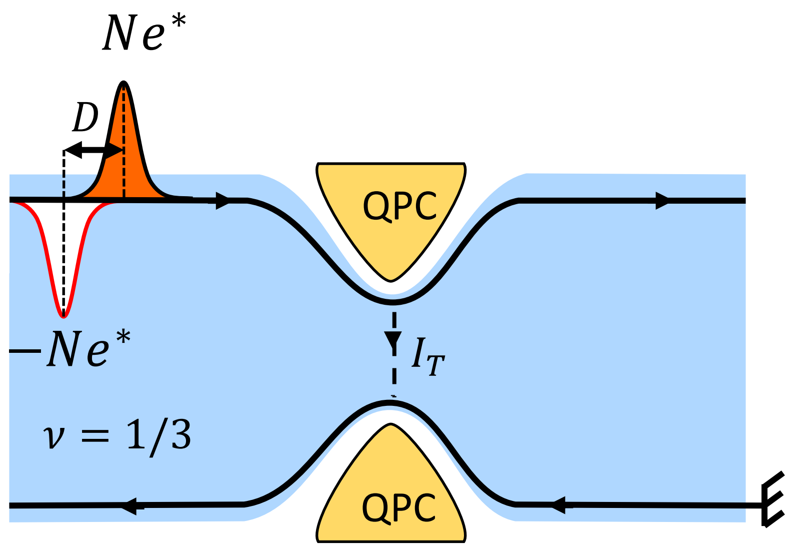




Tunable D, time delay
D = 0, zero-waveform
D>>W, two seperate pulses

T. Jonckheere, J. Rech, B. Grémaud and T. Martin, PRL **130**, 186203 (2023)
M. Ruelle et al., Science389,eadm7695(2025)

Sample and setup



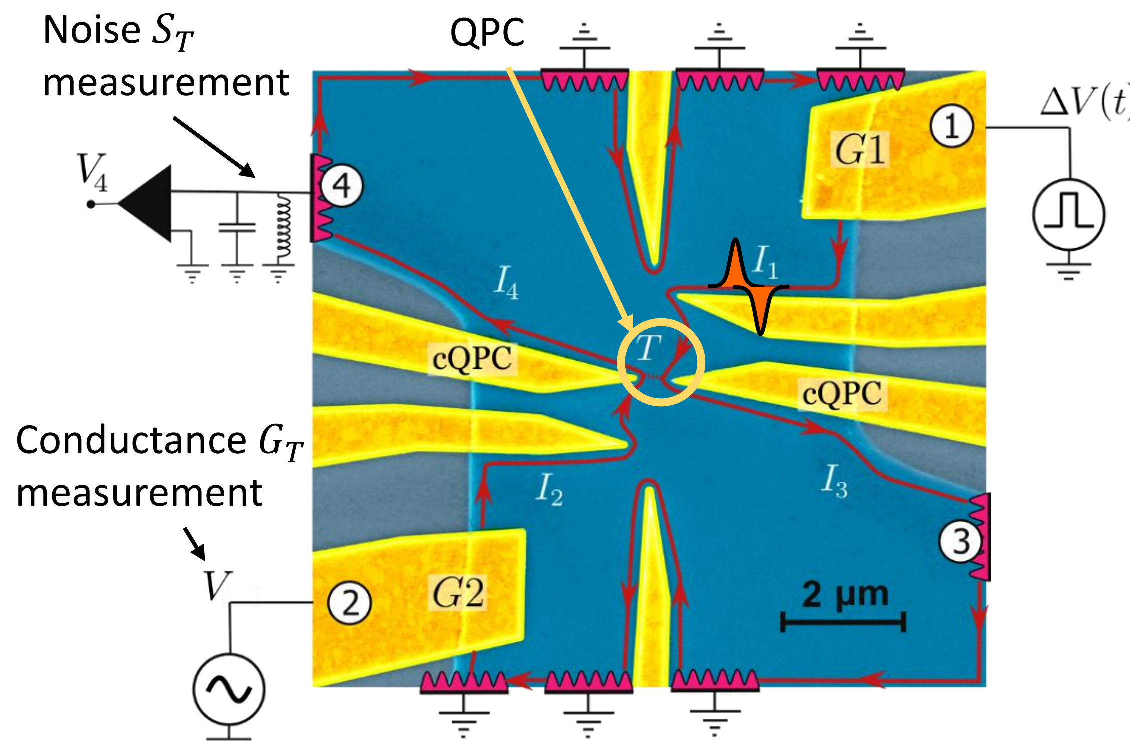
Measured quantities:

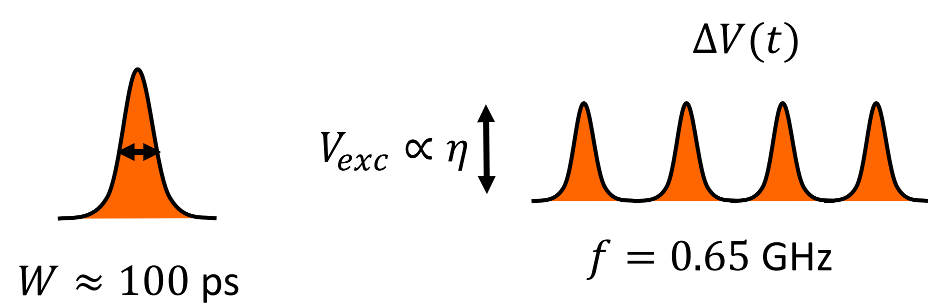
HOM noise and differential conductance

$$\boxed{\begin{aligned} S_T(D) &= 2(e^*)^2(\overline{\Gamma_+} + \overline{\Gamma_-}) \\ G_T(D) &= 2e^*(\overline{\Gamma_+} - \overline{\Gamma_-})/V \end{aligned}}$$

$$\Delta S_T(D) \propto \int_0^{+\infty} d\tau \left[\cos\left(\theta \Delta N(D)\right) - 1\right] Re\left(G_\delta(\tau)^2\right)$$

$$\Delta G_T(D) \propto \int_0^{+\infty} d\tau \left[\cos\left(\theta \Delta N(D)\right) - 1\right] Im\left(\tau G_\delta(\tau)^2\right)$$

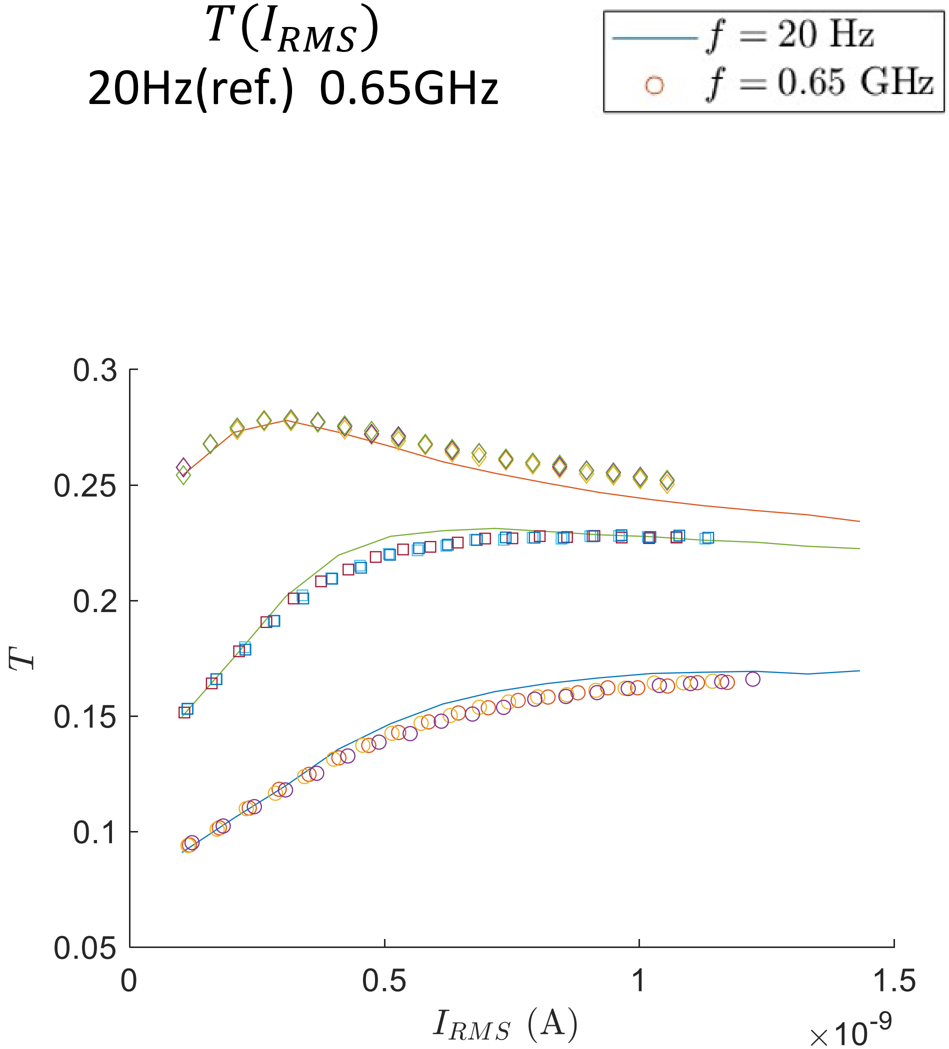




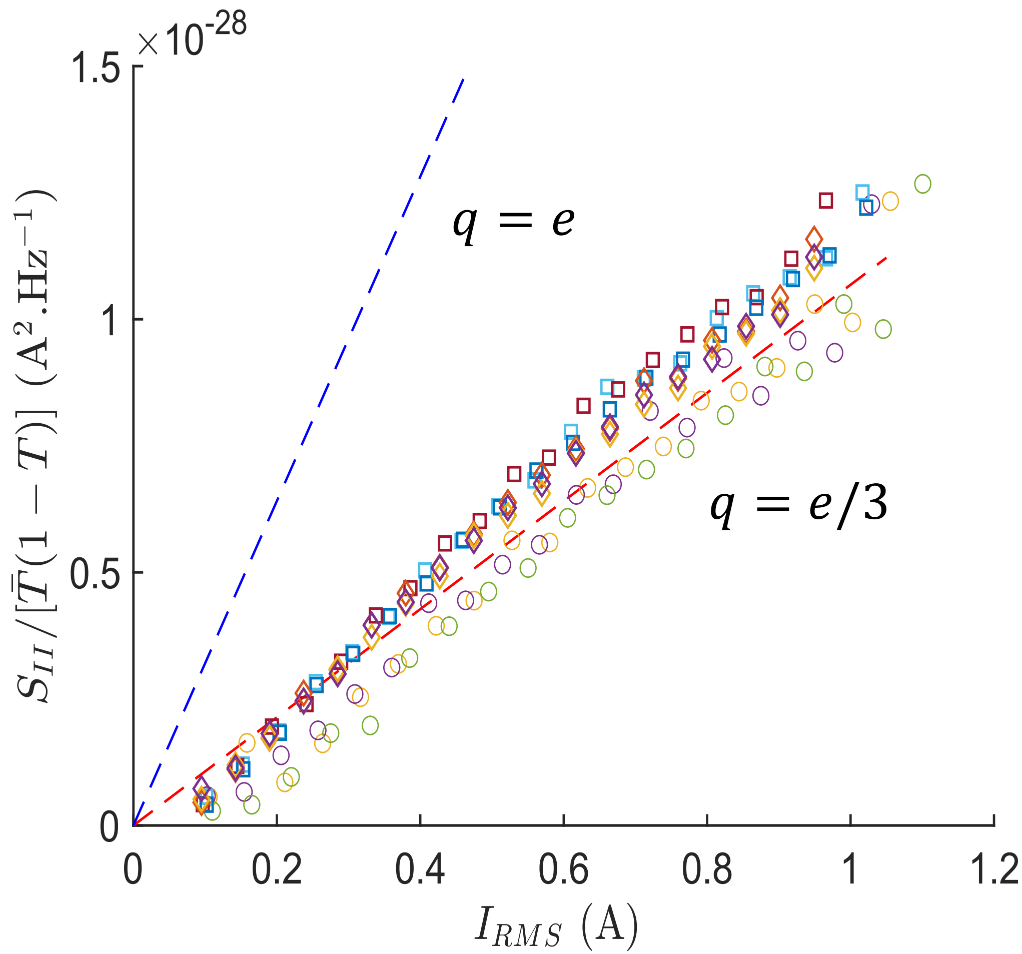
Calibration of the capacitance

$T(I_{RMS})$

20Hz(ref.) 0.65GHz

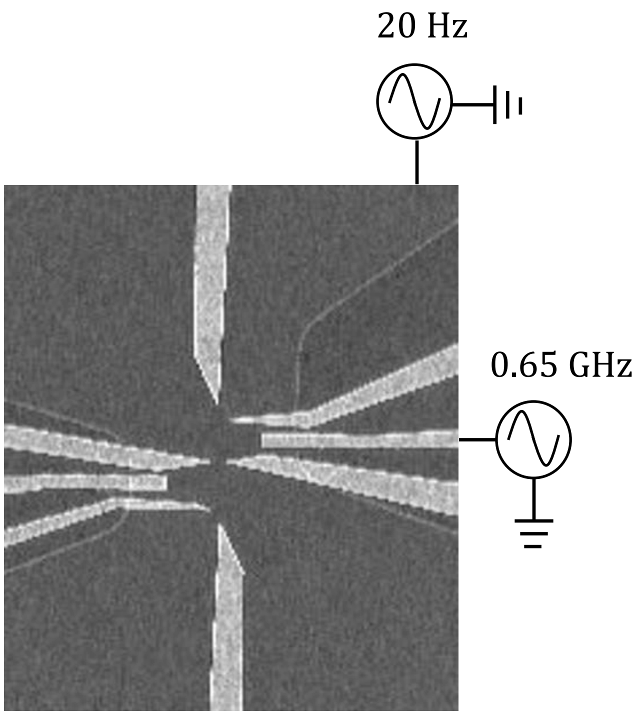


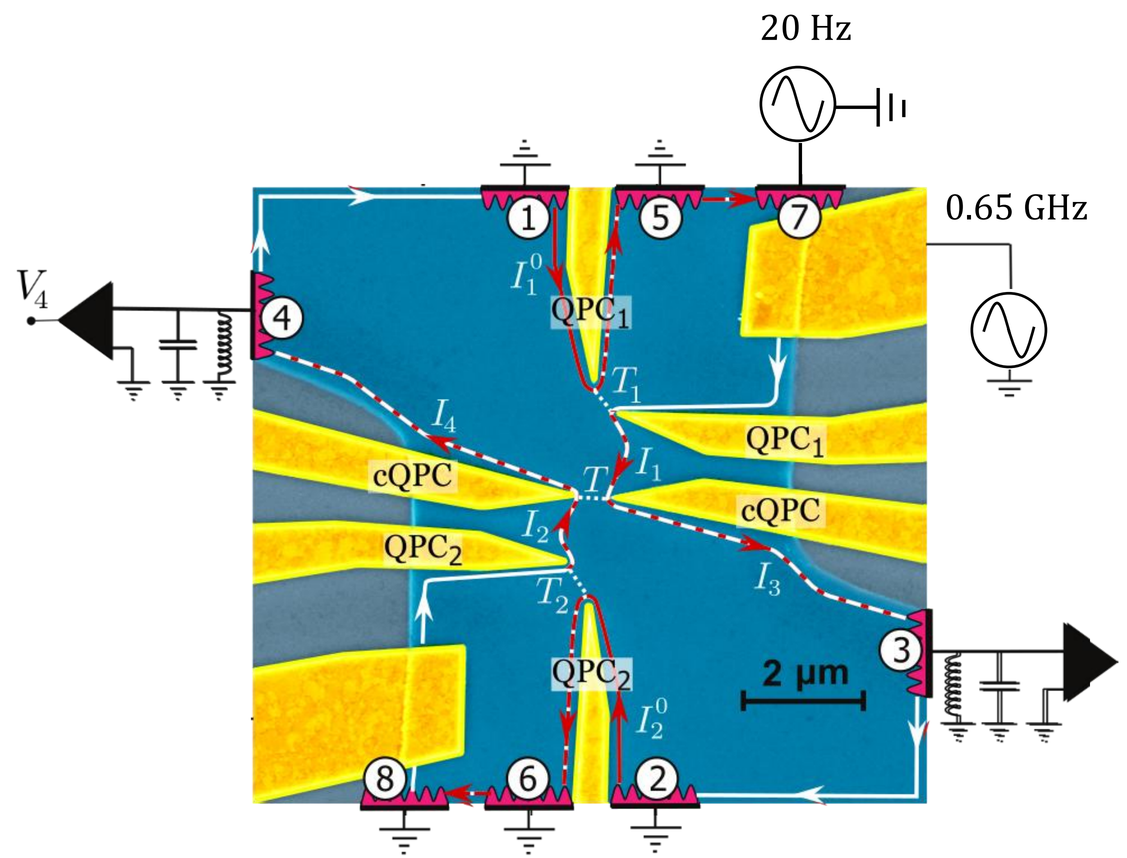
$C_{eff} = 0.5\ fF$



$S_{II} = 2qI_{B,AC}(1-T)$

HOM experiment with anyon pulses

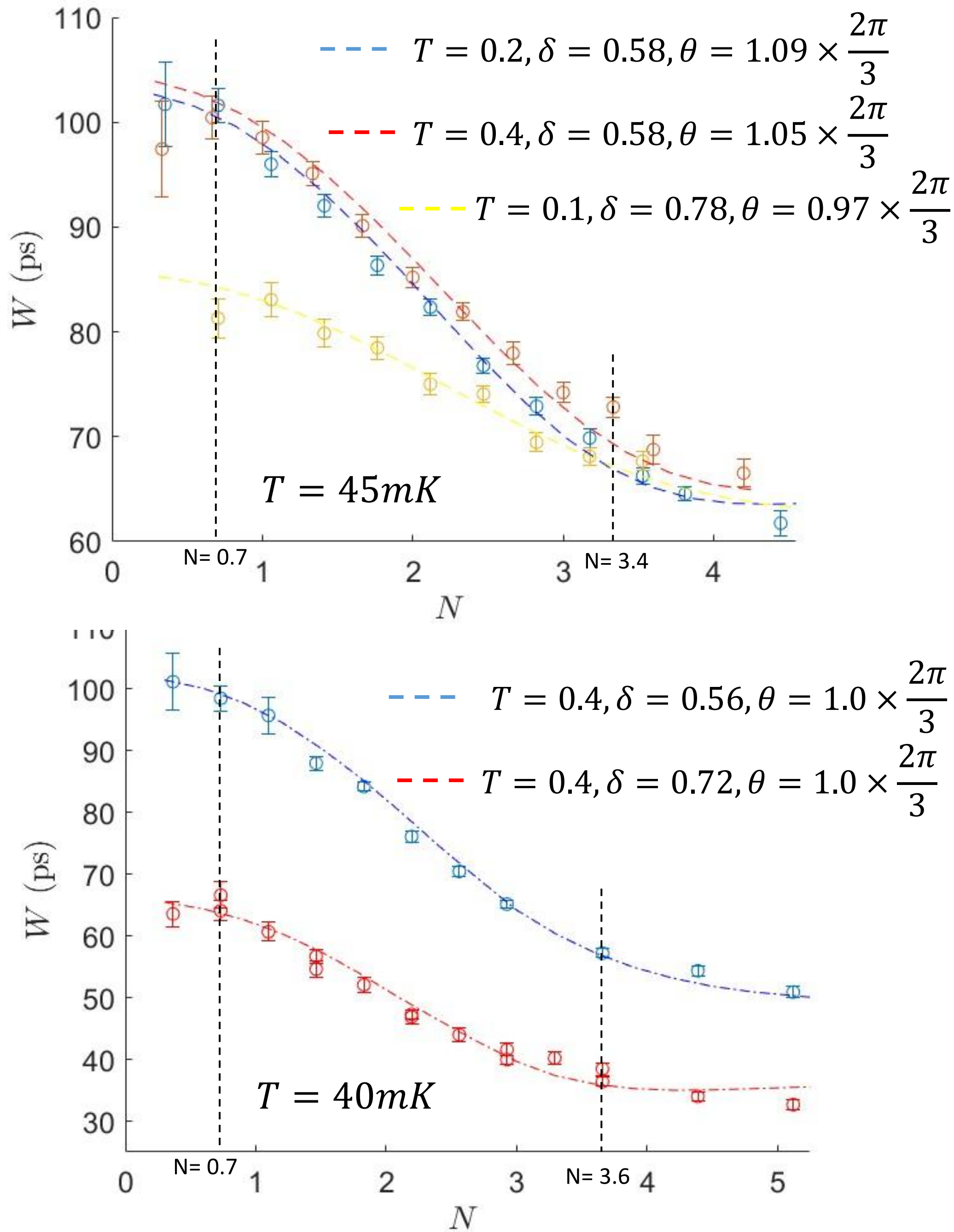


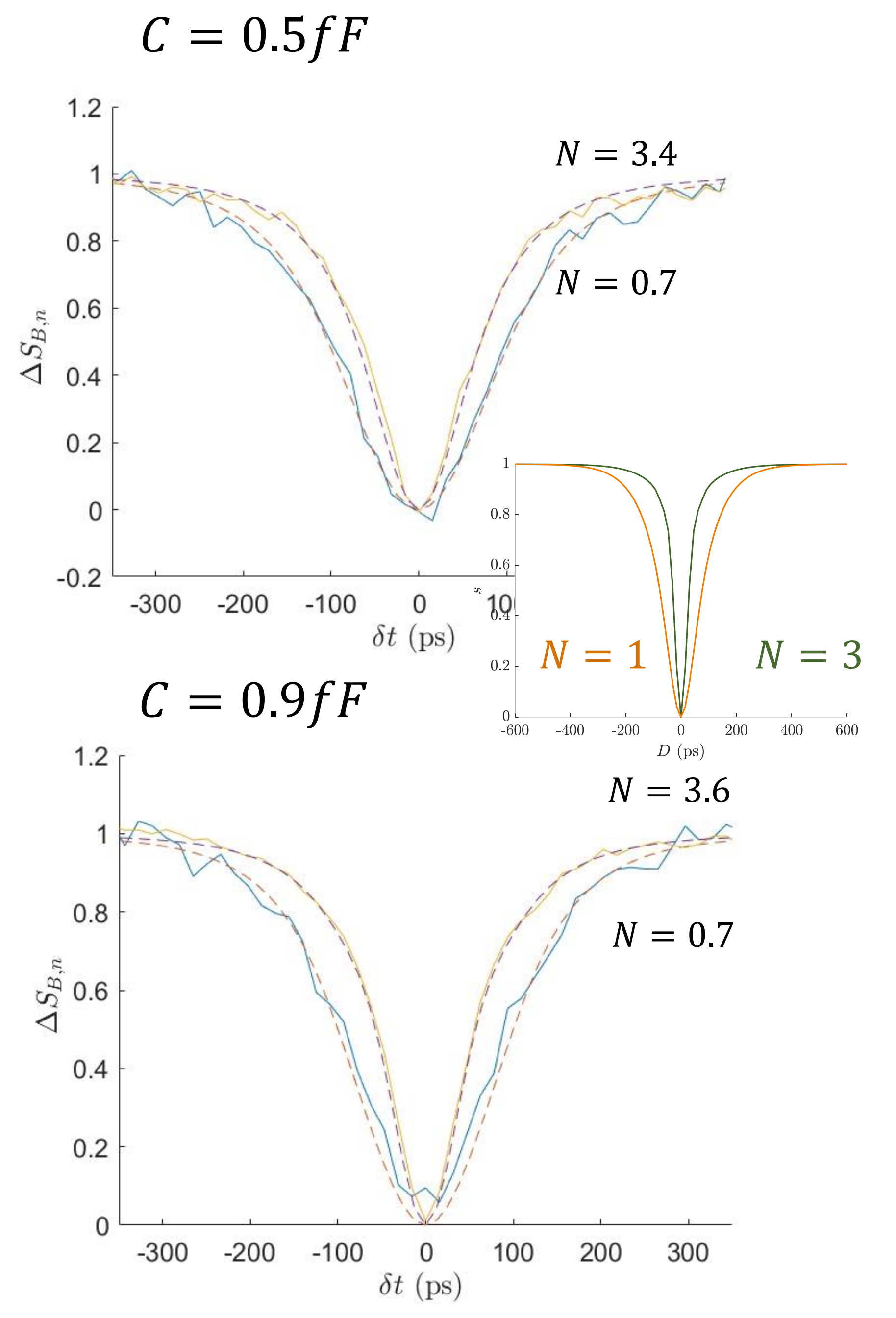


$C \approx 0.5\ fF$

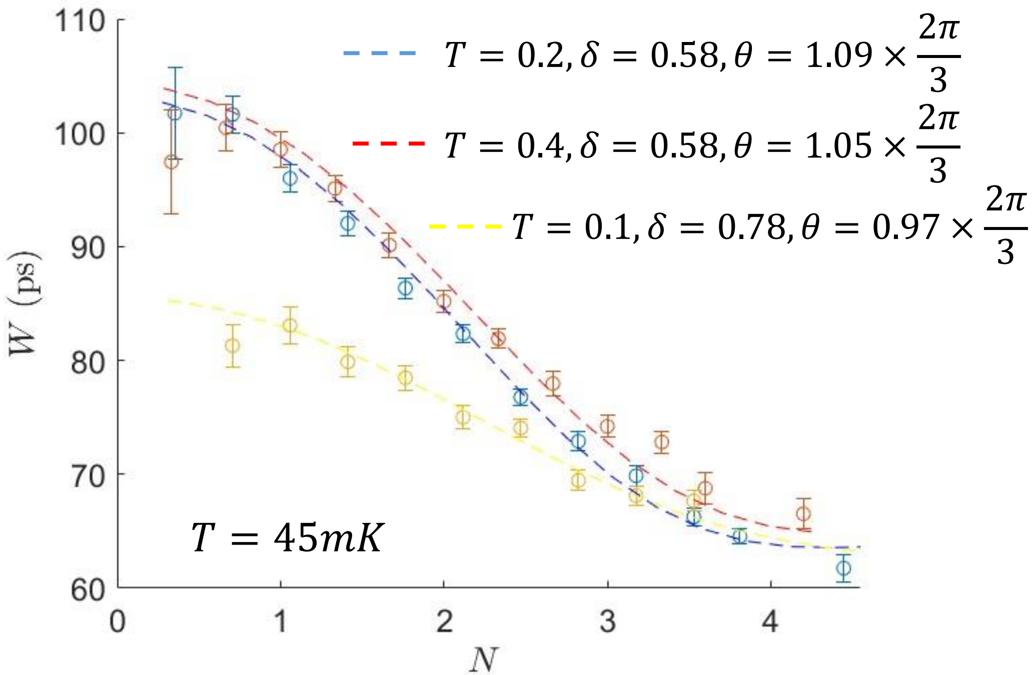
$C \approx 0.9\ fF$

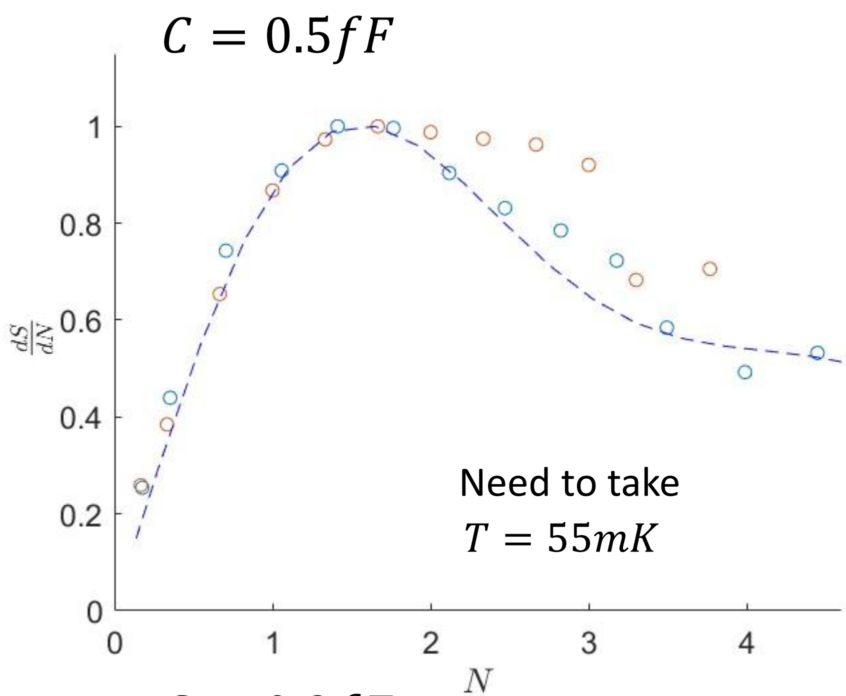
HOM experiment with anyon pulses

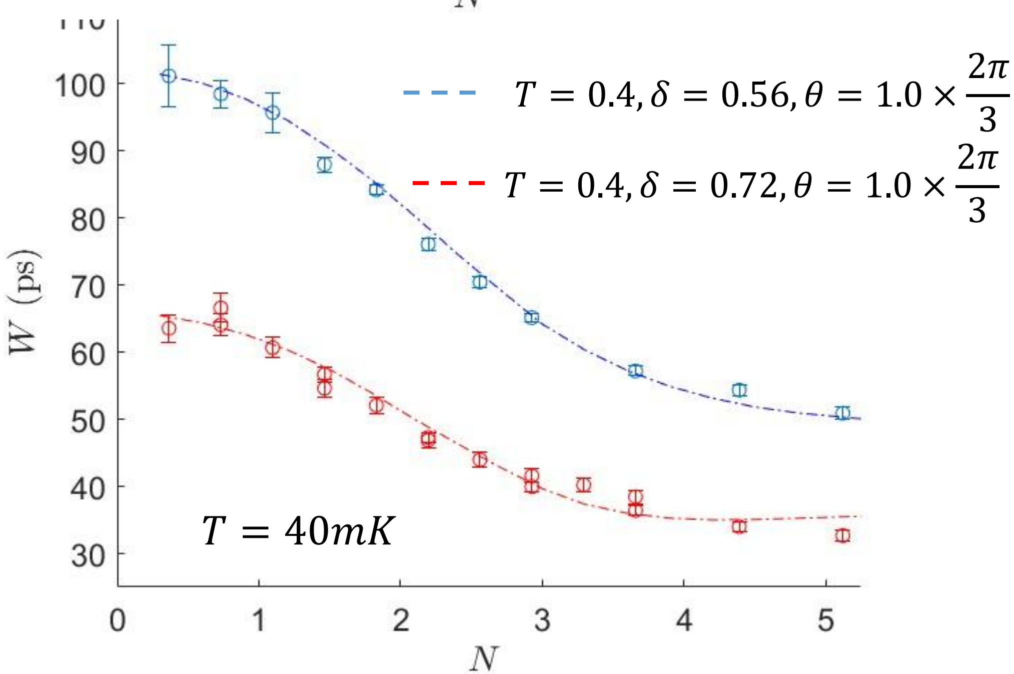


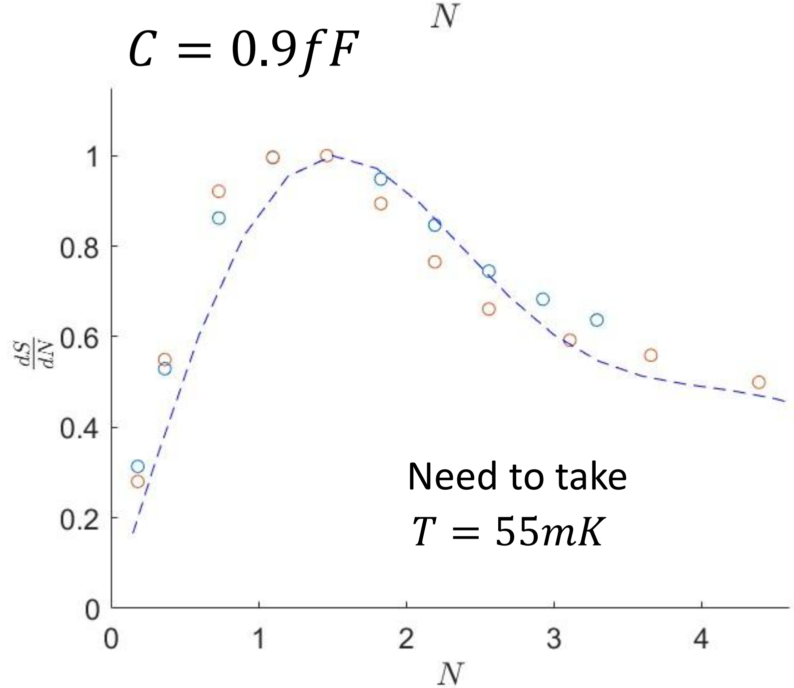


HOM experiment with anyon pulses

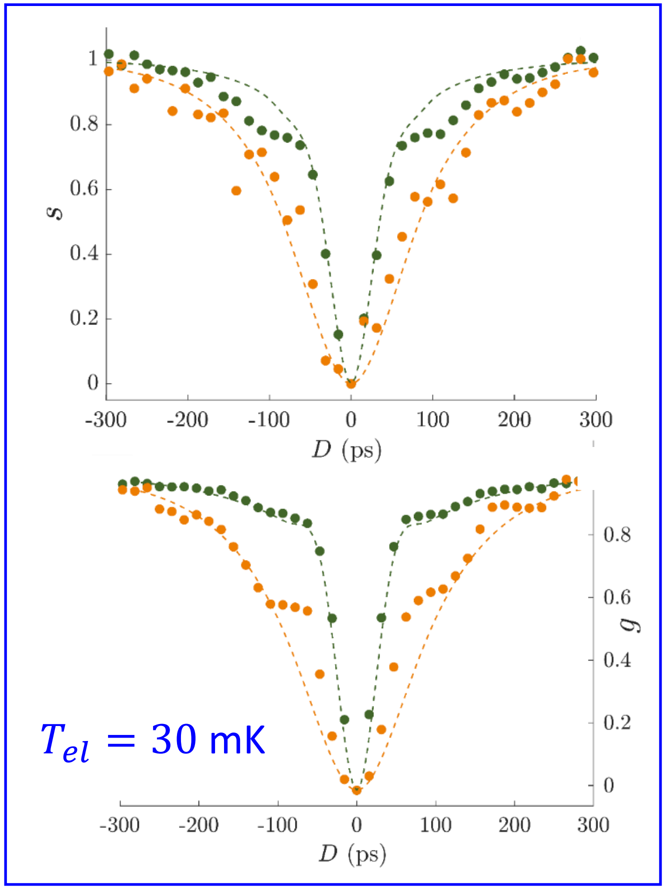


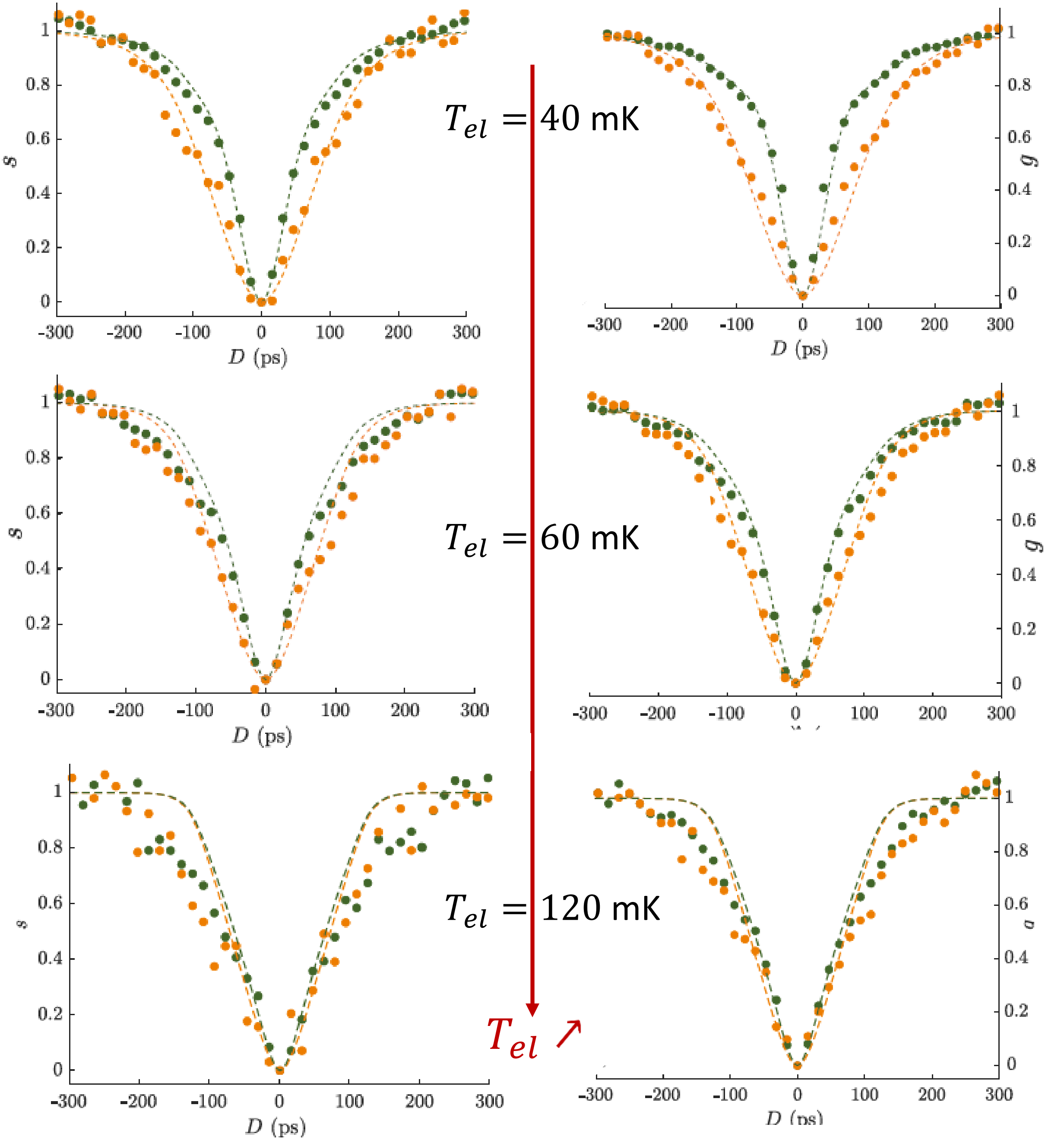






Observation of braiding in the time-domain





Influence of electronic temperature T_{el}:

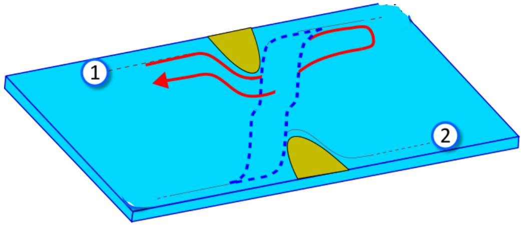
$$\tau_\delta = \frac{\hbar}{k_B T_{el} \pi \delta}$$

Effects of braiding vanish when increasing T_{el}

M. Ruelle et al., Science389,eadm7695(2025)

Conclusion

Two-particle interferometry reveals anyon braiding at a QPC:



θ_{rb}: mutual braiding phase between blue and red particles

Anyon collider experiments: $\nu = 1/3$ $\theta_{rb} = 2\pi/3$

Anyon pulses: $\nu = 1/3$ $\theta_{rb} = N \times 2\pi/3$

Braiding modifies tunneling dynamics

M. Ruelle et al.,Science389,eadm7695(2025)

T. Jonckheere et al., PRL **130**, 186203 (2023)

Anyon physics in the integer regime: charge fractionalization $\theta_{rb} = \frac{q}{e} \times 2\pi$

P. Glidic et al., Nature Communications **15**, 6578 (2024)

Fabry-Perot interferometry:

J. Nakamura et al., Nature Physics **16** 931 (2020).

J. Kim et al., Nat. Nanotechnol. 19, 1619–1626 (2024)

T. Werkmeister et al., Science388,730-735(2025)

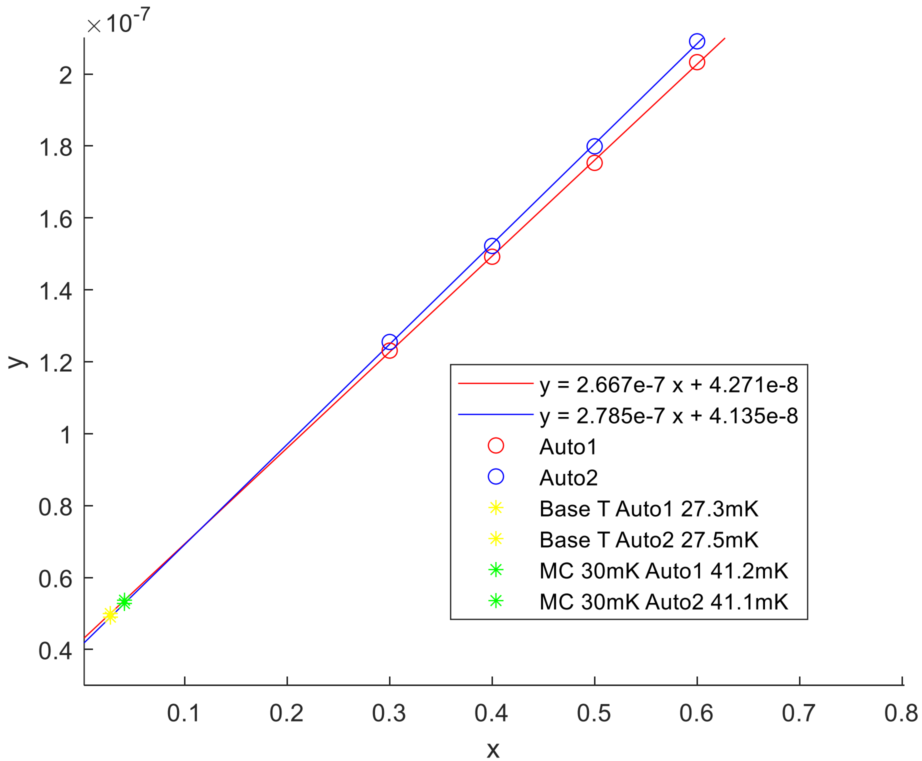
N.L. Samuelsson et al., Phys. Rev. X 16, 011062(2025)

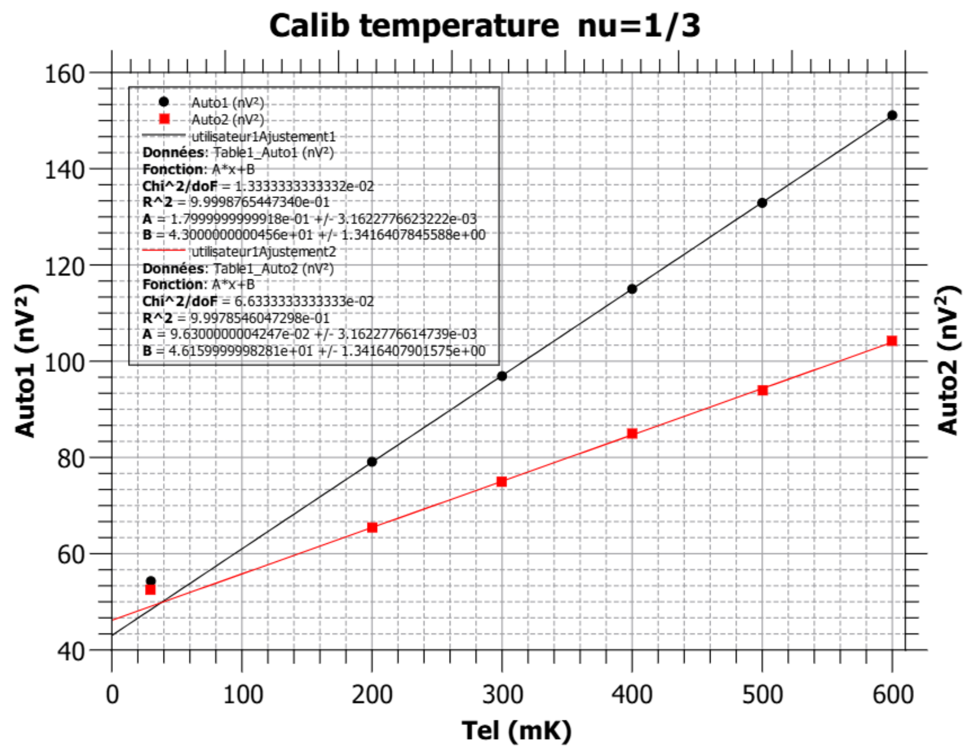
Scaling dimension: A. Veillon et al., Nature **632**, 517–521 (2024). $\delta = 1/3$

Recent work: R. Guerrero-Suarez et al., Nat. Phys. 21, 1787–1793 (2025) …

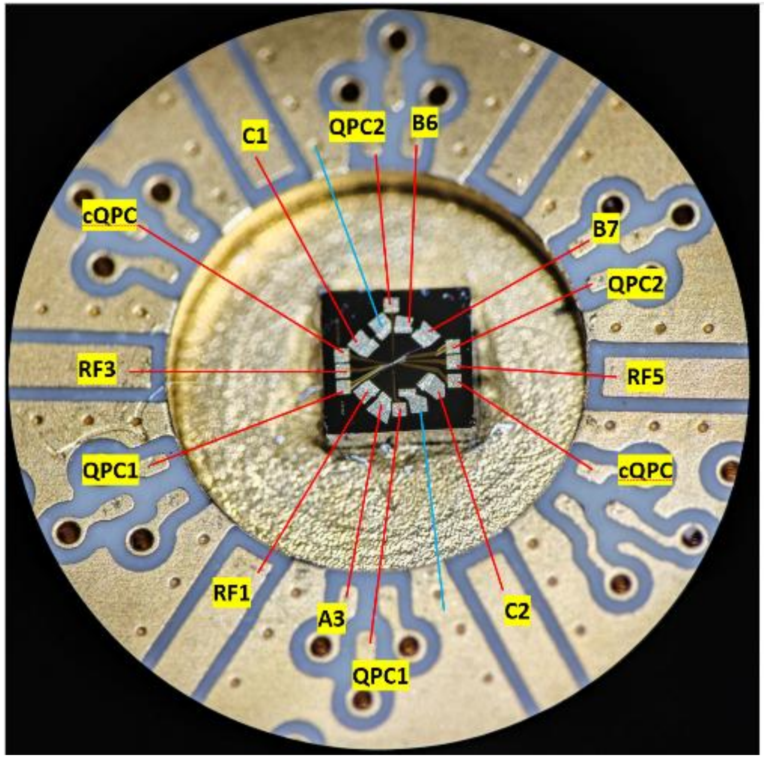
Temperature Calibration

$$
\begin{aligned}
< \delta V_{ii}^2 > &= 4k_b T G_i^2 \int d\omega \Re e(Z) \\
&= 4k_b T G_i^2 \int d\omega \frac{|Z|^2}{R} \\
&= 4k_b \frac{1}{R} \gamma_i T
\end{aligned}
$$

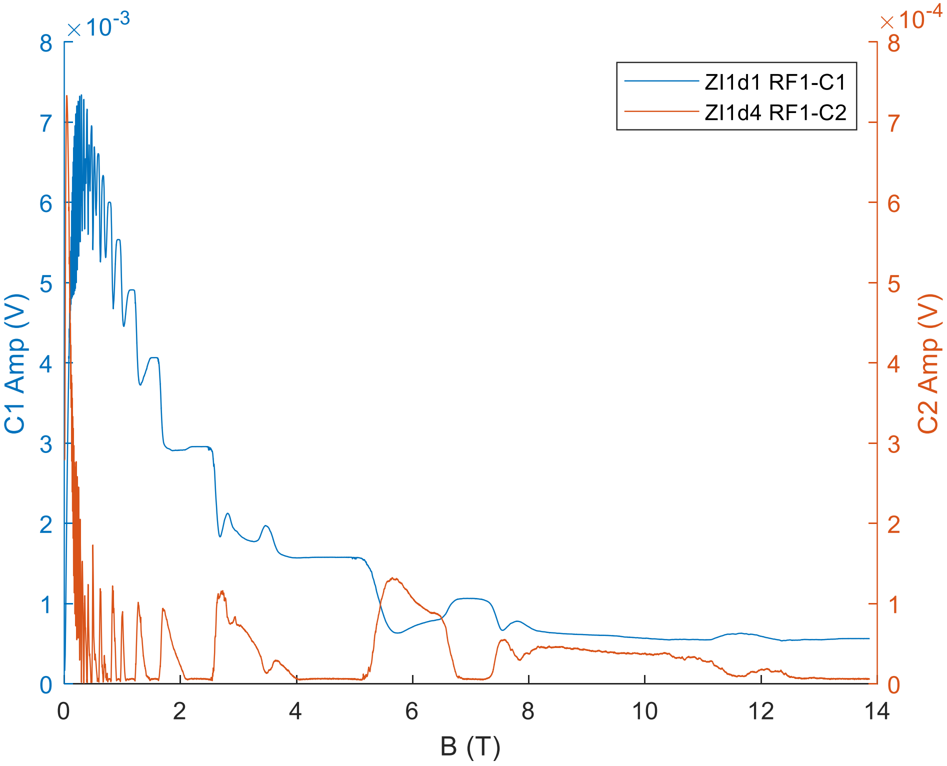




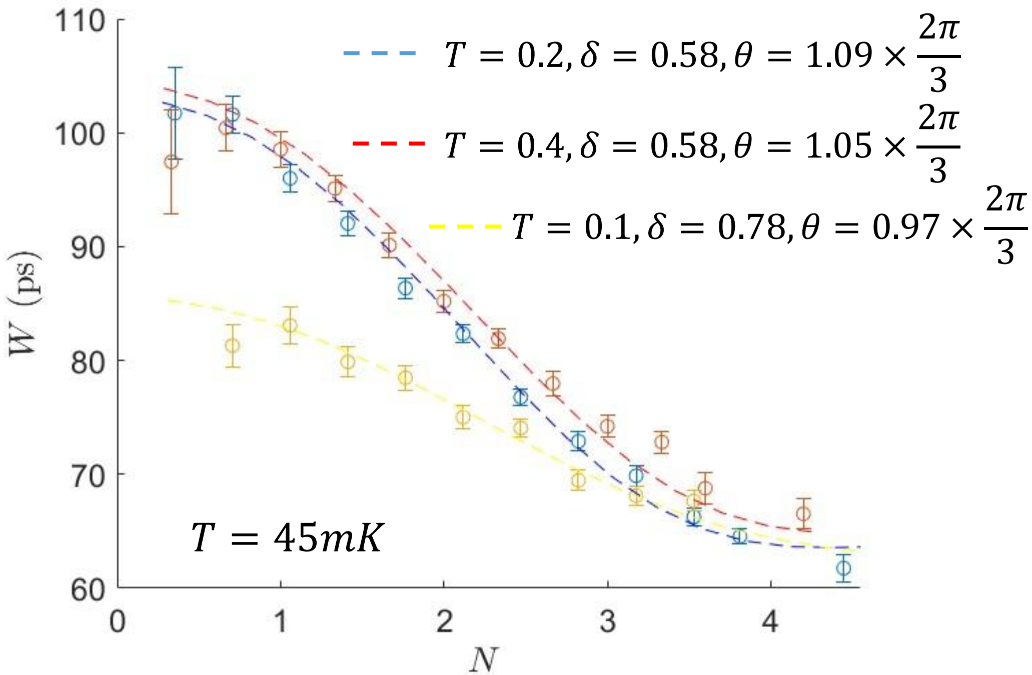
Sample



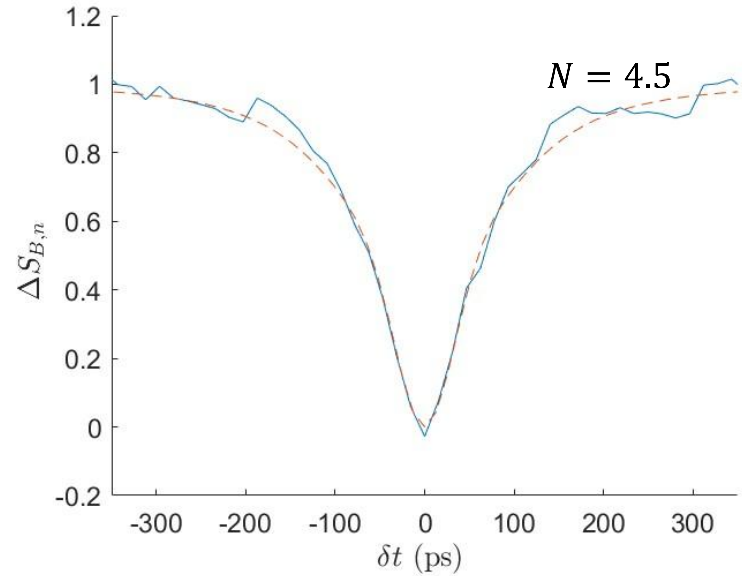
17MA03
AnyonCollider2
20251114

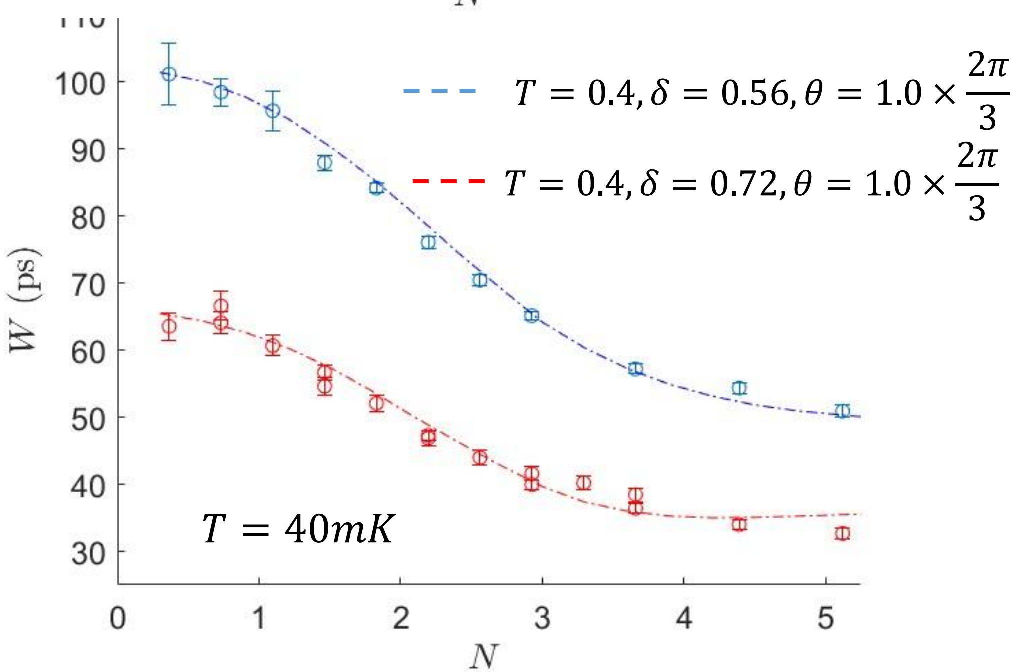


HOM experiment with anyon pulses

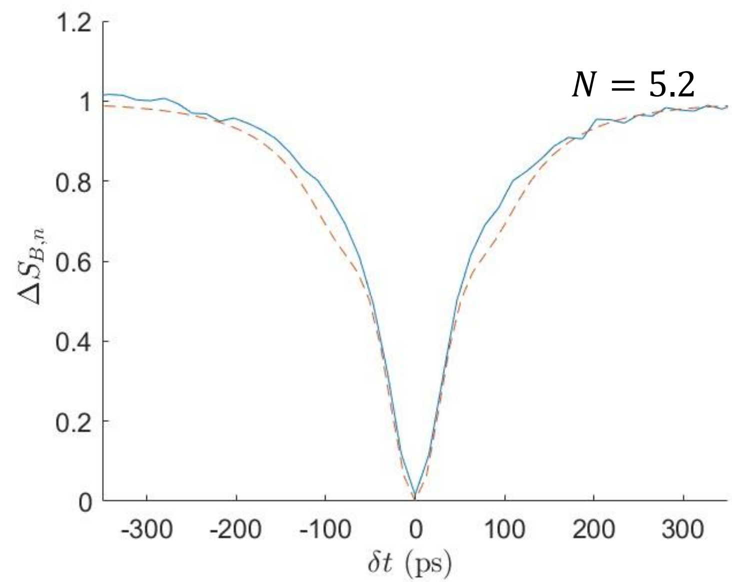


$C = 0.5fF$





$C = 0.9fF$



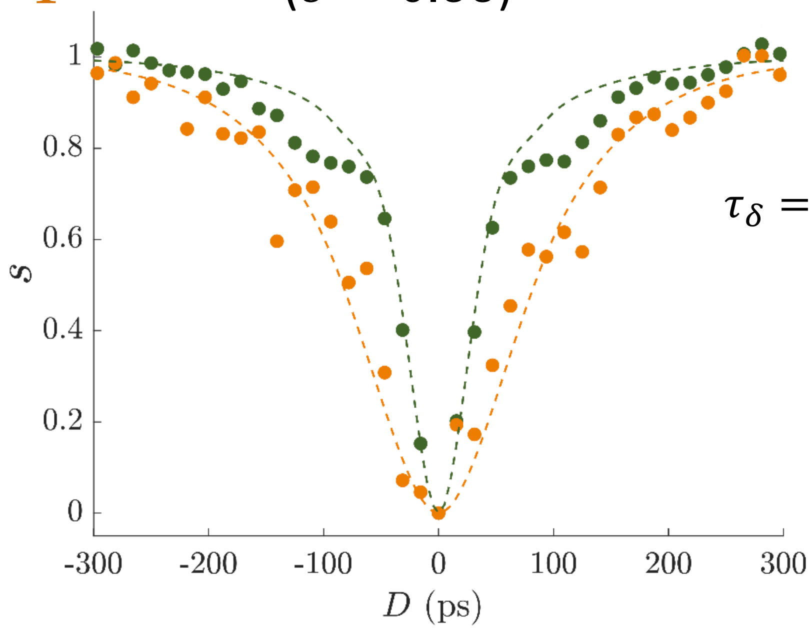
Observation of braiding in the time-domain

$N = 3$

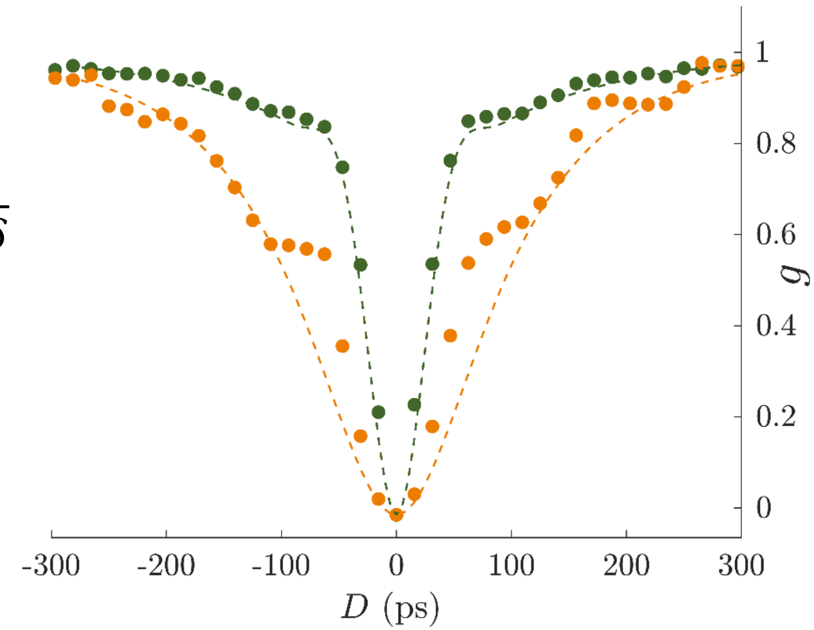
$N = 1$

simulations ($\delta = 0.66$)

$T_{el} = 30$ mK



$$\tau_\delta = \frac{\hbar}{k_B T_{el} \pi \delta}$$



$$S_T(D) = 2(e^*)^2(\overline{\Gamma_+} + \overline{\Gamma_-})$$

$$G_T(D) = 2e^*(\overline{\Gamma_+} - \overline{\Gamma_-})/V$$

normalized excess noise and conductance $s(D)$, $g(D)$

$$s(D) = \frac{S(D) - S(D=0)}{S(D=\frac{T}{2}) - S(D=0)}$$

Widening of the HOM dip when going from an electron pulse $N = 3$ to an anyon pulse $N = 1$

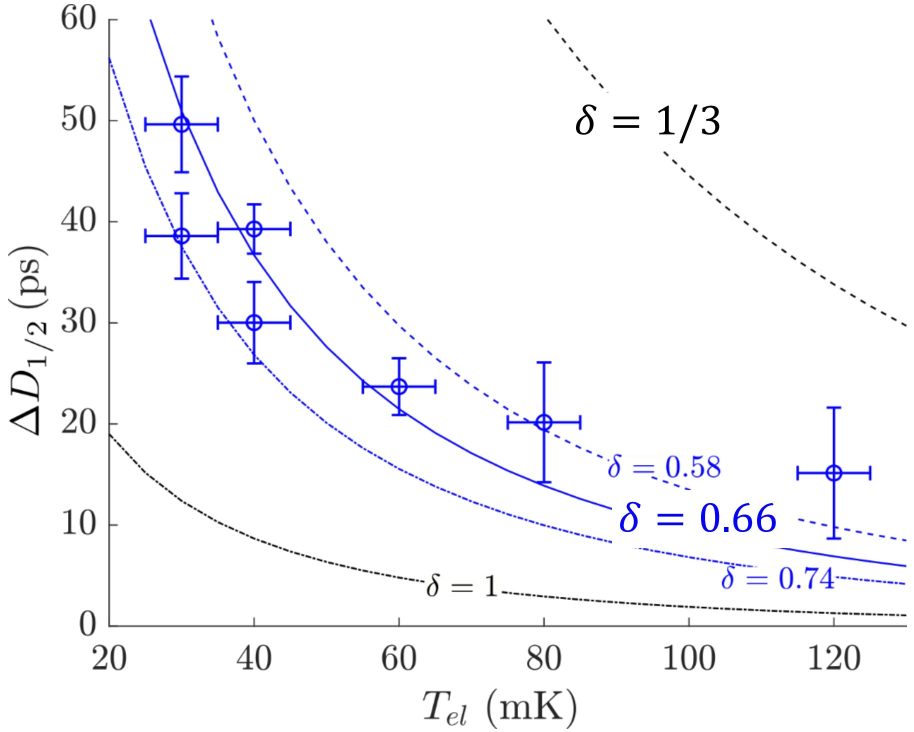
anyon tunneling dynamics is affected by braiding

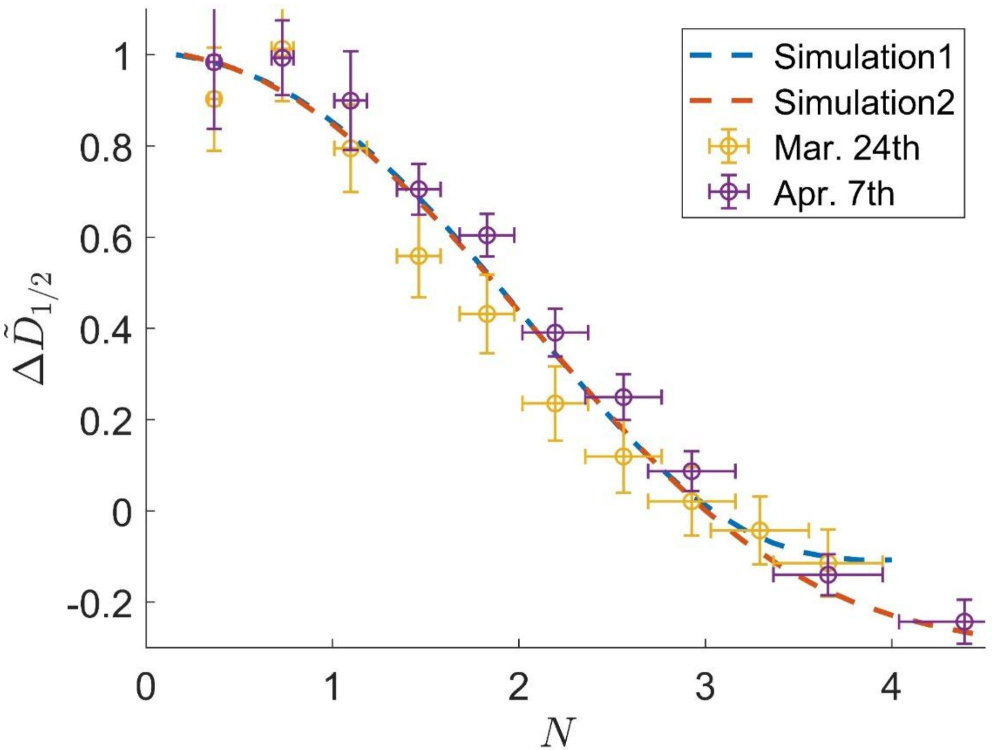
M. Ruelle et al., arXiv:2409.08685

T_{el} dependence and scaling dimension δ

$$\boxed{\tau_\delta = \frac{\hbar}{k_B T_{el} \pi \delta}}$$

$$\boxed{\theta_N = \theta \times N}$$





$\delta = 0.66 \pm 0.08$

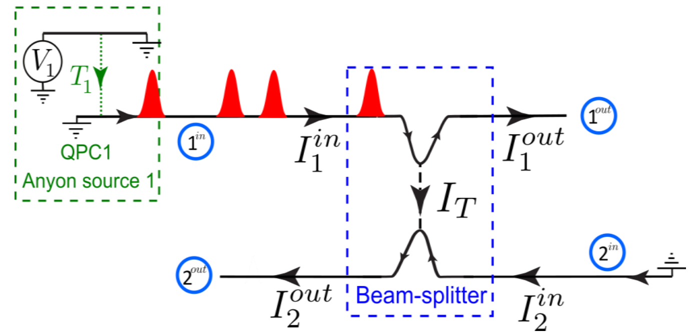
$\theta = 2\pi\delta$?

$q\, N = C\, V_{exc}$

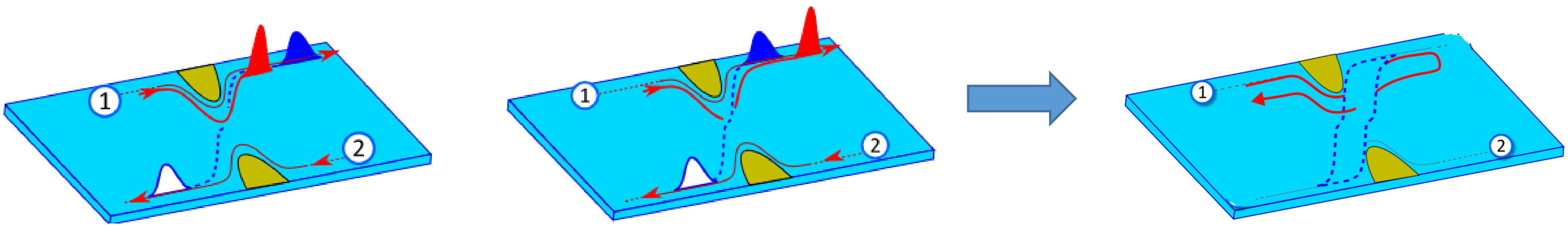
M. Ruelle et al., arXiv:2409.08685

Anyon tunneling at a QPC, random anyon source

Source S1: random emission, $N_1(t,t')$ anyons incoming on the QPC between times t' and t



Γ_-:



emitted at $t' < t_0$

emitted at $t > t_0$

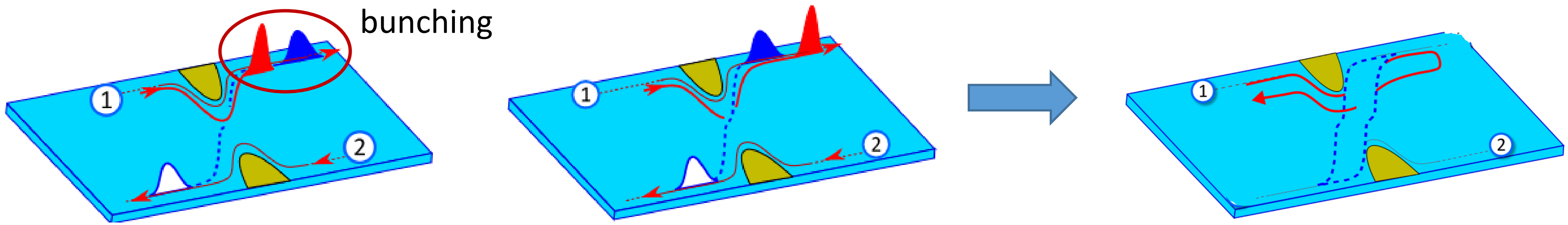
$$\Gamma_\pm(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

N_1 random poissonian variable

Morel et al., PRB **105**, 075433 (2022),
Lee et al., Nat. Commun. **13**, 6660 (2022)
Mora, arXiv:2212.05123 (2022)
Schiller et al., PRL **131** 186601 (2023)

Anyon tunneling at a QPC, random anyon source

Γ_{-}:

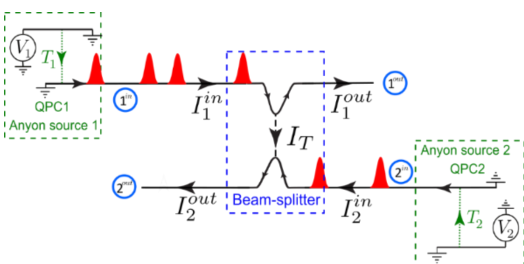


/ emitted at $t' < t_0$ / emitted at $t > t_0$

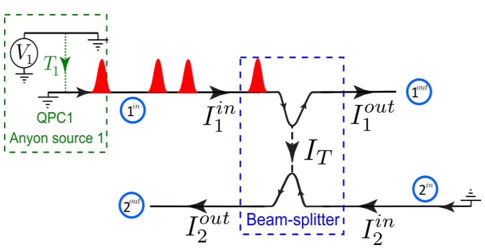
$$\Gamma_{\pm}(t) \propto Re\left[\int_{-\infty}^{t} e^{\pm i\theta N_1(t,t')} G_\delta(t-t')^2 dt'\right]$$

N_1 random poissonian variable

$\Gamma_{-} \neq 0$: bunching mechanism



Negative cross-correlations: $P(\theta = 2\pi/3) = \frac{S_{12}^{out}}{2qTI_+} = -2 < 0$

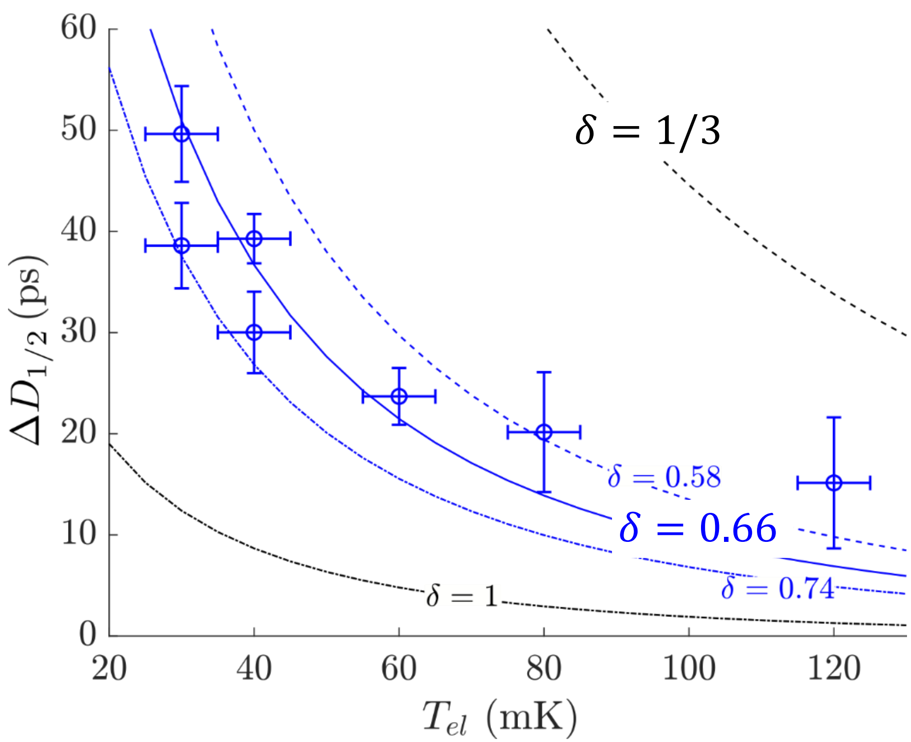


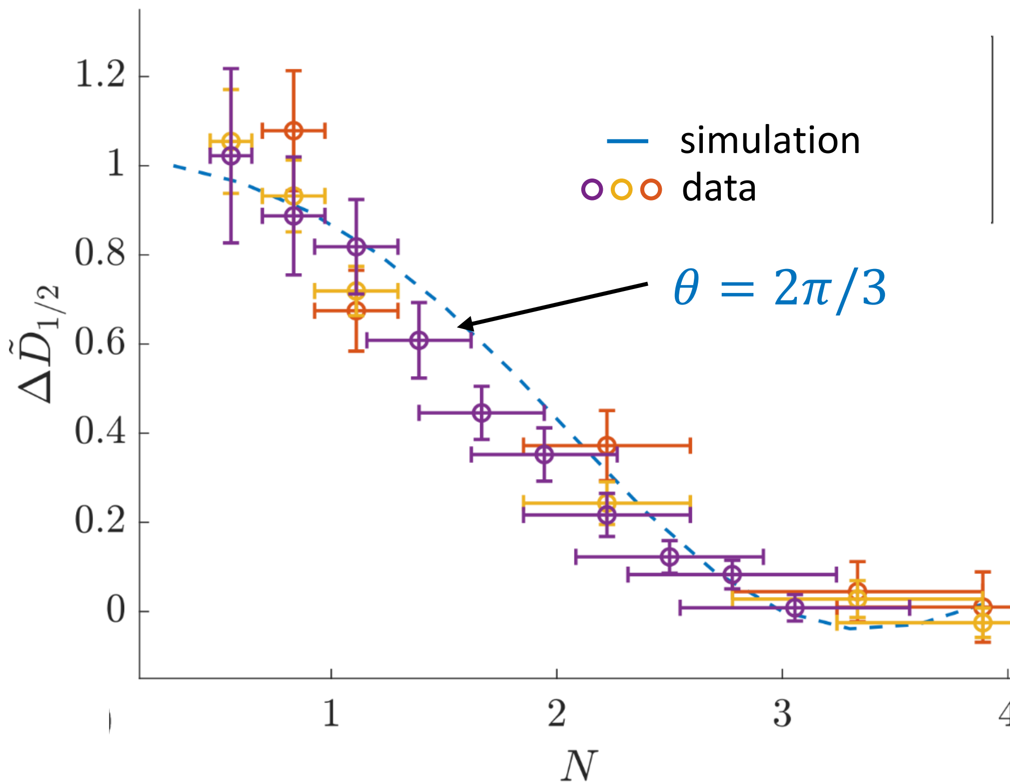
Super-Poissonian auto-correlations $F(\theta = 2\pi/3) = \frac{S_{I_T}}{2qI_T} = 3.27 > 1$

T_{el} dependence and scaling dimension δ

$$\boxed{\tau_\delta = \frac{\hbar}{k_B T_{el} \pi \delta}}$$

$$\boxed{\theta_N = \theta \times N}$$

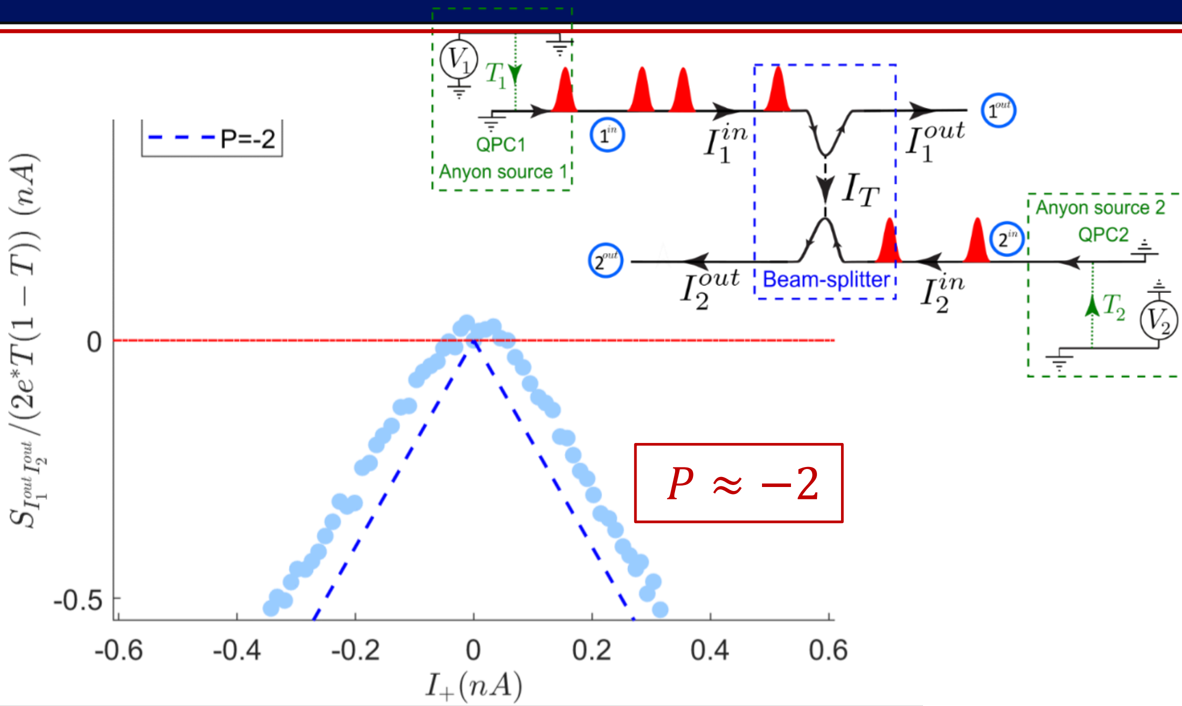




$$\delta = 0.66 \pm 0.08$$

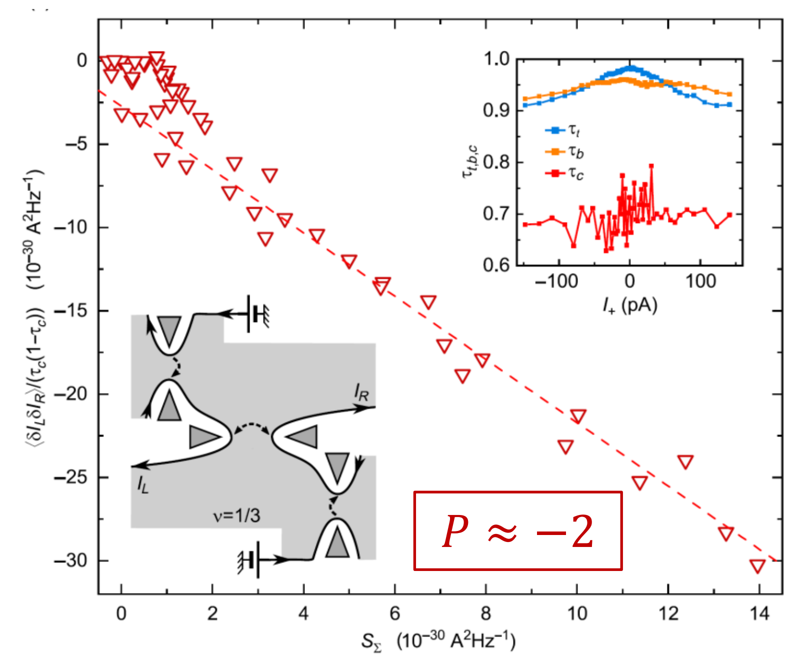
M. Ruelle et al., arXiv:2409.08685

Partitioning of diluted anyon beams, $\nu = 1/3$

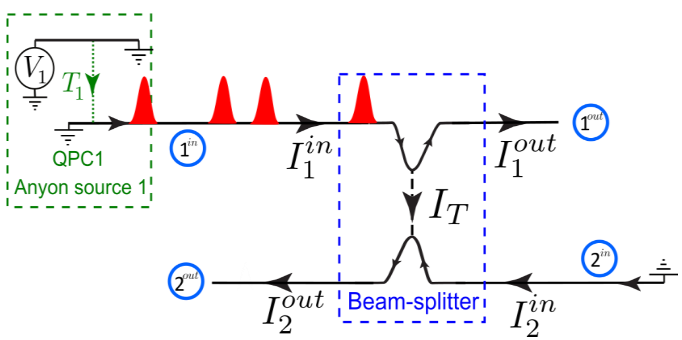


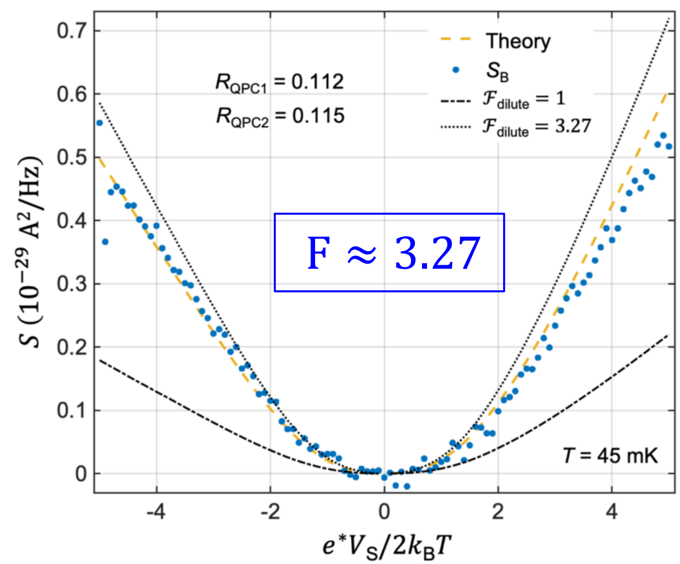
H. Bartolomei et al., Science **368** 173 (2020)

M. Ruelle et al., PRX **13**, 011031 (2023)



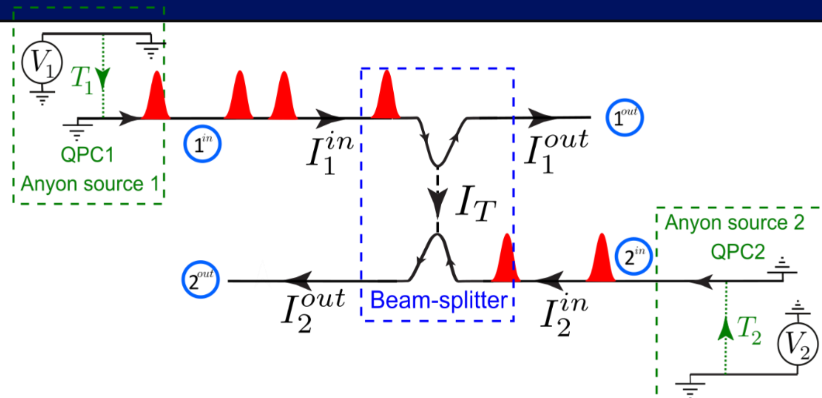
P. Glidic et al., PRX **13**, 011030 (2023).

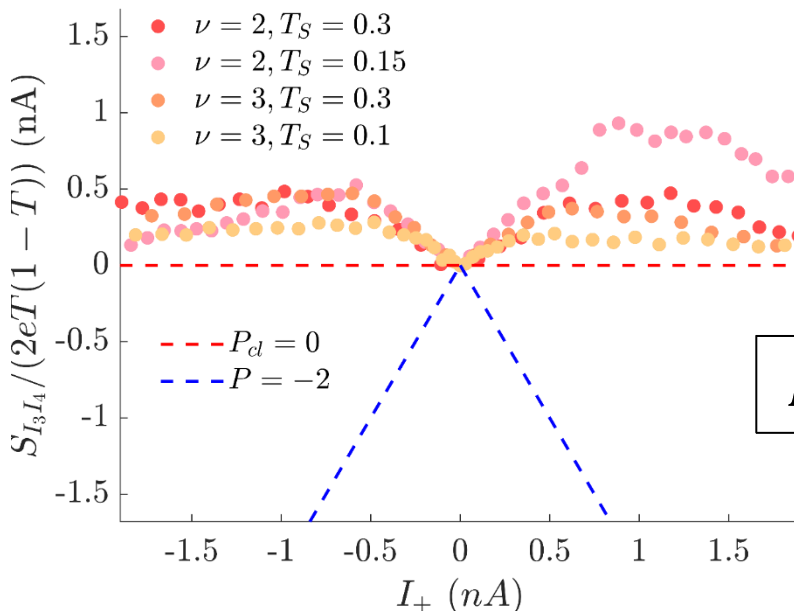




J.Y.M. Lee et al., Nature **617**, 277 (2023).

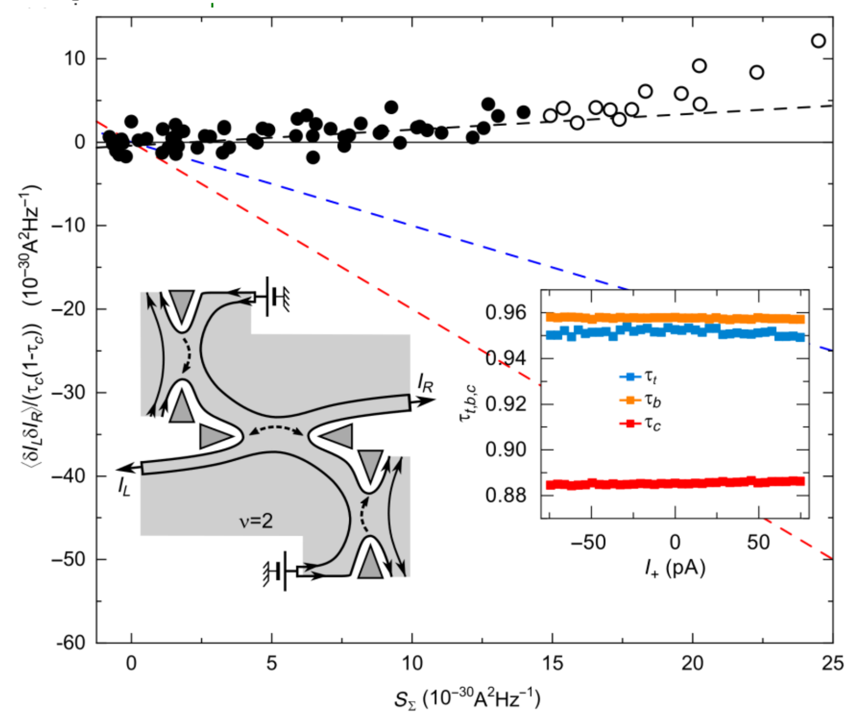
Partitioning of diluted electron beams, $\nu = 2, 3$





$P \approx 0$

H. Bartolomei et al., Science **368** 173 (2020)
M. Ruelle et al., PRX **13**, 011031
(2023)

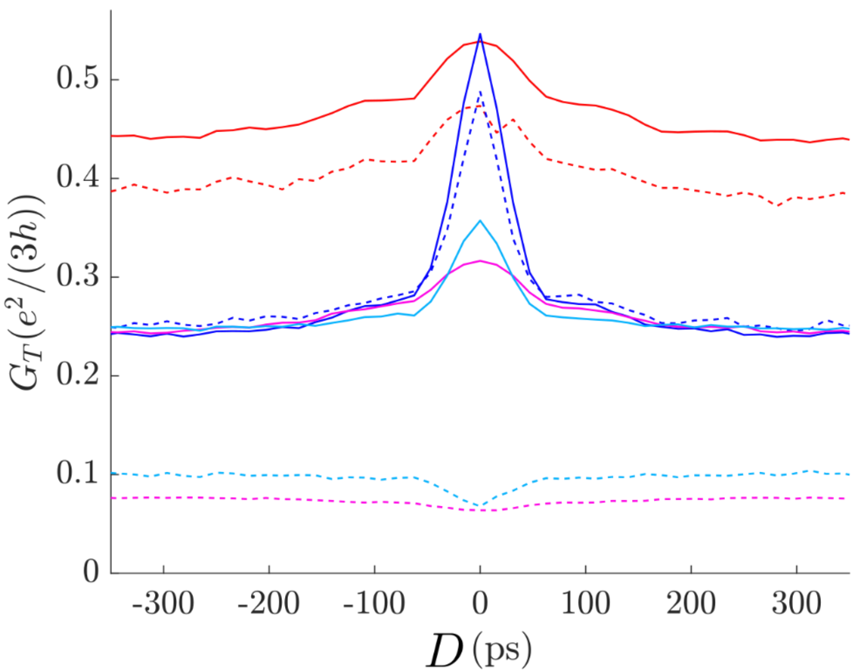


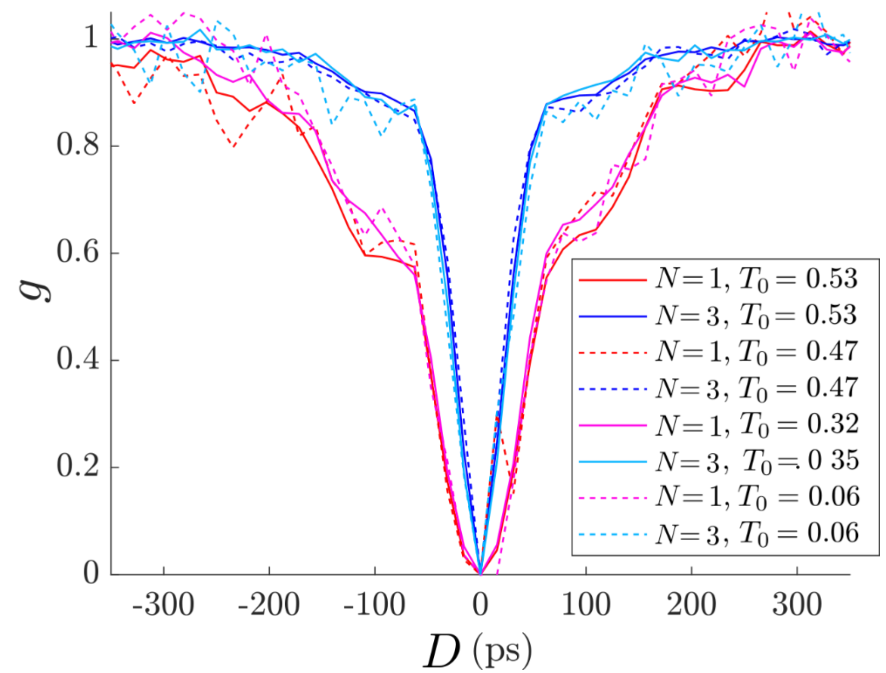
P. Glidic et al., PRX **13**, 011030 (2023).

<u>Fermionic statistics leads to suppression of output cross-correlations</u>

Scaling dimension δ

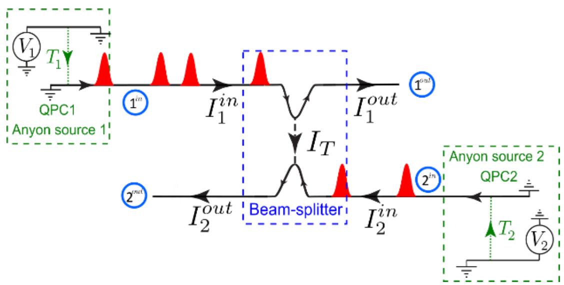
Evolution of the scaling dimension δ with the QPC backscattering probability:

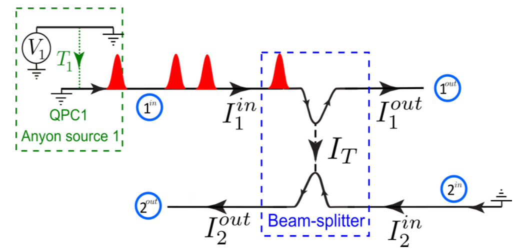


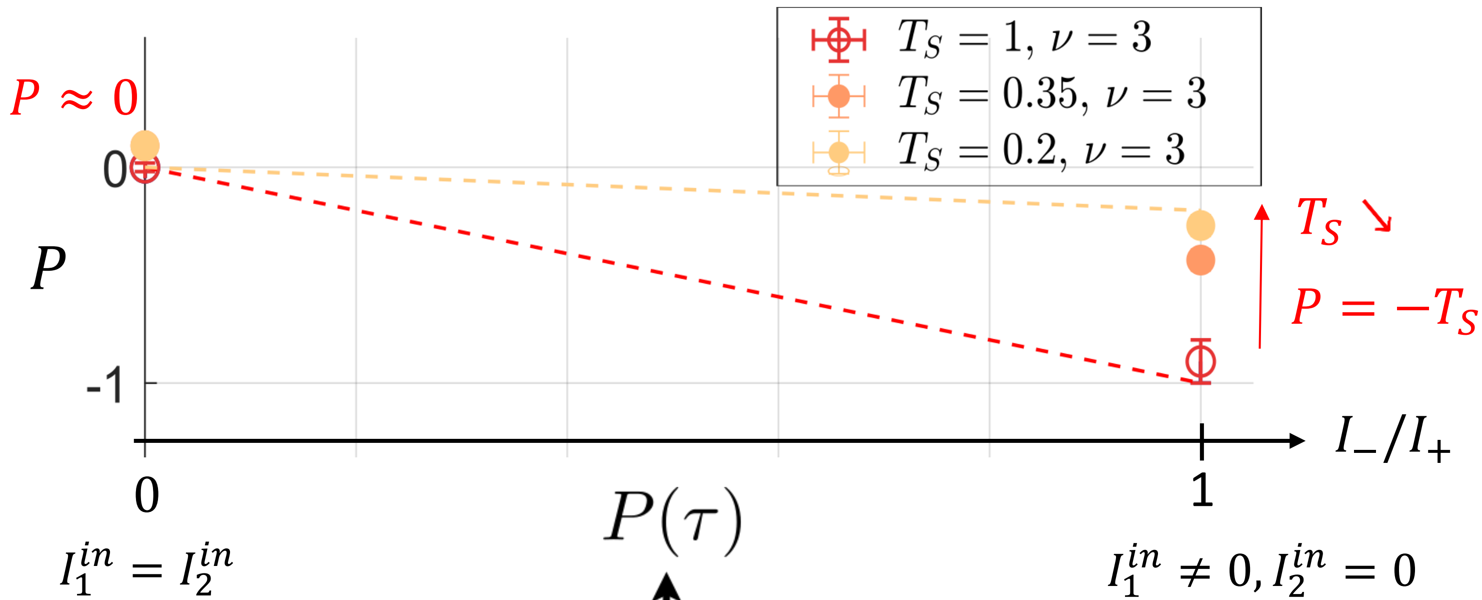


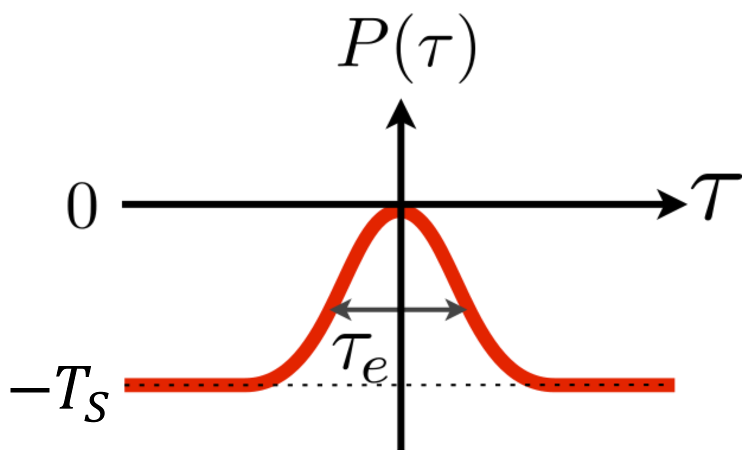
All measurements fall back on the same normalized curve: the widening of the dip (e.g. δ) does not depend on the QPC backscattering probability

Anyon/Fermion collisions, from balanced to unbalanced case





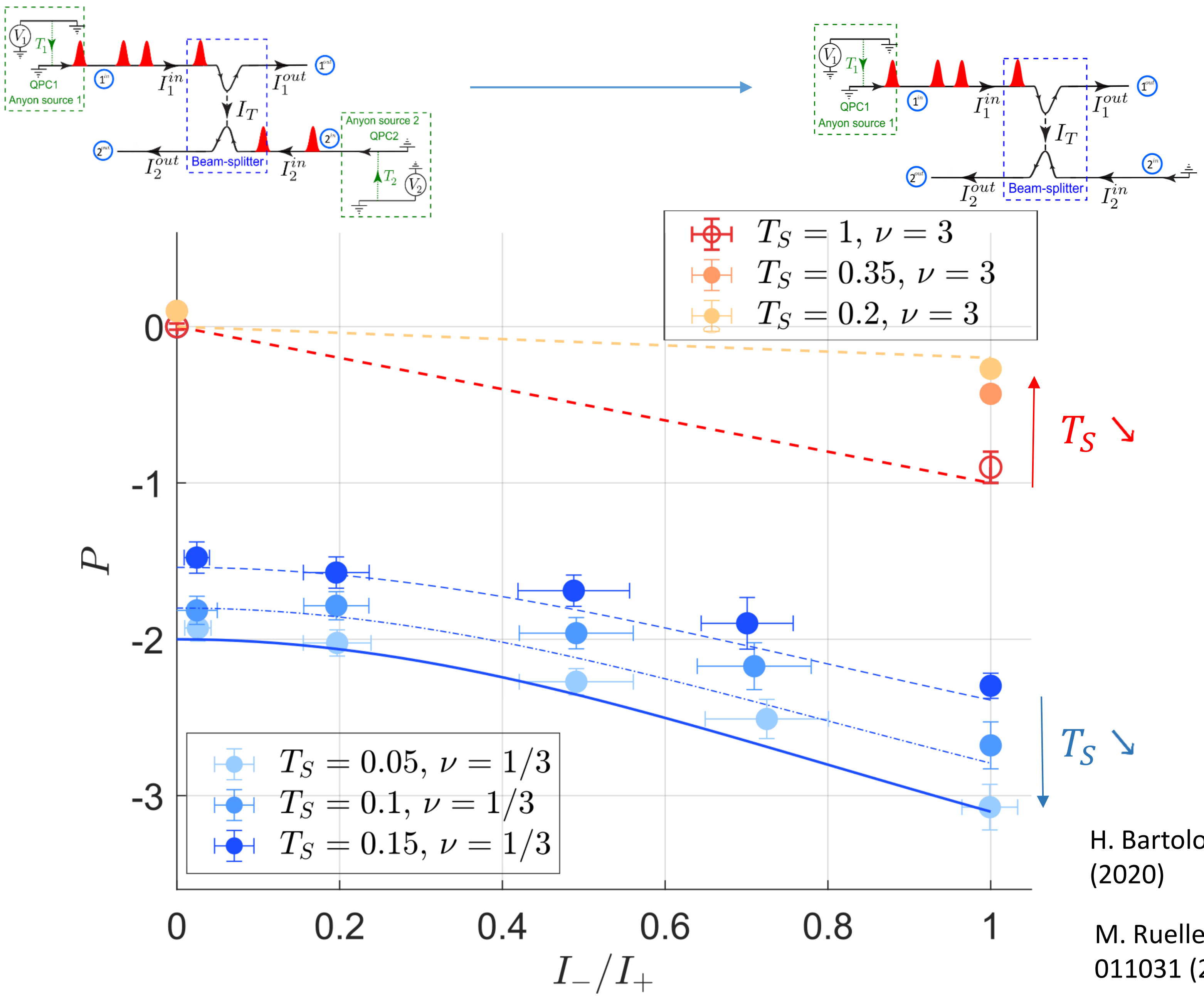




H. Bartolomei, M. Kumar et al., Science **368** 173 (2020)

M. Ruelle et al., PRX **13**, 011031 (2023)

Anyon/Fermion collisions, from the balanced to the unbalanced case



H. Bartolomei et al., Science **368** 173 (2020)

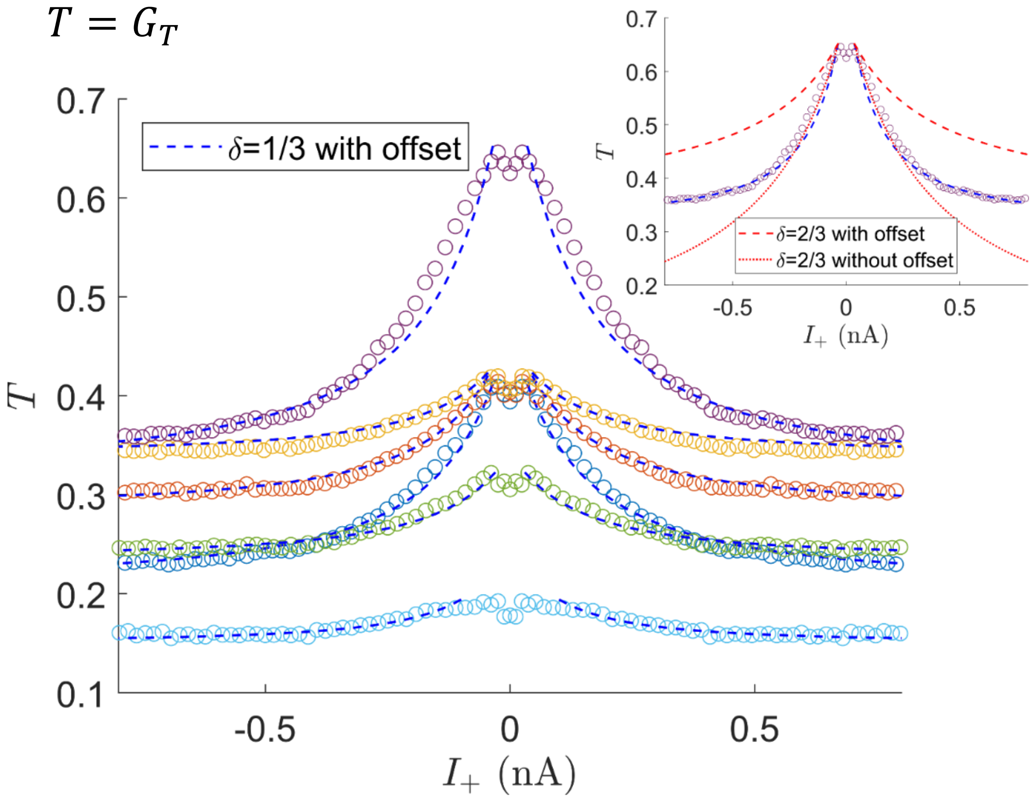
M. Ruelle et al., Phys. Rev. X **13**, 011031 (2023)

See also P. Glidic et al., PRX **13**, 011030 (2023).

Extracting the scaling dimension δ

Other method to measure δ:
IV measurements on a QPC, power law

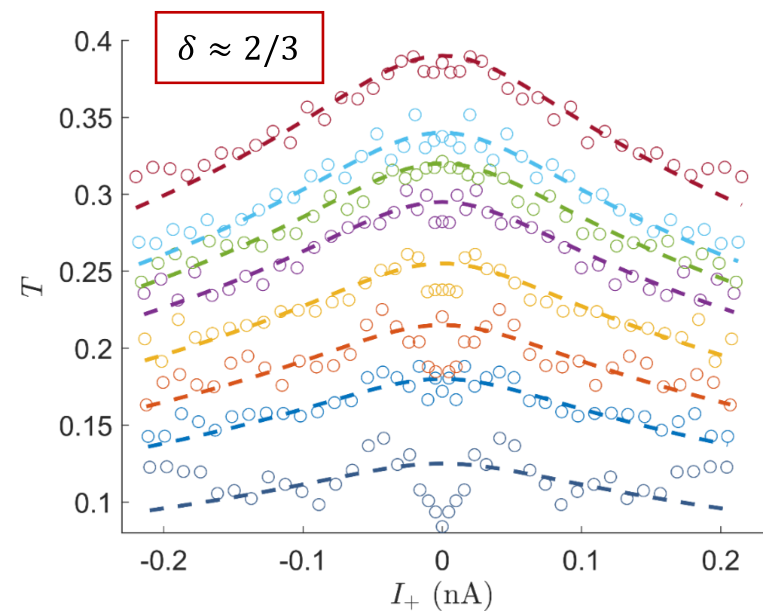
$$T = G_T$$



M. Ruelle et al., PRX **13**, 011031 (2023)

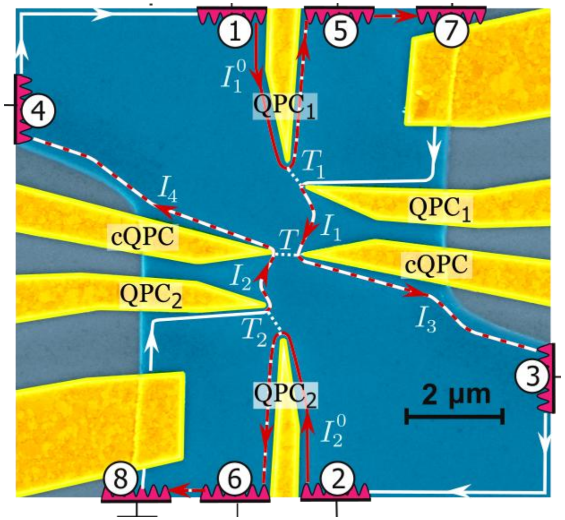
$$T = A \times \frac{\Gamma(\delta + \xi_+)}{\Gamma(1 - \delta + \xi_+)} [\psi(\delta + \xi_+) - \psi(1 - \delta + \xi_+)]$$

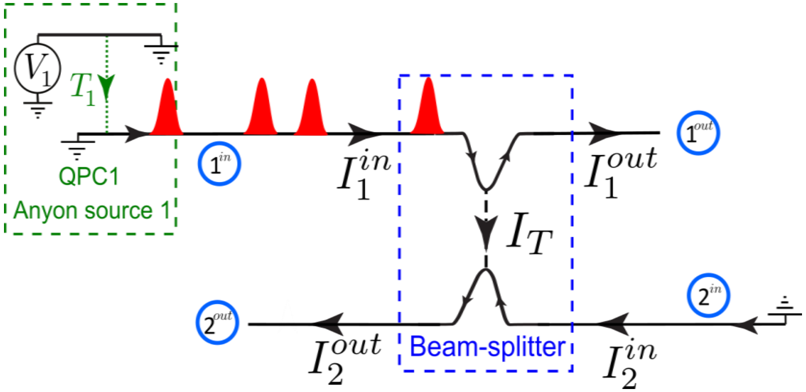
$$\xi_+ = \frac{I_+}{2\pi e^* \tau_{th}} (1 - \cos 2\varphi)$$

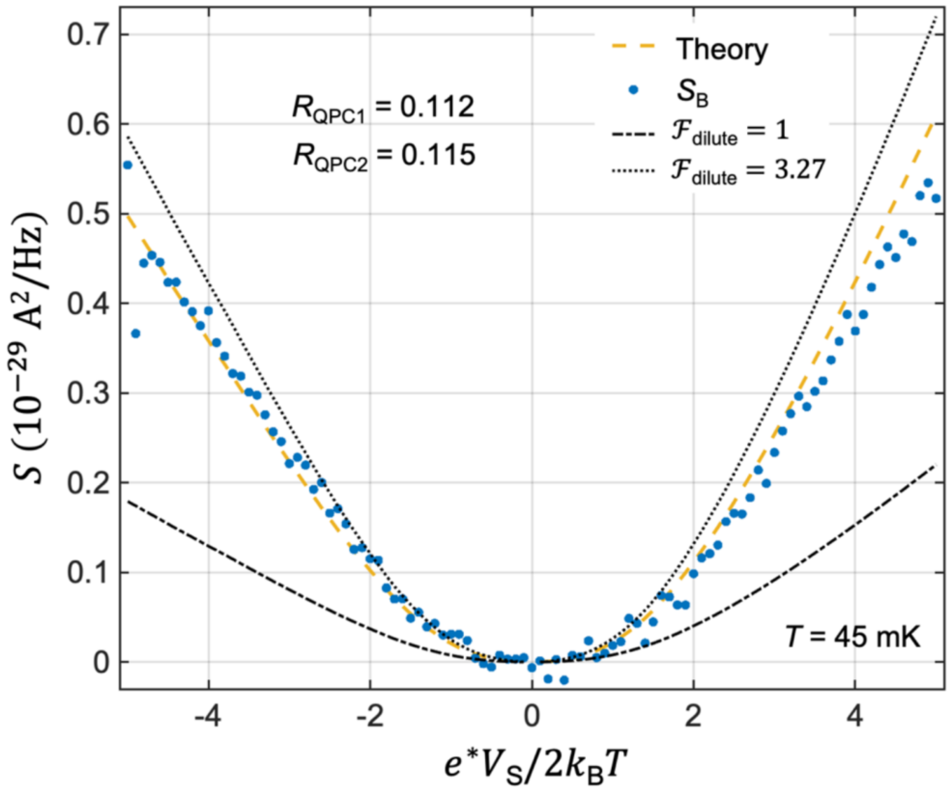


See also for measurements of δ with power laws with an offset:
S. Roddaro et al., PRL **90**, 046805 (2003)
S. Baer et al., PRB **90**, 075403 (2014)

Unbalanced collider, $F(I_2^{in} = 0)$: Lee et al. Nature (2023)



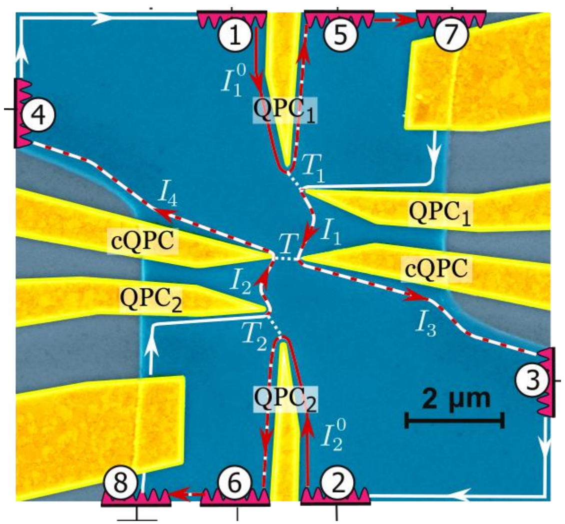


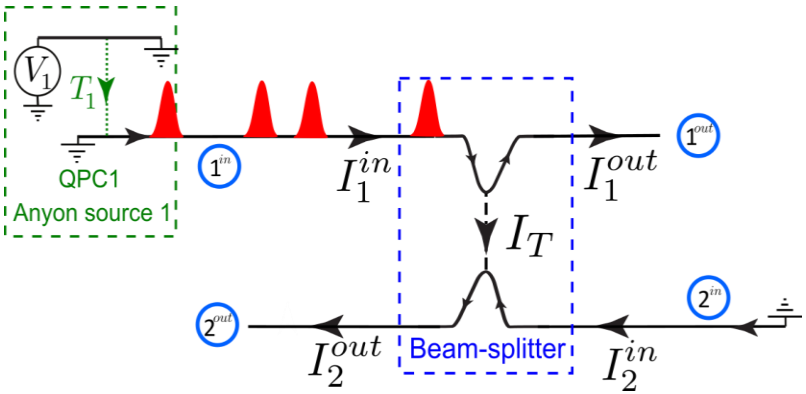


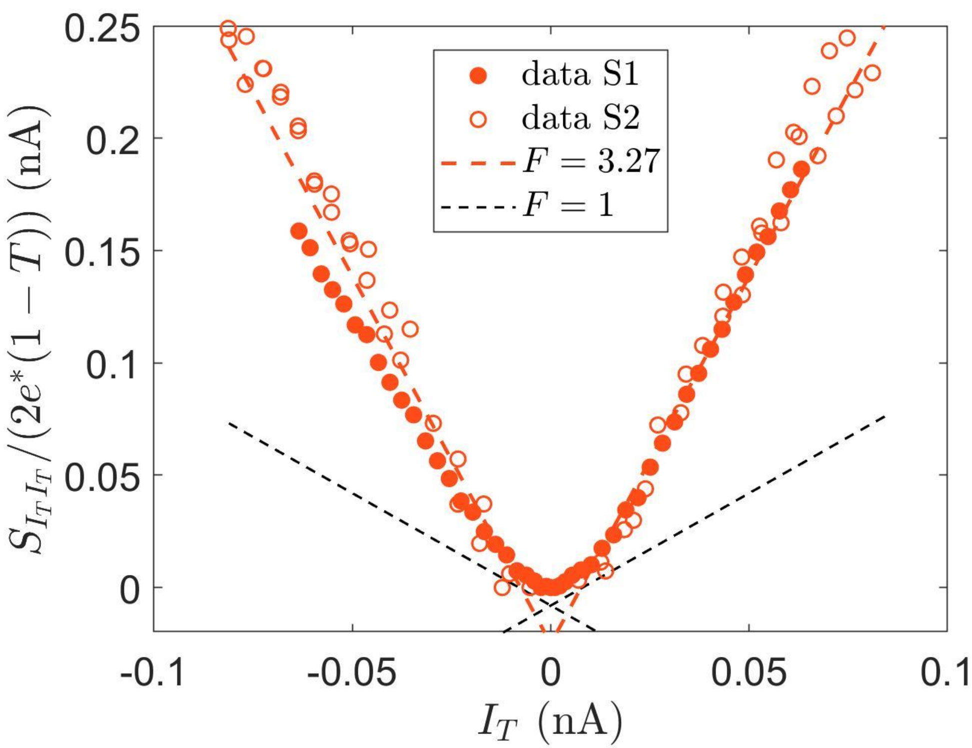
$F(I_- = I_+) \approx +3.27$

Lee et al., Nature (2023), https://doi.org/10.1038/s41586-023-05883-2

Unbalanced collider, $F(I_2^{in} = 0)$: our data







$F(I_- = I_+) \approx +3.27$

Comparison Rosenow et al./ Lee et al.

B. Rosenow, I.P. Levkivskyi, B. Halperin PRL **116**, 156802 (2016) Arbitrary imbalance I_-/I_+

$$\langle I_T\rangle = \frac{e^\star}{i}|\zeta|^2\int_{-\infty}^{\infty} dt \frac{\sin\left[\frac{I_-}{e^\star} t\sin 2\pi\lambda\right](\tau_c)^{2\delta}}{\exp\left[\frac{I_+}{e^\star}|t|(1-\cos 2\pi\lambda)\right](\tau_c - it)^{2\delta}} = C\sin(\pi\delta)\mathrm{Im}\left(I_+ + \frac{iI_-}{\tan\pi\lambda}\right)^{2\delta-1}[1+O(\tau_c)], \quad (7)$$

$$\frac{\langle\delta I_T^2\rangle_{\omega=0}}{(e^\star)^2} = |\zeta|^2\int_{-\infty}^{\infty} dt \frac{2\cos\left[\frac{I_-}{e^\star} t\sin 2\pi\lambda\right]\tau_c^{2\delta}}{\exp\left[\frac{I_+}{e^\star}|t|(1-\cos 2\pi\lambda)\right](\tau_c - it)^{2\delta}} = \frac{C}{e^\star}\cos(\pi\delta)\mathrm{Re}\left(I_+ + \frac{iI_-}{\tan\pi\lambda}\right)^{2\delta-1}[1+O(\tau_c)].$$

$$F = \frac{S_{I_T}}{2qI_T} = -\cot(\pi\delta)\frac{\mathrm{Re}\left[\left(1+\frac{iI_-/I_+}{\tan\pi\lambda}\right)^{2\delta-1}\right]}{\mathrm{Im}\left[\left(1+\frac{iI_-/I_+}{\tan\pi\lambda}\right)^{2\delta-1}\right]}$$

Lee et al., Nature **617**, 277–281 (2023) Completely unbalanced case I_-/I_+=1

$$\mathcal{F}_{\text{dilute}} = \frac{W^{\text{braid}}_{2\to3}+W^{\text{braid}}_{3\to2}}{W^{\text{braid}}_{2\to3}-W^{\text{braid}}_{3\to2}} = -\cot\pi\delta\frac{\mathrm{Re}\left[\left(-\log\left(1+R_{\text{QPC1}}\left(e^{-i2\theta}-1\right)\right)\right)^{2\delta-1}\right]}{\mathrm{Im}\left[\left(-\log\left(1+R_{\text{QPC1}}\left(e^{-i2\theta}-1\right)\right)\right)^{2\delta-1}\right]}. \quad (7)$$

Dilute limit: $R_{QPC1} \ll 1$

$$F = -\cot(\pi\delta)\frac{\mathrm{Re}\left[\left(e^{-2i\theta}-1\right)^{2\delta-1}\right]}{\mathrm{Im}\left[\left(e^{-2i\theta}-1\right)^{2\delta-1}\right]} = -\cot(\pi\delta)\frac{\mathrm{Re}\left[\left(1+\frac{i}{\tan\theta}\right)^{2\delta-1}\right]}{\mathrm{Im}\left[\left(1+\frac{i}{\tan\theta}\right)^{2\delta-1}\right]}$$

Identical taking $\theta = \pi\lambda = \pi/3$