Andreev physics: from transport to qubits

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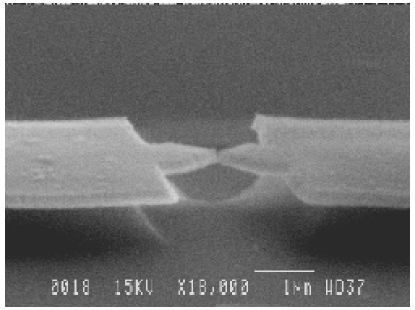
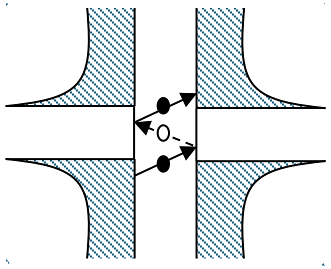
EXCELENCIA SEVERO OCHOA

20th Capri Spring School on Transport in Nanostructures
April 13-18 (2026)

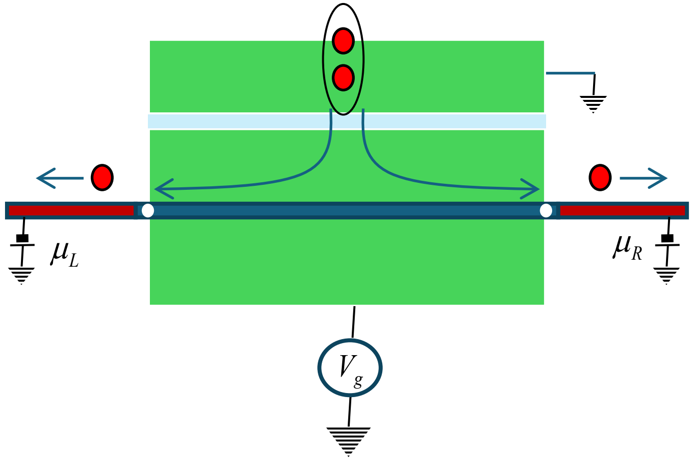
Andreev physics in previous Capri schools

2007:
Atomic size conductors

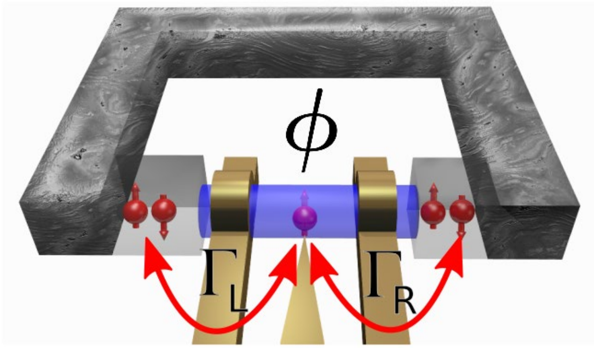


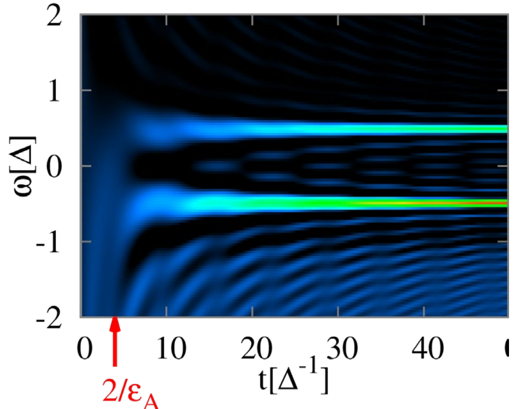


2014:
From atomic contacts to Majorana wires



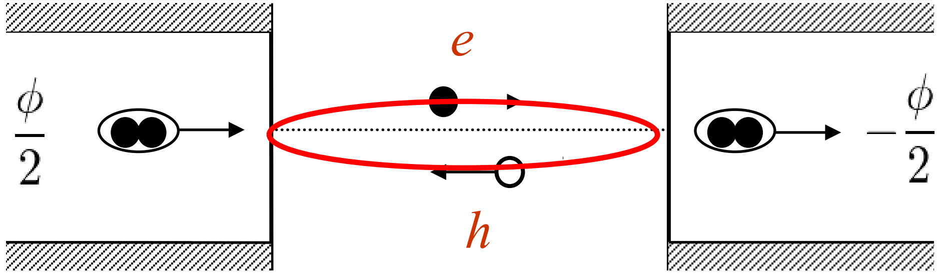
2018:
Hamiltonian approach to transport



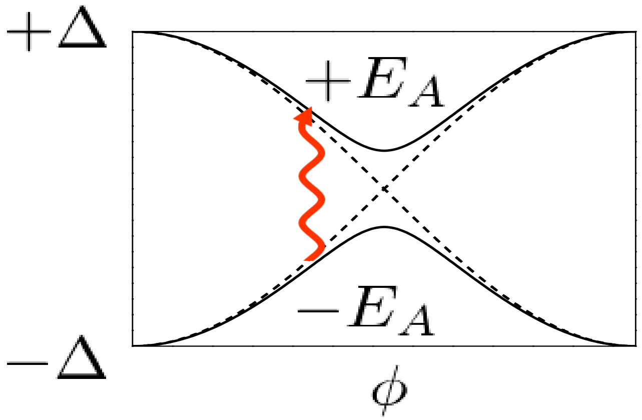


Andreev physics and qubits

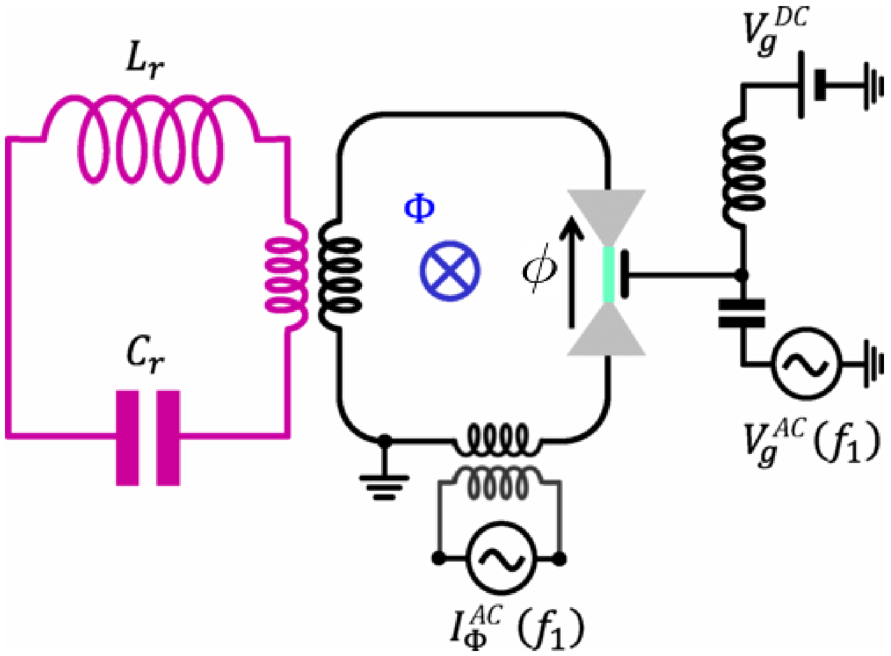
Josephson effect mediated by **ABS** in weak links



Andreev qubits



Thy: Despósito (2001), Zazunov (2003), Chtchelkatchev (2003), Park (2017)



Exps: Janvier (2015), Tosi (2019), Hays (2021), Pita-Vidal (2023)

FET-Open project

AndQC

https://www.andqc.eu/

Outline:

1) Andreev physics in transport:

Emergent topology by Landau level mixing in quantum Hall-superconductor nanostructures

Y. Baba, P. Burset and ALY, arXiv:2507.14074 and PRL (to be published)

2) Andreev physics and qubits:

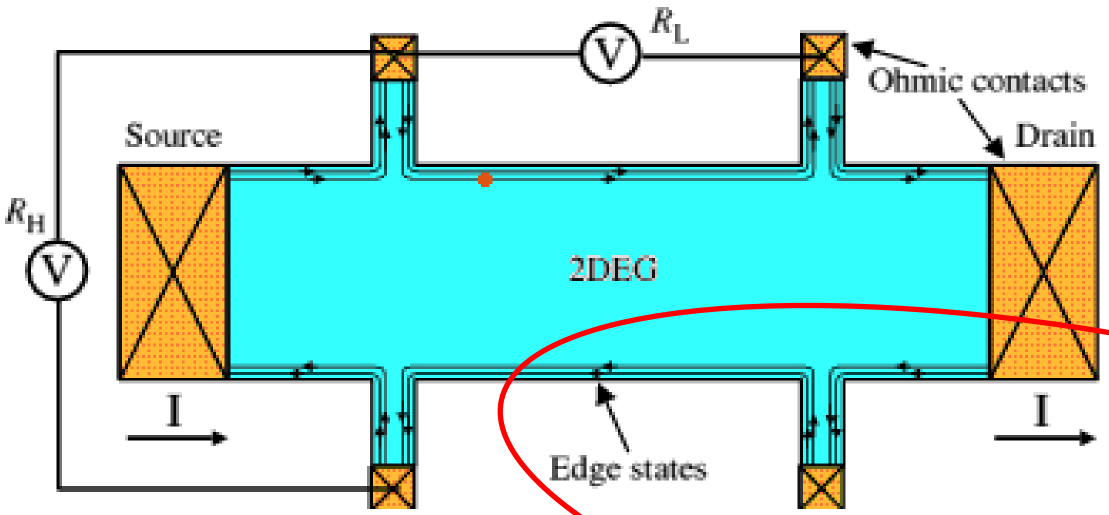
Quantum circuits with multiterminal Josephson-Andreev junctions

F.J. Matute-Cañadas, L. Tosi, and ALY, PRX Quantum 5, 020340 (2024)

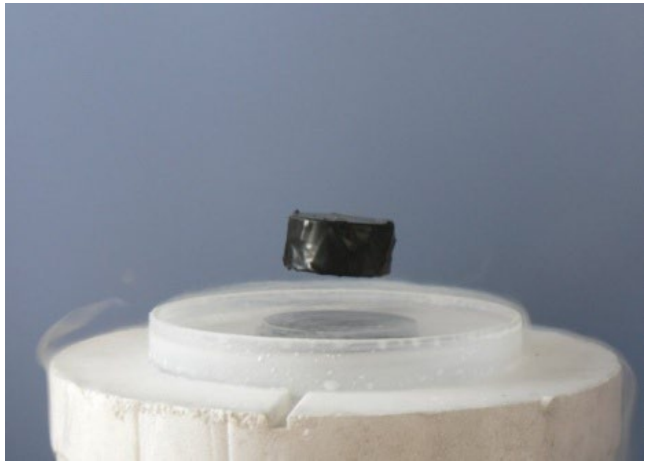
FerBo, a noise resilient qubit hybridizing Andreev and fluxonium states

J.J. Cáceres, F.J. Matute-Cañadas, D. Sanz Marco, J. Ortuzar, E. Flurin, ALY, C. Urbina, H. Pothier and M. Goffman, arXiv:2604.01145

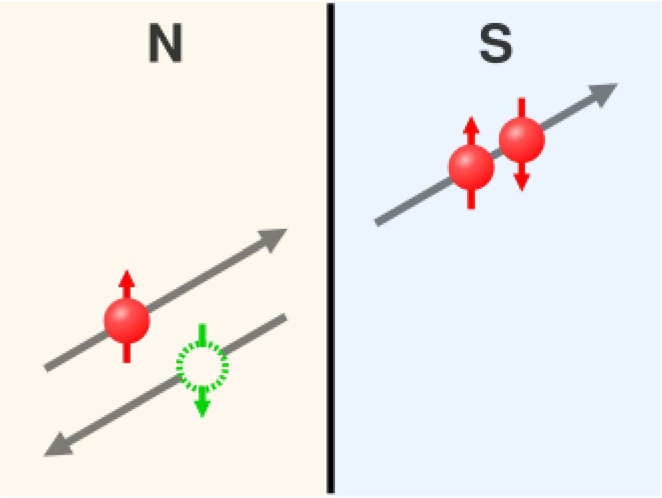
Quantum Hall Effect + Superconductivity



+

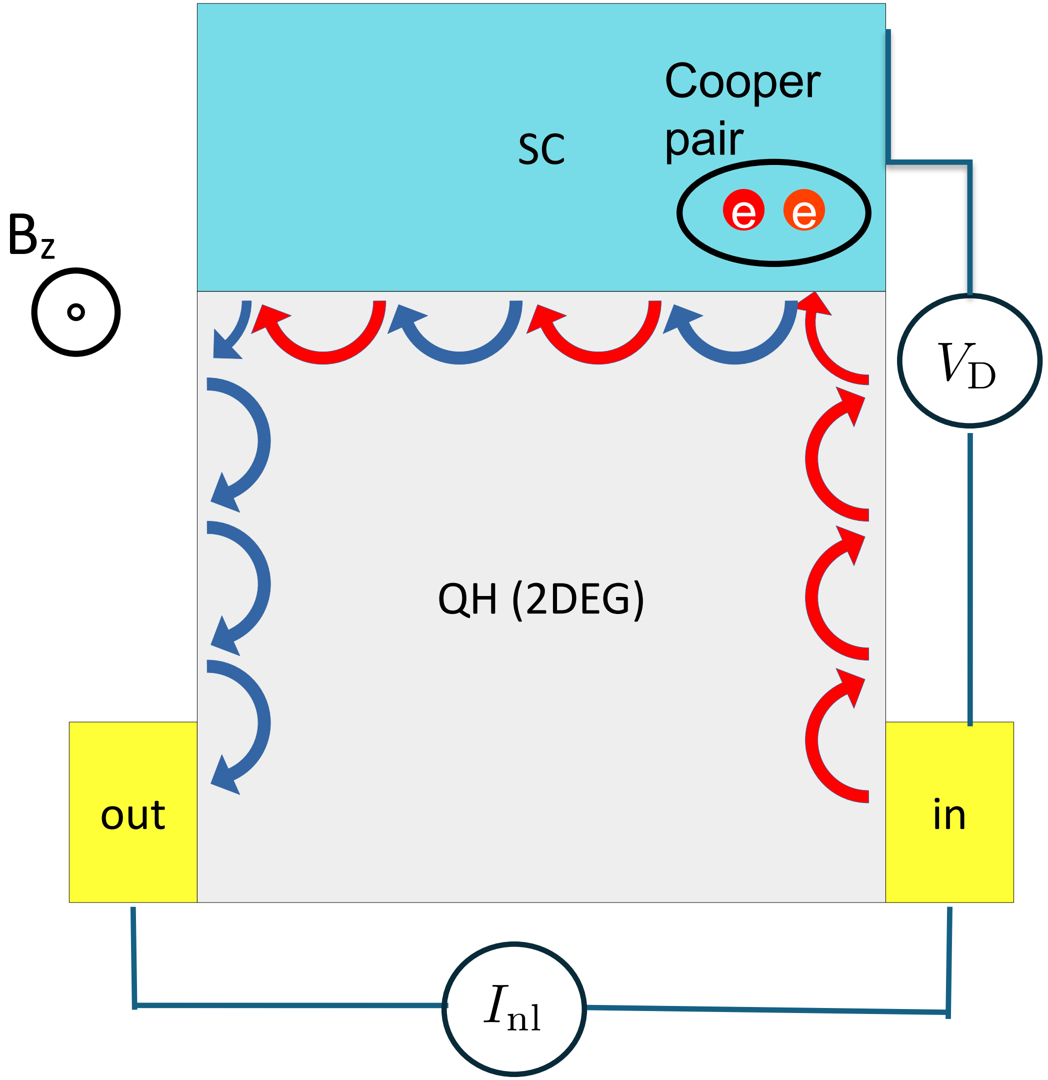


Andreev reflection

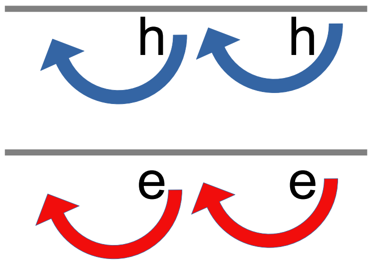


Old dream in Condensed Matter:
Ma & Zyuzin, EPL (1993)
Takayanagi & Akazaki, Physica B (1998)
Hoppe et al. PRL (2000)
…

Non-local transport: skipping orbits picture



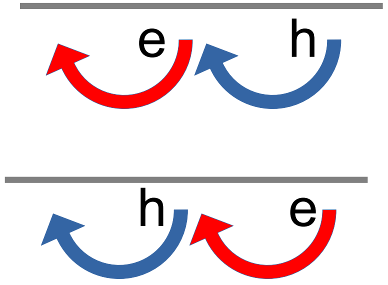
Normal reflections:



T_{EC}

EC: Electron cotunnelling (e→ e)

Andreev reflections:



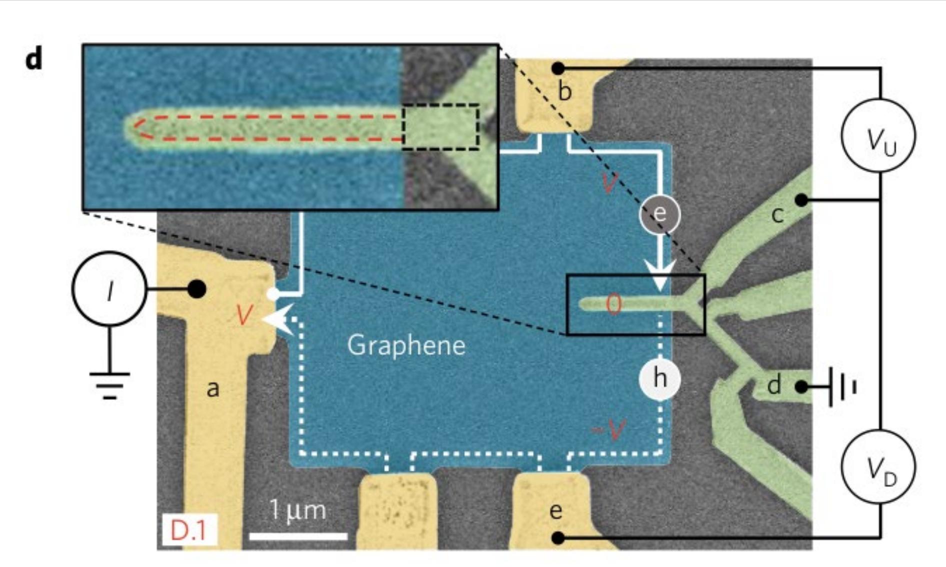
T_{CAR}

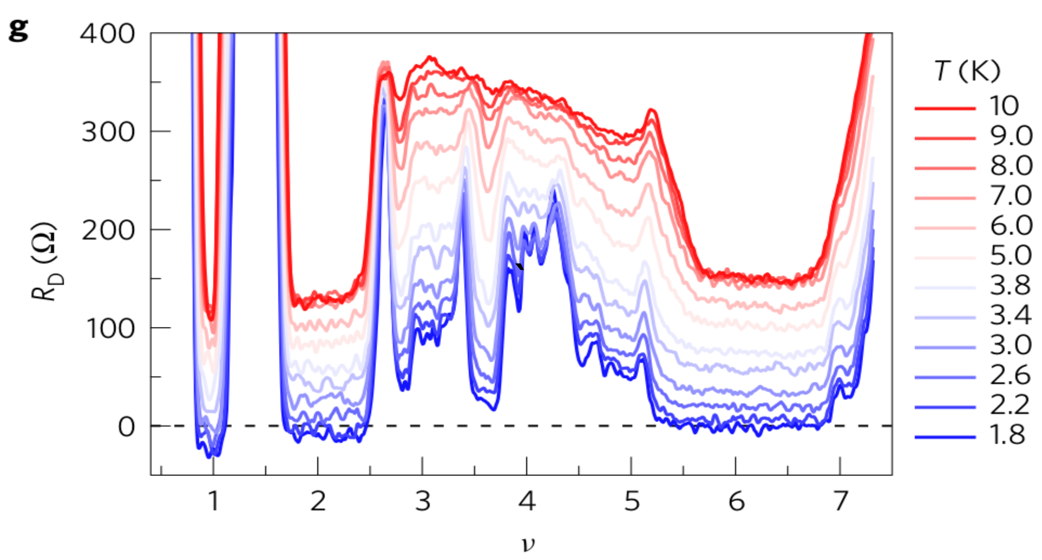
CAR: Crossed Andreev reflection (e→ h)

$$G_{nl} = \frac{I_{nl}}{V_D} = \frac{e^2}{h}(T_{EC} - T_{CAR})$$

Experimental platforms: graphene

- Very strong proximity effect
- Quantized QH at low magnetic field



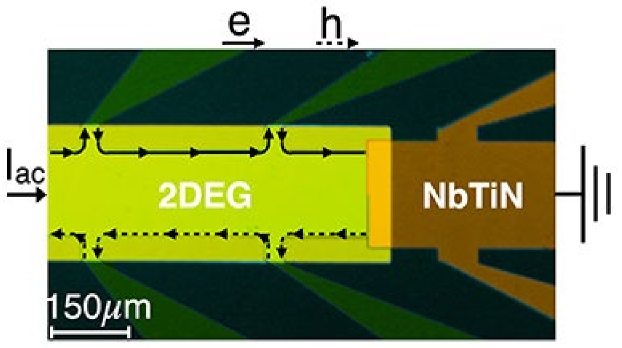


G. Lee, … P. Kim, *Nat. Phys.* **13**, 7 (2017)
O. Gül,... P. Kim, *Phys Rev X* **12**, 021057 (2022)

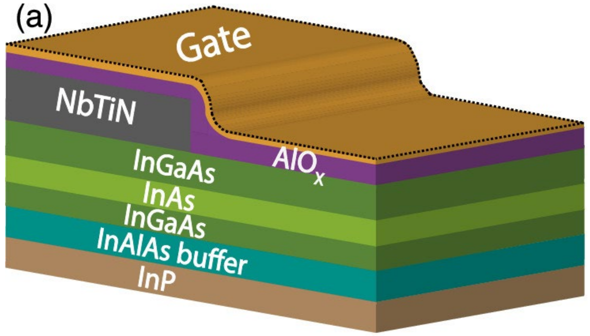
finger geometry: 100 nm

Long graphene interface:
Zhao,…, Finkelstein *Nat. Phys*. **16**, 862 (2020)
Zhao,., Finkelstein *Phys Rev Lett*,**131**,176604 (2023)

Experimental platforms: proximitized 2DEGs

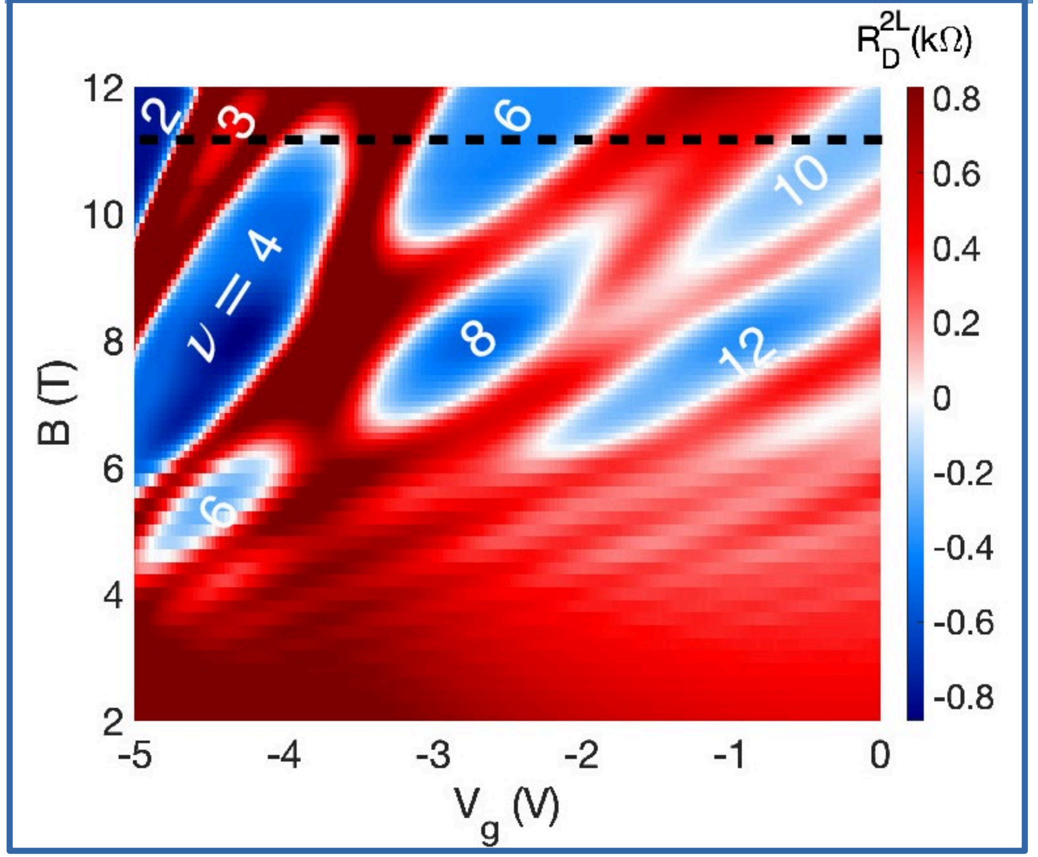


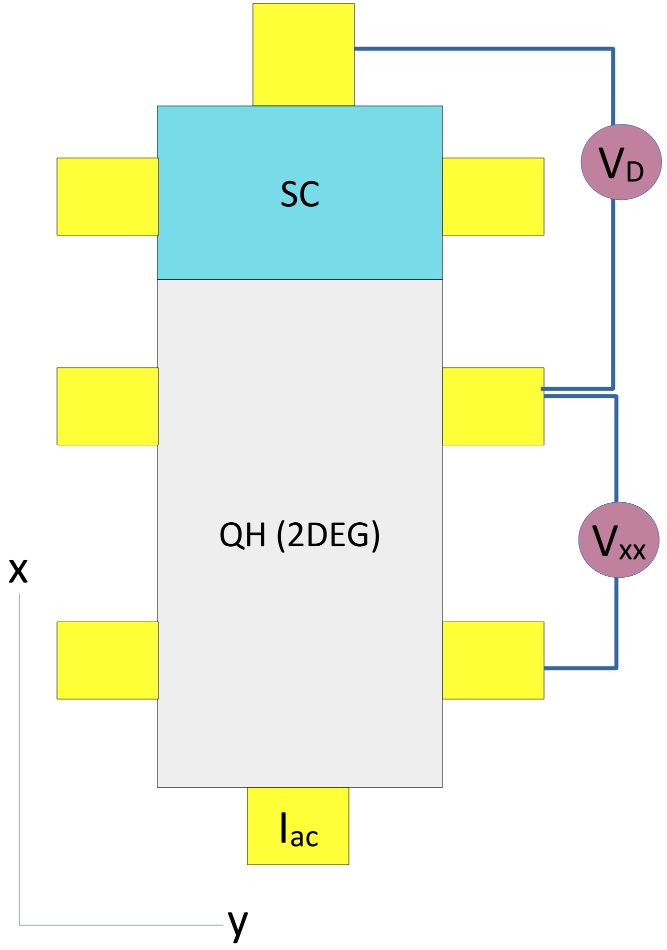
Epitaxial contact:



Extremely long interface:
100 um

Negative downstream resistance at the LL plateaus:



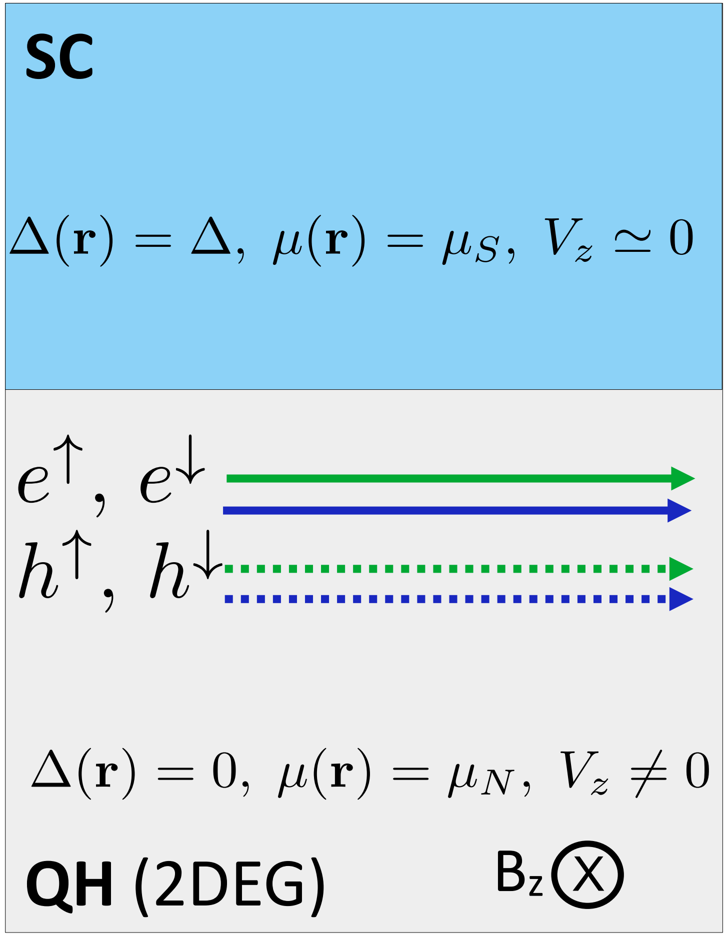


M. Hatefipour, ... J. Shabani, *Nano Lett.* **22**, 15 (2023)

M. Hatefipour, ... J. Shabani, *PRB* **109**, 3 (2024)

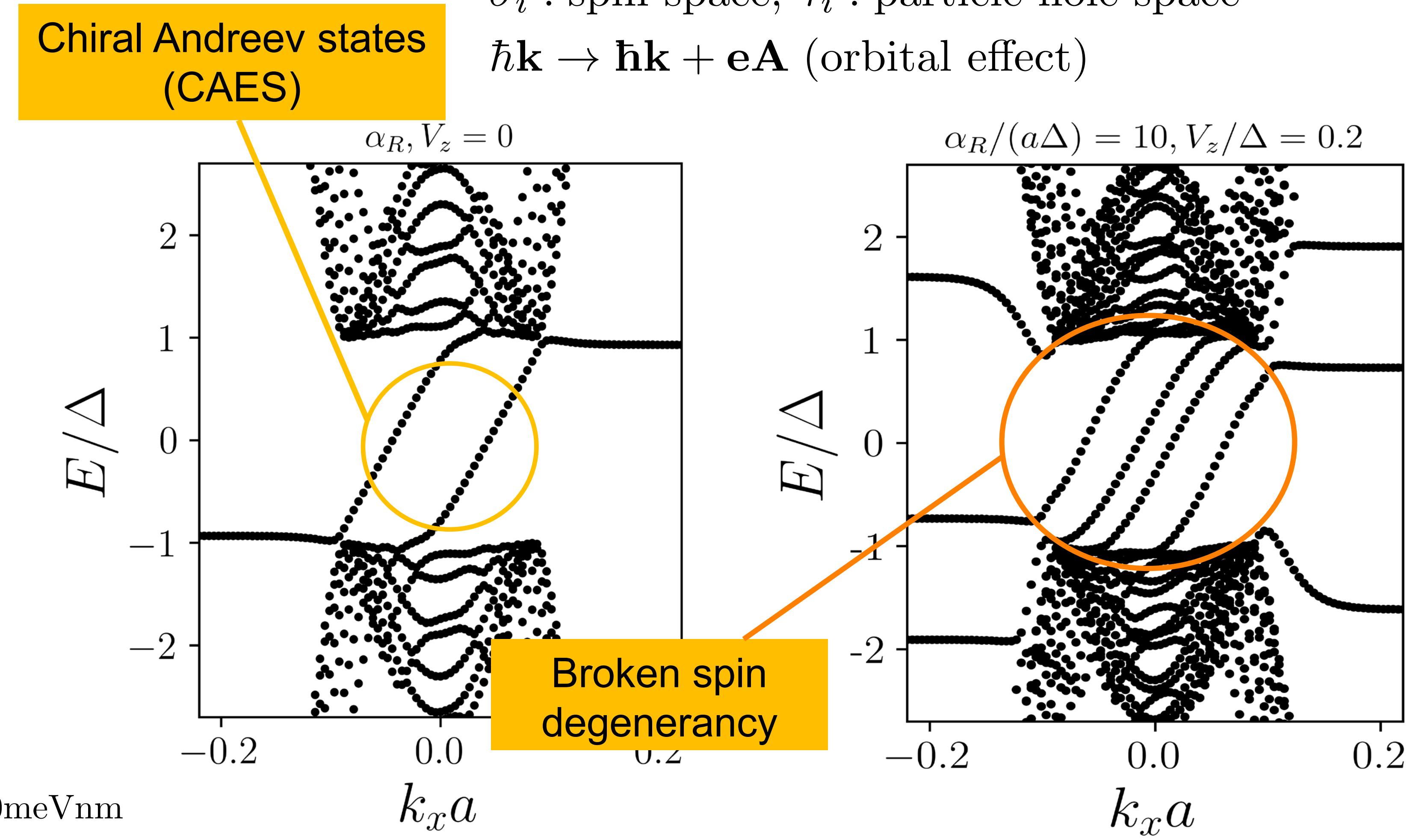
Model for a proximitized 2DEG

$$\mathcal{H} = \left(\frac{\hbar^2\mathbf{k}^2}{2m^*} - \mu(\mathbf{r})\right)\sigma_0\tau_z + V_z(\mathbf{r})\sigma_z\tau_0 + \alpha_R(\mathbf{r})\left(k_x\sigma_y\tau_0 - k_y\sigma_x\tau_0\right) - \Delta(\mathbf{r})\tau_x\sigma_x$$



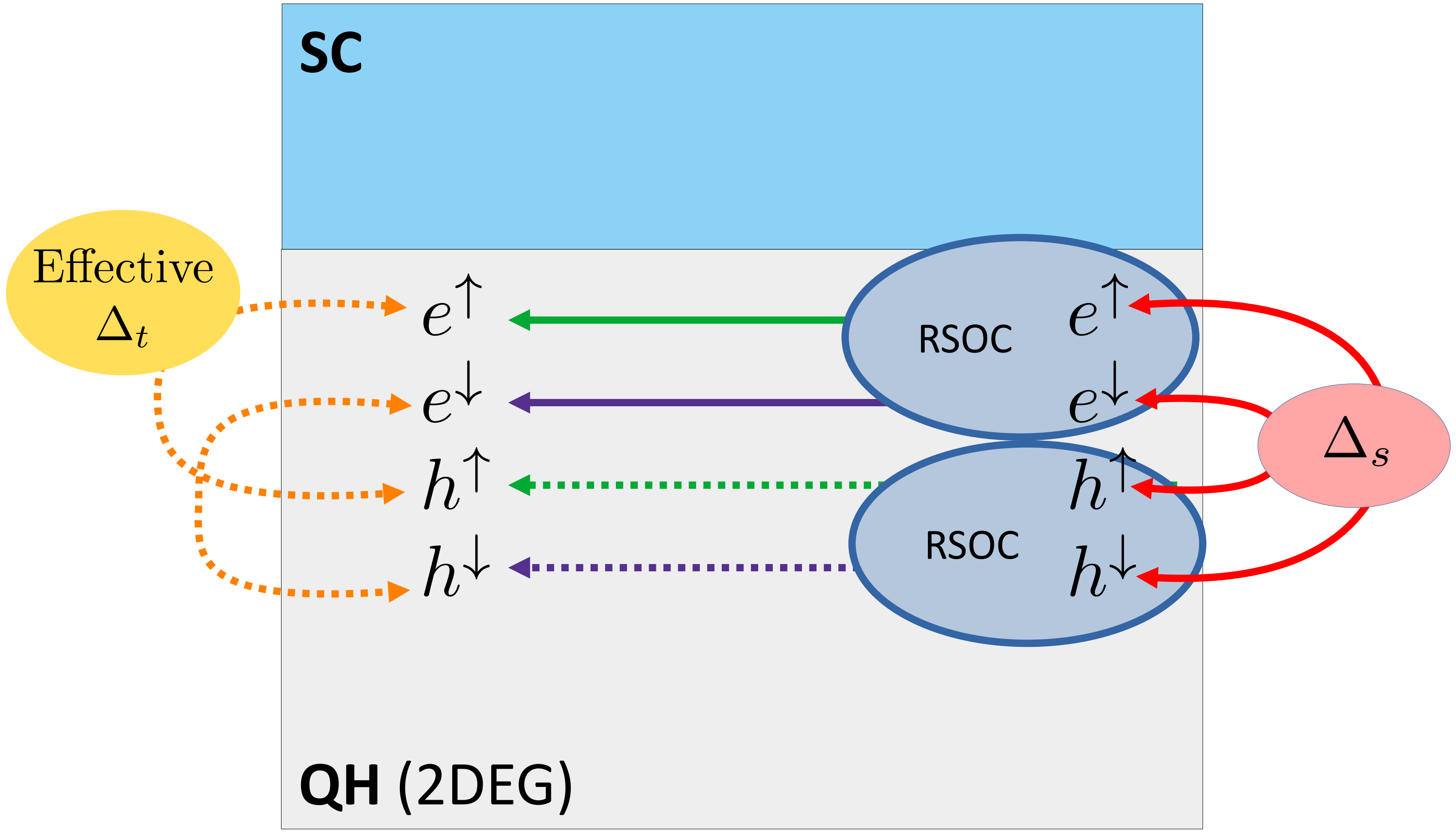
σ_i : spin space; τ_i : particle-hole space

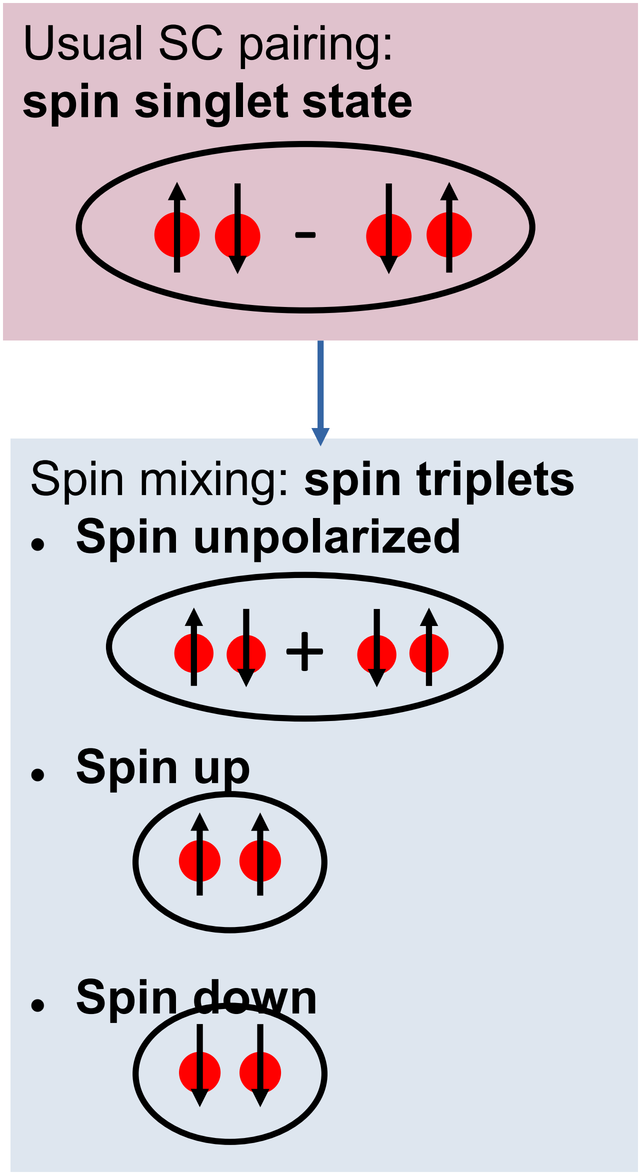
$\hbar\mathbf{k} \rightarrow \hbar\mathbf{k} + e\mathbf{A}$ (orbital effect)



$\Delta \simeq 1\text{meV},\ m^*/m_e \simeq 0.07,\ g \simeq 8,\ \alpha_R \simeq 20\text{meVnm}$

Proximitized 2DEG: triplet pairing



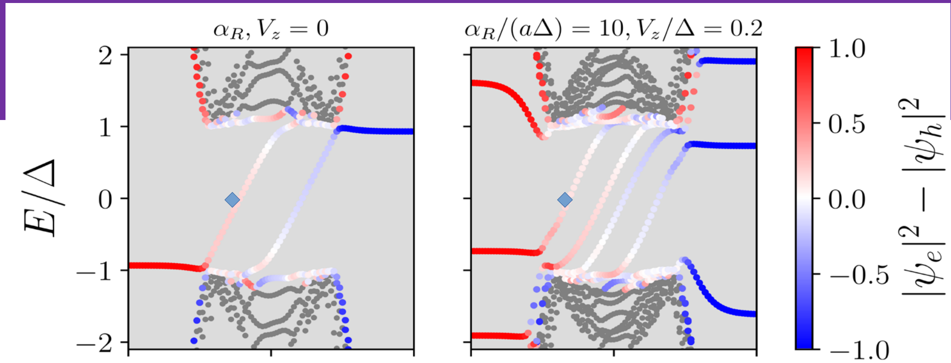


J.A.M. Van Ostay, A.R. Akhmerov and C.W.J. Beenaker, PRB (2011)

A.B. Michelsen, P. Recher, B. Braunecker and T.L. Schmidt, PRB (2023)

Triplet pairing

- 1D system with PBC in x



Local triplet correlations:

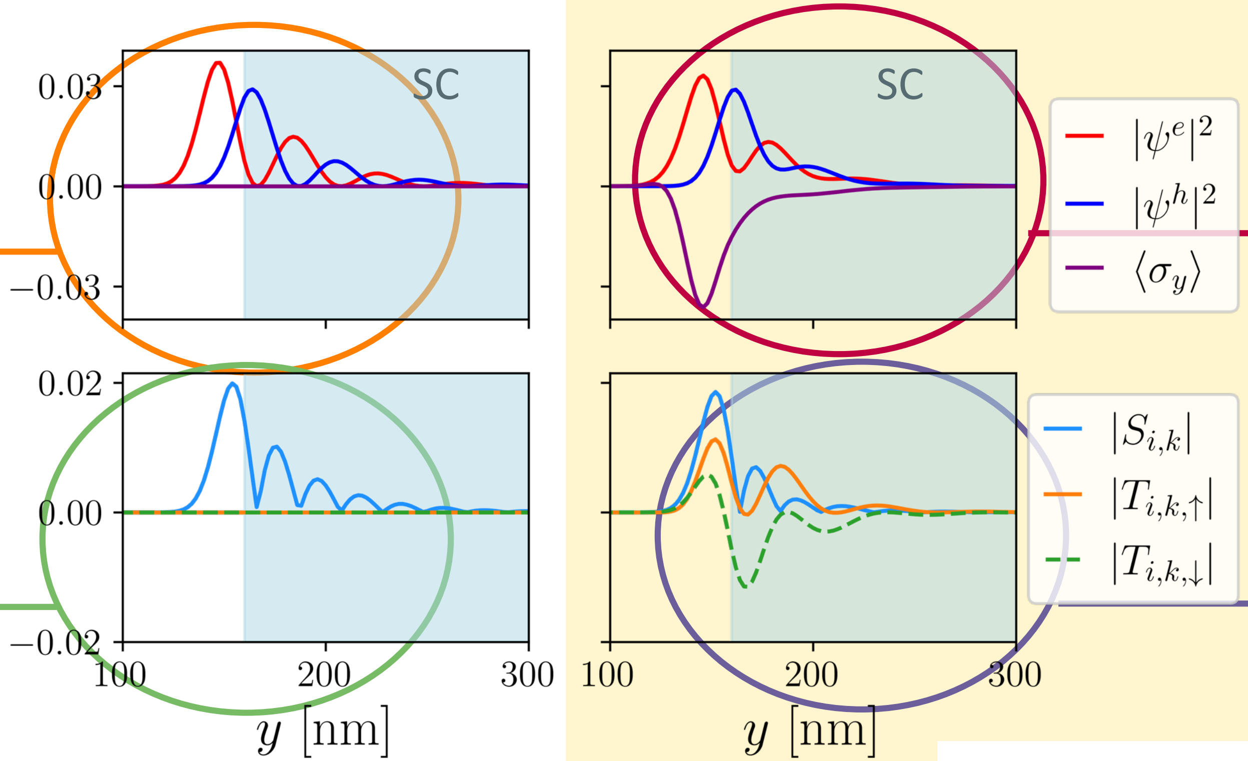
$$T_{i,k,\uparrow} \equiv \langle c_{i,k,\uparrow} c_{i,-k,\uparrow} \rangle \ ,$$
$$T_{i,k,\downarrow} \equiv \langle c_{i,k,\downarrow} c_{i,-k,\downarrow} \rangle \ .$$

Without RSOC and Zeeman splitting

Including RSOC and Zeeman splitting

Quasi e- h superposition

Only singlet pairing



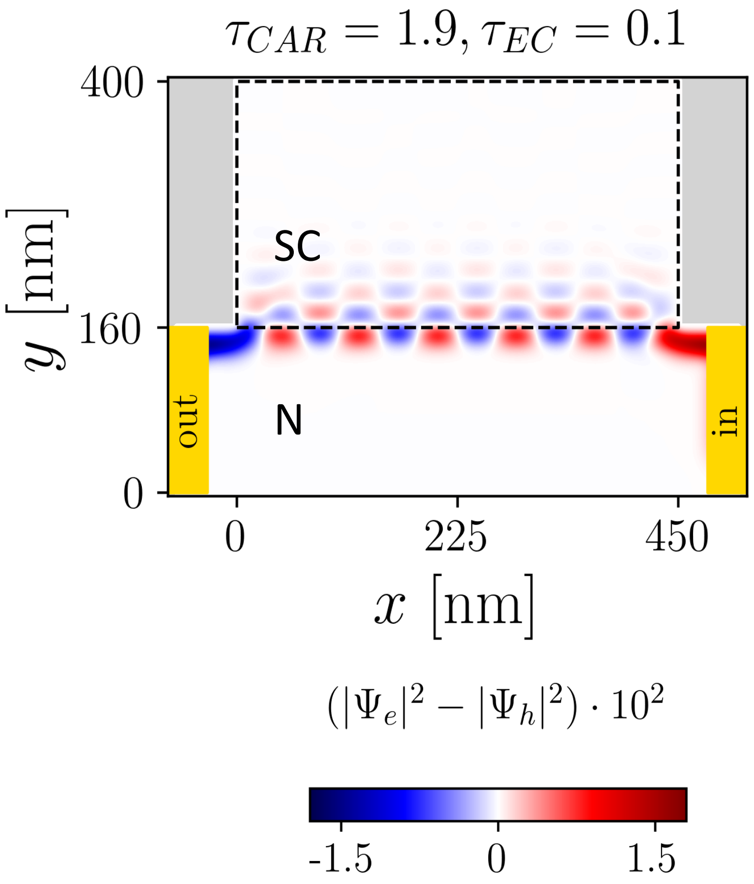
In-plane spin polarized

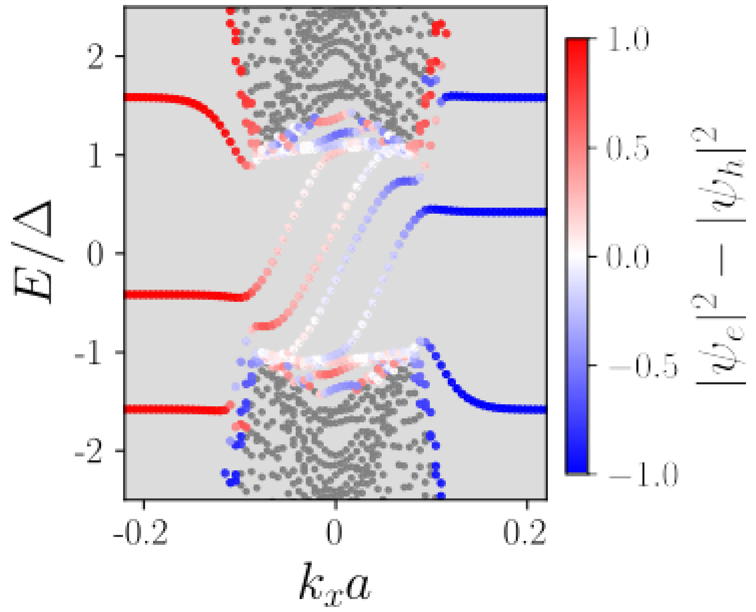
Triplet pairings

L. Arrachea, A. Levy Yeyati, C. Balseiro. PRB 2024

Non-local transport calculations

- **CAES:** chiral Andreev edge state
- **Triplet pairing** due to the interplay of the RSOC and the *singlet* pairing.
- **Tuning of the CAR signal** as a function of the parameters of the system:
  - magnetic field, interface length, chemical potential, …



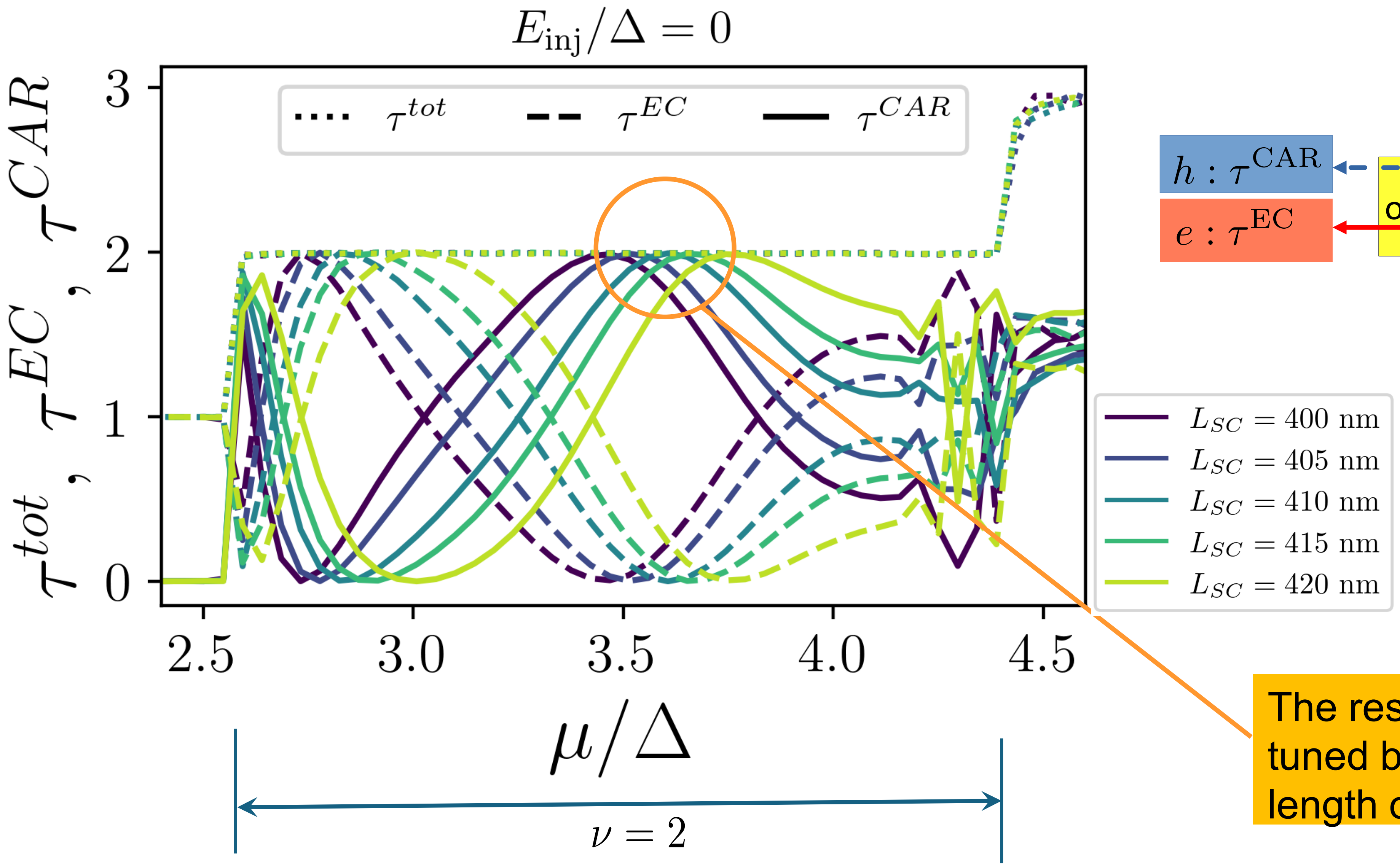


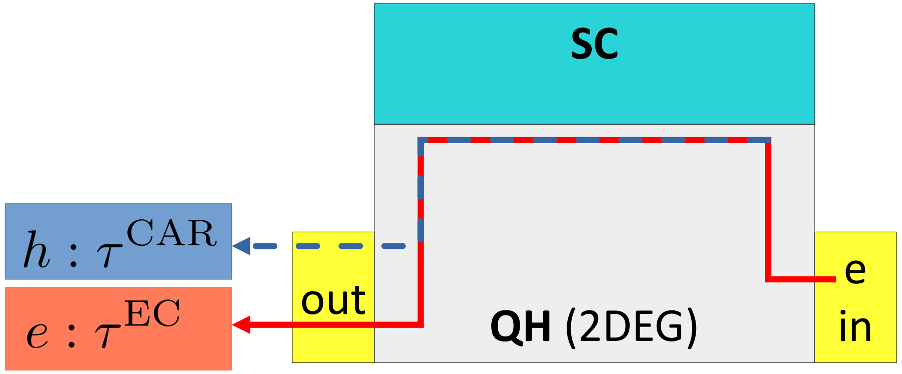
kwant

Groth, *et al, New J. Phys.* **16** 063065 (2014)

David, Meyer, Houzet, *Phys Rev B*, **107**, 125416 (2023)

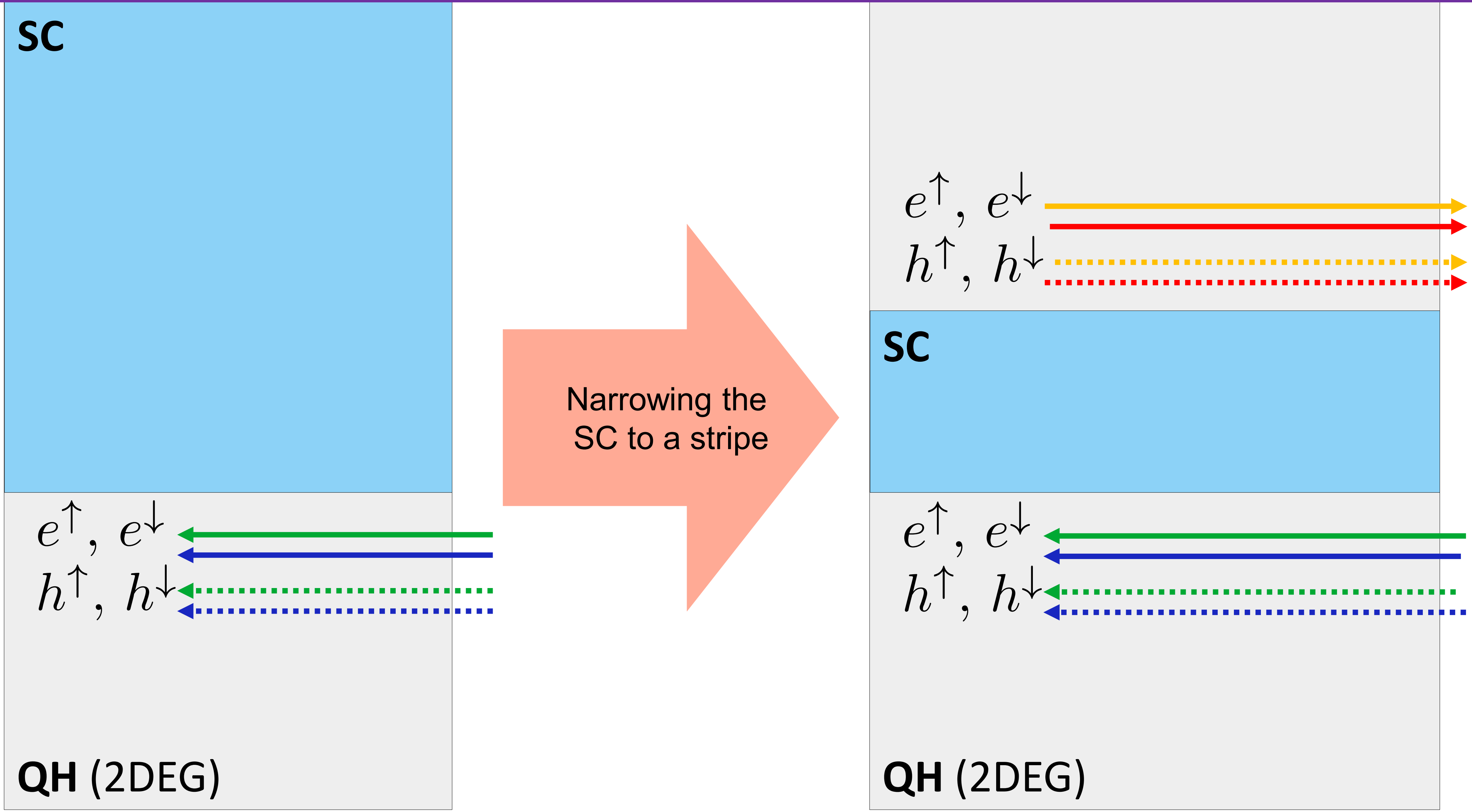
Non-local transport: tuning the CAR/EC signal



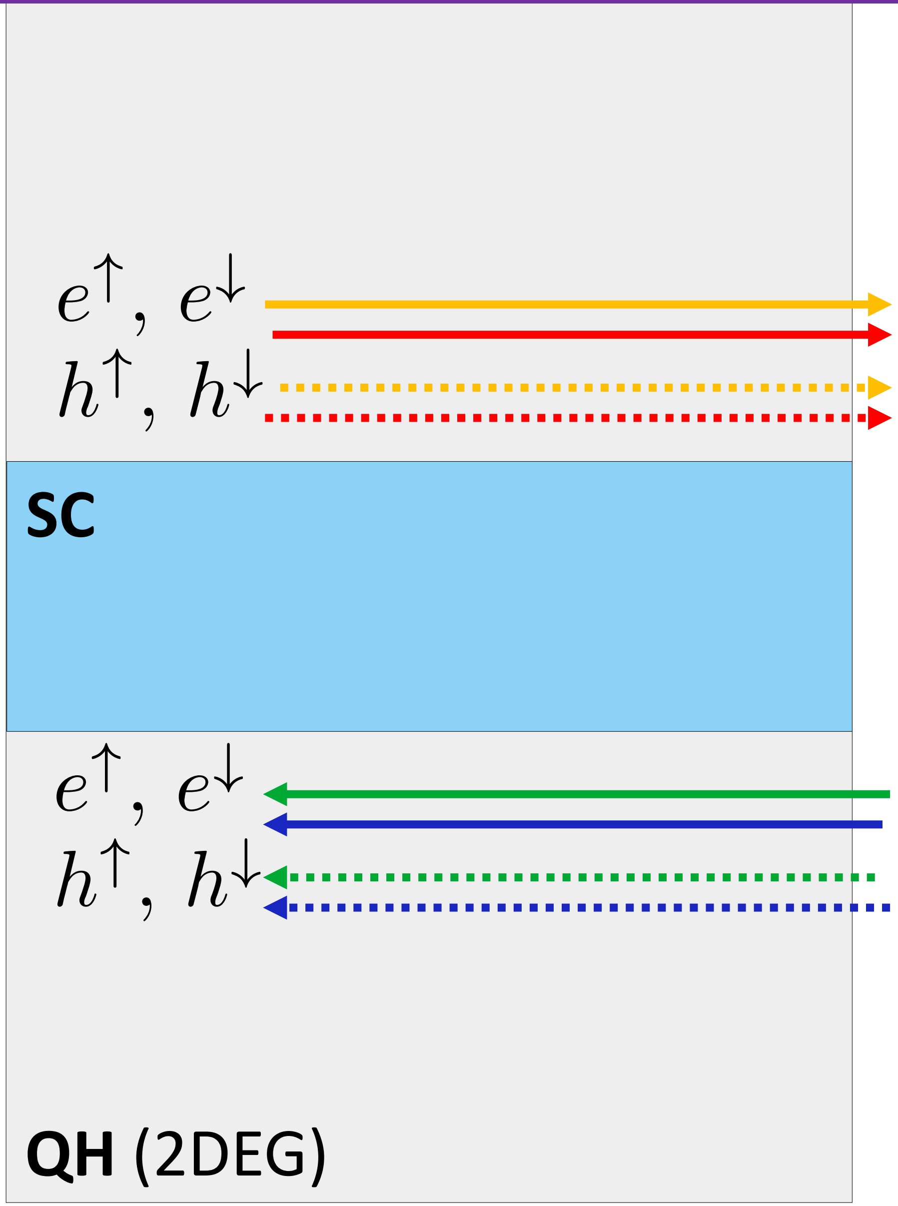


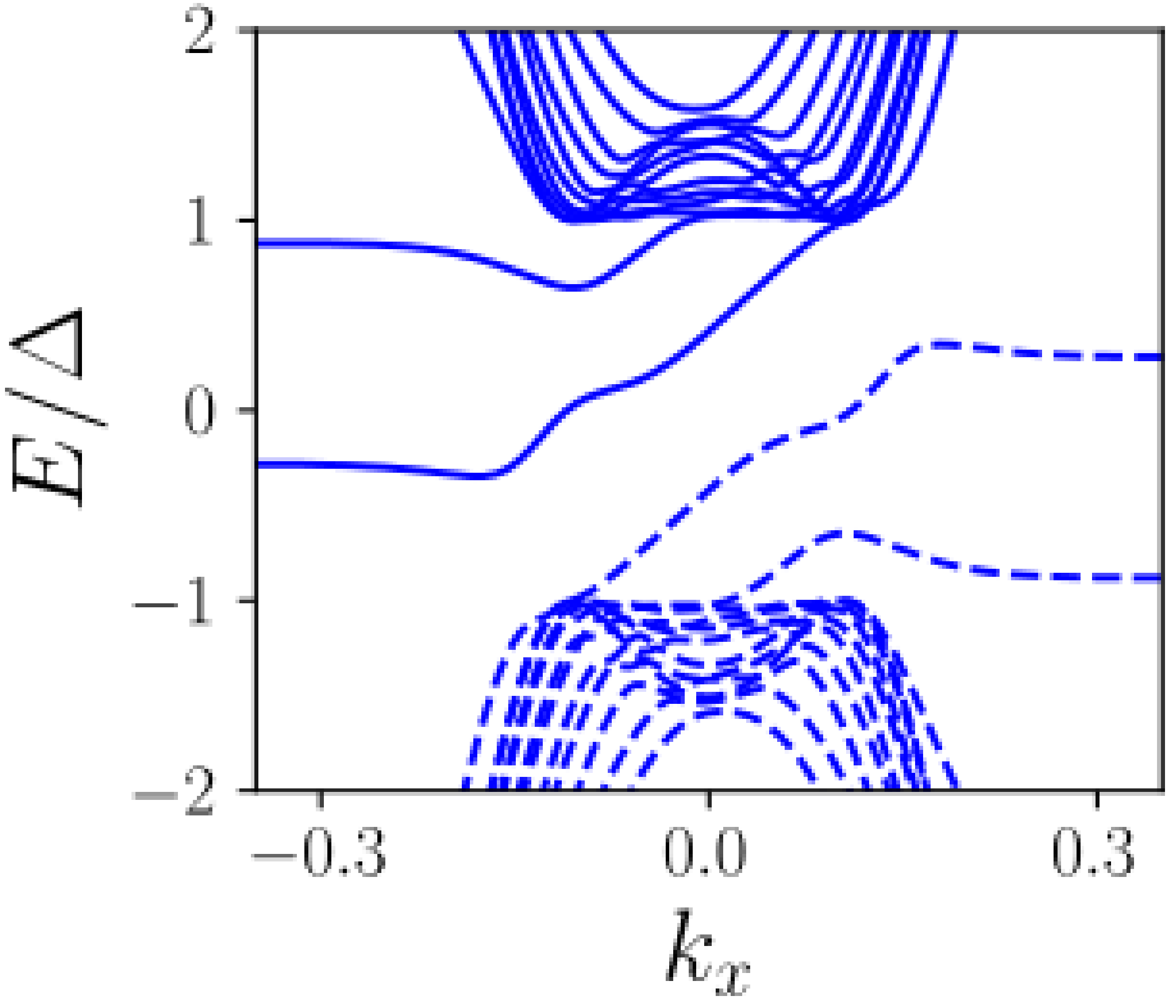
The resonance can be tuned by changing the length of the interface

Stripe geometry

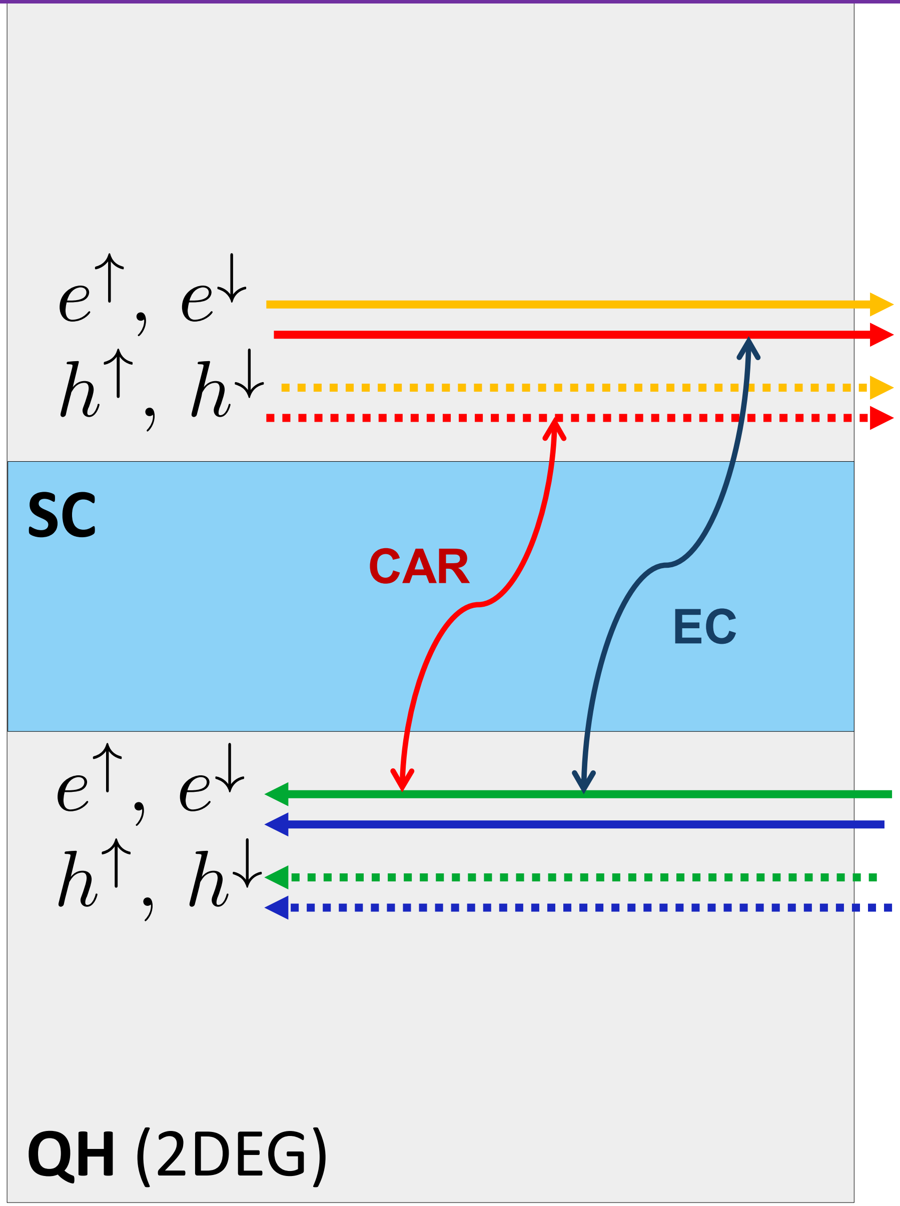


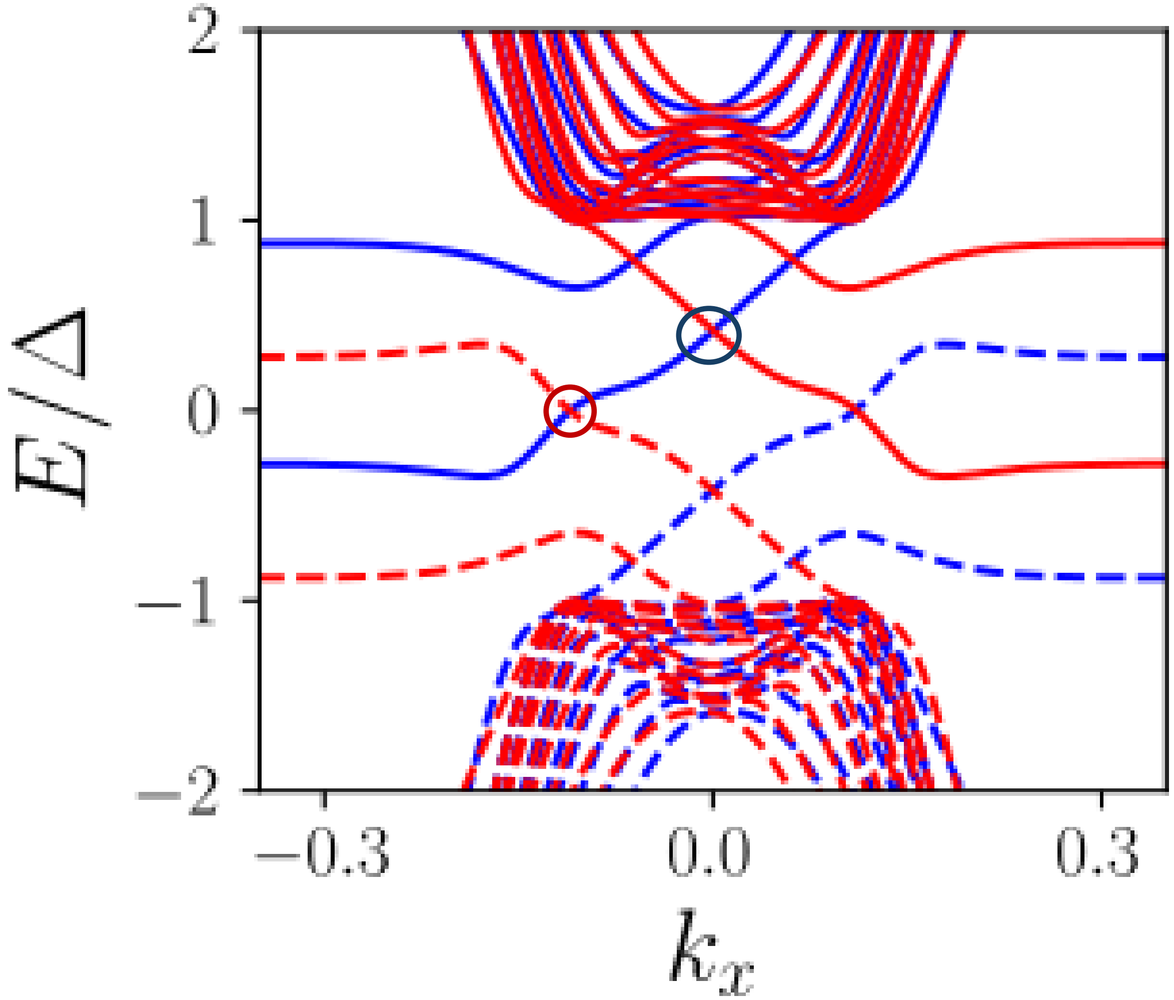
Stripe geometry



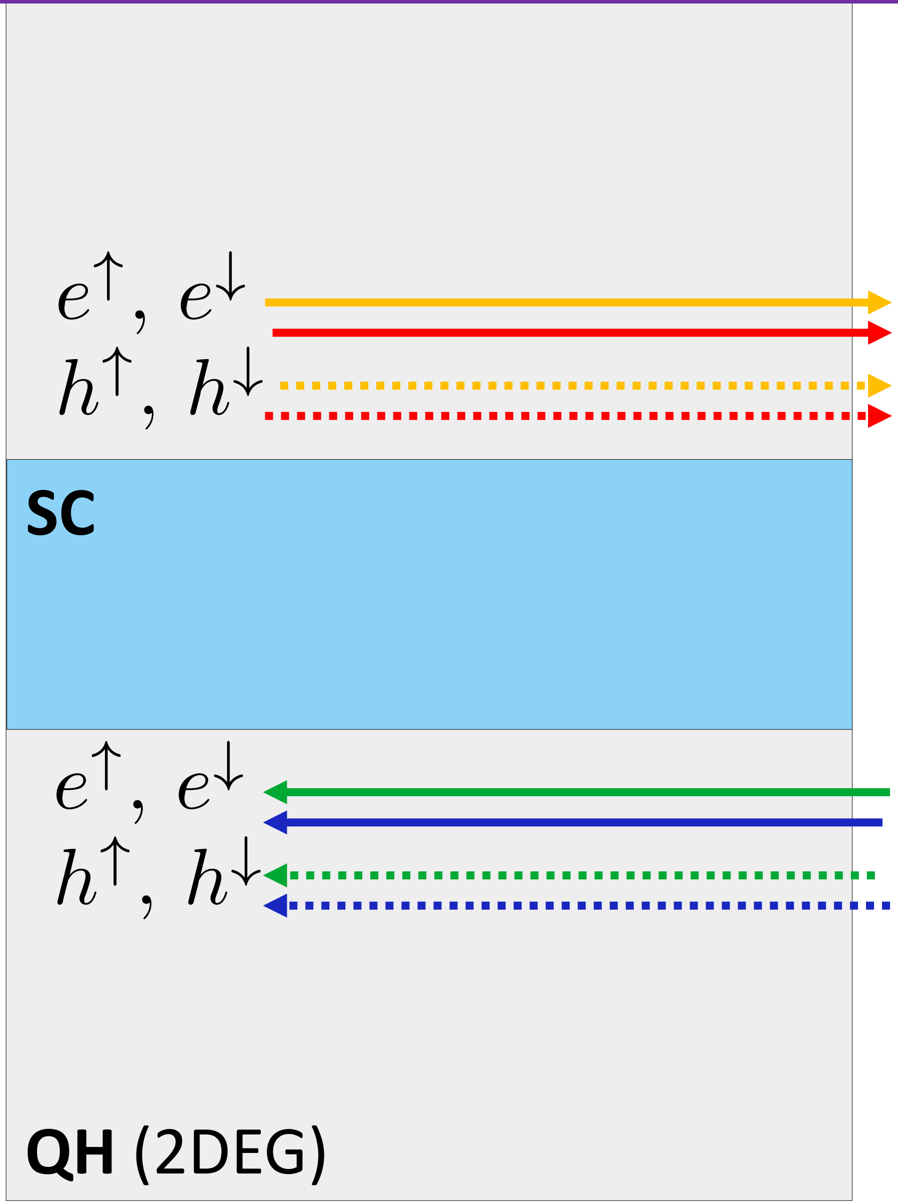


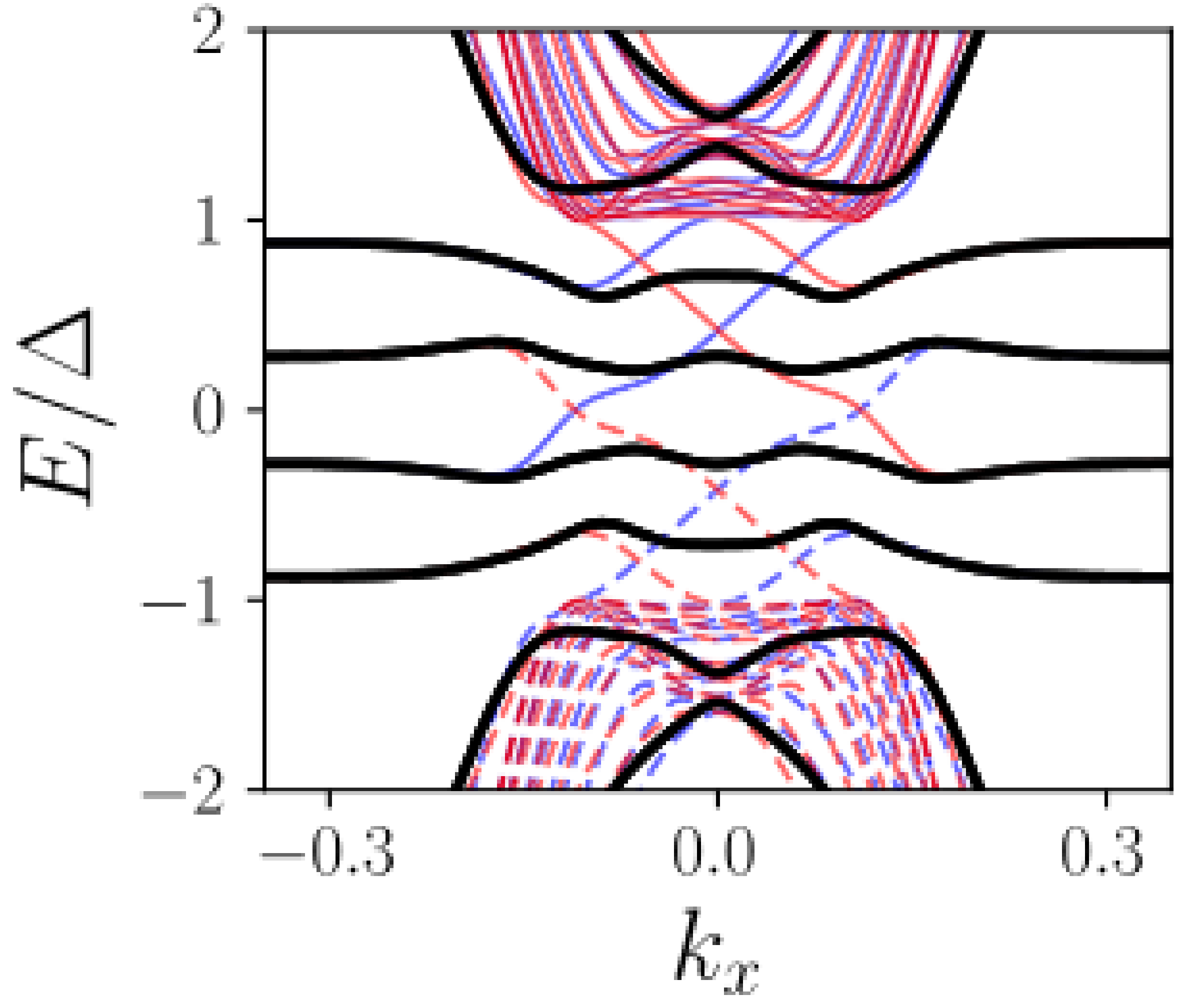
Stripe geometry: coupling L/R CAES



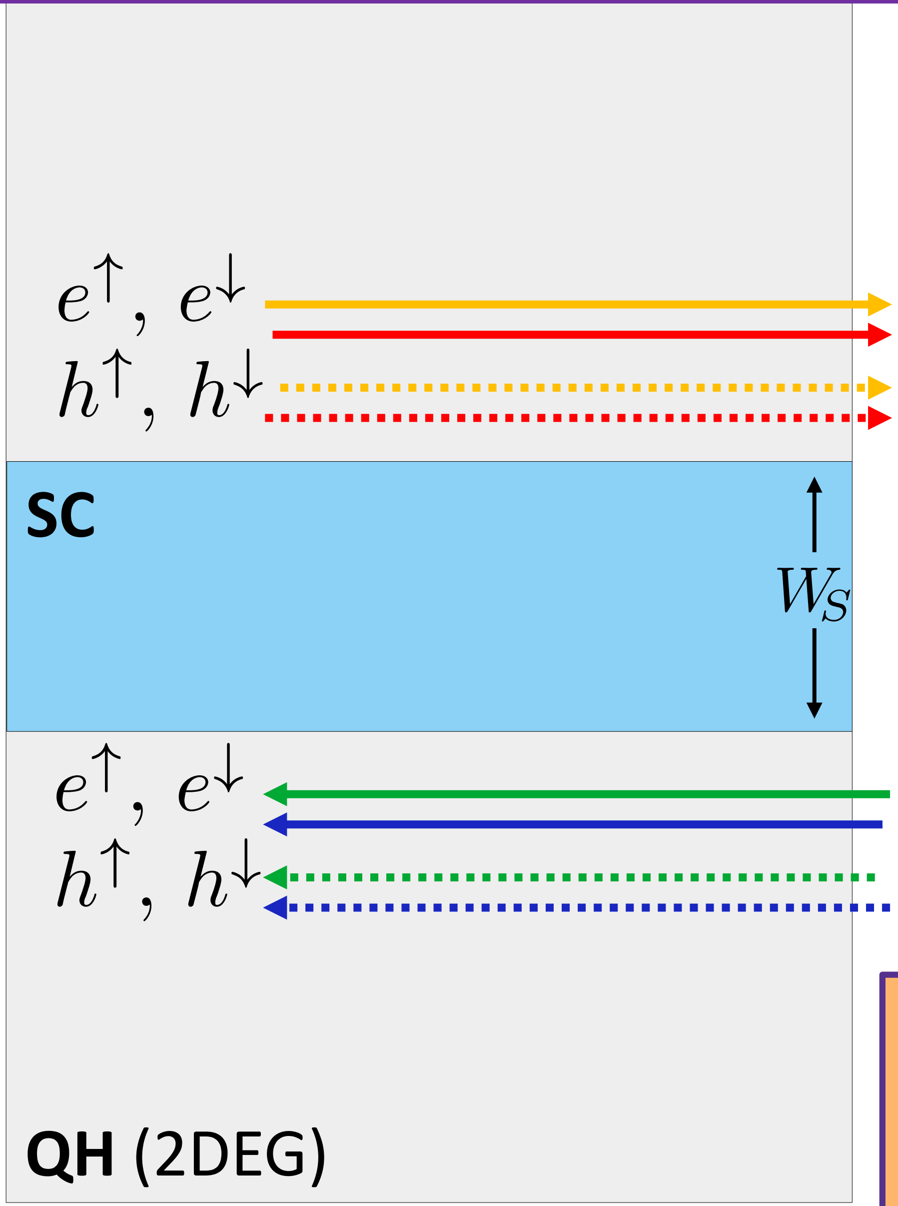


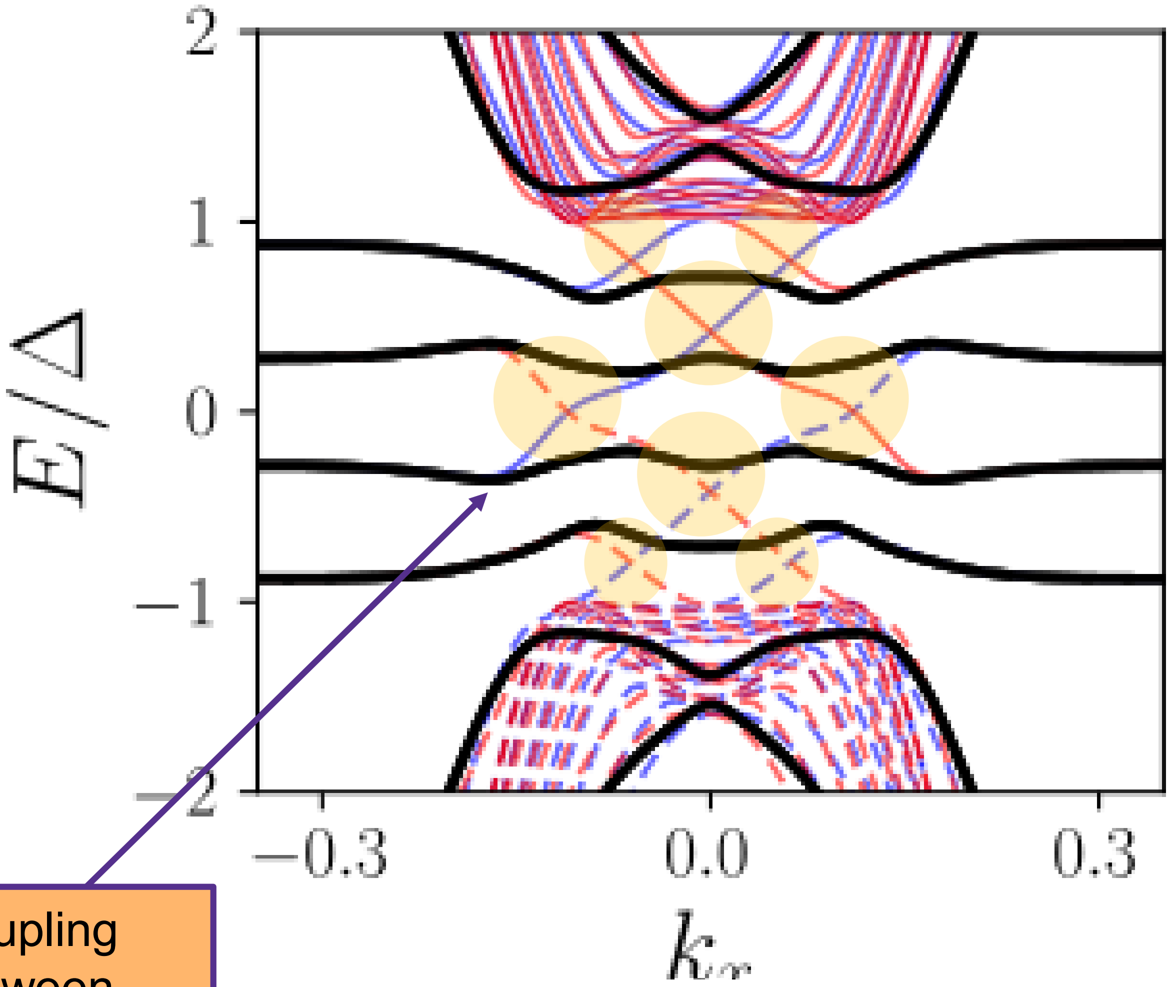
Stripe geometry: hibridized CAES bands





Stripe geometry: emergent topology





Coupling between opposite-side CAESs

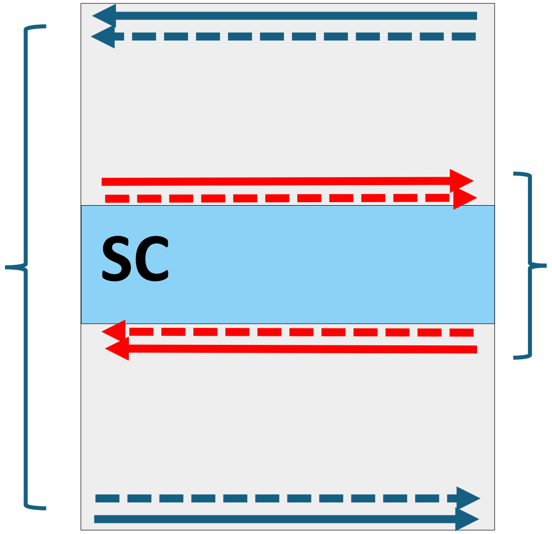
Finocchiaro, Guinea, San-Jose, *PRL* **120**, 116801 (2018)
Kurilovich, Glazman, *Phys Rev X*, **13**, 031027 (2023)
Galambos,...Klinovaja, *Phys Rev B*, **106**, 075410 (2022)

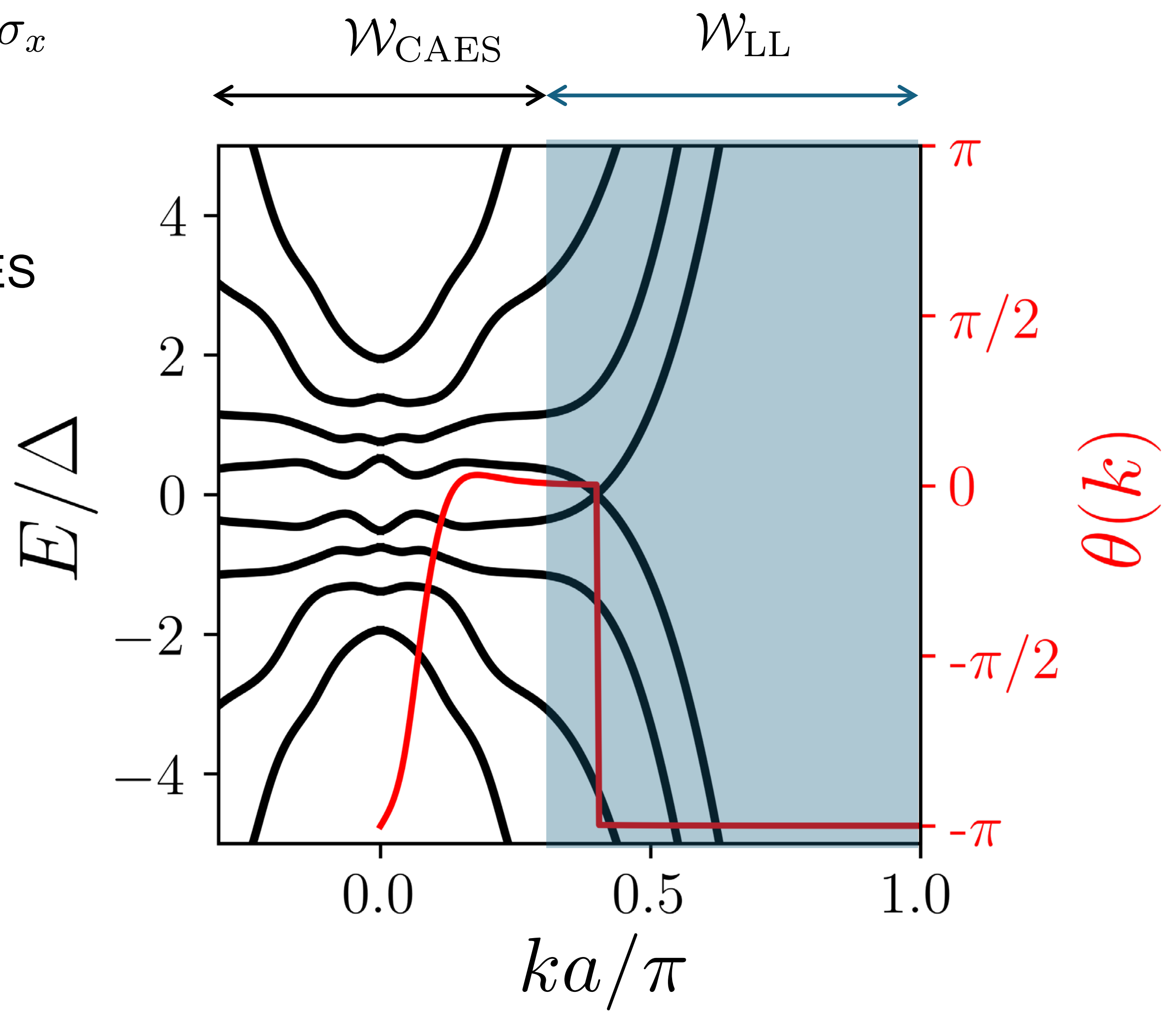
Stripe geometry: topological invariants

$$\mathcal{Q}_{\mathrm{T}} = \mathrm{sign}\left\{\mathrm{Pf}\left[\mathcal{H}(k=0)U_C\right]/\mathrm{Pf}\left[\mathcal{H}(k=\pi/a)U_C\right]\right\}$$ (1D, D class)

Chiu et al, RMP (2016)

$$U_c = \tau_x \sigma_x$$





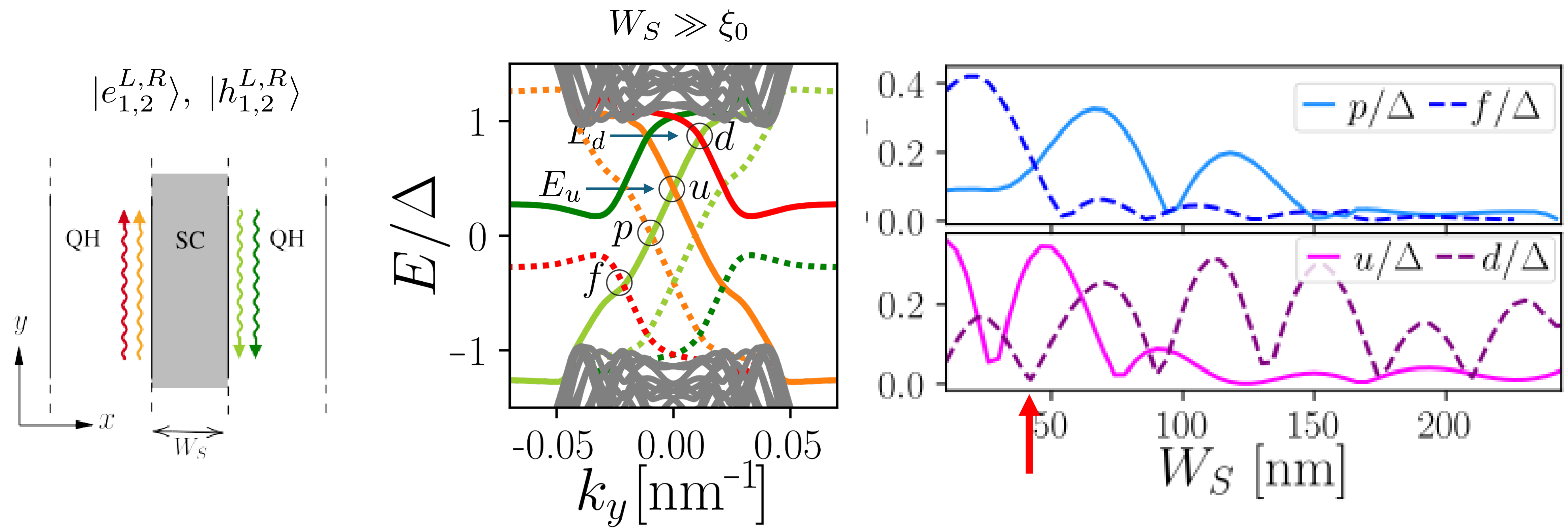
$$\mathcal{Q}_{\mathrm{T}} = \mathrm{e}^{i\pi\mathcal{W}}$$

$$\mathcal{W} = \mathcal{W}_{\mathrm{CAES}} + \mathcal{W}_{\mathrm{LL}}$$

$$\mathcal{Q}_T = \mathcal{Q}_S \mathcal{Q}_N = \mathcal{Q}_S(-1)^{\nu}$$

$$\boxed{\mathcal{Q}_{\mathrm{S}} = \mathrm{sign}\left\{\frac{\mathrm{Pf}\left[\mathcal{H}(k=0)U_C\right]}{\mathrm{Pf}\left[\mathcal{H}(k=\pi/a)U_C\right]}\right\}(-1)^{-\nu}}$$

Stripe geometry: topological invariant in minimal model

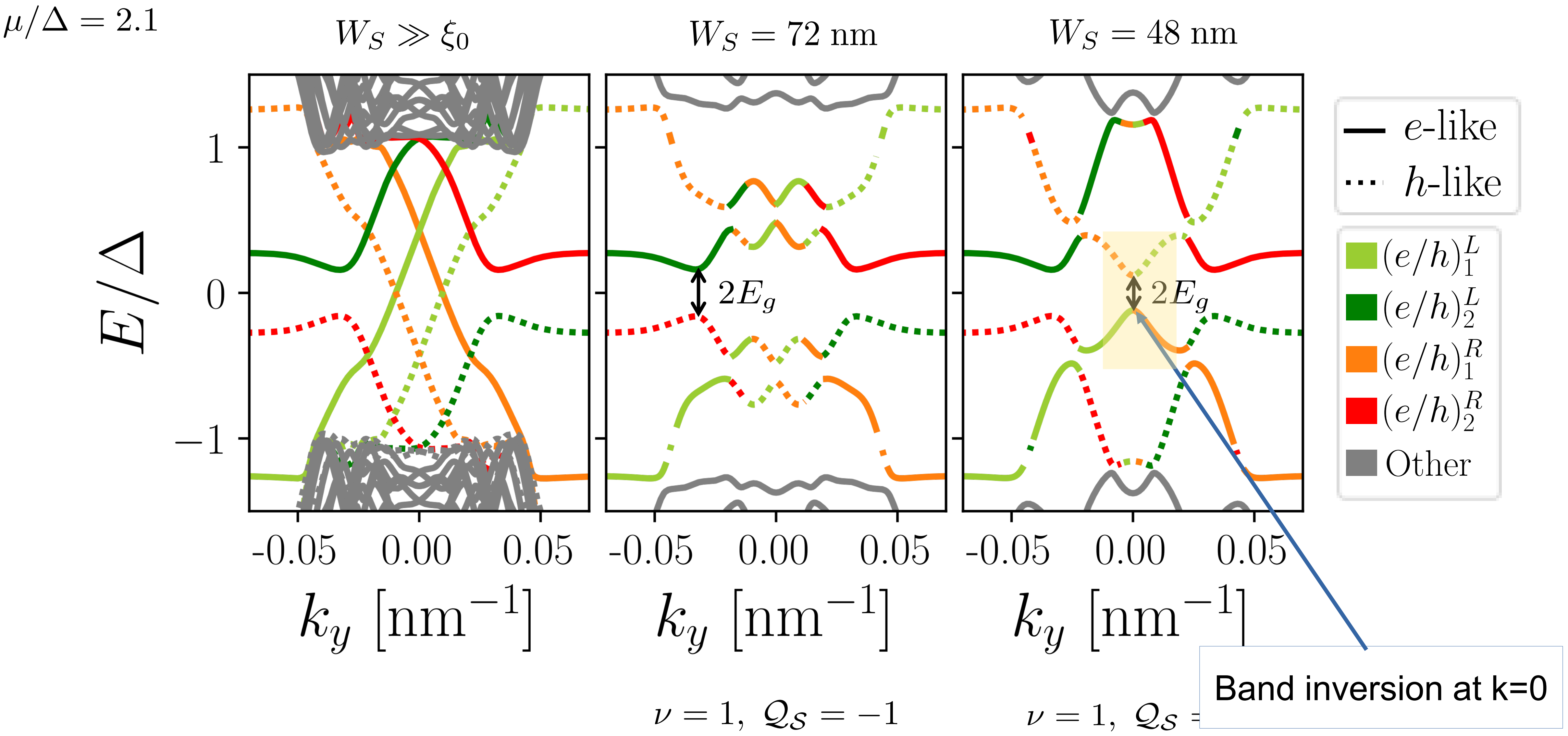


$$\mathcal{Q}_\mathrm{S} = (-1)^\nu\,\mathrm{sign}\left[d^4 - 2d^2\left(E_u E_d + p^2 + u^2\right) + \left(E_d^2 + p^2 - u^2\right)\left(E_u^2 + p^2 - u^2\right)\right]$$

for $d = 0 \rightarrow$ change sign for $E_u^2 < u^2 - p^2$

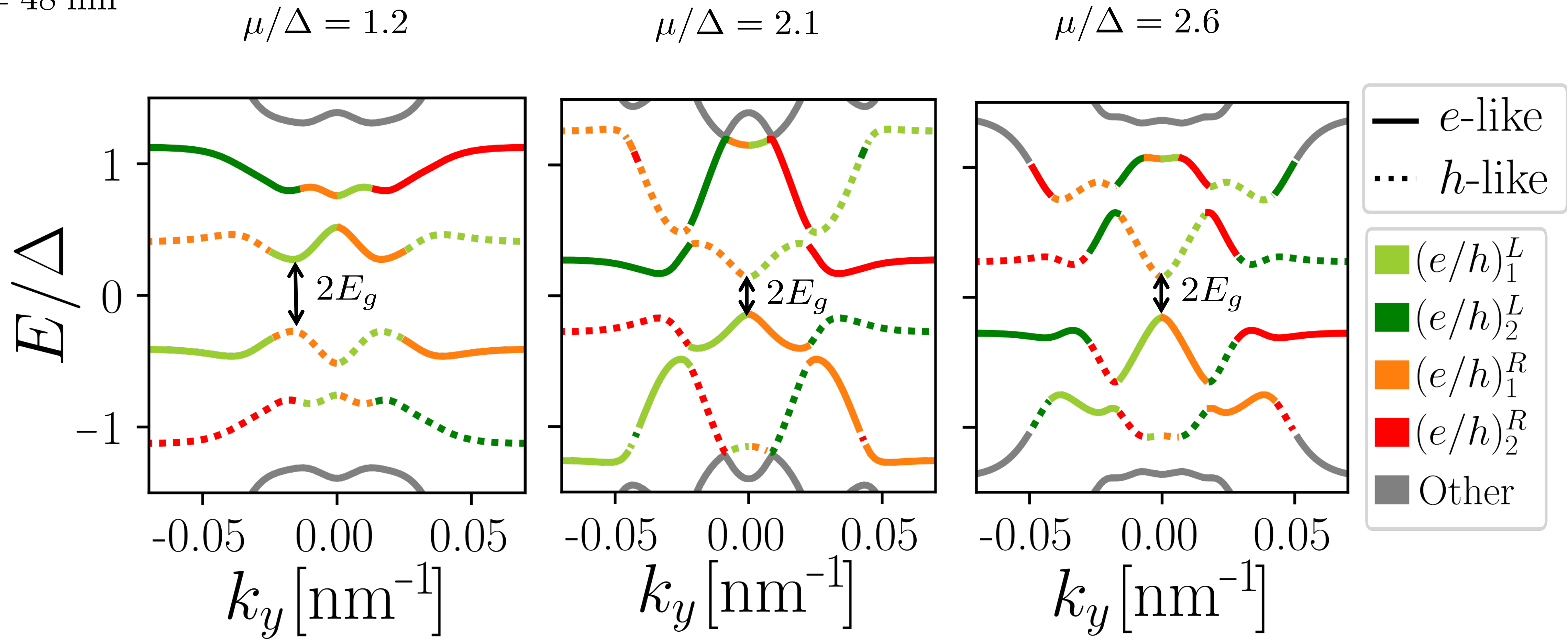
EC/CAR competition!

Stripe geometry: hybridized CAES bands



Stripe geometry: hibridized CAES bands in different phases

$W_S = 48$ nm

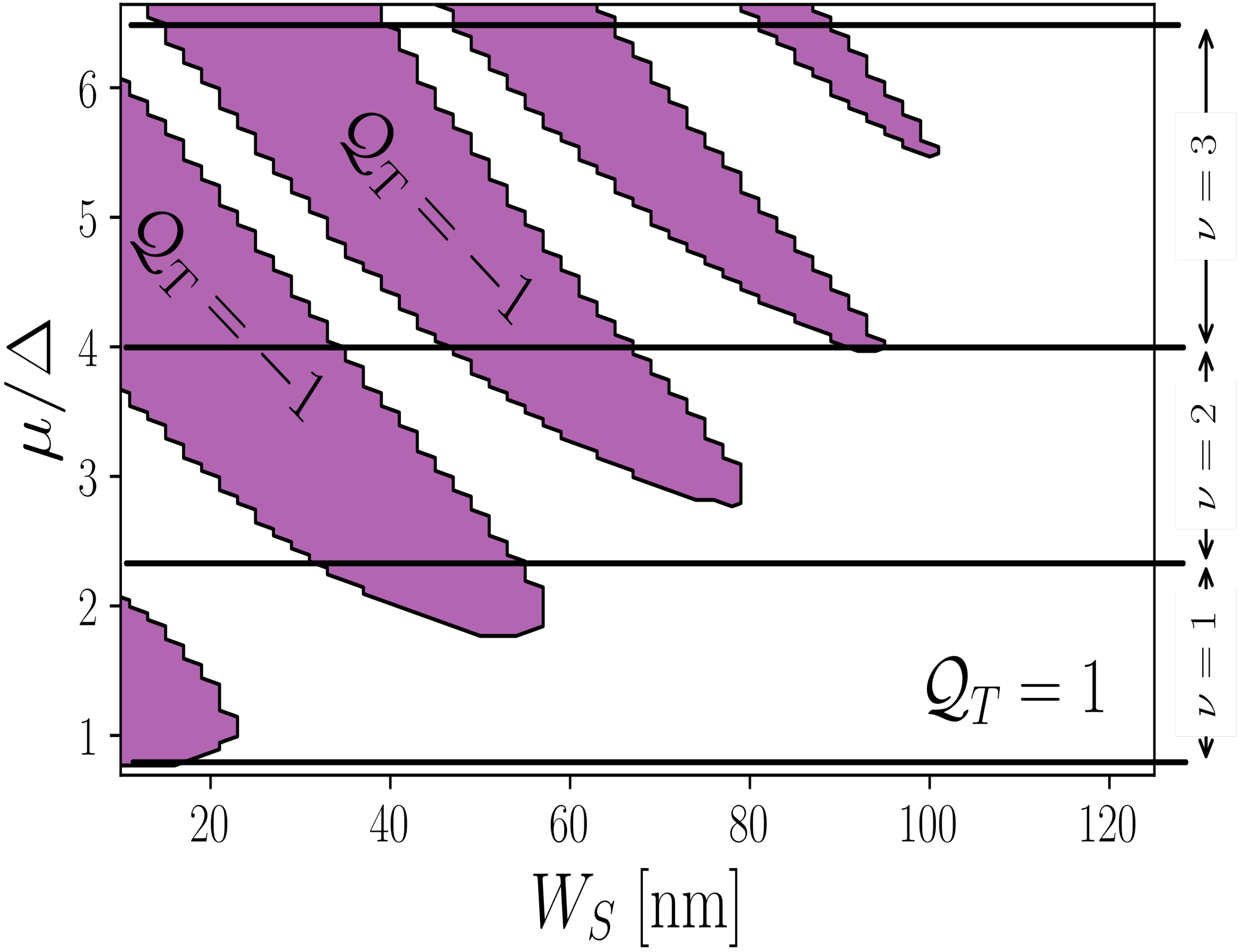


$\nu = 1,\ \mathcal{Q}_S = -1$

$\nu = 1,\ \mathcal{Q}_S = +1$

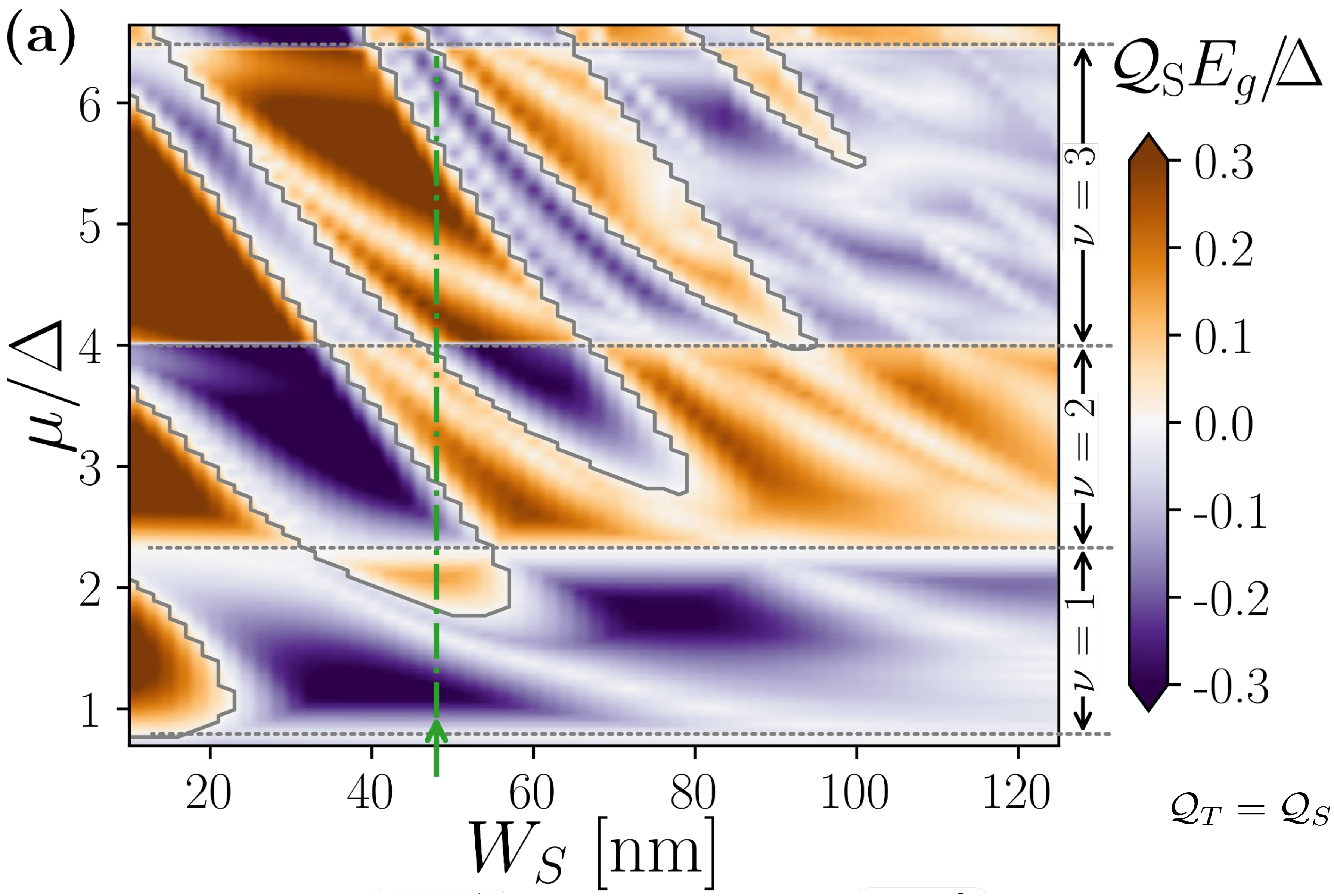
$\nu = 2,\ \mathcal{Q}_S = -1$

Stripe geometry: phase diagram



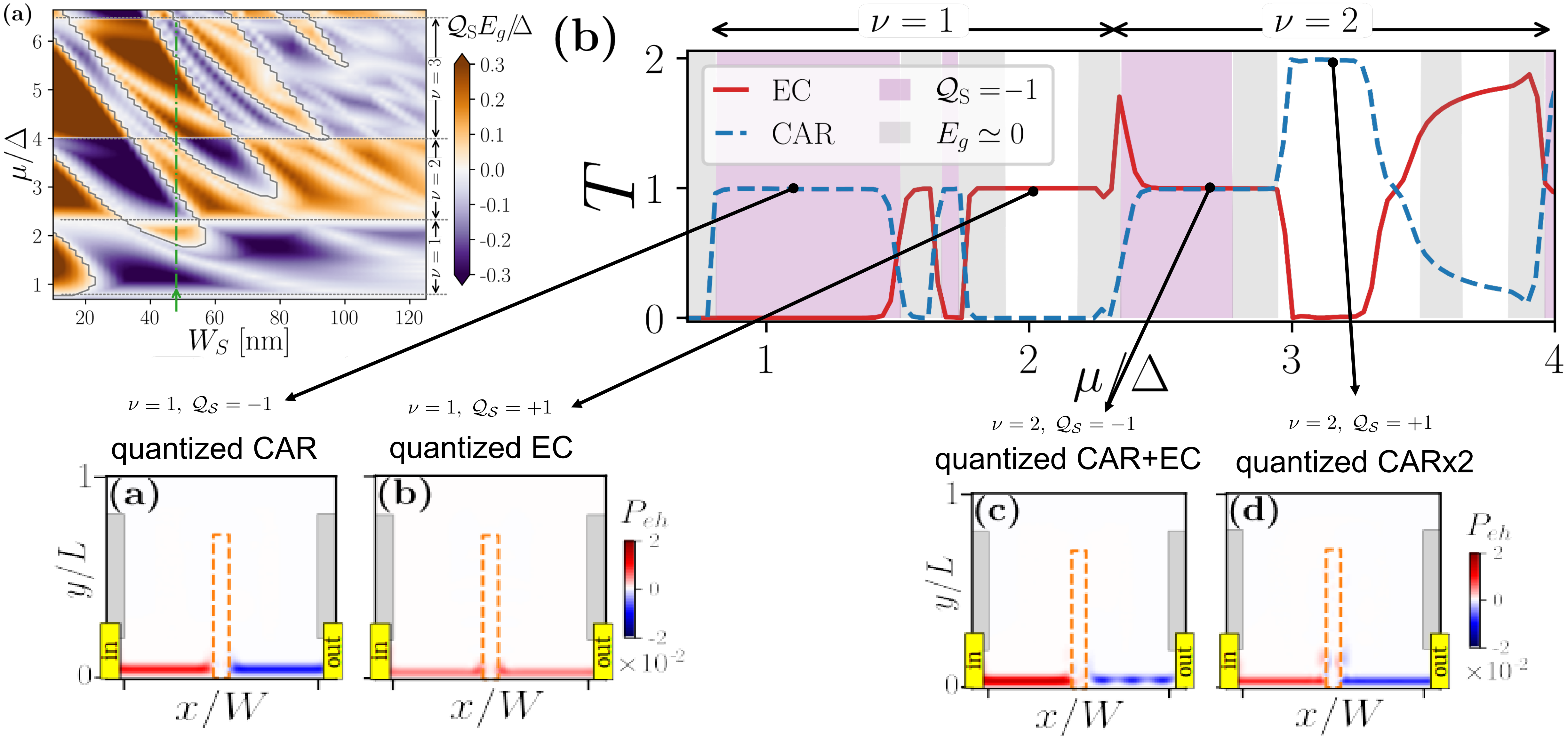
$\mathcal{Q}_T = \mathcal{Q}_S \mathcal{Q}_N = \mathcal{Q}_S(-1)^\nu$

Stripe geometry: phase diagram

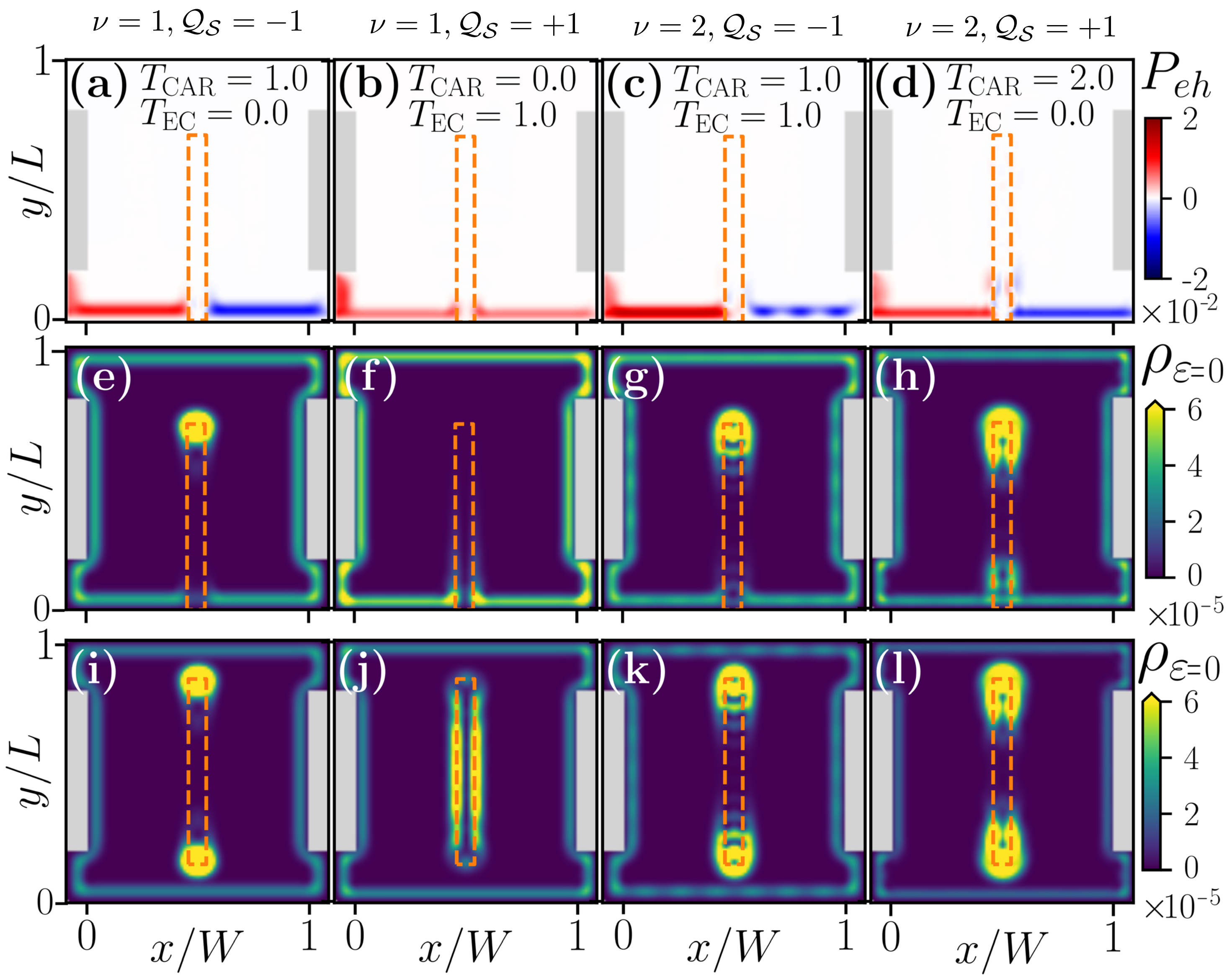


$$\mathcal{Q}_T = \mathcal{Q}_S \mathcal{Q}_N = \mathcal{Q}_S (-1)^\nu$$

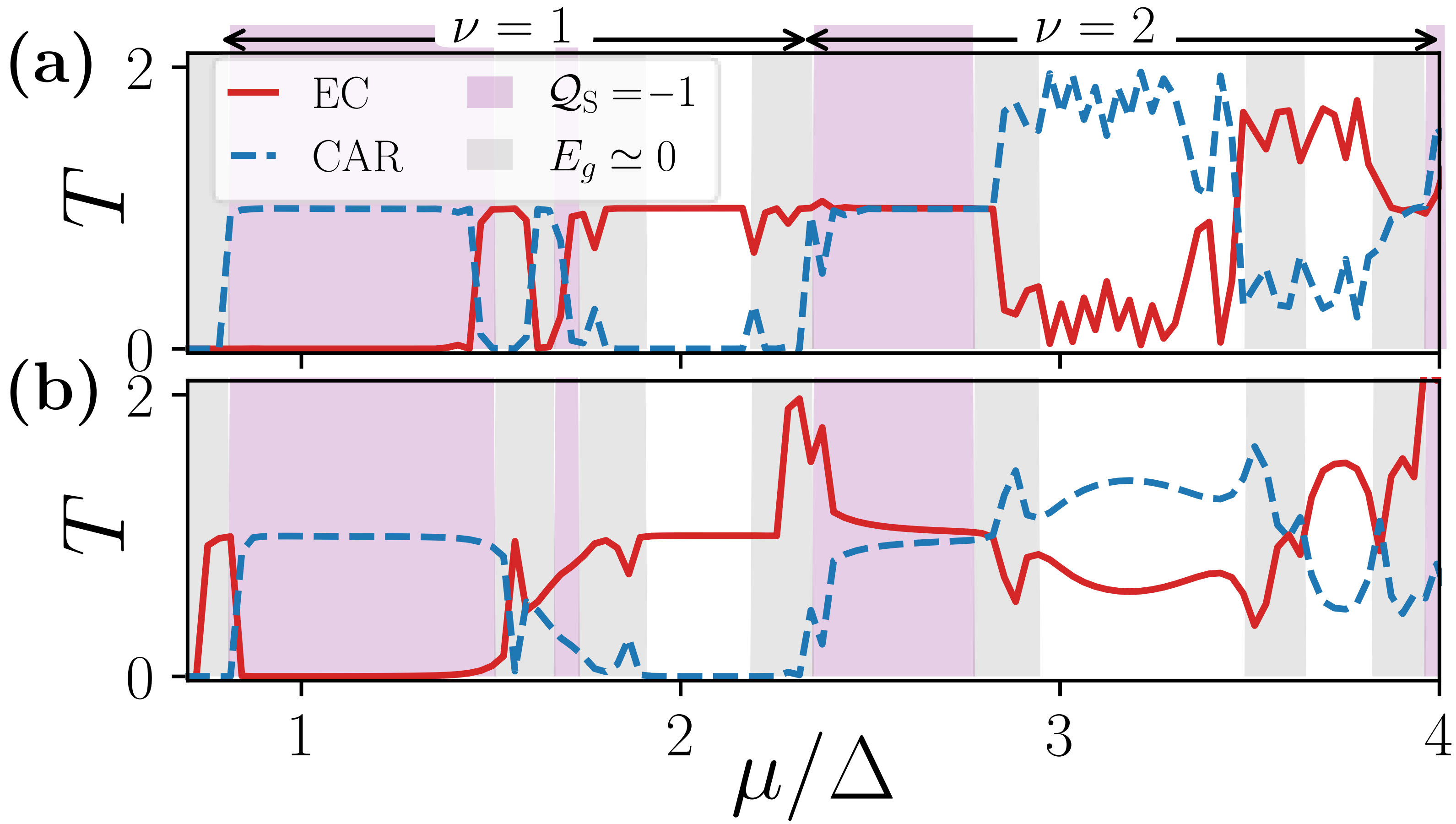
Non-local transport across phase diagram



Transport-LDOS correspondence



Non-local transport: effect of disorder and injection energy

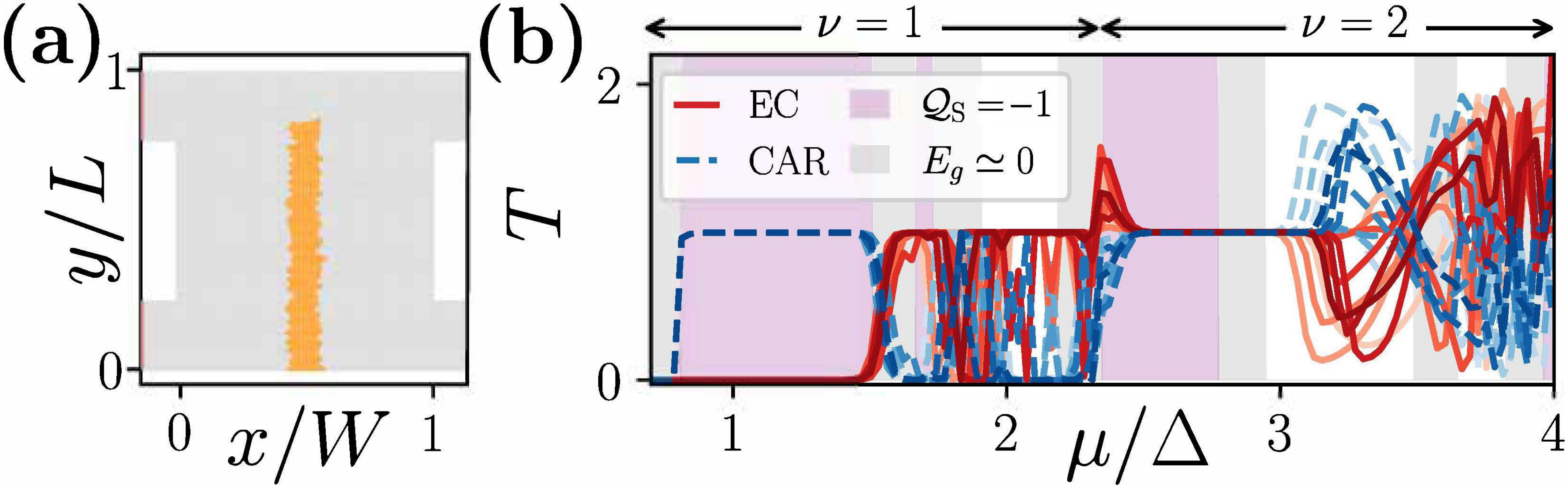


$E_{\mathrm{inj}}/\Delta \simeq 0.0$

+ disorder $\omega_A/\Delta = 10$

$E_{\mathrm{inj}}/\Delta = 0.05$

Non-local transport: effect of surface roughness



Conclusions (1st part):

- **Richer phase diagram due to LL mixing in stripe geometry**
- Peculiar transport signals: quantized EC at $\nu=1$ and quantized EC+CAR at $\nu=2$
- Appearance at low filling factors requires RSOC dominant over Zeeman
- Mixing allows topological phases at higher filling factors, i.e. lower fields

Conclusions (1st part):

- **Richer phase diagram due to LL mixing in stripe geometry**
- **Peculiar transport signals: quantized EC at ν=1 and quantized EC+CAR at ν=2** $\Rightarrow G_{\mathrm{nl}} = 0$

  Excitation of a neutral mode
- Appearance at low filling factors requires RSOC dominant over Zeeman
- Mixing allows topological phases at higher filling factors, i.e. lower fields

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Emergent topology by Landau level mixing in quantum Hall-superconductor nanostructures

Y. Baba, P. Burset and ALY, [arXiv:2507.14074]() and PRL (to be published)

2) Andreev physics and qubits:

Quantum circuits with multiterminal Josephson-Andreev junctions

F.J. Matute-Cañadas, L. Tosi, and ALY, PRX Quantum 5, 020340 (2024)

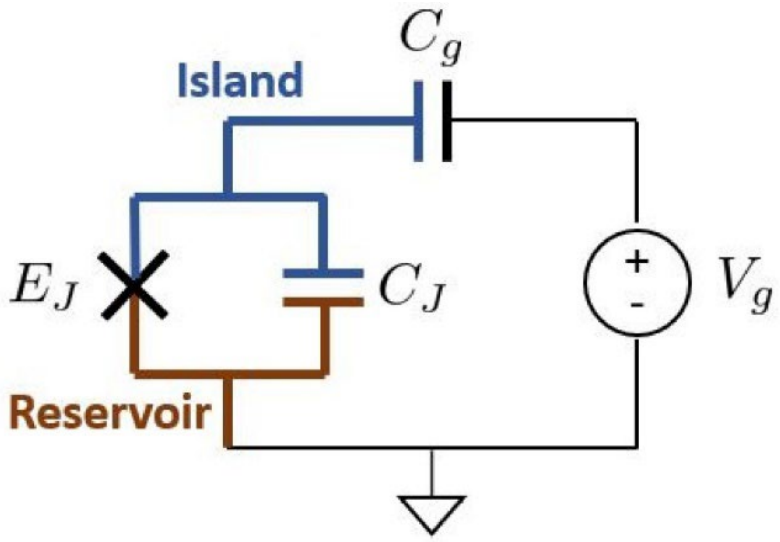
FerBo, a noise resilient qubit hybridizing Andreev and fluxonium states

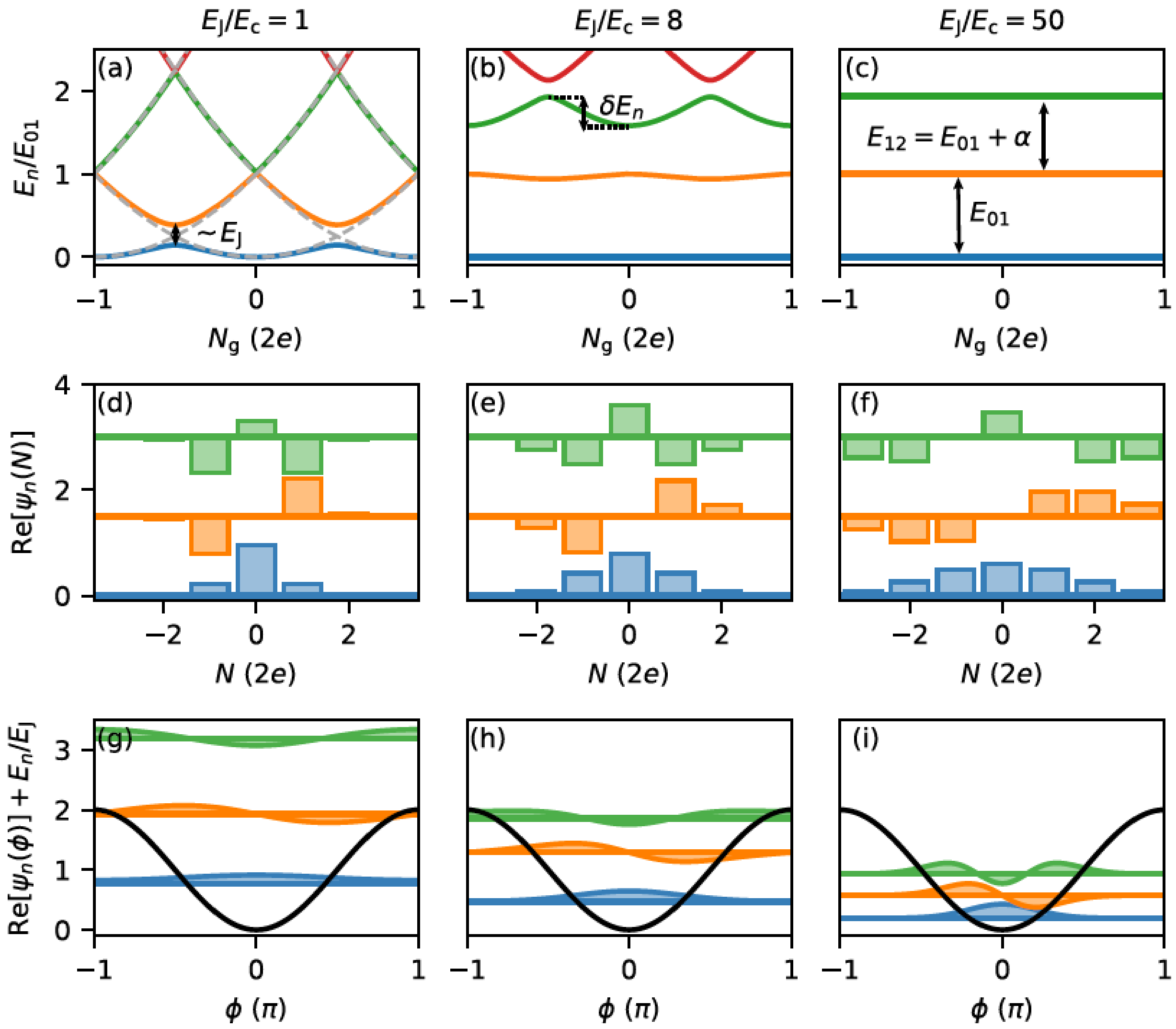
J.J. Cáceres, F.J. Matute-Cañadas, D. Sanz Marco, J. Ortuzar, E. Flurin,
ALY, C. Urbina, H. Pothier, M. Goffman, [arXiv:2604.01145]()

Conventional quantum circuits: from Cooper pair box to transmon qubit

$$H = E_c(\hat{N} - N_g)^2 - E_J \cos \hat{\varphi}$$

$$\left[\hat{\varphi}, \hat{N}\right] = i$$

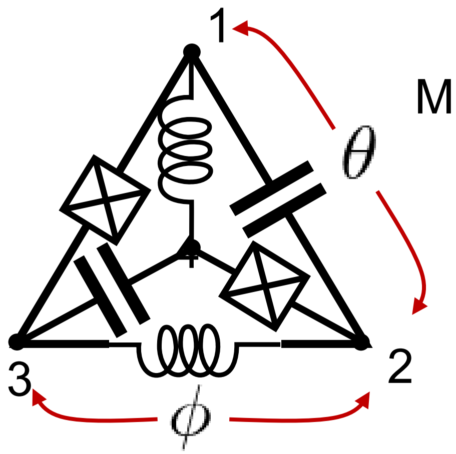




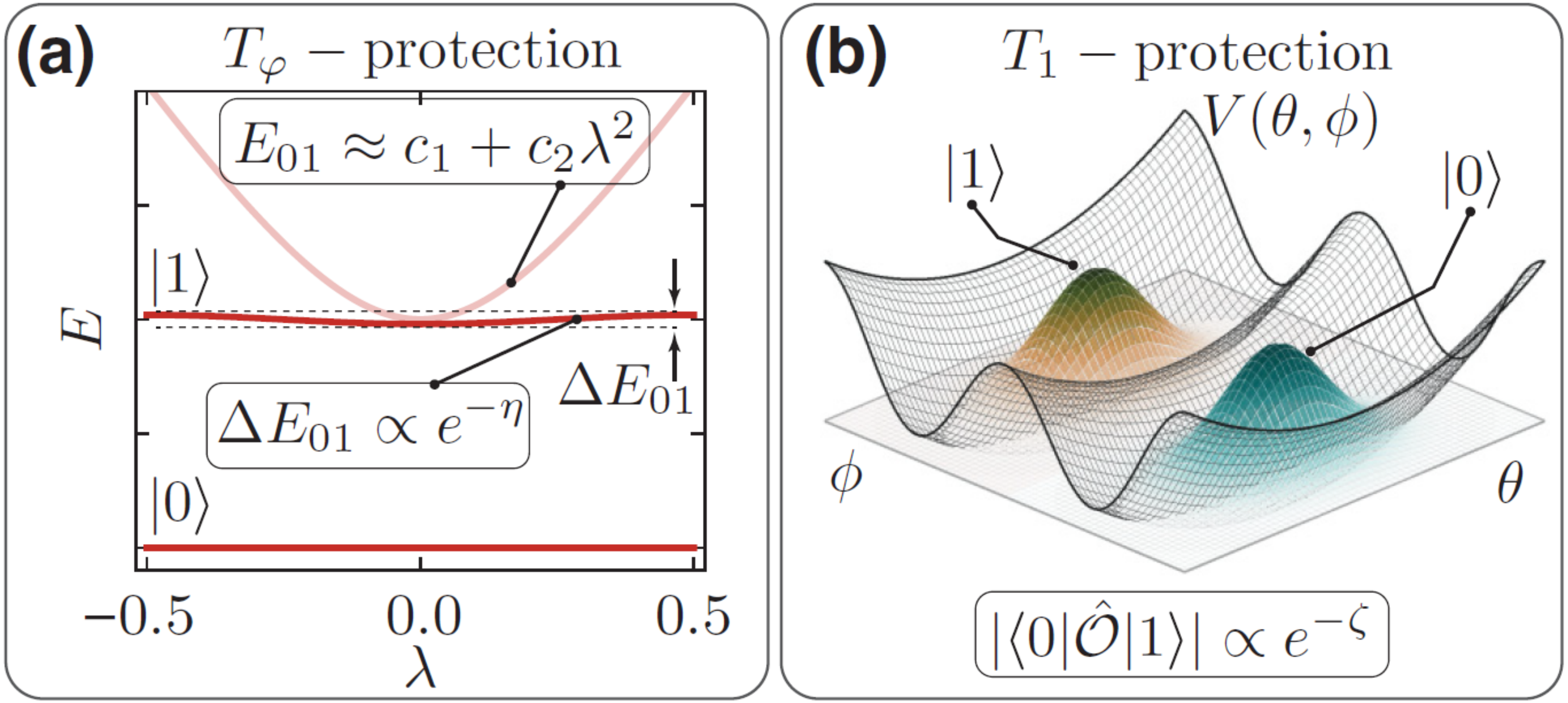
Koch et al., PRA (2007)

A. Bargebos, PhD. Thesis (Delft, 2023)

Conventional protected qubits

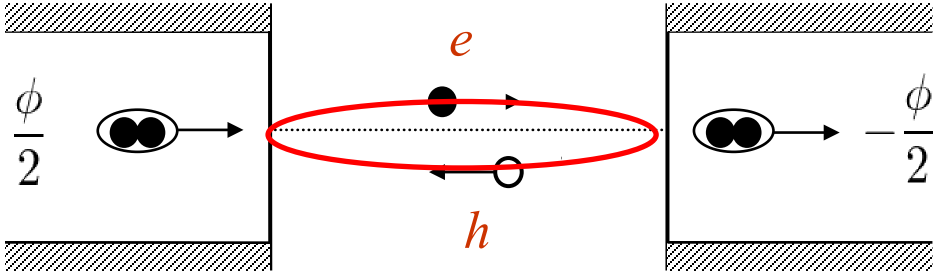


Multimode circuit, e.g. $V(\theta,\phi) = -E_J \cos\theta\cos\phi + E_L\left(\phi - \pi\phi_{\text{ext}}\right)^2$



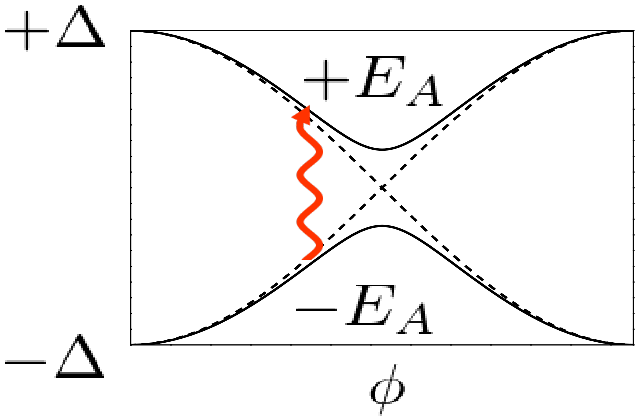
A. Gyenis, PRX QUANTUM **2**, 010339 (2021)

Josephson effect beyond tunnel limit: Josephson-Andreev junctions



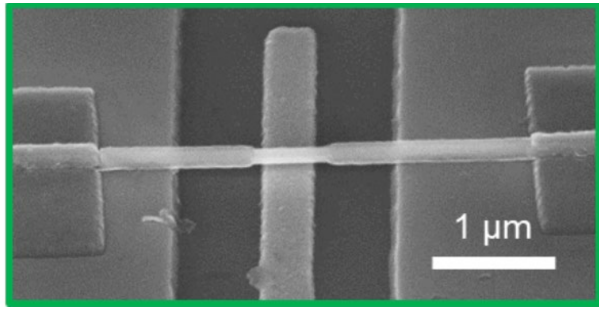
Andreev bound states (**ABS**)

Andreev qubits

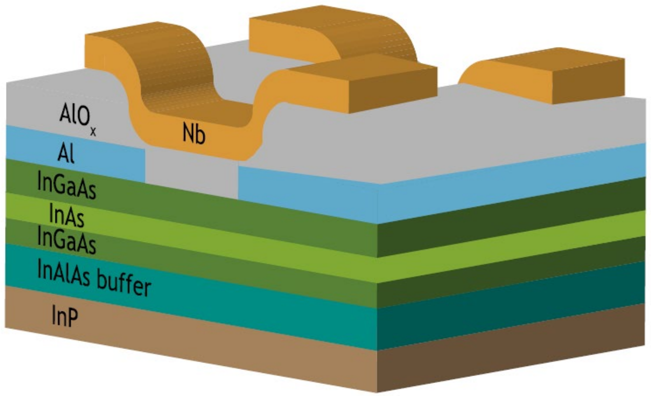


Thy: Despósito (2001), Zazunov (2003), Chtchelkatchev (2003)

Nanowire JAnctions



2DEG JAnctions

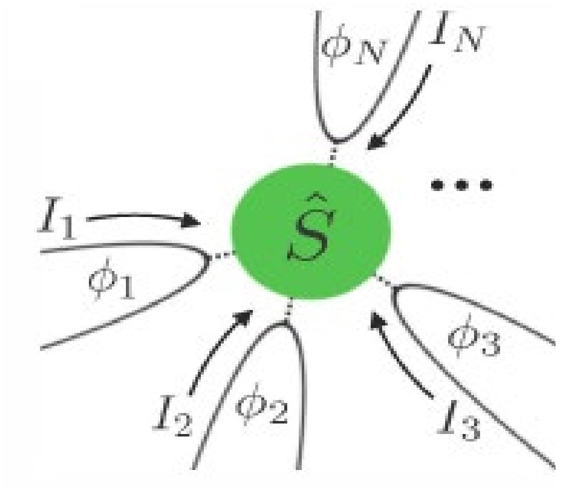


Advantages:
reduced size (compared to transmon)
tunability (gatemons)
tunable coupling *e.g. Pita-Vidal (2024)*

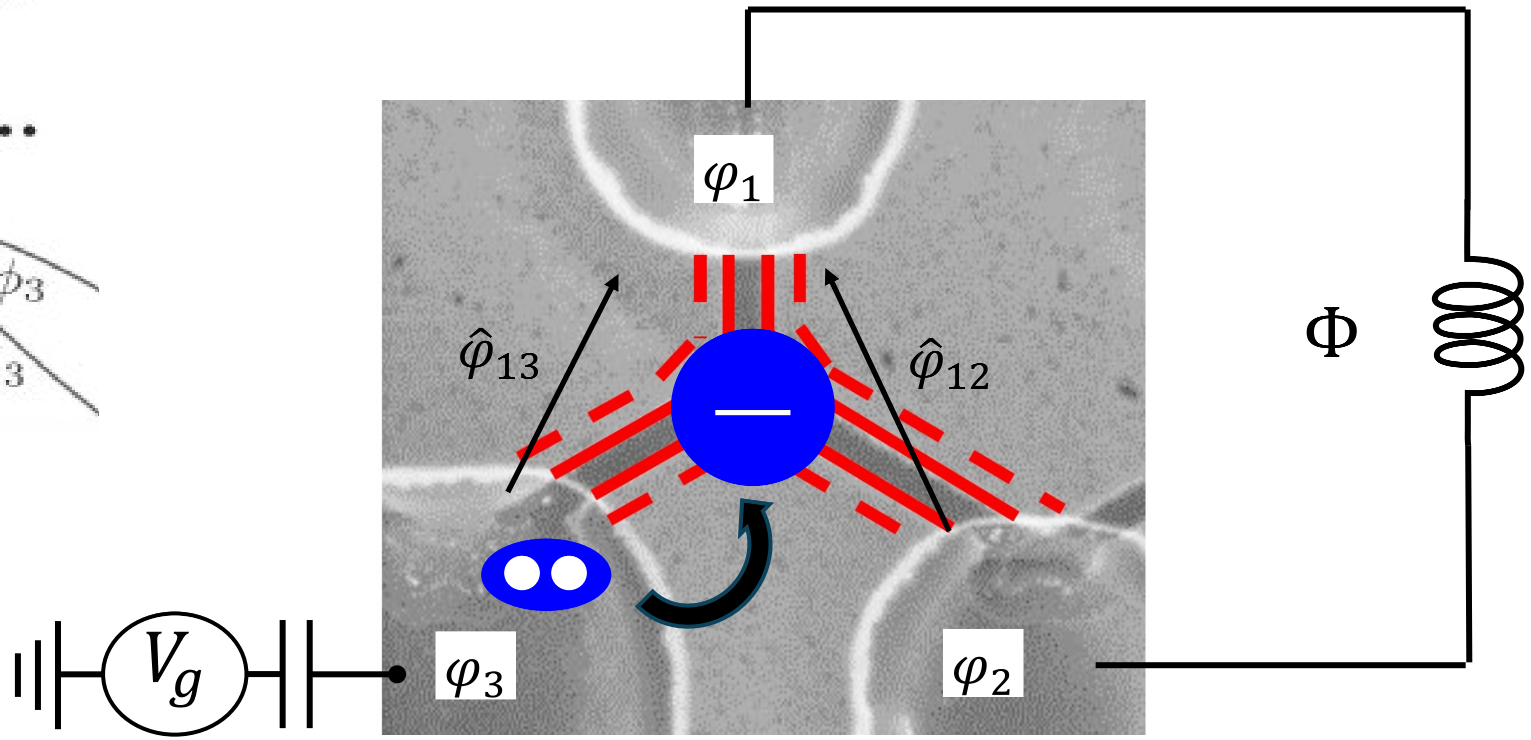
Problems:
Limited coherence $T_2 \sim 50 - 500$ ns

Janvier (2015), Hays (2021)

Multimode circuits based on multiterminal JAnctions?



Riwar et al (2016)



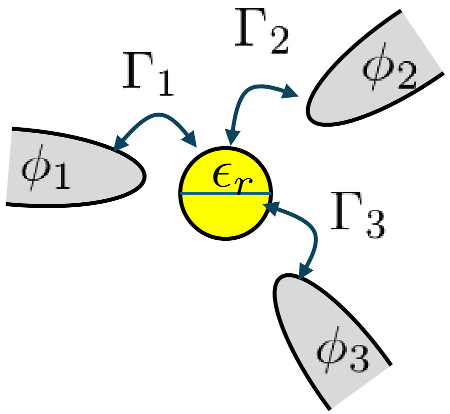
Tunable phases provide extra control parameters

Can be combined with other circuit components

Internal fermionic (Andreev) structure provides additional d.o.f.

Model and circuit configurations

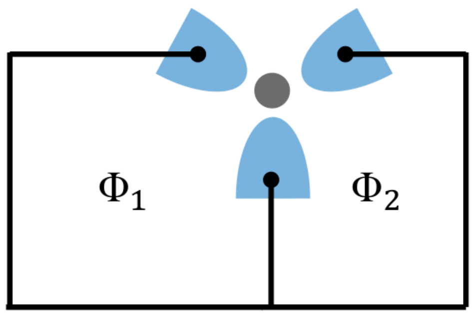
$$H = \sum_{\sigma} \epsilon_r d_\sigma^\dagger d_\sigma + \sum_{k\sigma\nu} t_\nu d_\sigma^\dagger c_{k\sigma\nu} + h.c. + \sum_{\nu} H_\nu$$



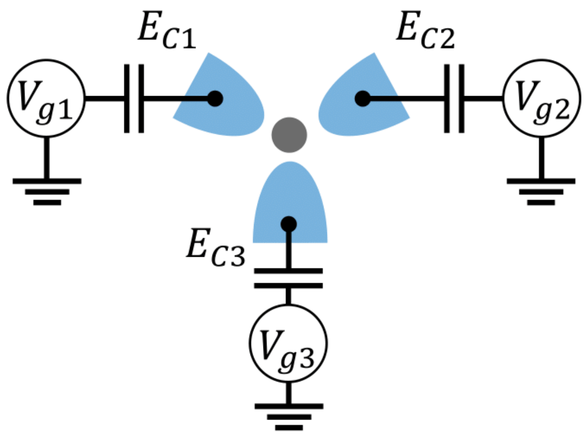
Integrate out leads $\Delta_\nu \to \infty$ $\quad \Gamma_\nu = \frac{t_\nu^2}{W}$

$$H_{\text{eff}} = \sum_{\sigma} \epsilon_r d_\sigma^\dagger d_\sigma + \sum_{\nu} \Gamma_\nu e^{-i\phi_\nu} d_\uparrow^\dagger d_\downarrow^\dagger + \text{h.c.}$$

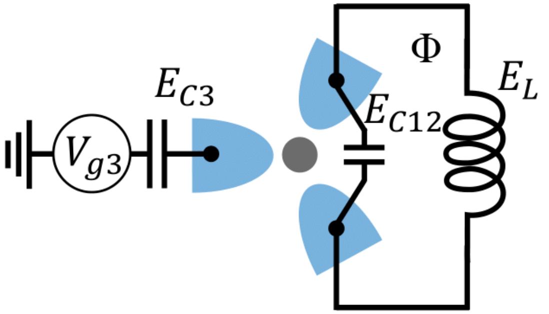
(d) Classical phase-biasing limit



(e) Charge-island configuration



(f) Charge-Flux hybrid configuration

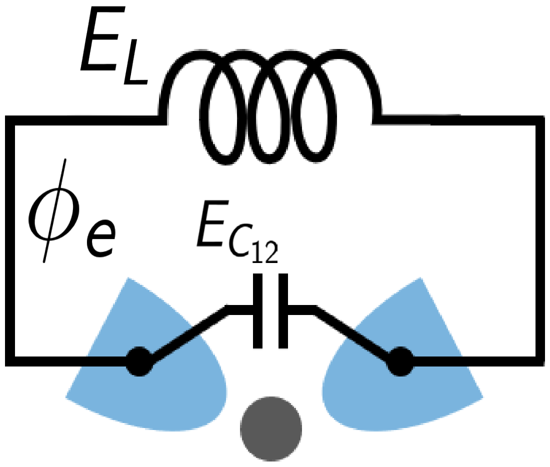


JAnction in a loop with large inductance

Kinetic term

$$H_C = E_{C_{12}} \hat{N}_{12}^2$$

$N_{12} = N_1 - N_2$ charge imbalance between the leads



Potential energy

$V(\hat{\phi}_{12})$

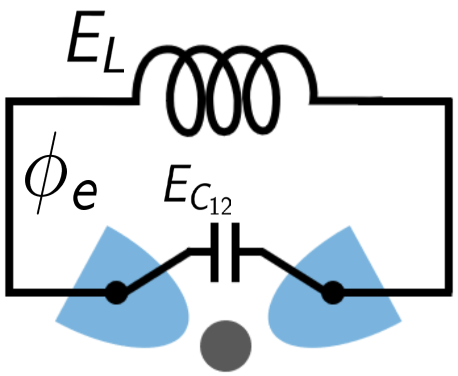
$$H_L = E_L(\hat{\phi}_{12} - \phi_e)^2 \quad E_L = \frac{\Phi_0^2}{2L}$$

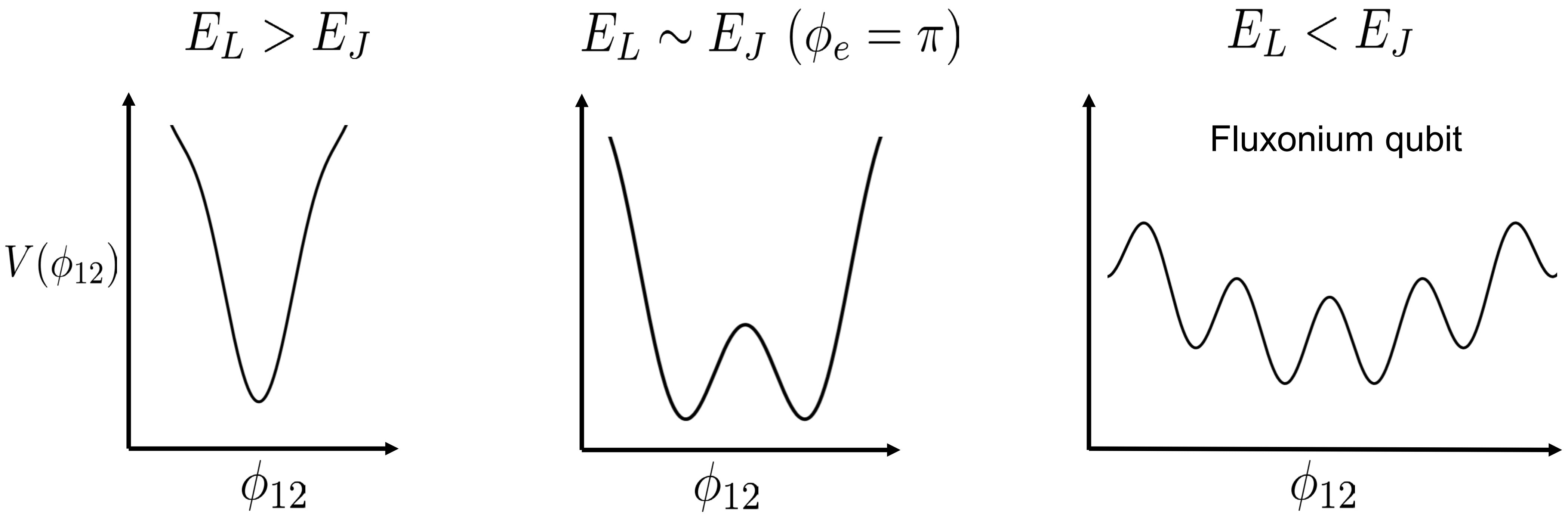
$$H_J = \Gamma_1 e^{-i\hat{\phi}_{12}/2} d_\uparrow^\dagger d_\downarrow^\dagger + \Gamma_2 e^{-i\hat{\phi}_{12}/2} d_\downarrow d_\uparrow + h.c. + \sum_\sigma \epsilon_r d_\sigma^\dagger d_\sigma$$

$$-E_J \cos \hat{\phi}_{12}$$ (tunnel limit)

JAnction in a loop with large inductance

Tunnel limit $$V(\hat{\phi}_{12}) = E_L(\hat{\phi}_{12} - \phi_e)^2 - E_J \cos\hat{\phi}_{12}$$



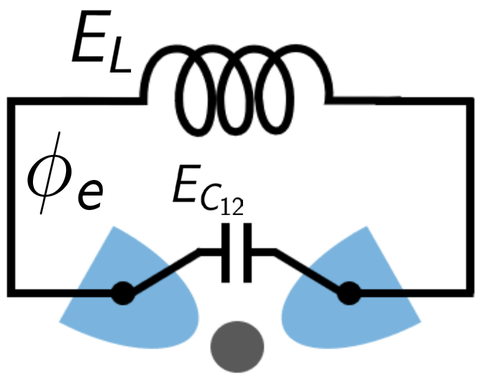


Manucharyan Science 09'

Randeria, PRX Quantum 2024 $T_\phi \sim 100\mu s$

JAnction in a loop with large inductance

Beyond tunnel limit (when Andreev structure plays a role)



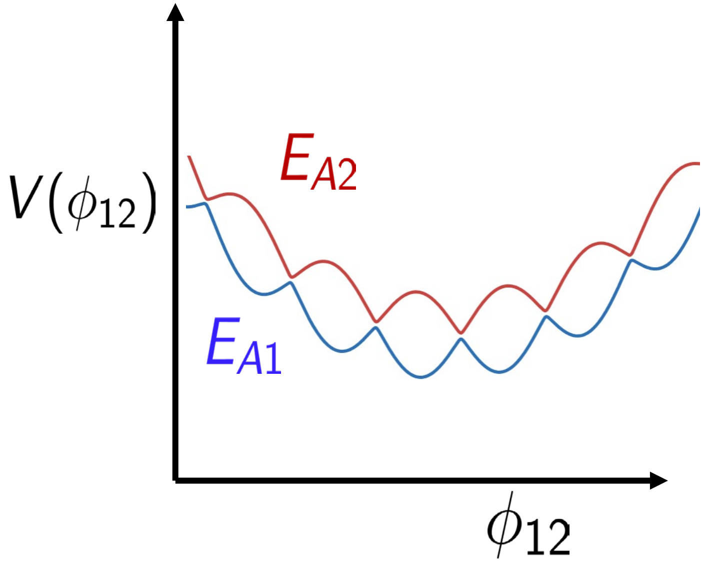
$$V(\hat{\phi}_{12}) = E_L(\hat{\phi}_{12} - \phi_e)^2 + \Gamma_1 e^{-i\hat{\phi}_{12}/2} d_\uparrow^\dagger d_\downarrow^\dagger + \Gamma_2 e^{-i\hat{\phi}_{12}/2} d_\downarrow d_\uparrow + h.c. + \sum_\sigma \epsilon_r d_\sigma^\dagger d_\sigma$$

For even parity this is a matrix in the space $\{|0\rangle, |\uparrow\downarrow\rangle\}$

$$\Gamma = \Gamma_1 + \Gamma_2$$
$$\delta\Gamma = \Gamma_1 - \Gamma_2$$

$$\Gamma \cos(\hat{\phi}_{12}/2)\hat{\sigma}_x - \delta\Gamma \sin(\hat{\phi}_{12}/2)\hat{\sigma}_y + \epsilon_r \hat{\sigma}_z$$

$E_L < \Gamma$



Perfect transmission limit

$\delta\Gamma, \epsilon_r \to 0$

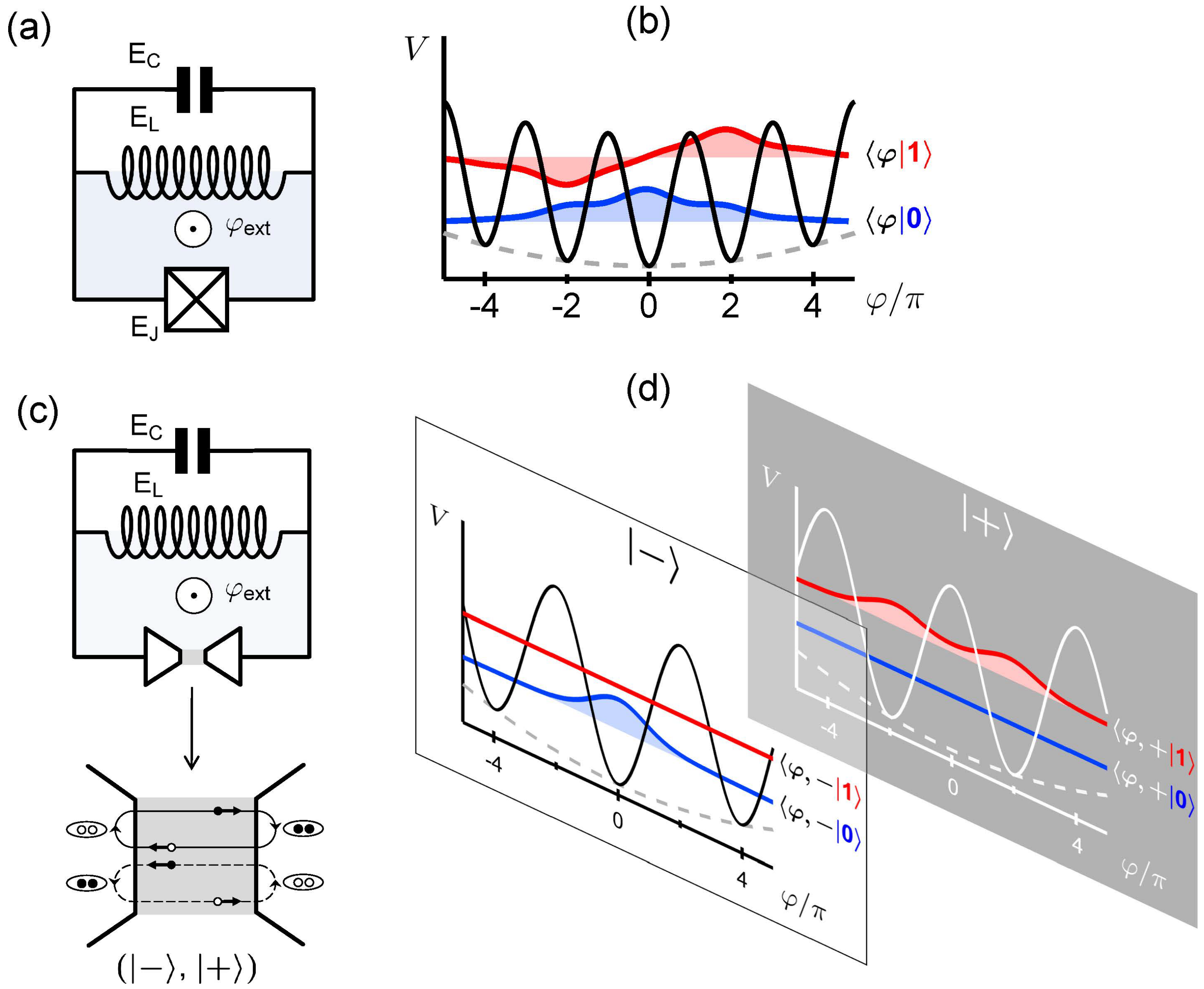
$$V(\hat{\phi}_{12}) \to E_L(\hat{\phi}_{12} - \phi_e)^2 \pm \Gamma \cos(\hat{\phi}_{12}/2)$$

The FERBO qubit: principle of protection

Large inductance quantum circuits

Tunnel limit:
Fluxonium qubit

Ballistic limit:
FERBO qubit

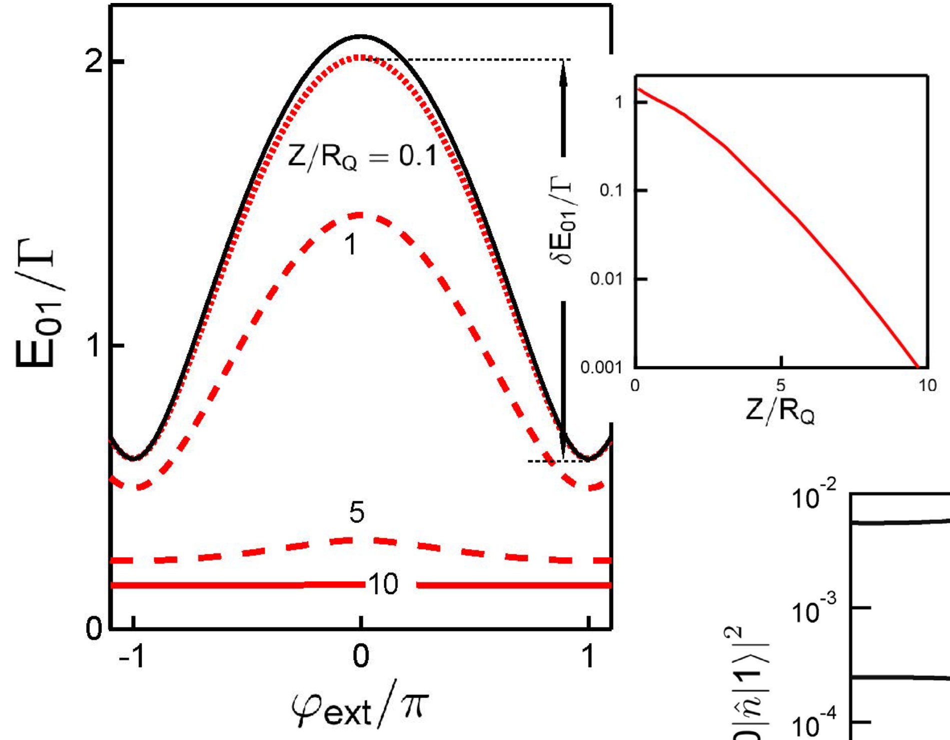


J.J. Cáceres et al, arXiv:2604.01145

FERBO qubit: dephasing and relaxation protection

→ Suppression of flux dispersión with increasing impedance

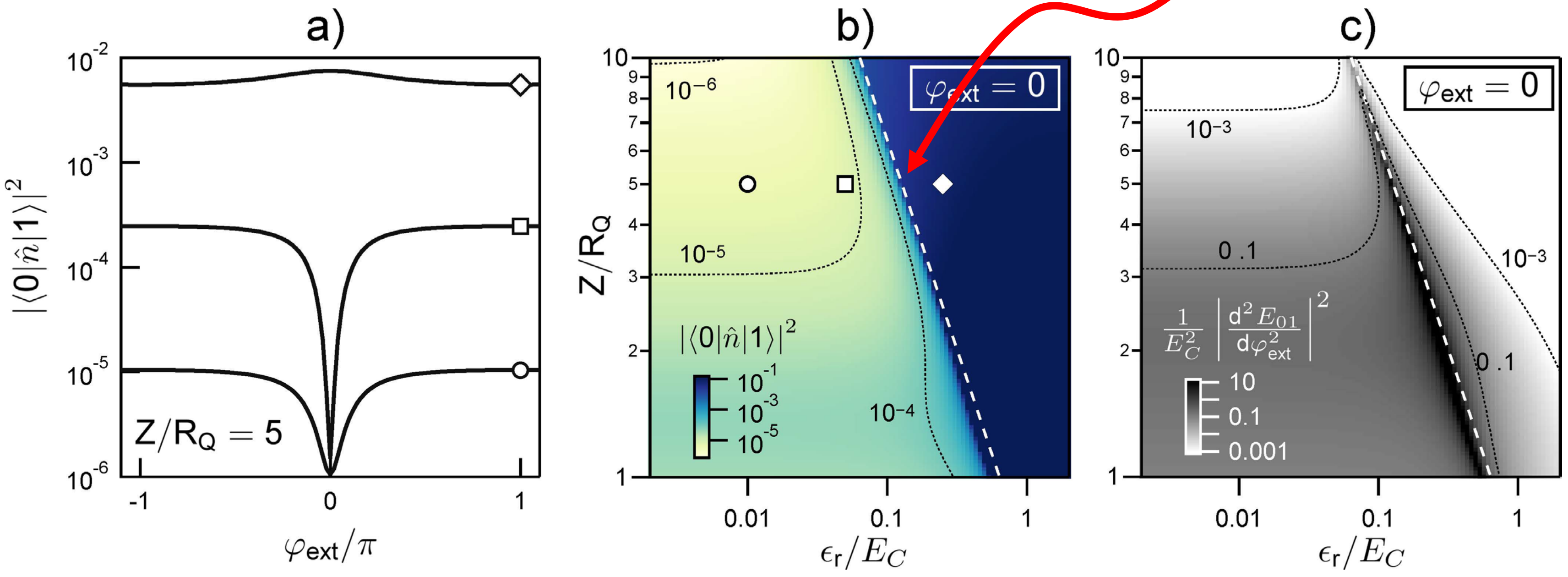
$E_C > \Gamma,\ \delta\Gamma/\Gamma = 0.01$



$$Z = (R_Q/\pi)\sqrt{2E_C/E_L}$$

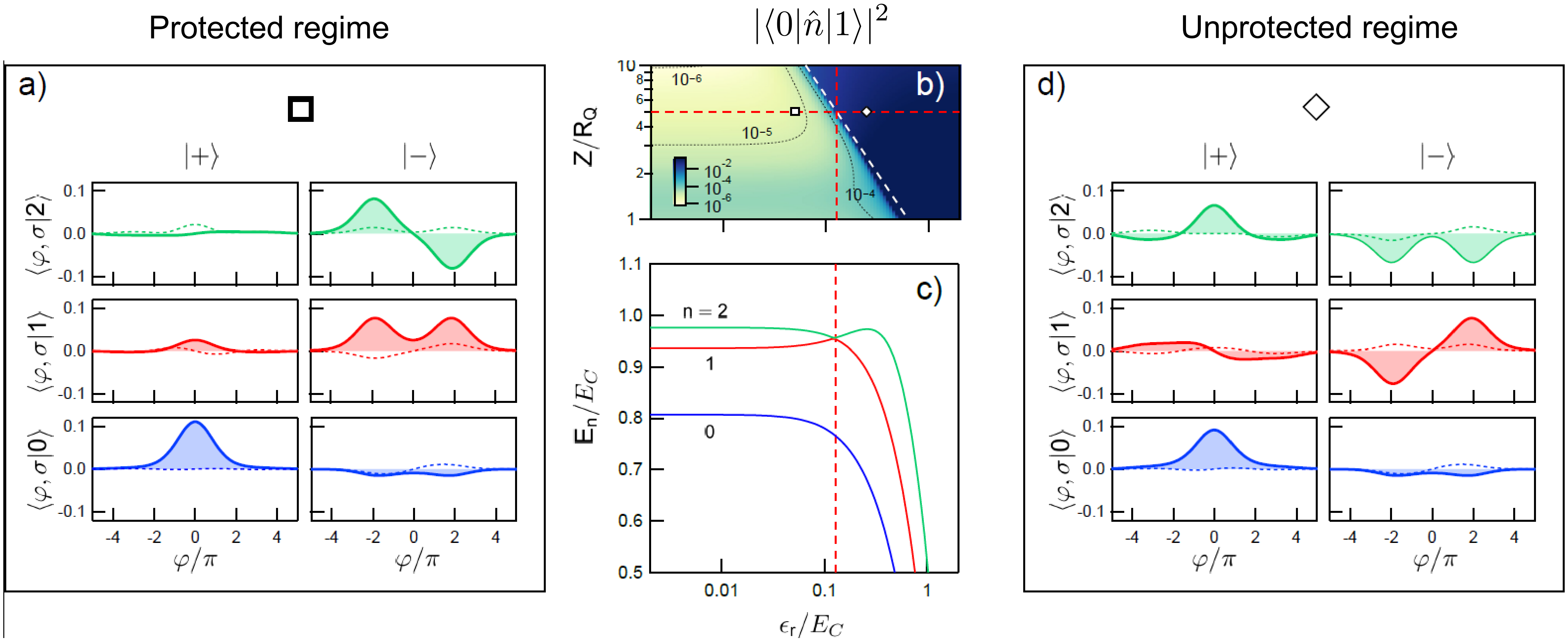
$$\boxed{Z/R_Q = 2E_C/\pi\epsilon_r}$$

→ Suppression of charge-noise susceptibility with increasing transmission



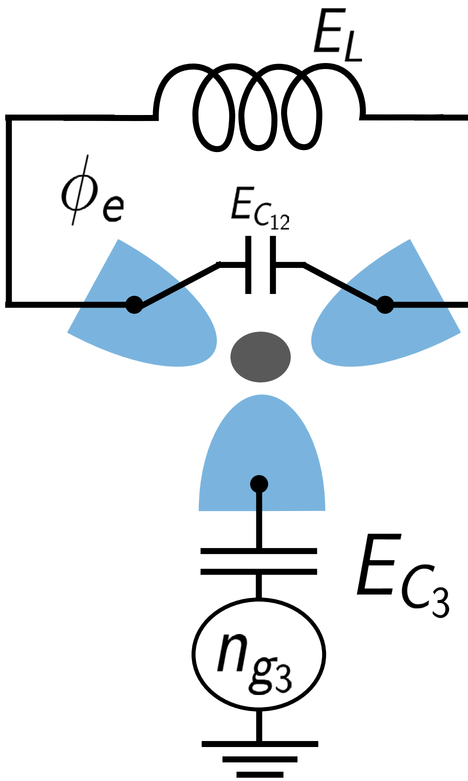
J.J. Cáceres et al, arXiv:2604.01145

FERBO qubit: relaxation protection



J.J. Cáceres et al, arXiv:2604.01145

Three terminal: hybrid charge-flux configuration



<u>Combination of two bosonic modes in a trijunction:</u>

- Flux mode in the loop $\phi_{12} \in \mathbb{R} \quad \Leftrightarrow \quad N_{12} \in \mathbb{R}$

- Charge mode in the island $\phi_3 \in [-\pi, \pi) \Leftrightarrow N_3 \in \mathbb{Z}$

States of the form $|\phi_{12}, N_3, A\rangle$

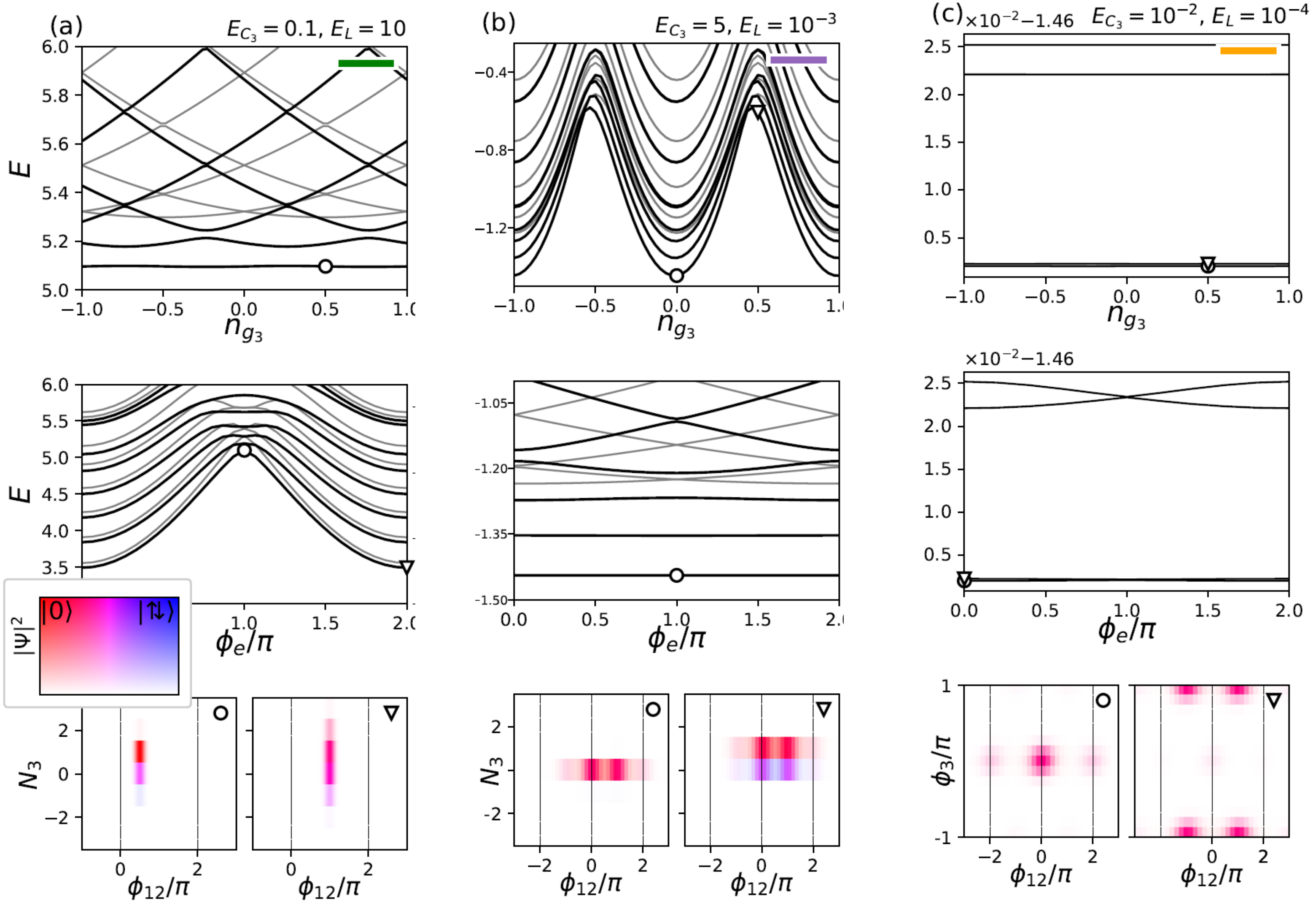
(for the calculation) $A \in \{0, \uparrow\downarrow\}$

$$H = E_{C_3}(N_3 - n_{g3})^2 + E_{C_{12}} N_{12}^2 + E_L(\phi_{12} - \phi_e)^2$$
$$+ \sum_\sigma \epsilon_r d_\sigma^\dagger d_\sigma + \Gamma_1 e^{-i\phi_{12}/2} d_\uparrow^\dagger d_\downarrow^\dagger + \Gamma_2 e^{-i\phi_{12}/2} d_\downarrow d_\uparrow + \Gamma_3 e^{-i\phi_3} d_\uparrow^\dagger d_\downarrow^\dagger + h.c.$$

F.J. Matute-Cañadas, L. Tosi, and ALY, PRX Quantum 5, 020340 (2024)

Three terminal: hybrid charge-flux configuration

$$\Gamma_1 = 0.4,\ \Gamma_2 = 0.6,\ \Gamma_3 = 1,\ \epsilon_r = 0.6$$



0-π HYBRID

PRX Quantum 5, 020340 (2024)

Conclusions (2nd part):

- **Mesoscopic JAnctions in quantum circuits: ABS excitations + collective modes**
- **Quantum circuit rules beyond tunnel limit**
- **Two terminal JAnctions: the *FERBO* qubit, protected for high transparency + high inductance**
- **Three terminal JAnctions: the 0-π hybrid qubit, protected even for asymmetric couplings**

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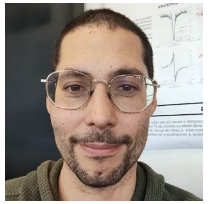

Saclay

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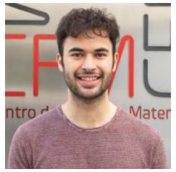
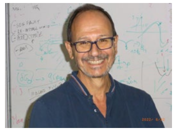
H. Pothier
M. Goffman





Thank you!