Magnetochiral anisotropy in the inductance

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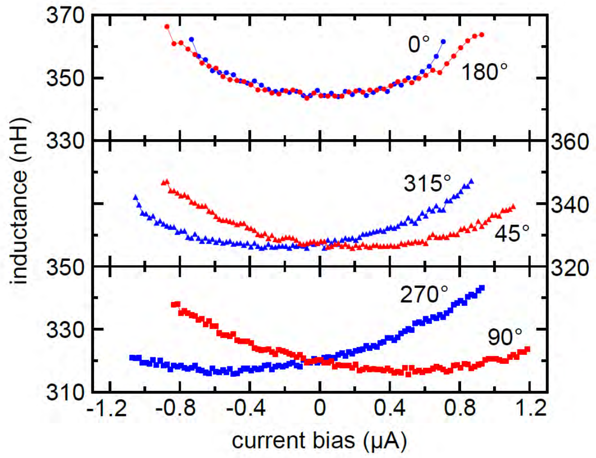
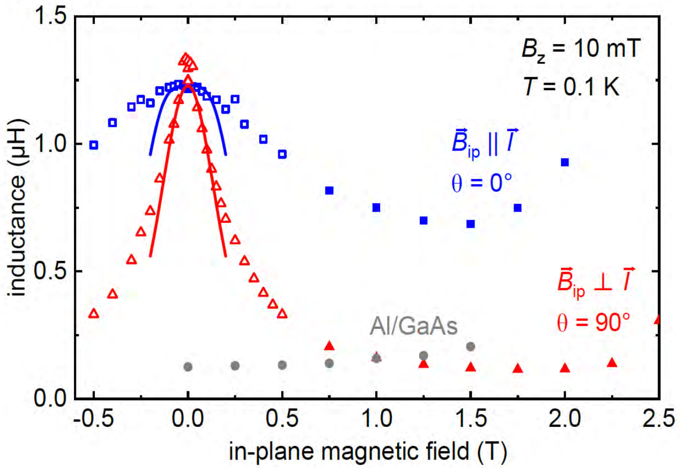
CRC 1277

Purdue University

University of Regensburg



analog of
non-reciprocal resistance?



How innocent are hybrid superconducting contacts?

Measurements


N. Paradiso


L. Tosi


S. Feyrer


J. Berger


S. Reinhardt


C. Baumgartner


L. Fuchs

University of Regensburg

Material growth


M. Manfra


S. Gronin


T. Lindemann

Purdue University

Theory support


A. Costa


P. E. Faria Junior


J. Fabian


D. Kochan

University of Regensburg


G. Rodríguez Ruiz

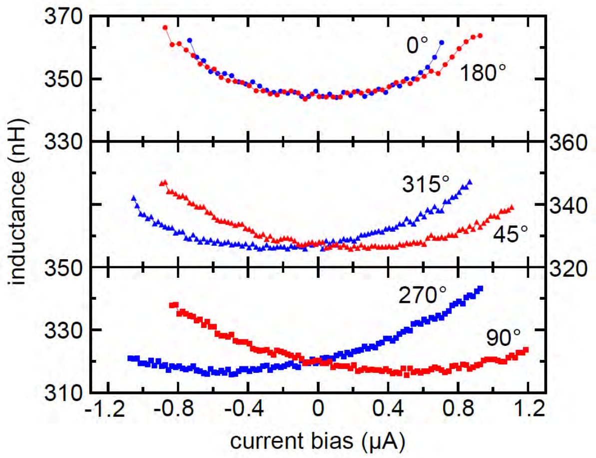

C. Balseiro


L. Arrachea

Centro Atómico Bariloche

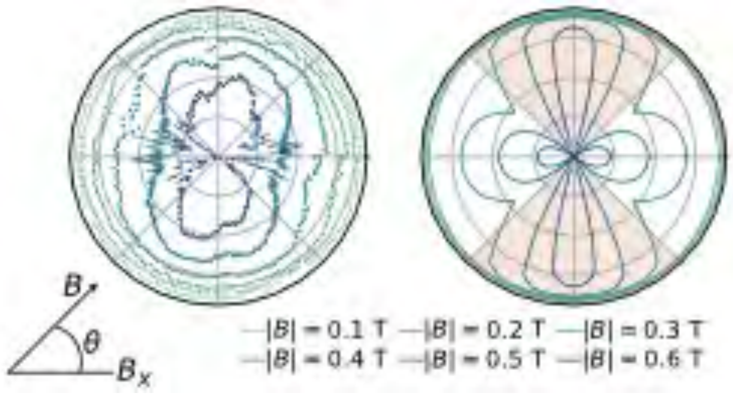
Outline:

- Non-reciprocal Josephson inductance
  - superconducting analog of non-reciprocal resistance ?
  - very sensitive method to measure tiny changes of CPR



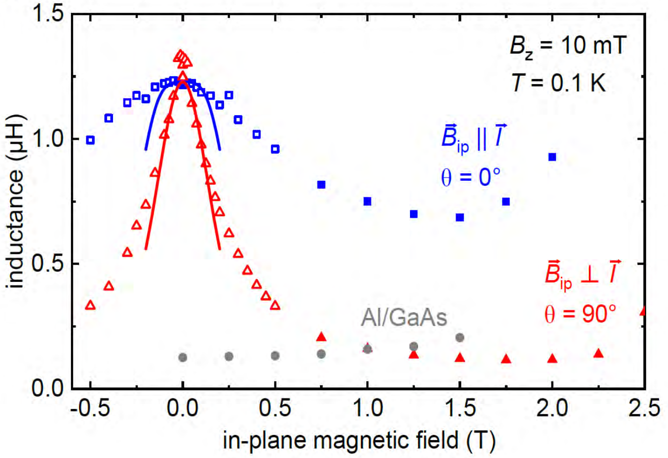
C. Baumgartner, C. S. et al., Nat. Nanotech. **17**, 39 (2022)

- Omit the junctions : kinetic inductance
  - SOI-induced superposition of s- and p-components of OP?
  - Bogoliubov Fermi surfaces?



D. Phan et al., PRL **128**, 107701 (2022)

- Vortex inductance
  - Rashba SOI at Ginzburg-Landau level: Lifshitz-invariants in free energy
  - anisotropic squeezing of vortex cores



L. Fuchs, C. S. et al., PRX **12**, 041020 (2022)

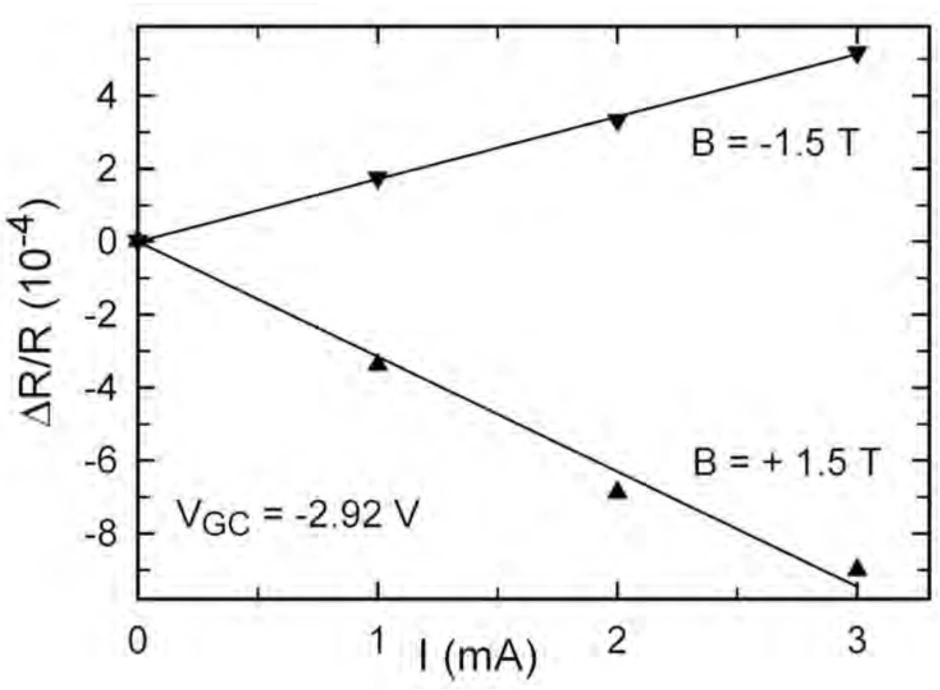
Non-reciprocal transport in homogeneous devices:

Requires simultaneous broken inversion and time-reversal symmetries

Competition between Zeeman and Rashba spin-orbit interaction: non-linear non-reciprocal resistance:

$$R = R_0[1 + \gamma \hat{e}_z(\vec{B} \times \vec{I})]$$

G. L. J. A. Rikken, J. Fölling, and P. Wyder, Phys. Rev. Lett. **87**, 236602 (2001)
G. L. J. A. Rikken and P. Wyder, Phys. Rev. Lett. **94**, 016601 (2005)



Magnetochiral anisotropy in the fluctuation regime of 2D superconductors:

Itahashi, Y. M. *et al., Science Advances 6, (2020)*
Ideue, T. *et al., Nature Physics 13, 578 (2017)*
Wakatsuki, R. et al., *Science Advances* 3, (2017)

Does inductance provide an analogue of the resistive magnetochiral anisotropy (at the lowest T) ?

$$L = L_0[1 + \gamma_L \hat{e}_z(\vec{B} \times \vec{I})]$$?

Tutorial: Ginzburg-Landau and Josephson equations

Normal electric currents are driven by electric potential $(\phi_{\rm el})$ differences :

$$\vec{j}_{\rm el} \;=\; -\sigma \;{\rm grad}\; \phi_{\rm el}$$

in superconductors: $\;{\rm grad}\; \phi_{\rm el} \;\equiv\; 0$ (in equilibrium)

So - what's drives a supercurrent?

$$\vec{j}_{\rm S} \;=\; -2e\,|\psi|^2\,\frac{\hbar}{m^*}\left({\rm grad}\;\varphi + \frac{2e}{\hbar}\vec{A}\right)$$

, where $\;\psi(\vec{r}) \;=\; |\psi(\vec{r})|^2 \exp i\varphi(\vec{r})\;$ complex order parameter

counterpart of current-phase-relation or CPR (1st Josephson equation)

Answer: supercurrents are driven by (gauge-invariant) phase differences!

electric potential differences generate a time evolution of the phase :

$$\frac{\partial}{\partial t}\Delta\varphi(\vec{r},t) \;=\; \frac{2e}{\hbar}\Delta\phi_{\rm el}(\vec{r},t)$$

2nd Josephson equation

Tutorial : London equations

A) Take curl of current-phase-relation:

$$\mathrm{rot}\,\vec{j}_\mathrm{S} = -2e\,|\psi|^2\,\frac{\hbar}{m^*}\left(\mathrm{rot\,grad}\,\varphi + \frac{2e}{\hbar}\mathrm{rot}\,\vec{A}\right) = -\underbrace{\frac{4e^2|\psi|^2}{m^*}}\,\vec{B}$$

2nd London equation

$$\Lambda_\mathrm{L}^{-1} = \frac{4e^2|\psi(T)|^2}{m^*} = \frac{\mu_0}{\lambda^2(T)}$$

London parameter
superfluid density
penetration depth
superfluid stiffness

Meissner – Effect !

hard to measure in thin films with thickness $d \lesssim \lambda(T)$

B) Take time derivative of current-phase-relation:

$$\frac{\partial\vec{j}_\mathrm{S}}{\partial t} = -2e\,|\psi|^2\,\frac{\hbar}{m^*}\left(\mathrm{grad}\,\frac{\partial\varphi}{\partial t} + \frac{2e}{\hbar}\frac{\partial\vec{A}}{\partial t}\right) = \Lambda_\mathrm{L}^{-1}\,\vec{E}$$

1st London equation

ideal conductor : $\sigma = \infty$

$$\frac{\partial I_\mathrm{S}}{\partial t} = \underbrace{\frac{wd}{\ell}\Lambda_\mathrm{L}^{-1}}_{L_\mathrm{kin}^{-1}}\cdot V$$

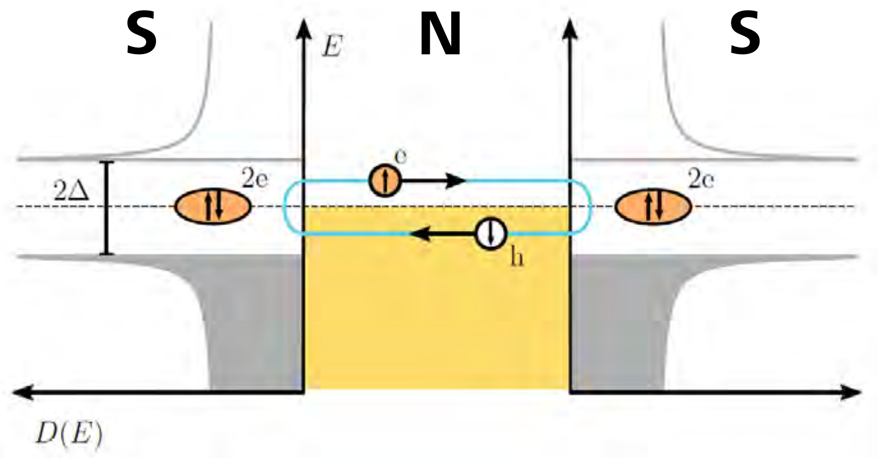
it is an induction law !

total inductance : $L = L_\mathrm{geo} + L_\mathrm{kin}$

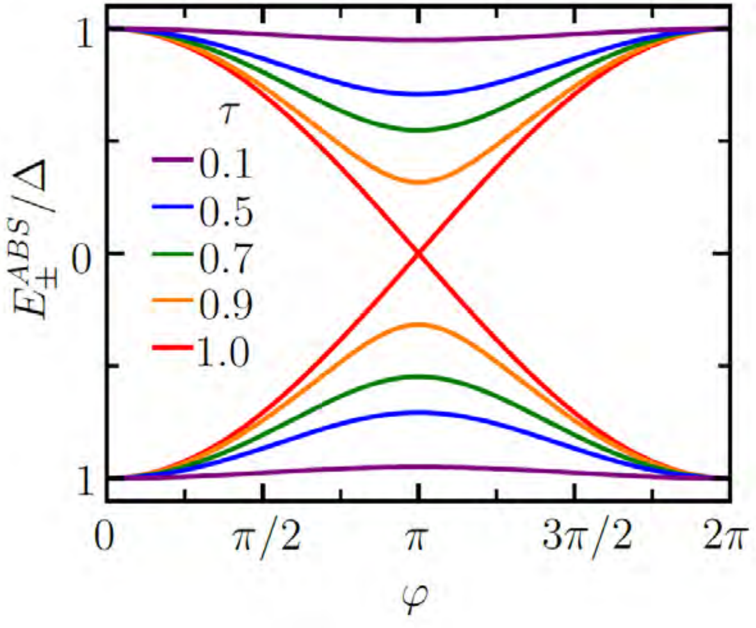
L_kin important for $d \lesssim \lambda(T)$: $= \mu_0\frac{w}{\ell}\left(d + \frac{\lambda^2}{d}\right)$

Josephson inductance:

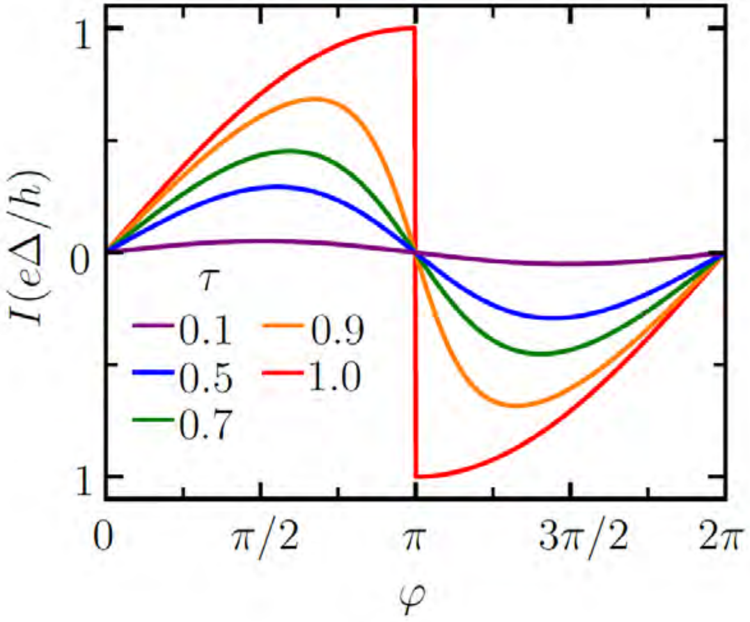
CPR I(φ) depends on ABS energy spectrum



supercurrent carried by
Andreev bound states (ABS)



$$I_{\pm}^{\mathrm{ABS}} = \frac{2e}{\hbar}\frac{dE}{d\varphi}$$



The Josephson inductance results from the Josephson equations:

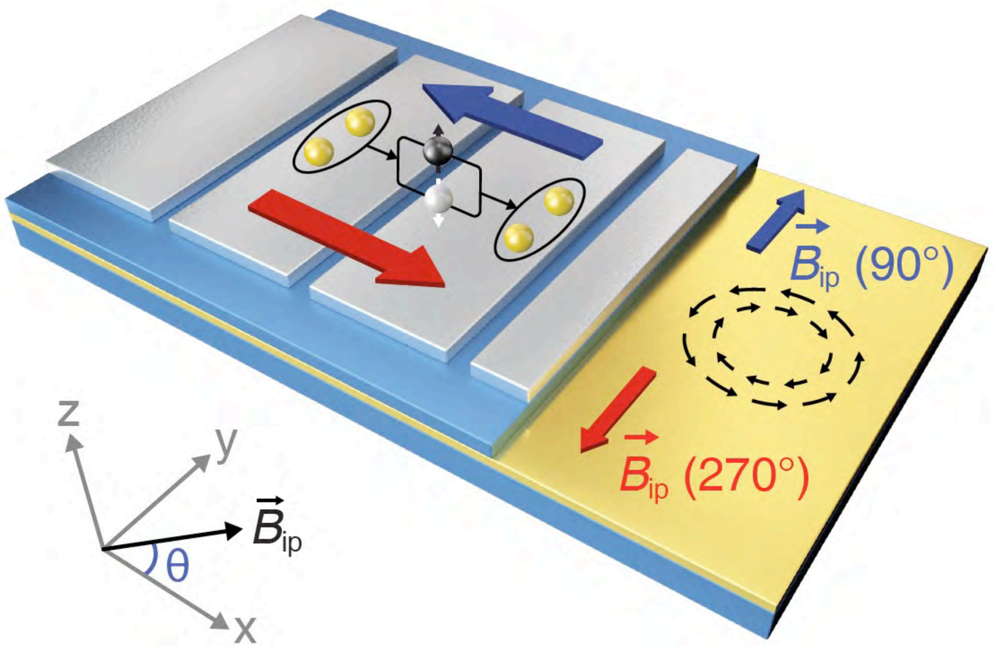
$$I = I_0 f(\varphi)$$

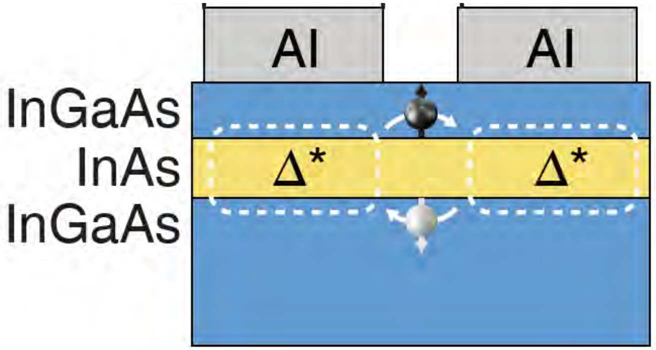
Current-phase relation (CPR)

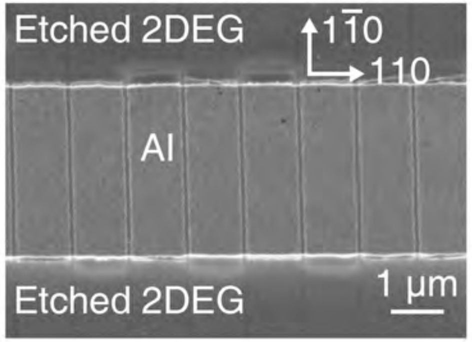
$$V = \hbar\dot{\varphi}/2e$$

$$L(\varphi) = \frac{V}{\frac{dI}{dt}} = \frac{\Phi_0}{2\pi I_0 f'(\varphi)}$$

Our device: a Josephson junction array







Analogue of

$$R = R_0[1 + \gamma \hat{e}_z(\vec{B} \times \vec{I})]$$

for superflow?

In bulk:
(superfluid density)$^{-1}$ ~ kinetic inductance

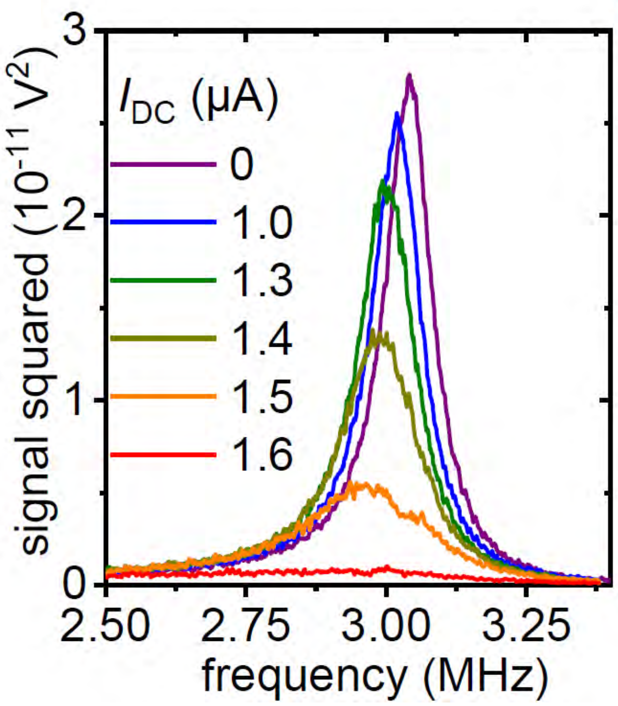
In Josephson junctions:
Josephson inductance

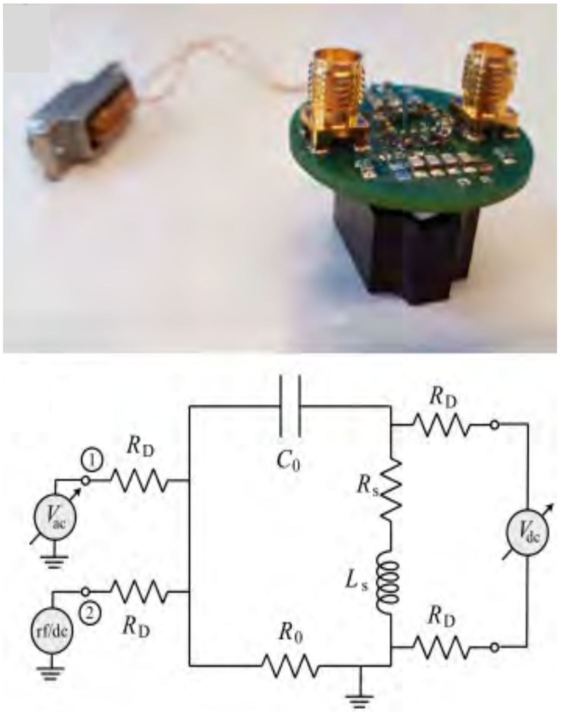
$$L = L_0[1 + \gamma_L \hat{e}_z(\vec{B} \times \vec{I})]$$

Non-linear (Josephson) inductance

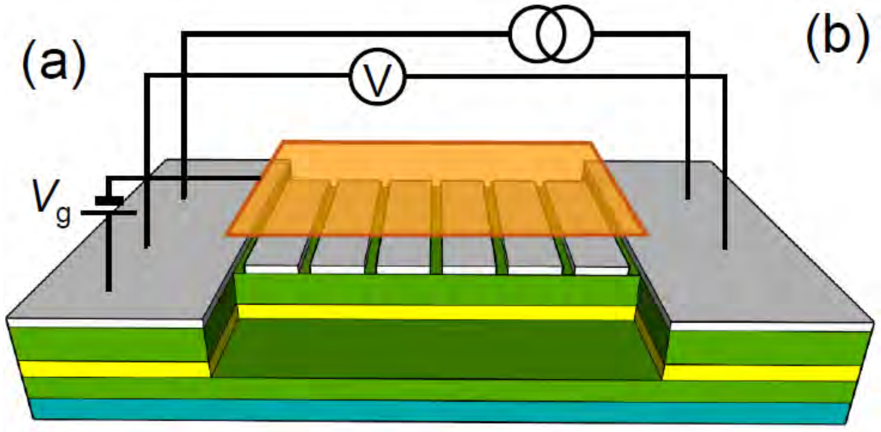
Frequency range:
few MHz ≪ all relevant frequencies

low-frequency limit ⟶ single junction physics





$L_{array}=L_{singleJJ} \times 2250$

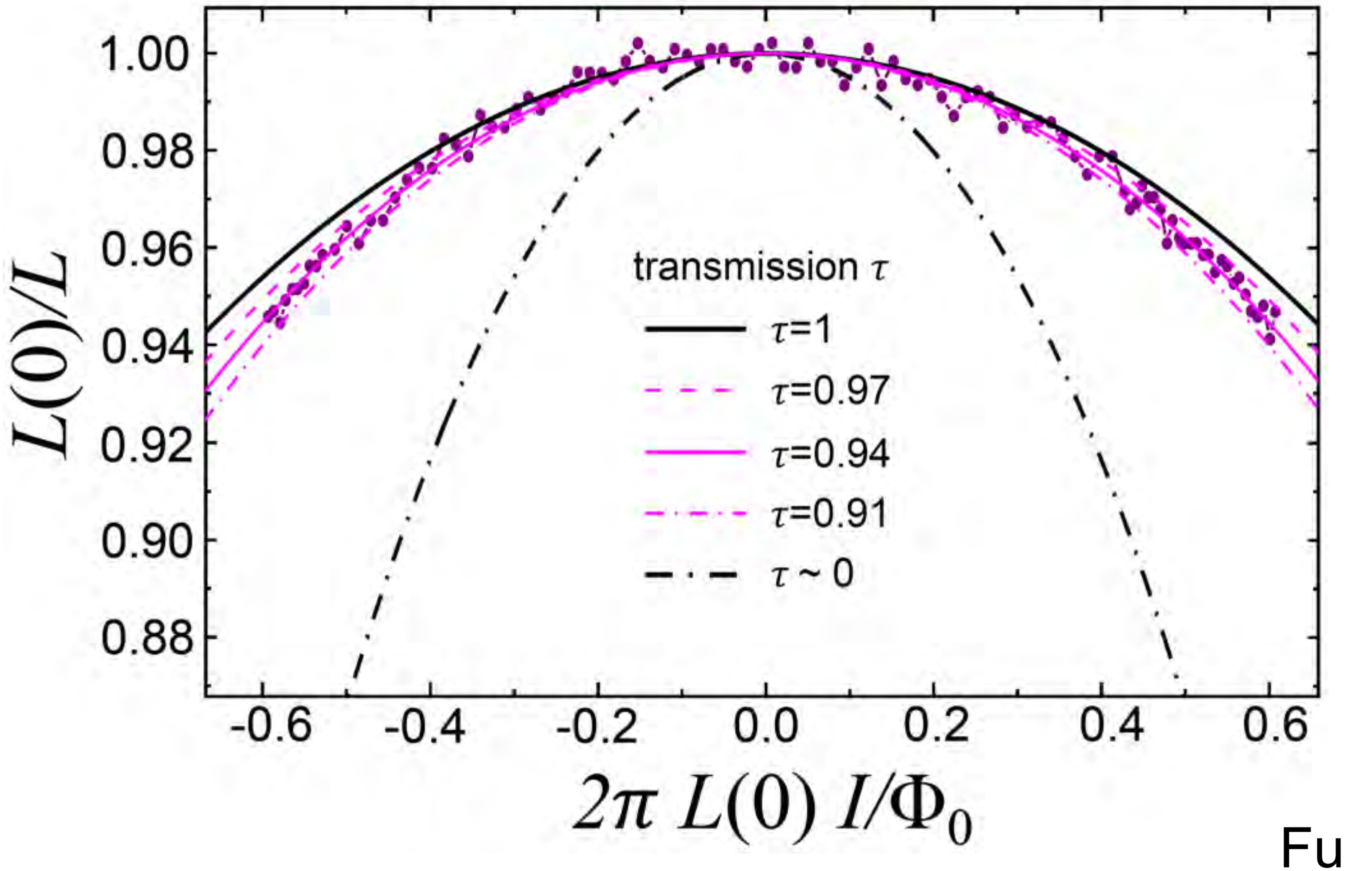


method demonstrated in
C. Baumgartner, CS, et al., PRL **126**, 037001 (2021)

much earlier:
R. Meservey and P. M. Tedrow, J. Appl. Phys. 40, 2028 (1969)

inductance allows for independent determination of (average) junction transparency τ = 94 % (!), number of transverse channels N_{ch} =190, and induced gap Δ^*= 130 μeV

Junction transparency τ from the *L(I)* curvature:



Normalized representation:
sensitive only to the *shape* of CPR –
i.e. to average τ

Result:
average transparency $\overline{\tau} = 0.94$

absolute value of $L(0)$ provides
number of channels *N* ~ 193

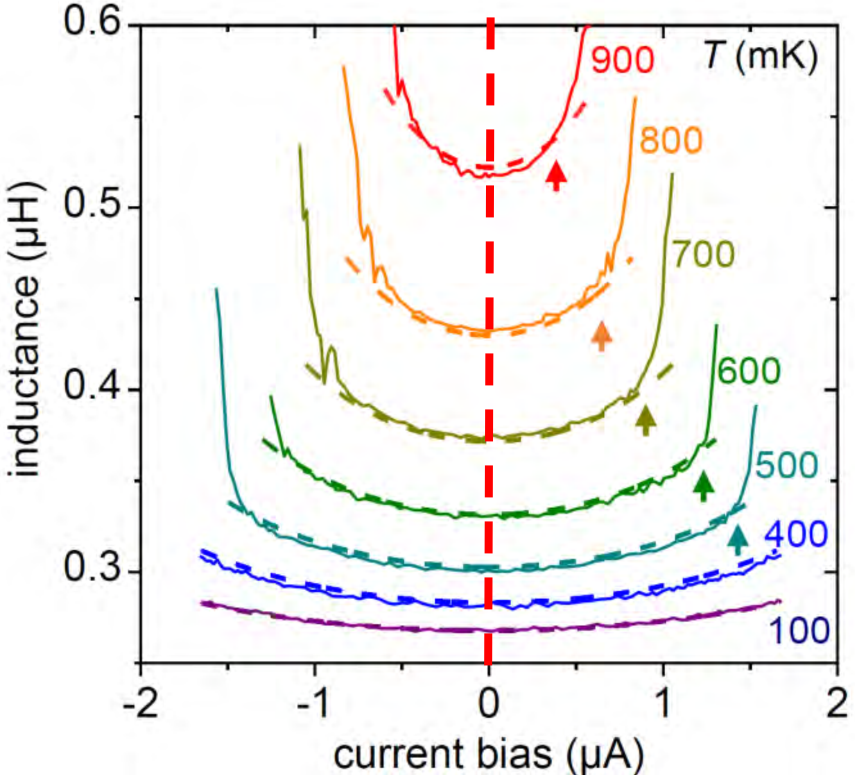
Furusaki-Beenakker formula:

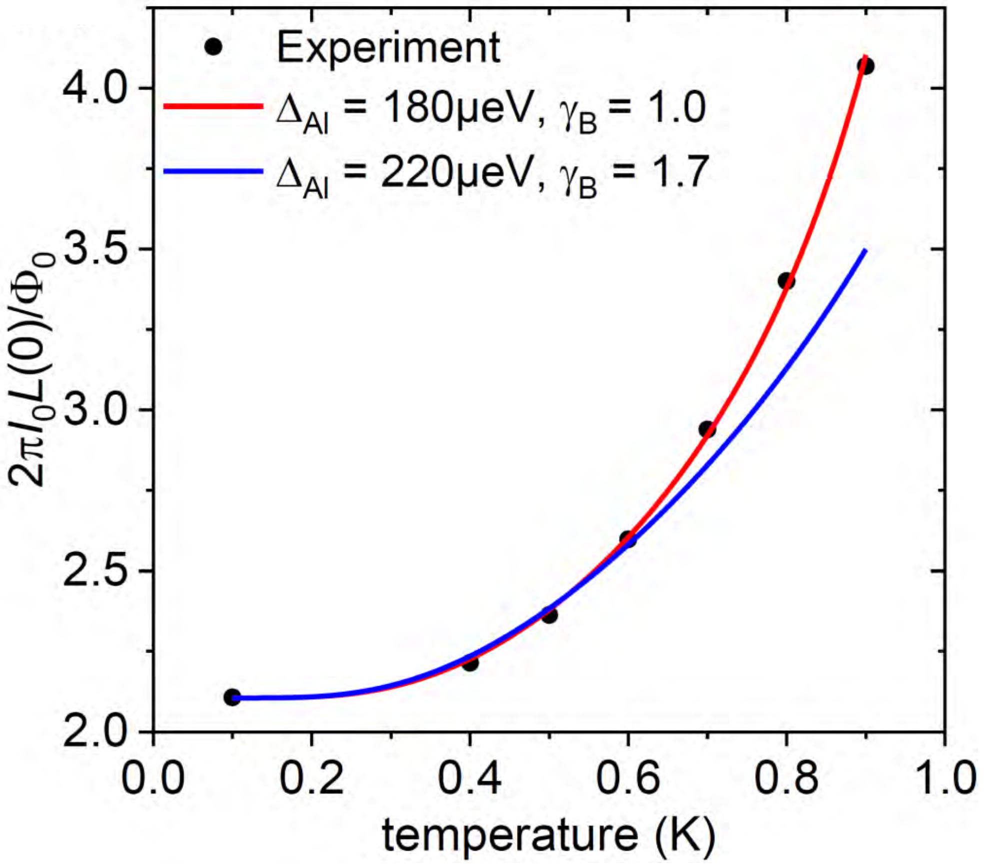
$$I(\varphi) = I_0 f(\varphi) = I_0 \frac{\overline{\tau}\sin\varphi \tanh\left[\frac{\Delta^*}{2k_B T}\sqrt{1-\overline{\tau}\sin^2(\frac{\varphi}{2})}\right]}{2\sqrt{1-\overline{\tau}\sin^2(\frac{\varphi}{2})}}$$

Proximity-induced gap from $L(0)$ vs T dependence

The temperature dependence of $L(0)$ depends on that of the proximity-induced gap Δ* (non-BCS)

By fitting * we obtain Δ*=130 μeV





$$* \quad \Delta^*(T) \approx \frac{\Delta_{\rm Al}(T)}{1+\gamma_B\sqrt{\Delta_{\rm Al}^2(T)-\Delta^{*2}(T)}/(\pi k_B T_c)}$$

[Kjaergaard *et al.* Phys. Rev. Appl. (2017) and Refs. therein]

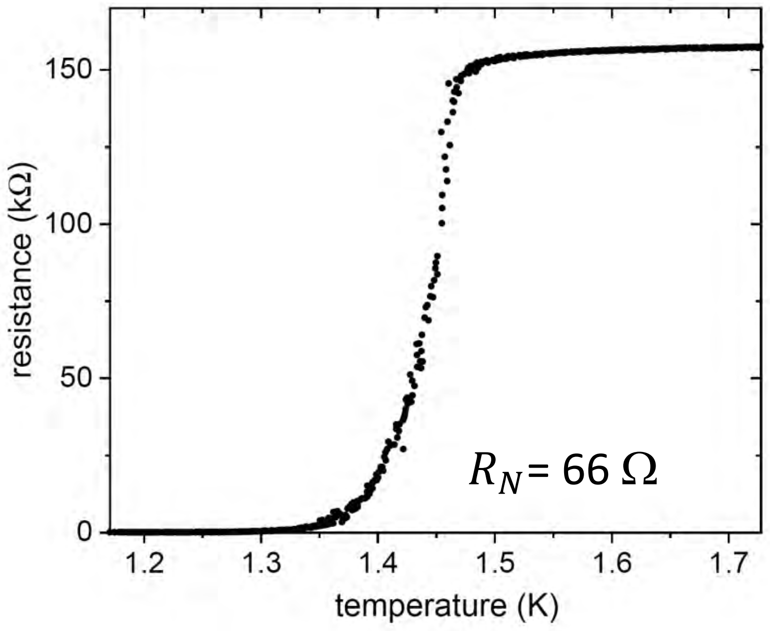
Proximity-induced gap Δ* from *L*(0) vs *T* dependence

The temperature dependence of *L*(0) depends on that of the Proximity-induced gap Δ*=130 μeV

compatible with $R_N I_c$ product (short ballistic case):

$$\Delta^* = eR_N I_c/\pi = 125\ \mu\text{eV}$$

Parameter	Value
$\overline{\tau}$	0.94
I_0	5.882 μA
I_C	4.41 μA
Δ^*	130 μeV
Δ_{Al}	220 μeV
γ_B	1.7
N	187



Number of channels *N* ~ 193
was independently extracted from Sharvin resistance

expectations from carrier density *n*:

N ~ 156 for *n* = 3.7·10^{11}/cm^2 without Al

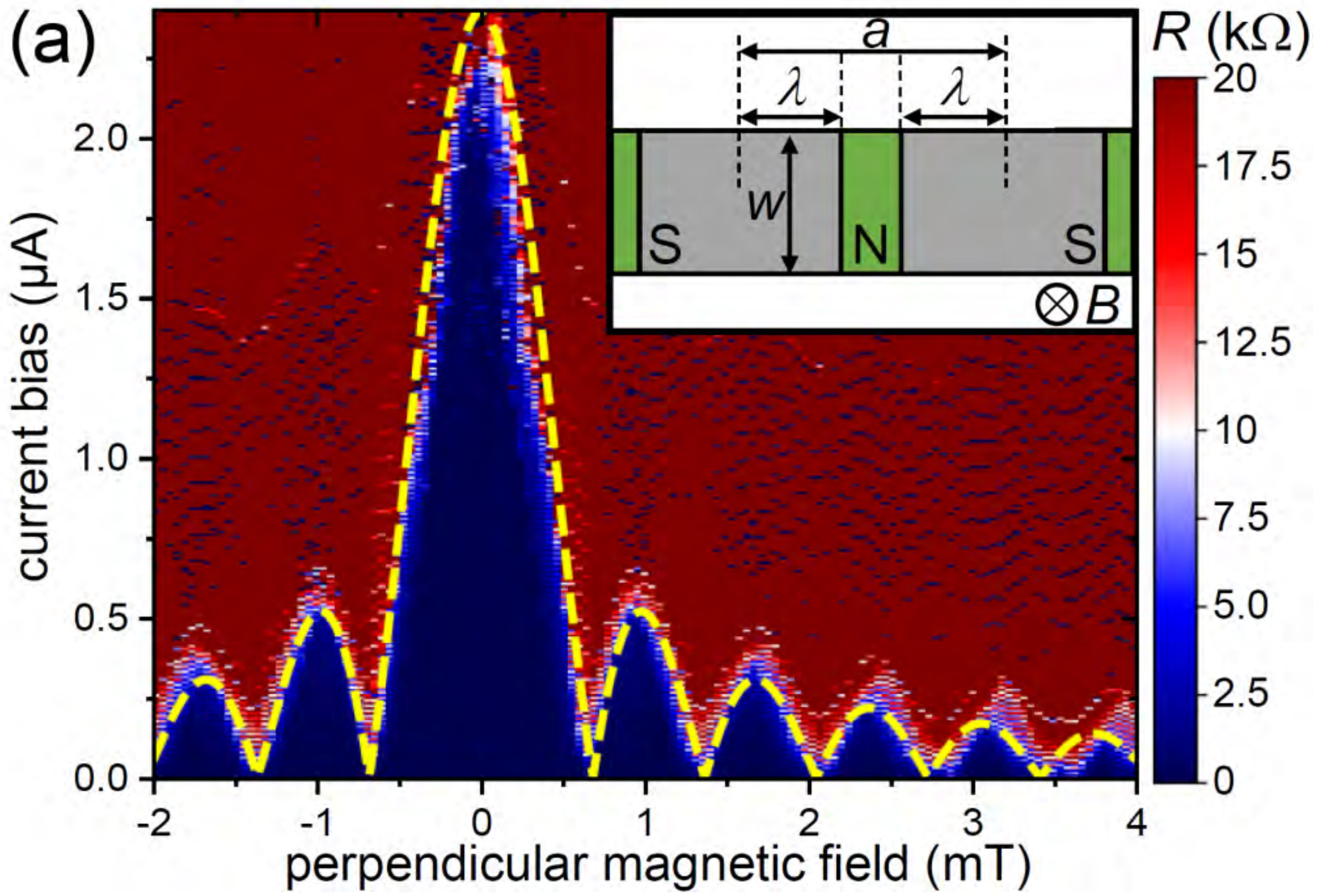
N ~ 220 for *n* = 8.5 ·10^{11}/cm^2 with Al

Complete characterization of junctions at $B_{||}$ = 0

Chauhan et al., APL 120, 142205 (2022)

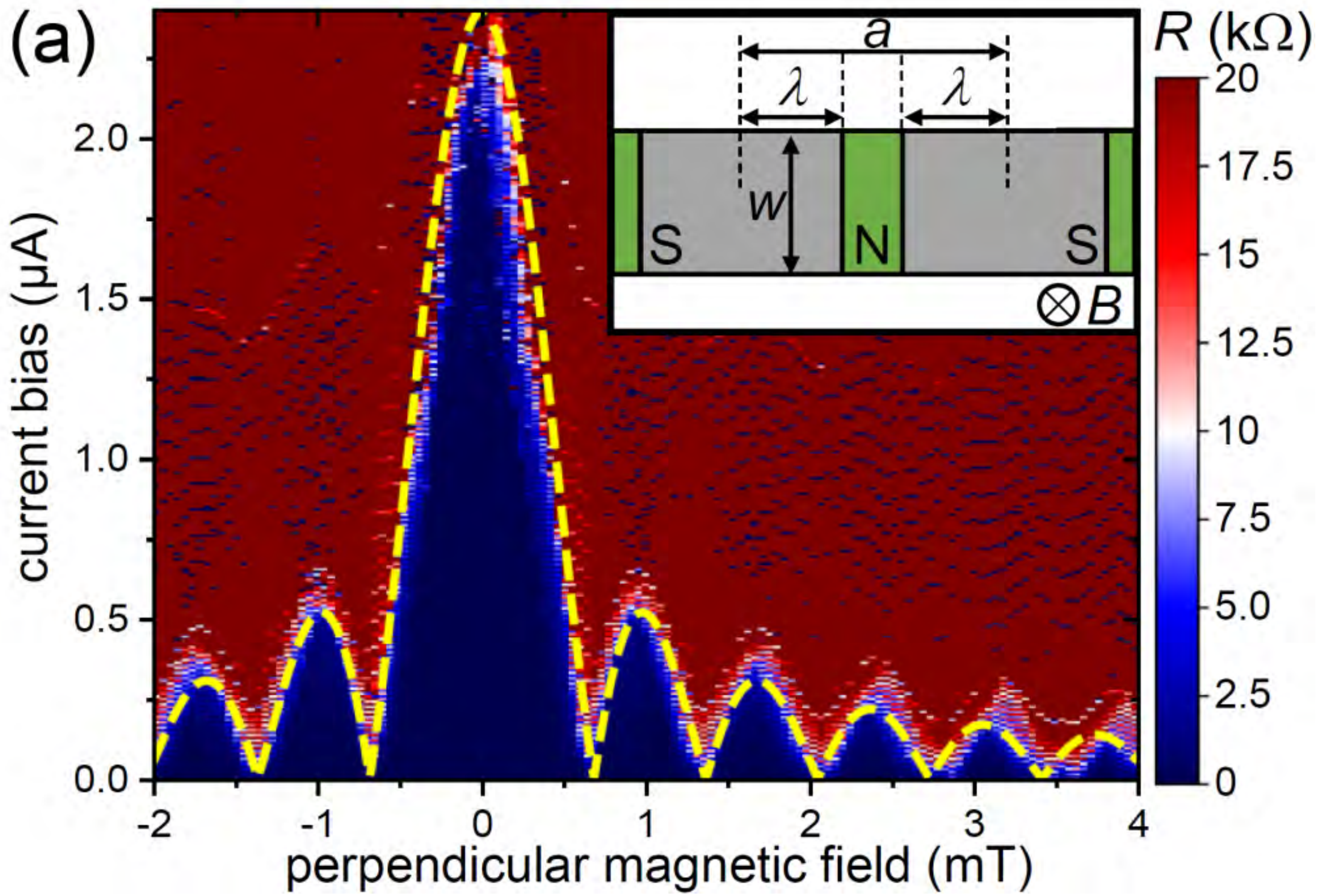
Fraunhofer diffraction pattern: DC transport

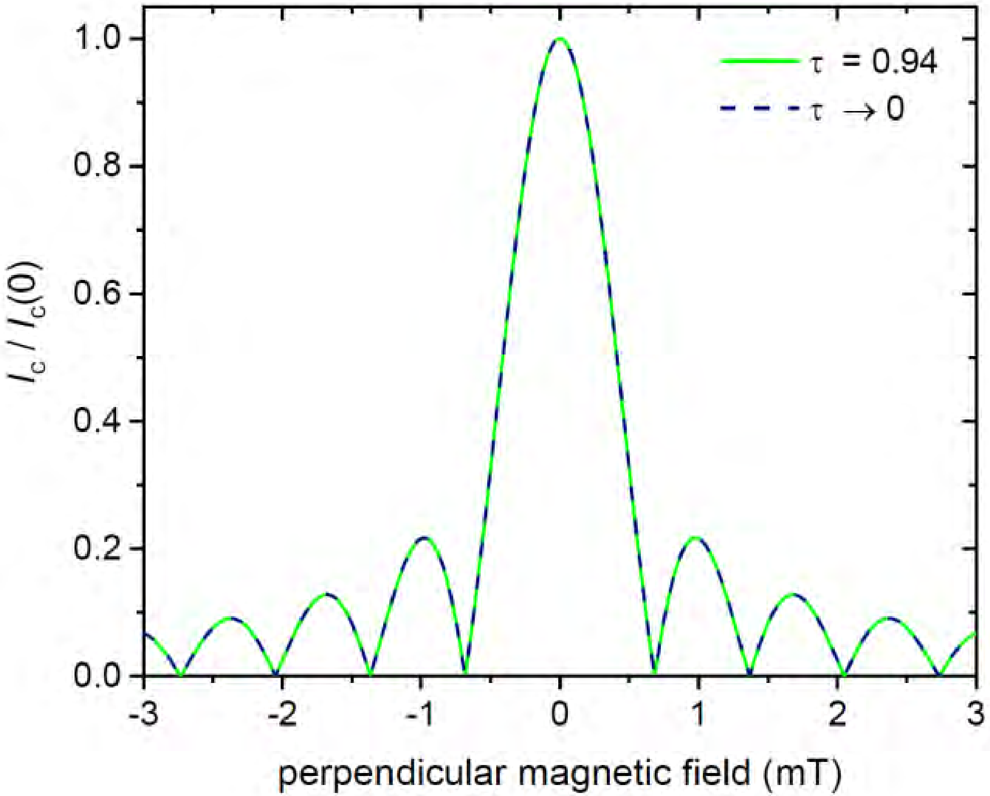
The series of 2250 junctions behaves (nearly) like a single ballistic junction:



Fraunhofer diffraction pattern: dc transport

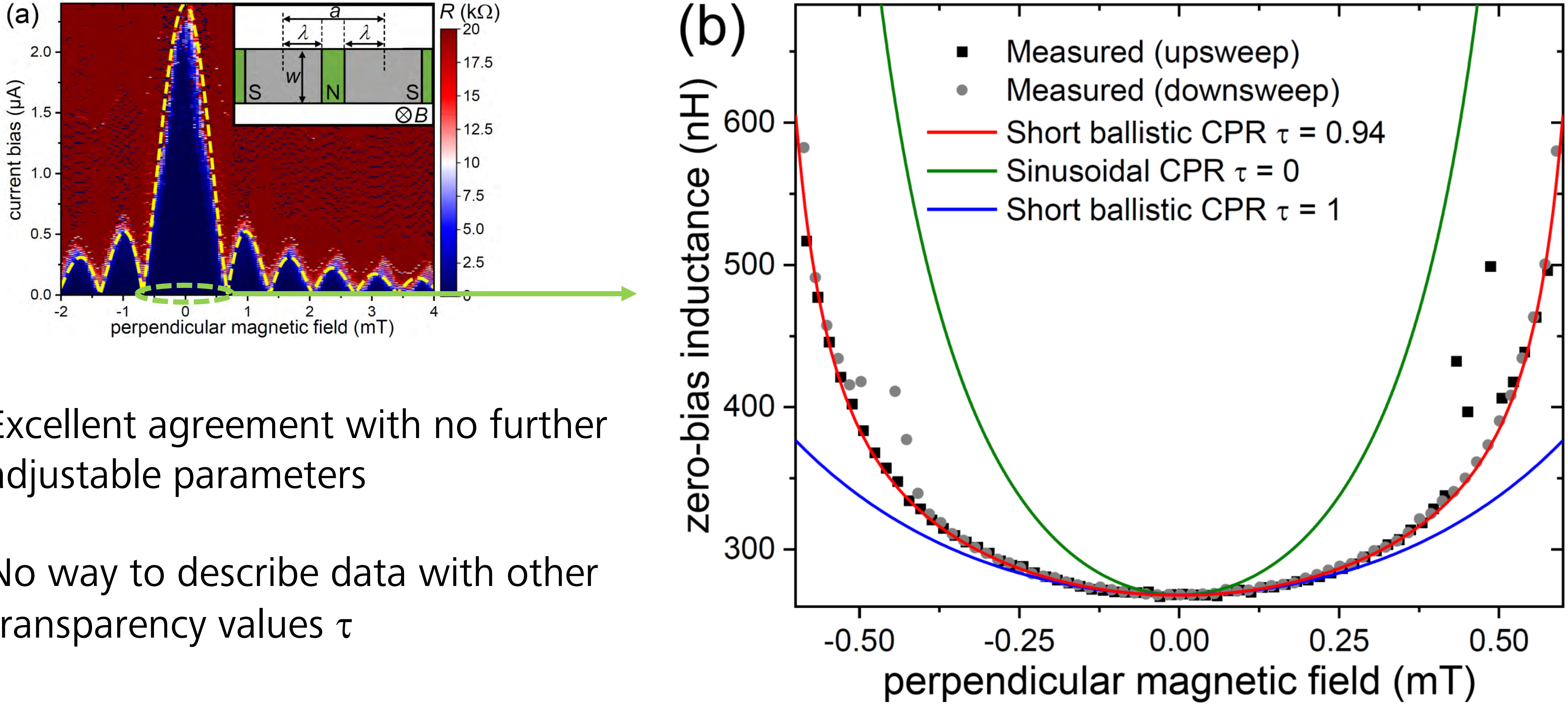
The series of 2250 junctions behaves (nearly) like a single one:





Interestingly, the short-ballistic case produces normalized diffraction patterns which are independent of τ

Fraunhofer diffraction pattern: inductance



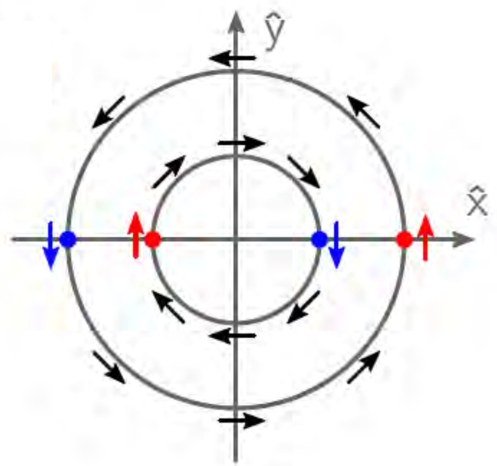
Excellent agreement with no further adjustable parameters

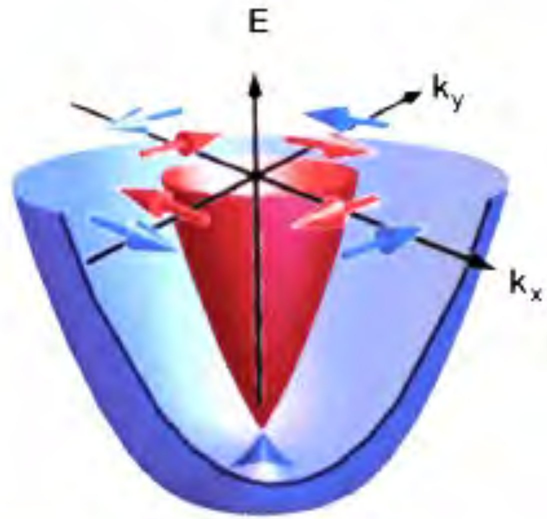
No way to describe data with other transparency values τ

C. Baumgartner, CS, et al., PRL **126**, 037001 (2021).

Josephson junctions are understood very well -

but, what about spin-orbit interaction ??



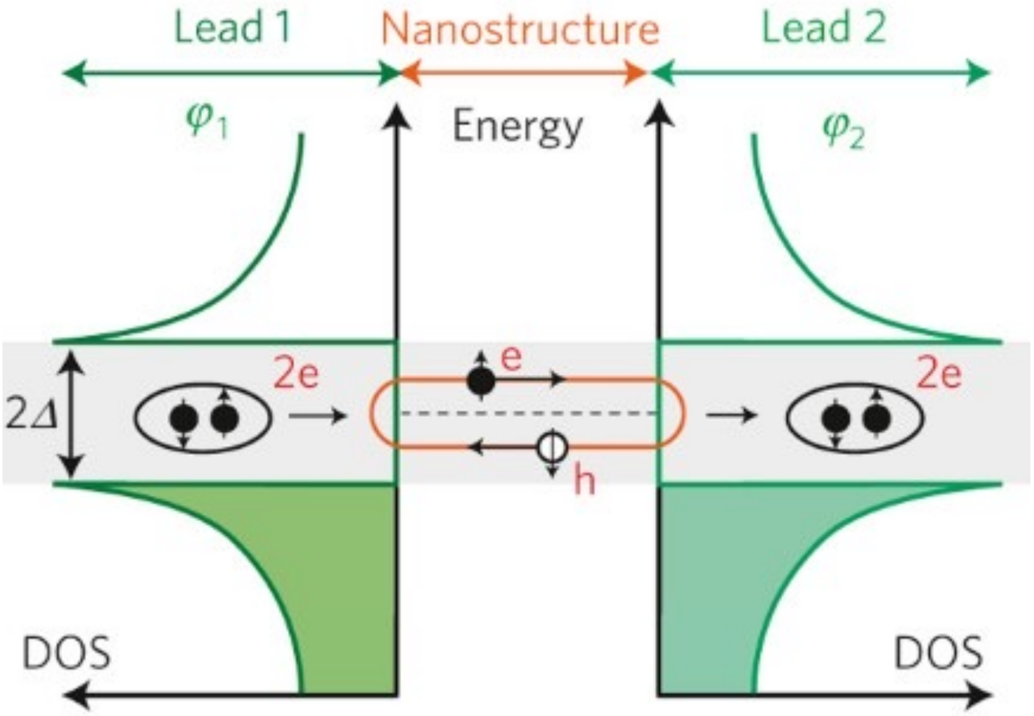


no effect,

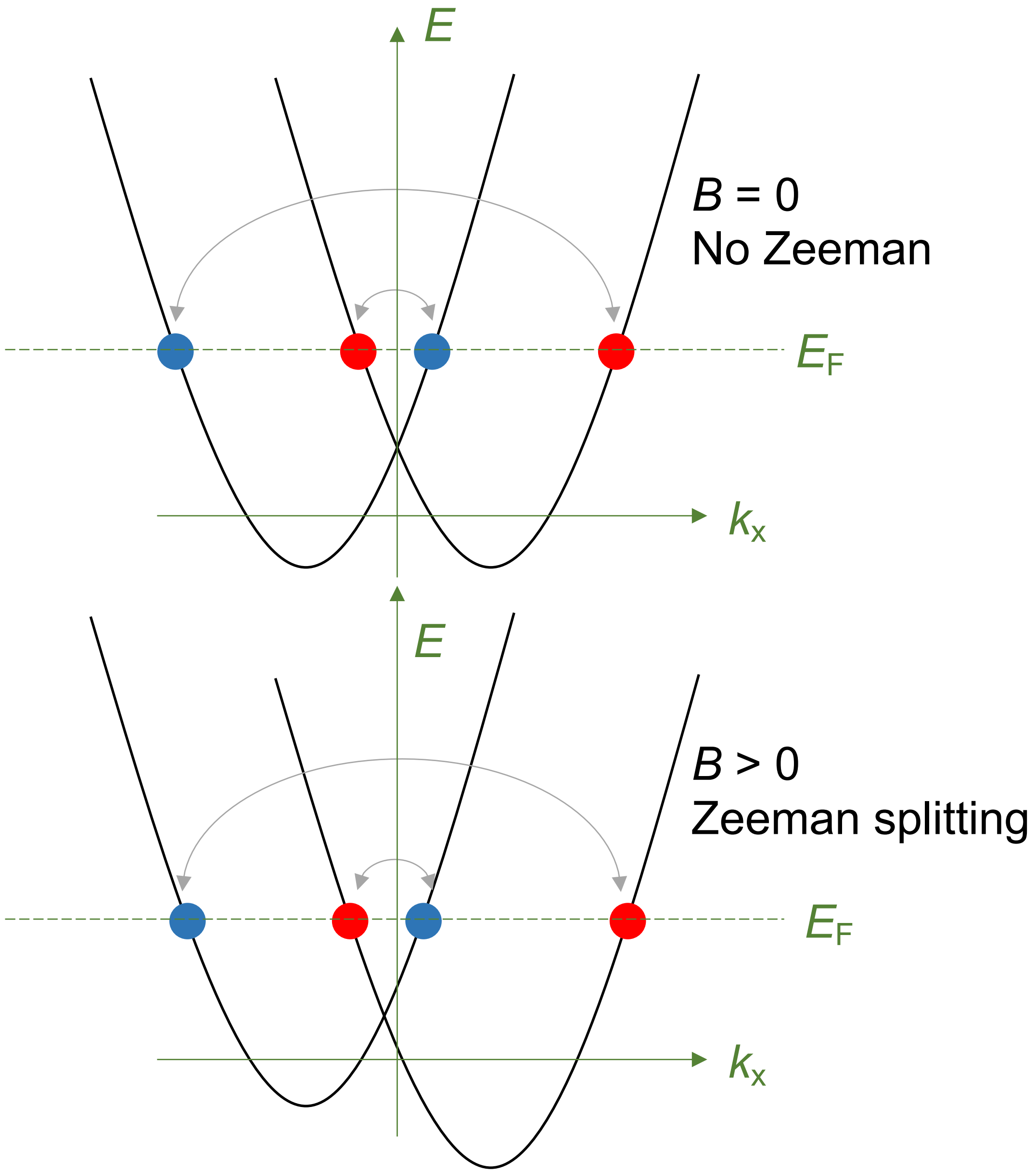
unless SOI is competing with external magnetic field!

Supercurrent in Rashba SNS junctions

How does a SNS junction carry a supercurrent?
Andreev bound states:



Difference in Fermi velocities is essential !



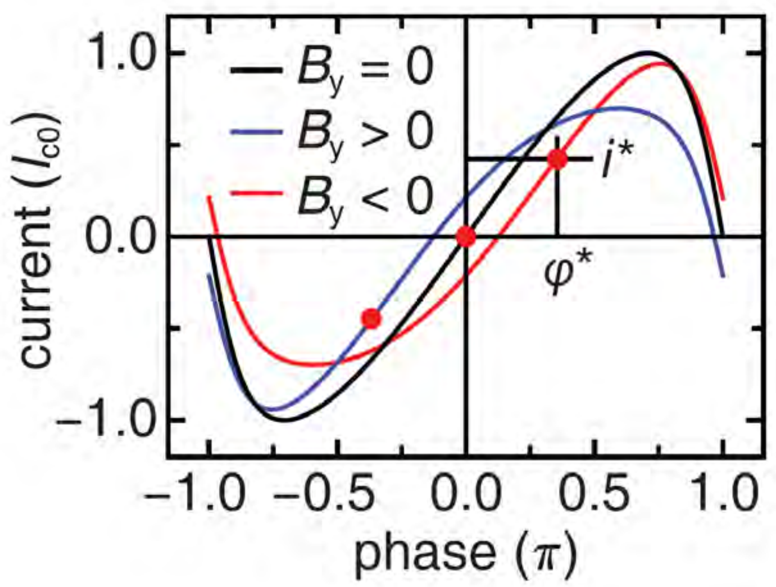
Impact of a transverse in-plane field on the CPR in presence of SOI

If the CPR is sinusoidal

$$a \sin\varphi + b\cos\varphi = c\sin(\varphi + \varphi_0) \rightarrow$$

$\rightarrow \varphi_0$ junction, but no diode

A. Buzdin, PRL **101**, 107005 (2008).
A. Reynoso, et al., PRL **101**, 107001 (2008).
A. Zazunov, et al. PRL **103**, 147004 (2009).

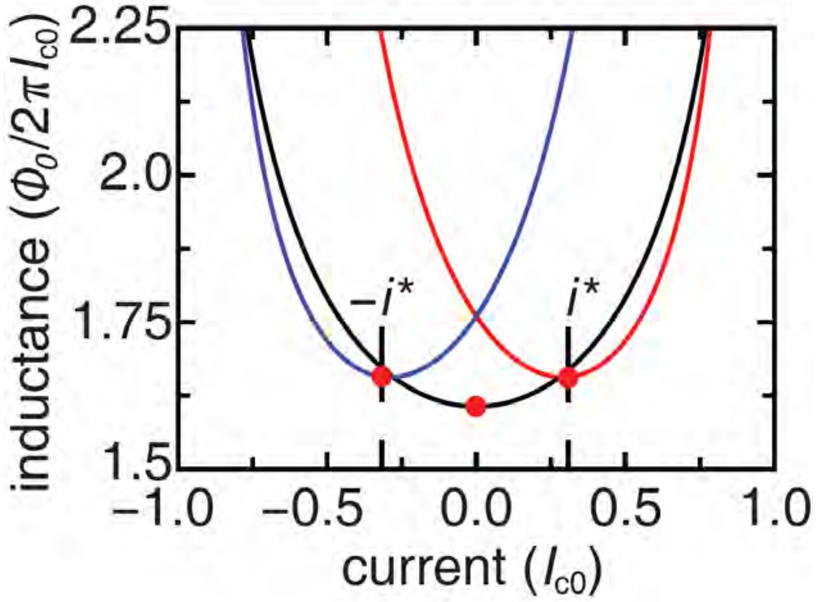


If the CPR is skewed (e.g. for short ballistic JJ)

$$I = I_0 f(\varphi) = a_1 \sin\varphi + a_2 \sin 2\varphi + b_1 \cos\varphi + \dots$$

$\rightarrow$ distorted CPR

$\rightarrow \varphi_0$ junction AND diode effect



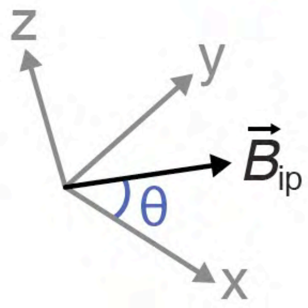
R. Grein, M. Eschrig *et al.*,
PRL **102**, 227005 (2009).

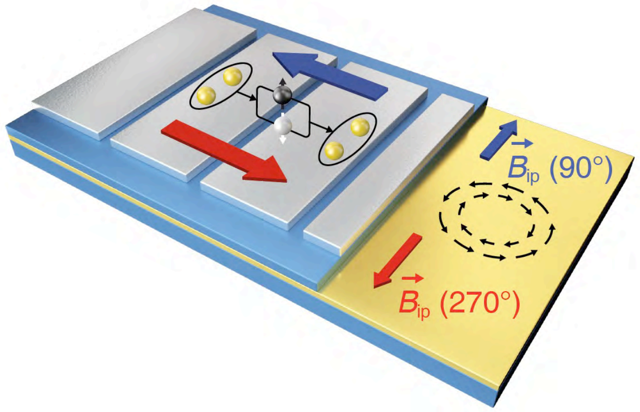
T. Yokoyama, M. Eto, Y. V. Nazarov,
PRB **89**, 195407 (2014).

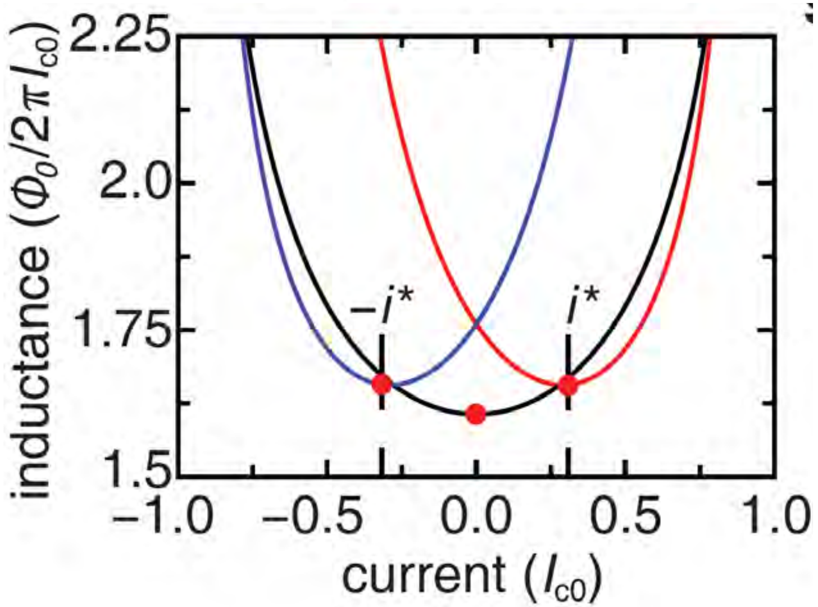
inflection point (φ^*, i^*) of CPR
corresponds to minimum of $L(I)$

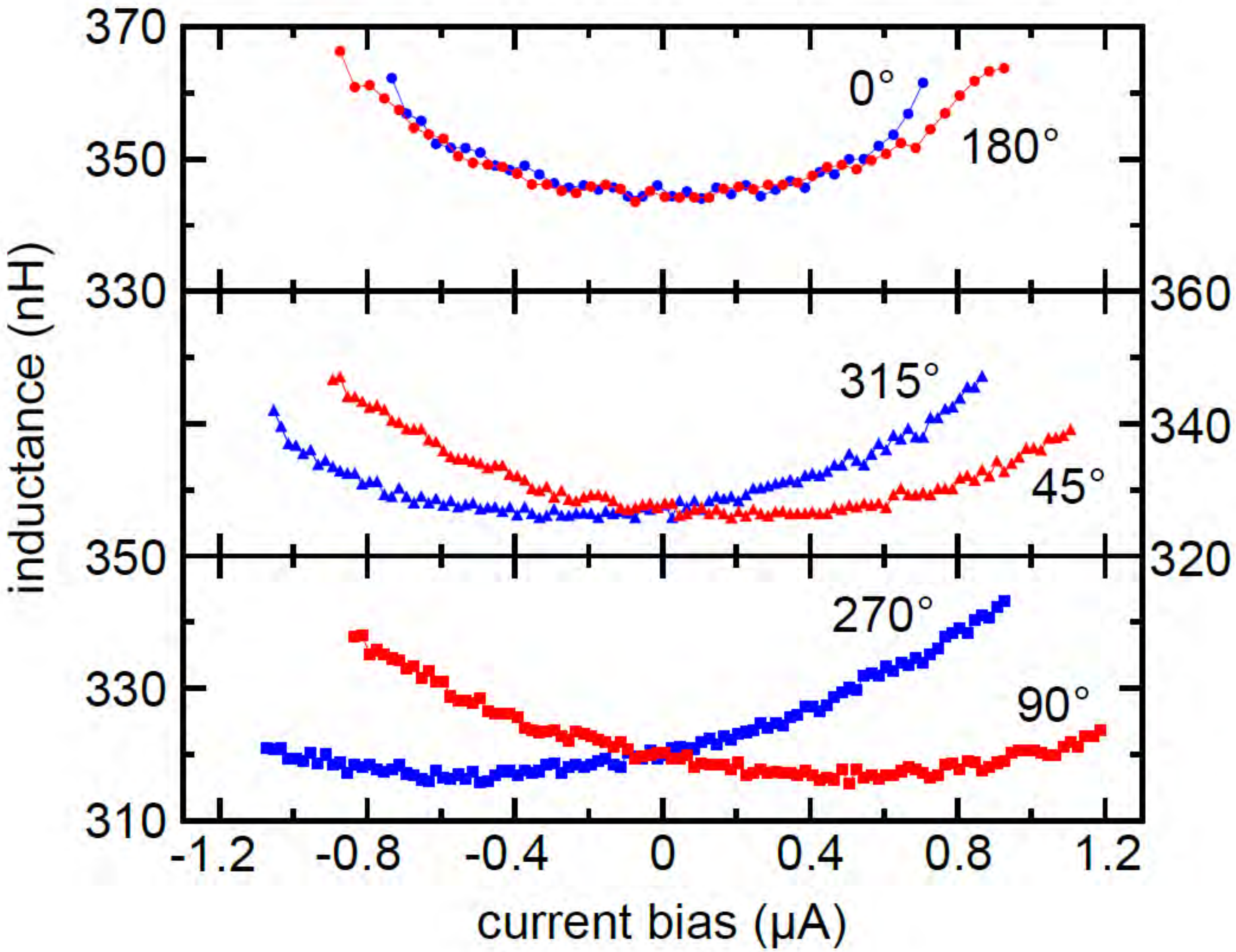
Asymmetric non-linear inductance reflects supercurrent diode effect

We measure non-linear inductance $L(I, \theta)$ @ finite $\vec{B}_{ip}$



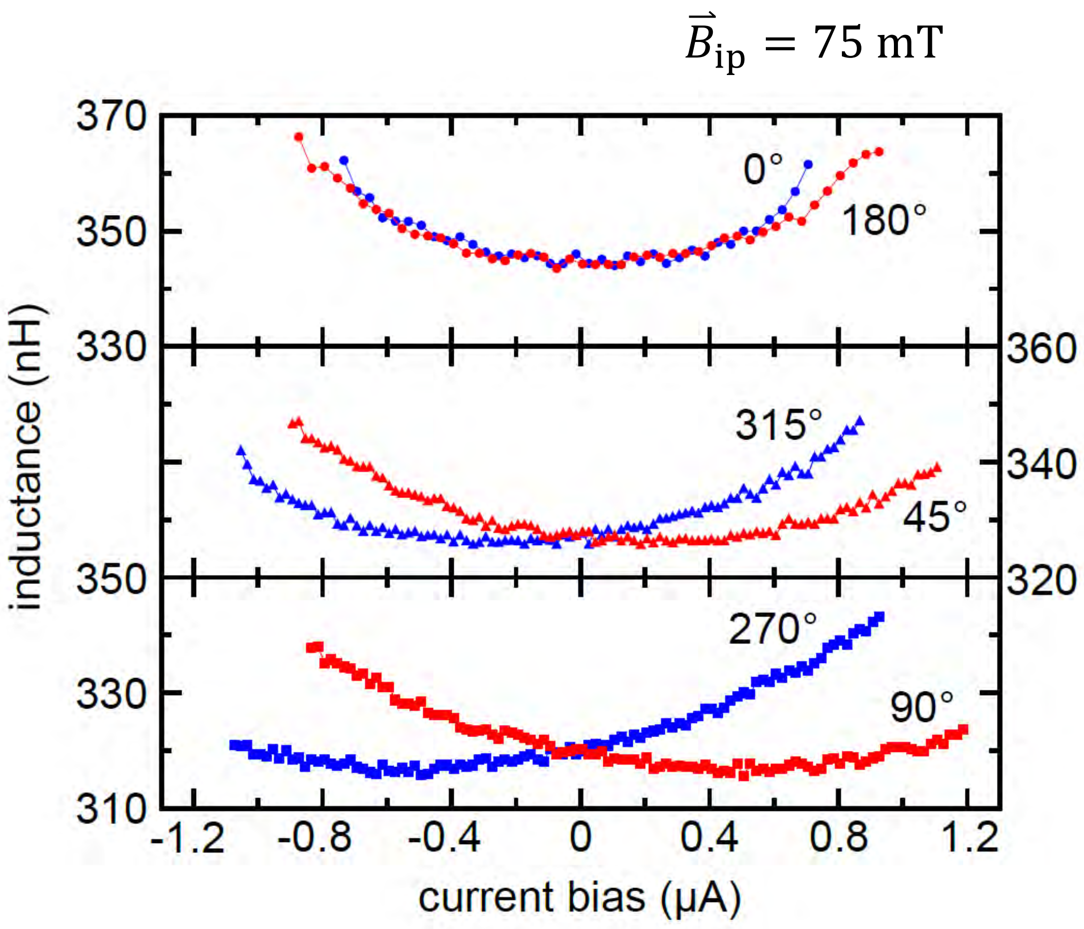


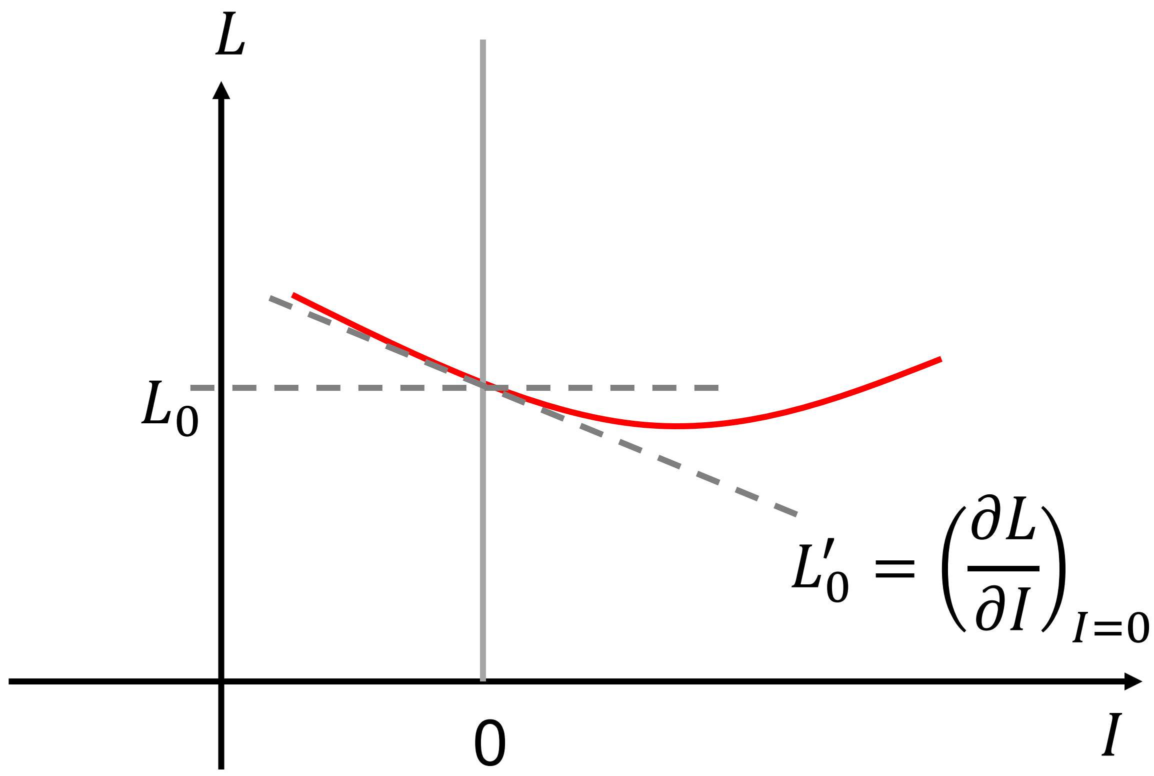




Asymmetric non-linear inductance reflects supercurrent diode effect

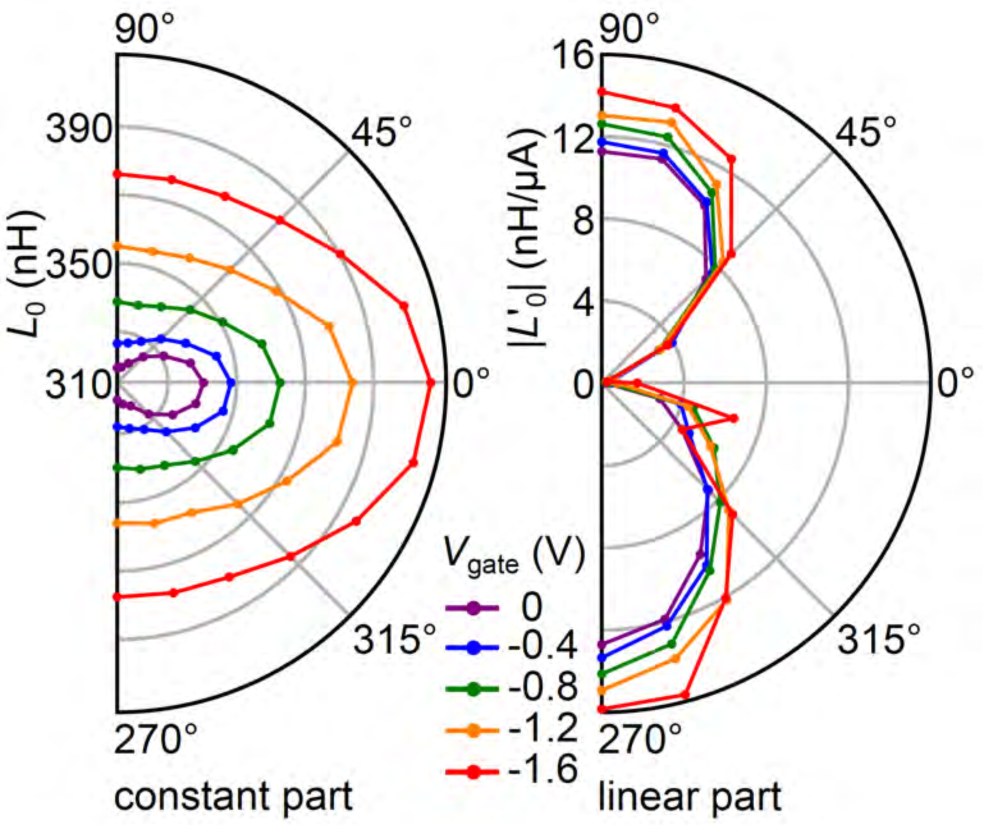
Polynomial expansion of $L(I) = L_0 + L'_0 I + L''_0 I^2 + \ldots$

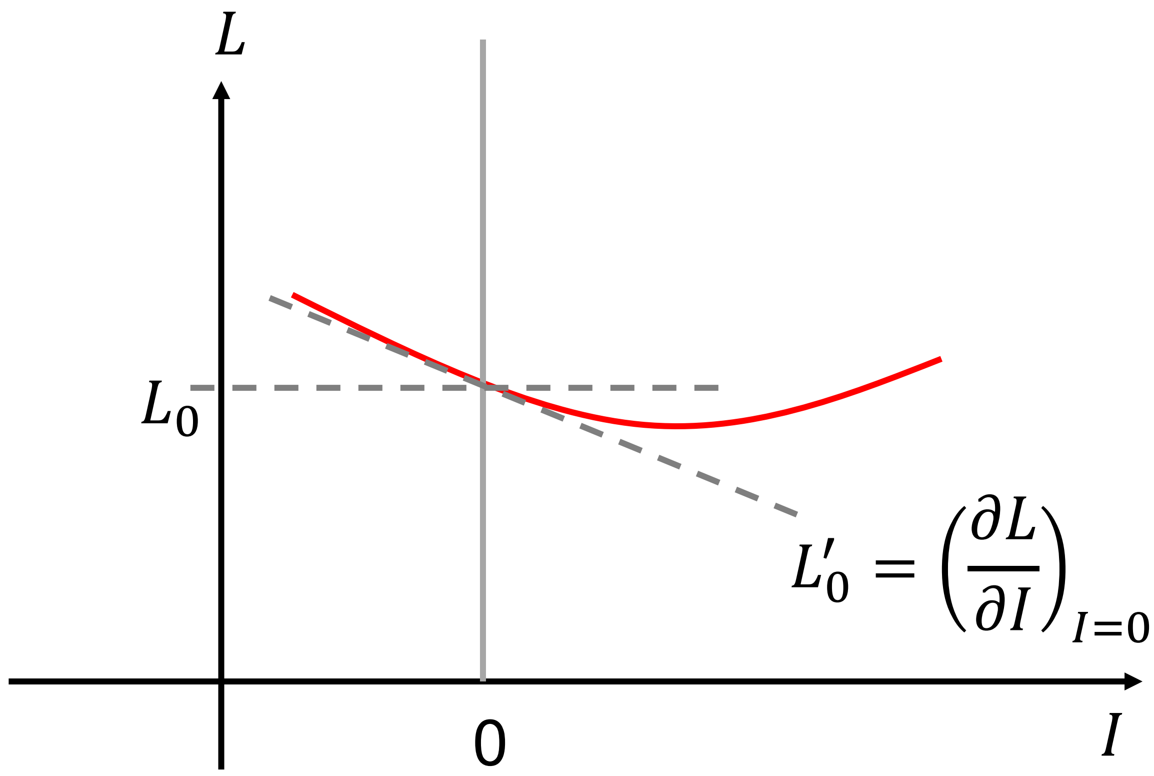




Asymmetric non-linear inductance reflects supercurrent diode effect

Polynomial expansion of: $L(I) = L_0 + L'_0 I + L''_0 I^2 + \ldots$

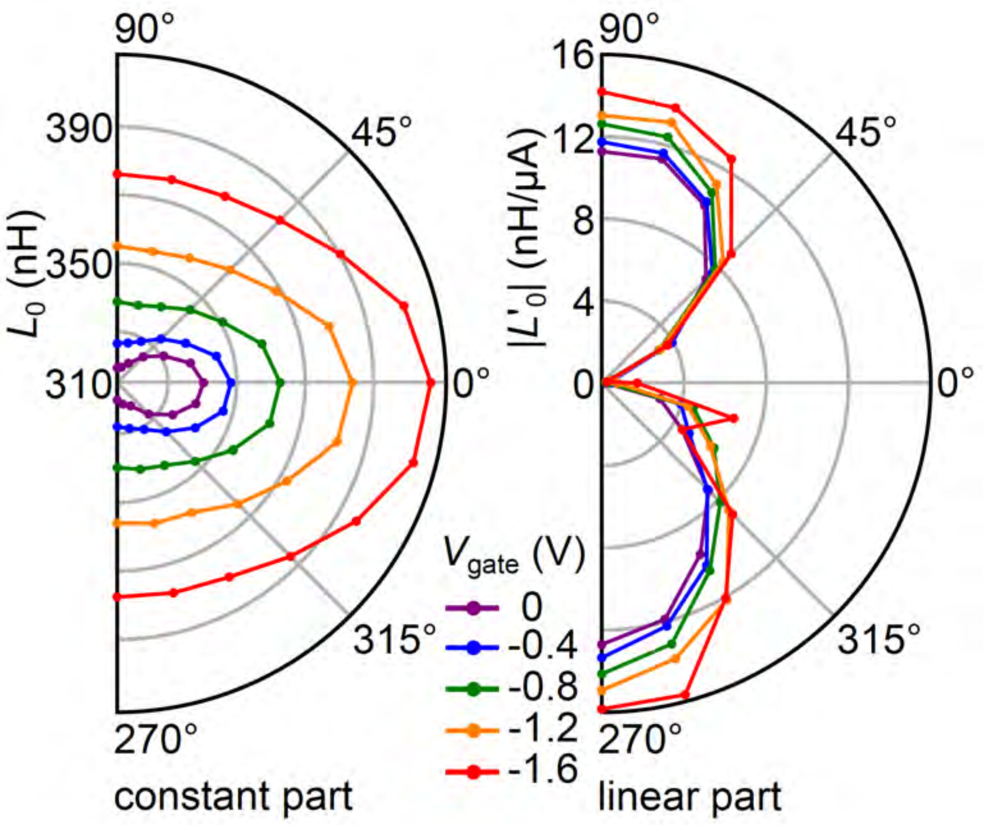


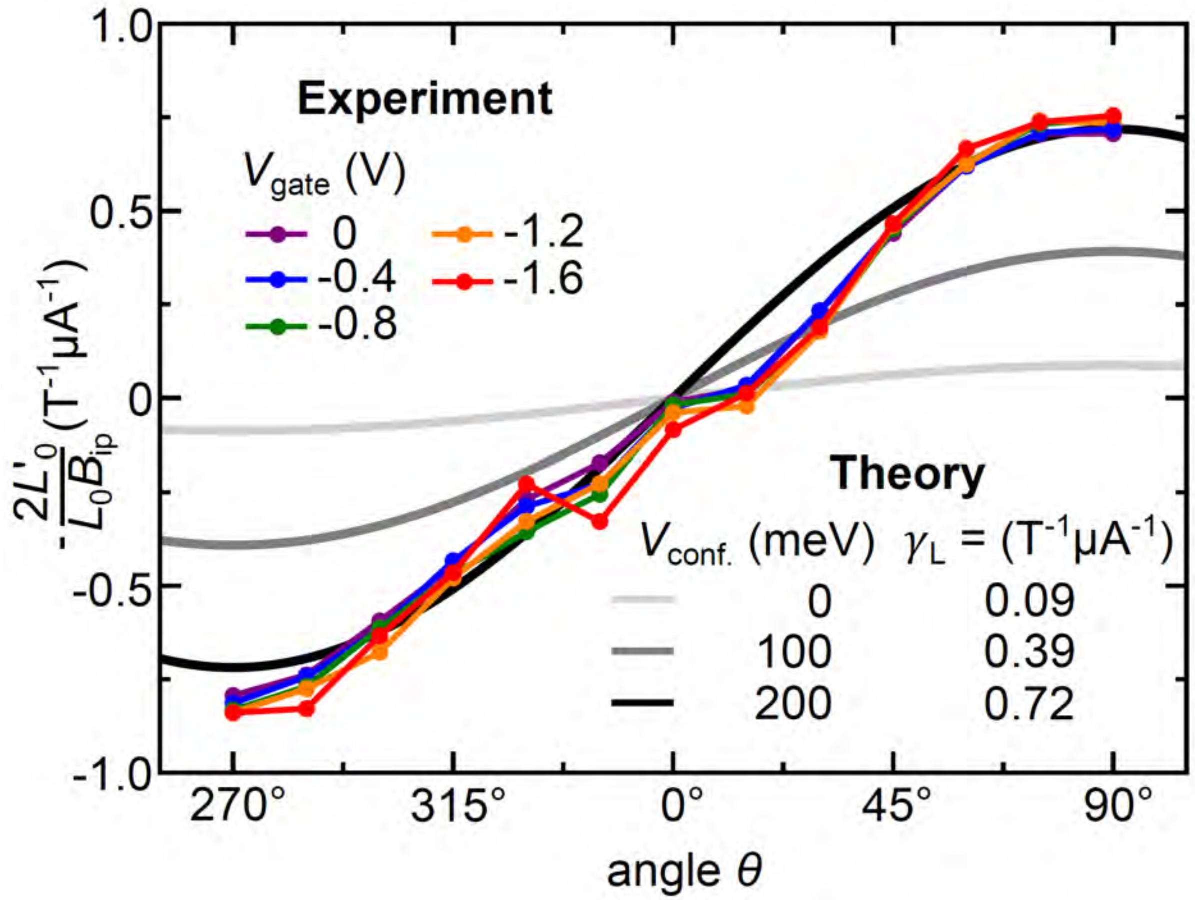


Magnetochiral anisotropy coefficient for the supercurrent

From L_0 and L'_0 → magnetochiral anisotropy coefficient $\gamma_L = 2L'/L_0B = 0.76 \cdot 10^6$ $T^{-1}A^{-1}$

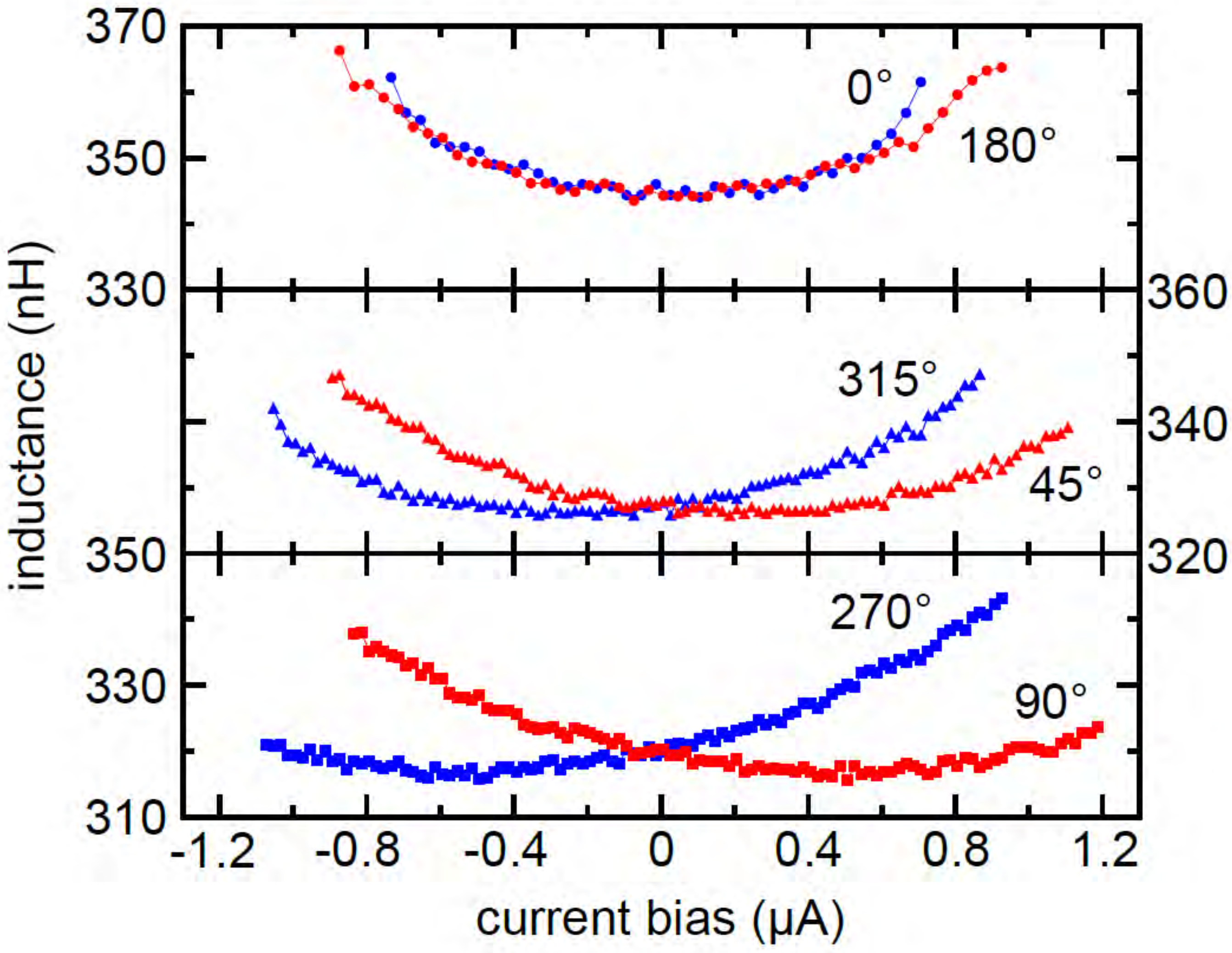
$$L = L_0[1 + \gamma_L \hat{e}_z(\vec{B} \times \vec{I})]$$





C. Baumgartner, CS, et al., Nat. Nanotech. **17**, 39 (2022)

AC supercurrent diode effect

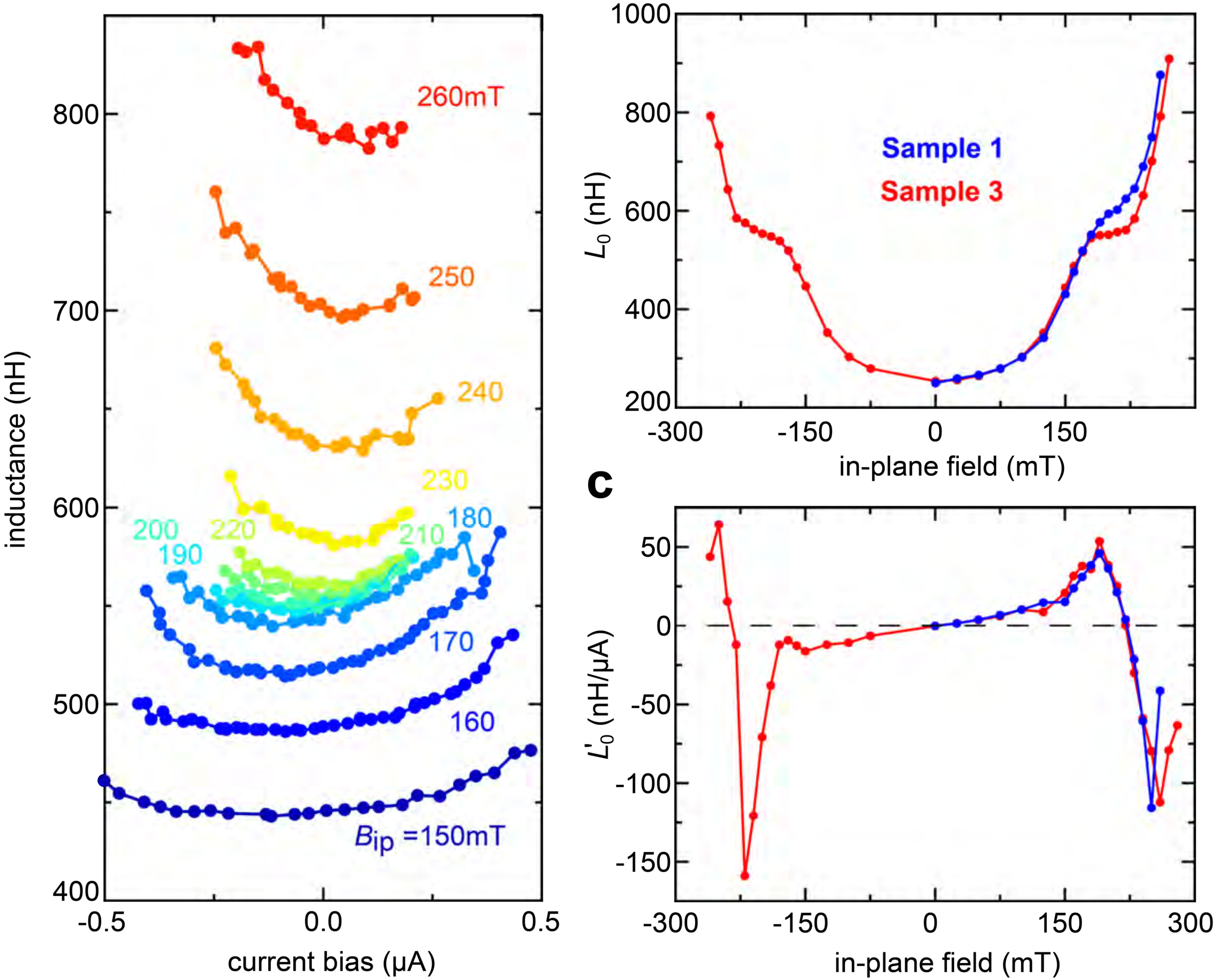


Reminder:

Asymmetry of differential inductance at low in-plane field:

$$\vec{B}_{\mathrm{ip}} = 75\ \mathrm{mT}$$

Sign reversal of Josephson diode effect at high in-plane field



- Asymmetry of nonlinear inductance changes sign at higher $B_{||}$
- Constant term L_0 exhibits shoulder near $B_{||}$ = 220 mT
- Linear term L_0' exhibits sign change near $B_{||}$ = 220 mT
- Looks different from "finite momentum pairing"

N. F. Q. Yuan, L. Fu,
PNAS 119, e2119548119 (2022),

B. Pal et al., Nat. Phys. **18**,1228 (2022).
A. Banerjee et al., arXiv:2301.01881.

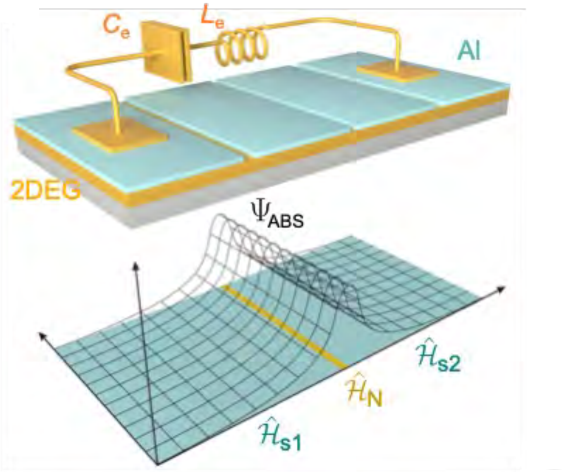
A. Costa, C.S. et al.,
Nat. Comm. **15**, 4413 (2024)

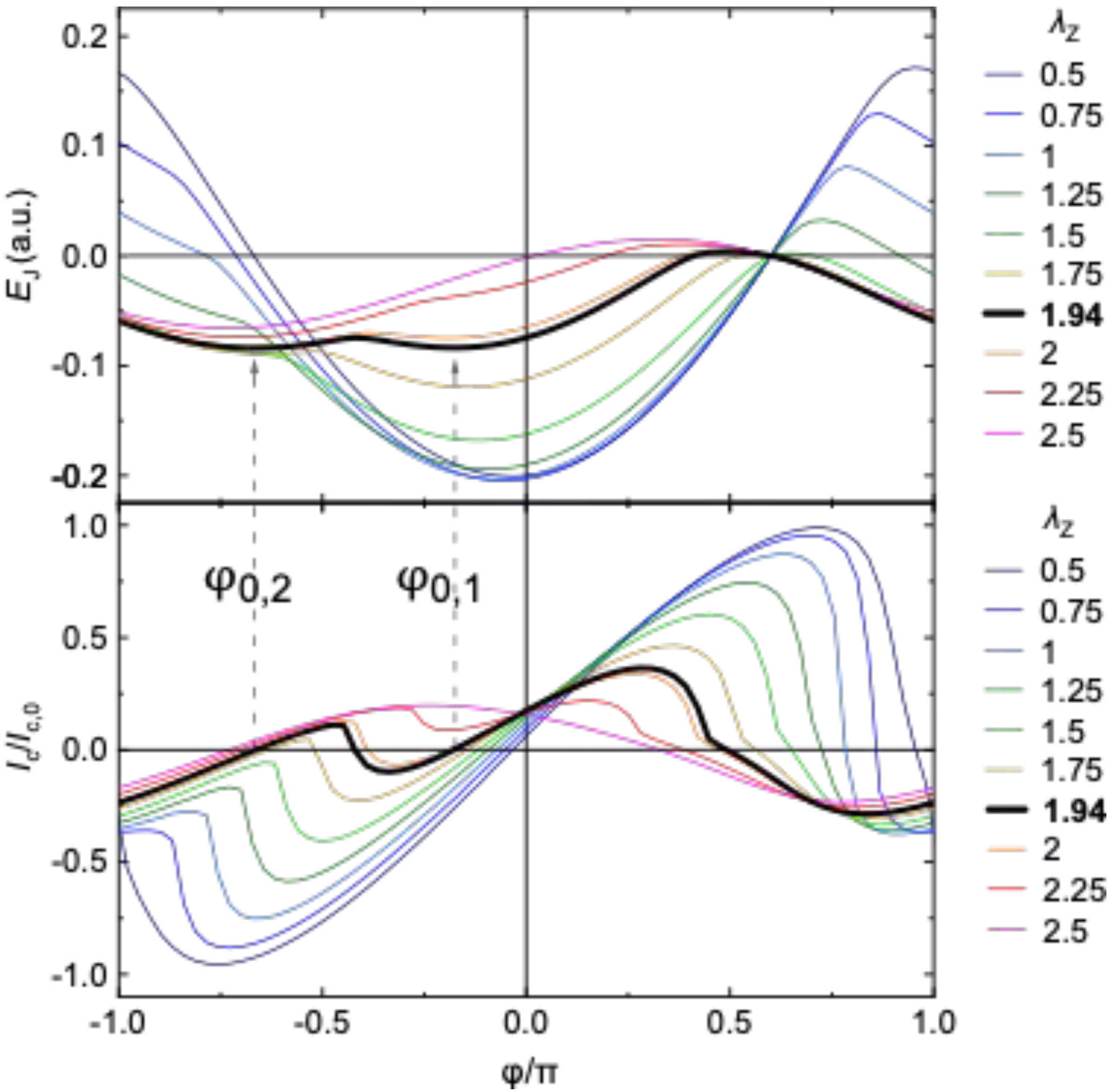
Sign reversal of Josepshson diode effect

minimal theoretical model:

- short junction limit,
- Rashba SOI
- phenomenological Zeemann coupling

(A. Costa, D. Kochan, J. Fabian)

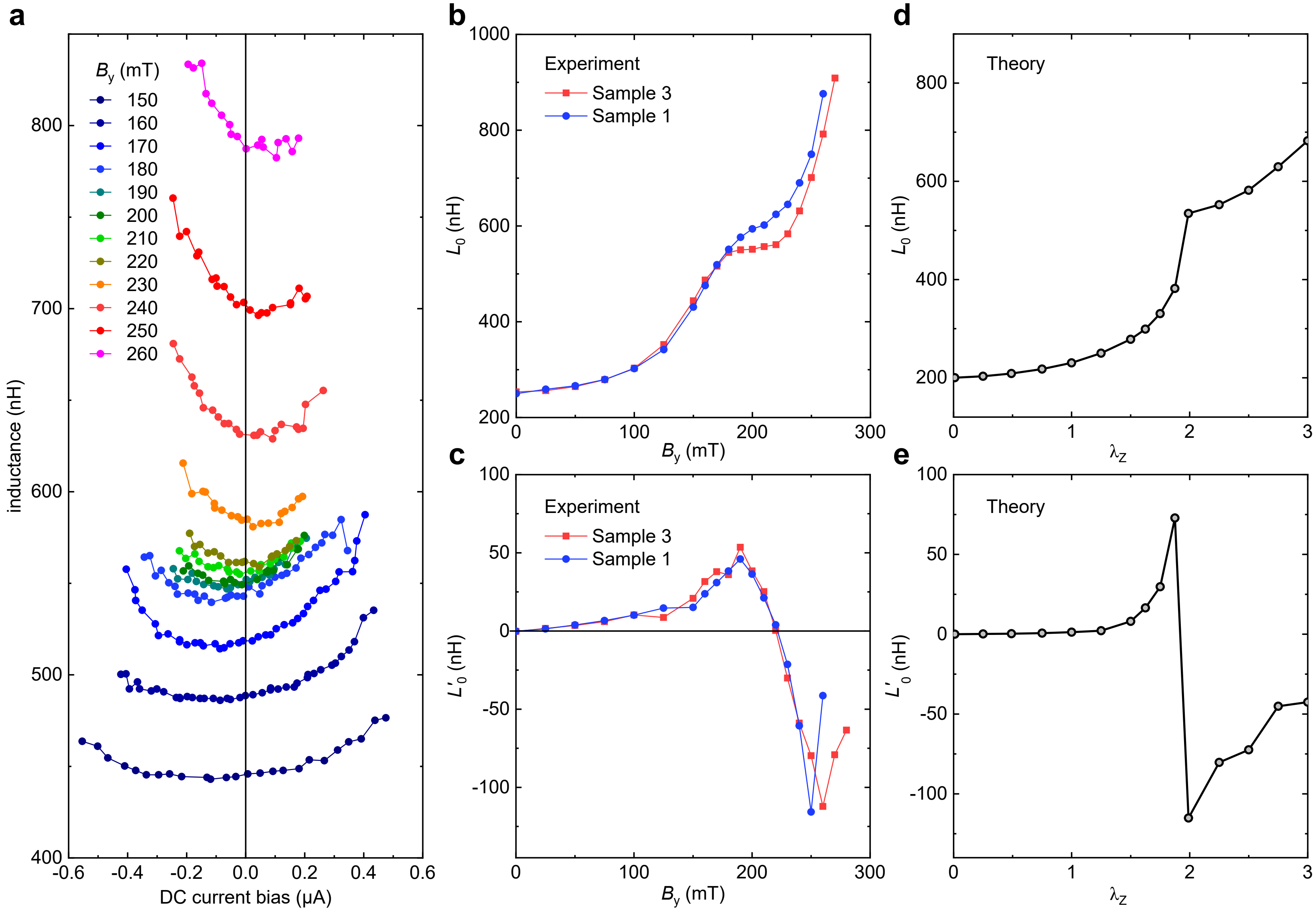




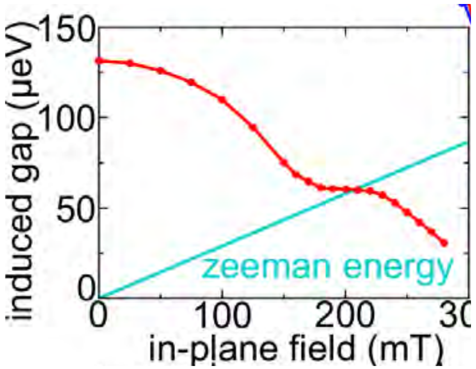
CPR undergoes drastic change with increasing $B_{||}$

- $\varepsilon_J(\phi_S)$ develops competing minimum near $B_{||}$
- cross-over between two phase offsets: φ_{01}, φ_{02}

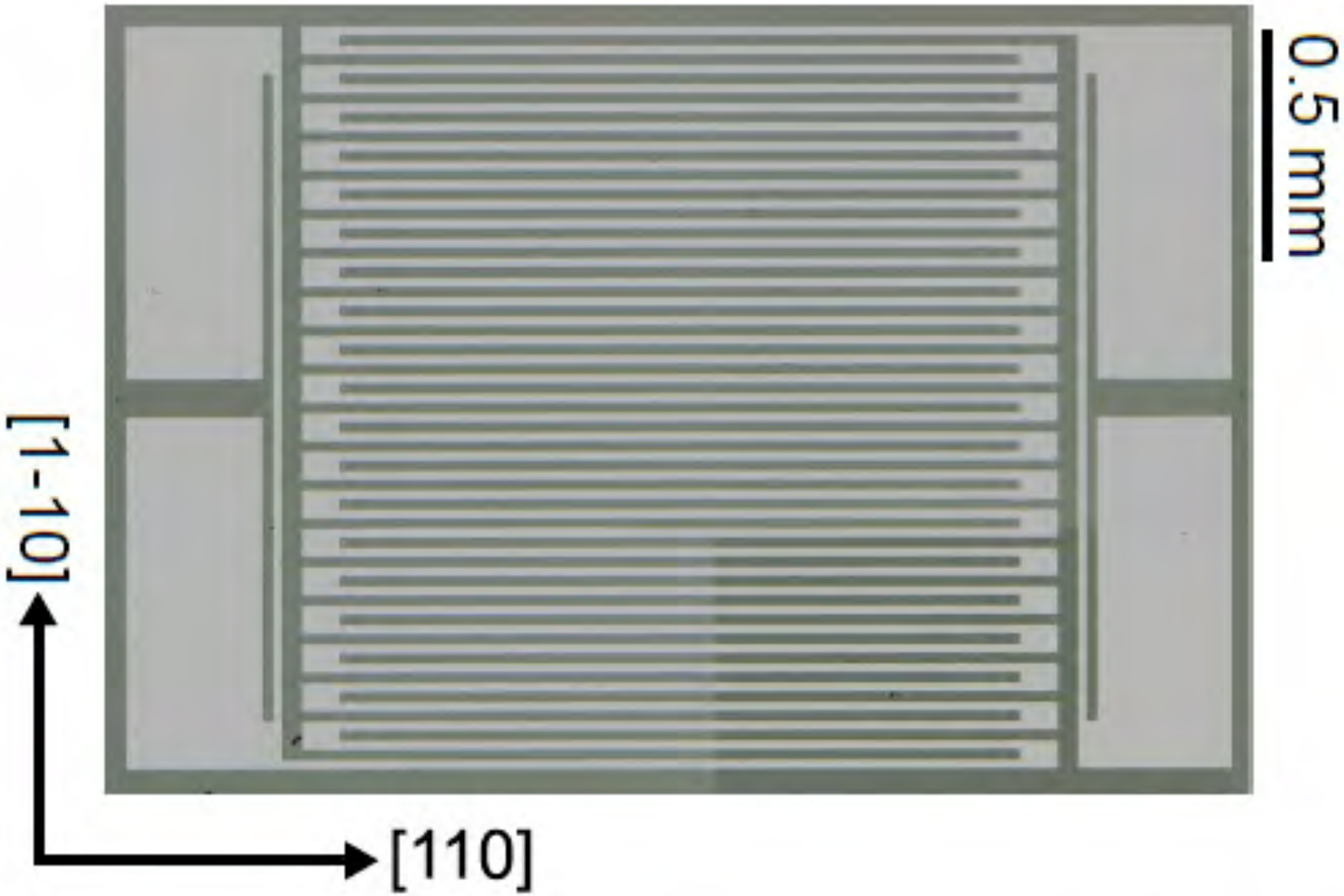
Sign reversal of Josephson diode effect



- Asymmetry of nonlinear inductance changes sign at higher $B_{||}$
- Constant term L_0 develops shoulder near $B_{||}$ = 220 mT
- Linear term L'_0 changes sign at $\Delta \cong E_Z$ near $B_{||}$ = 220 mT



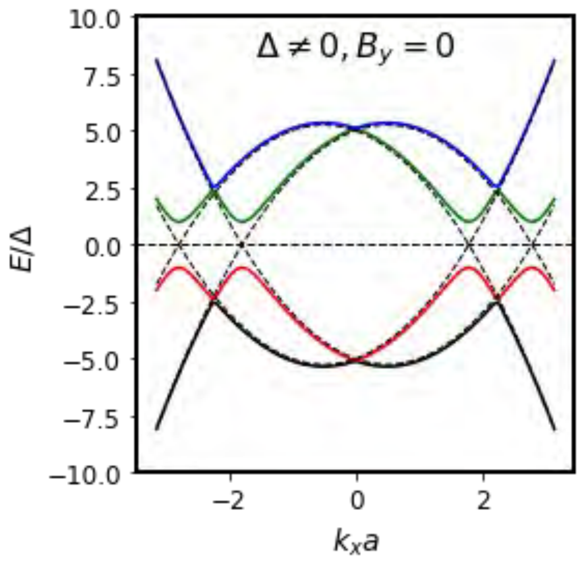
A. Costa, C.S. et al., Nat. Comm. **15**, 4413 (2024)

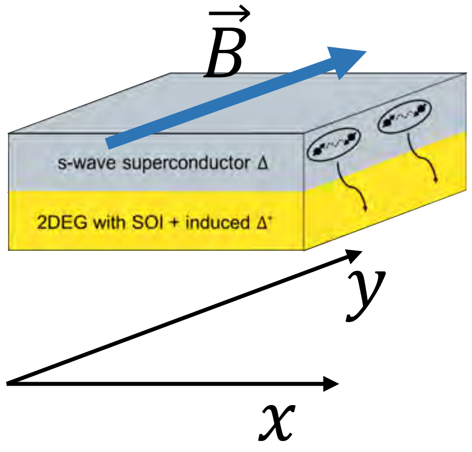


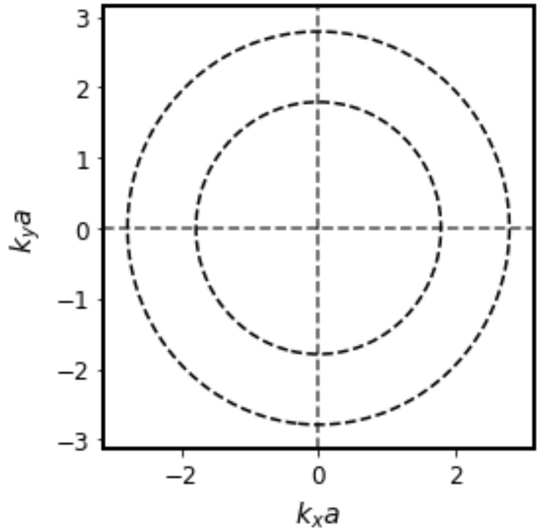
Let us omit the junctions…

Rashba + superconductivity

measure kinetic inductance
via resonator technique
(as before)

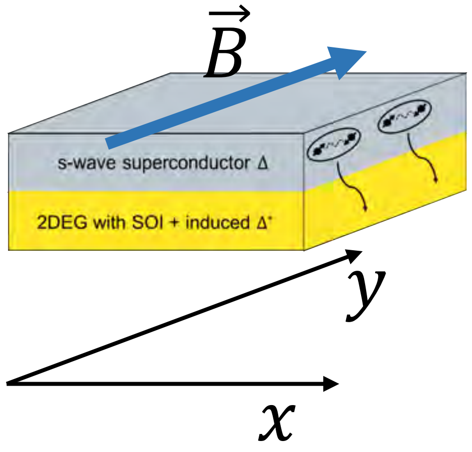


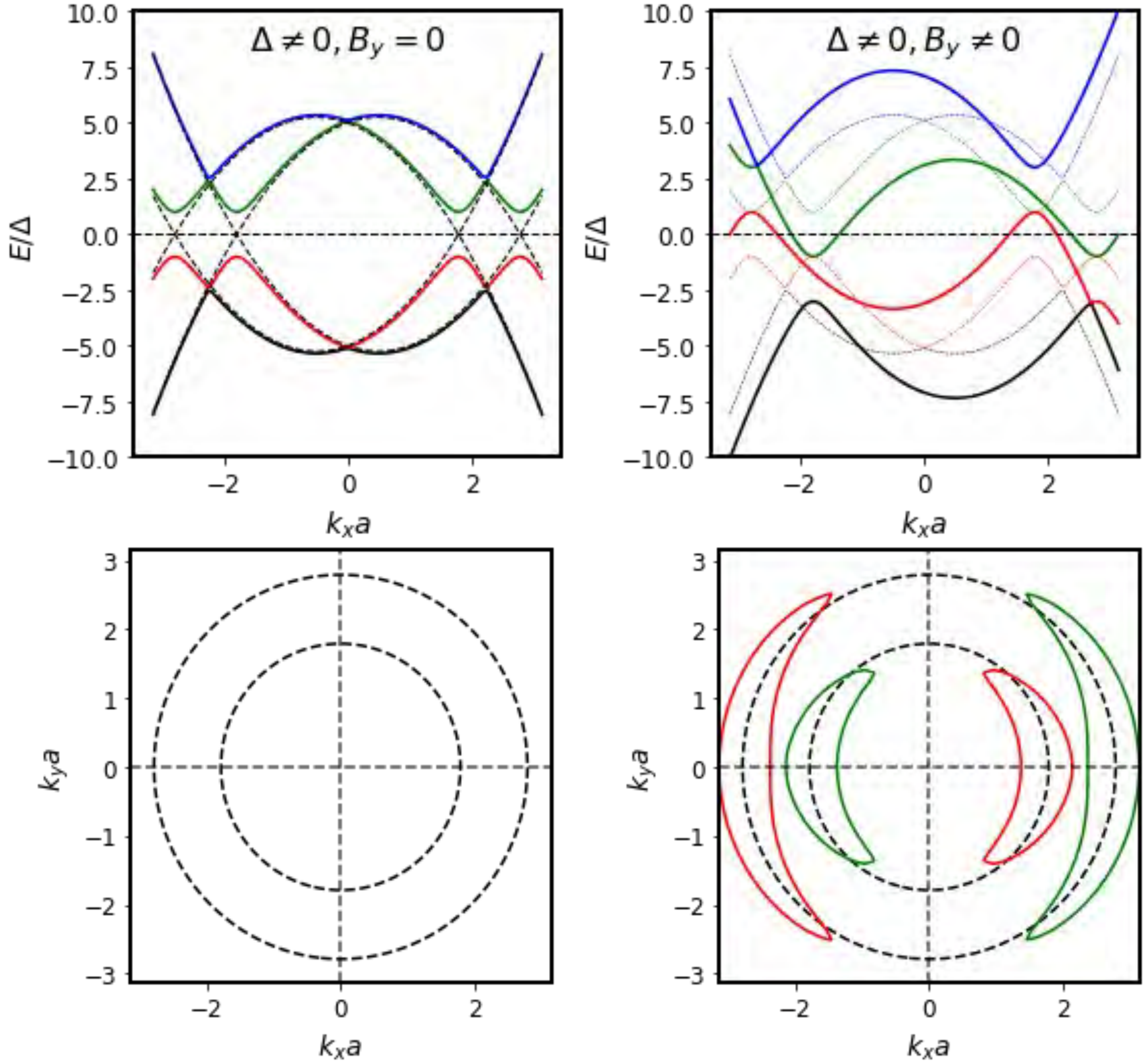




Rashba + superconductivity + Zeeman splitting

measure kinetic inductance
via resonator technique
(as before)



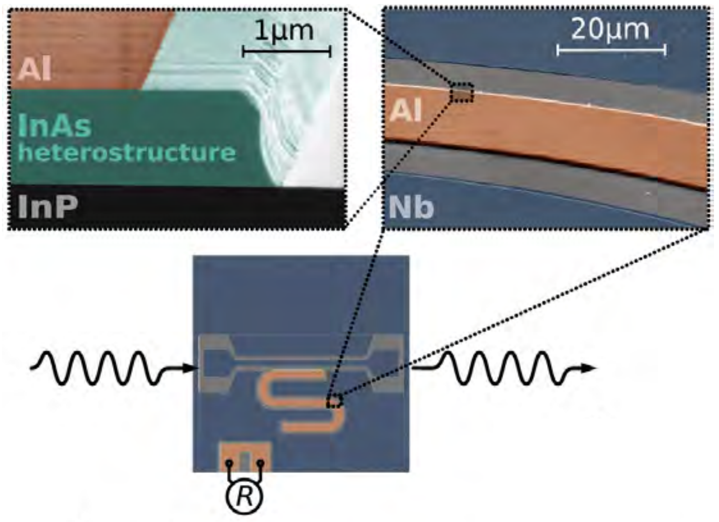


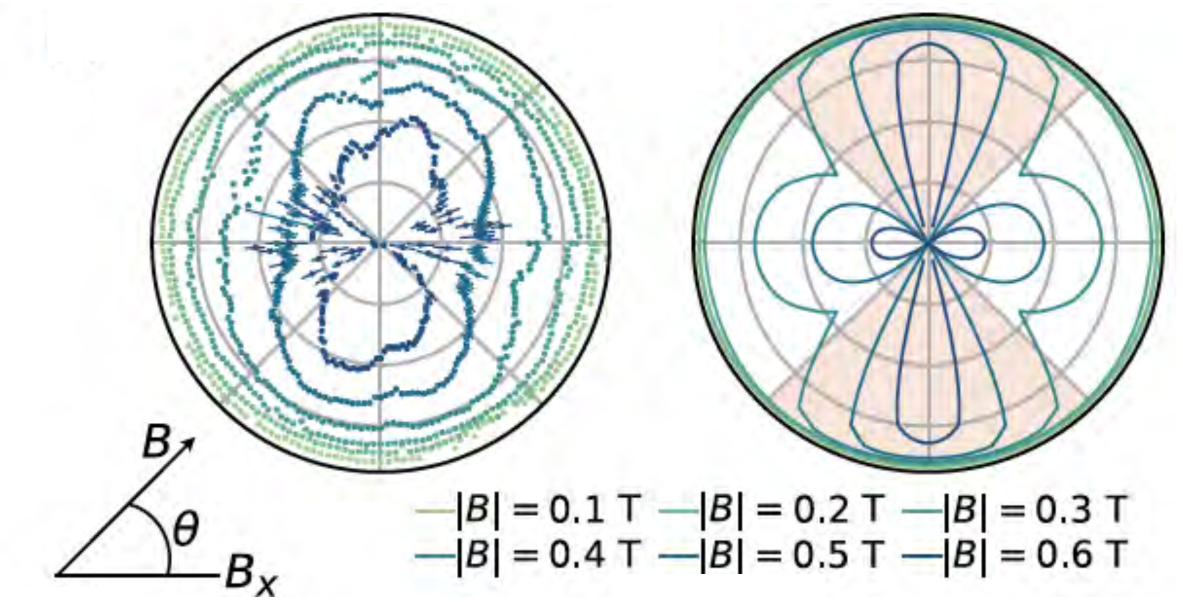
Bogoliubov Fermi surfaces

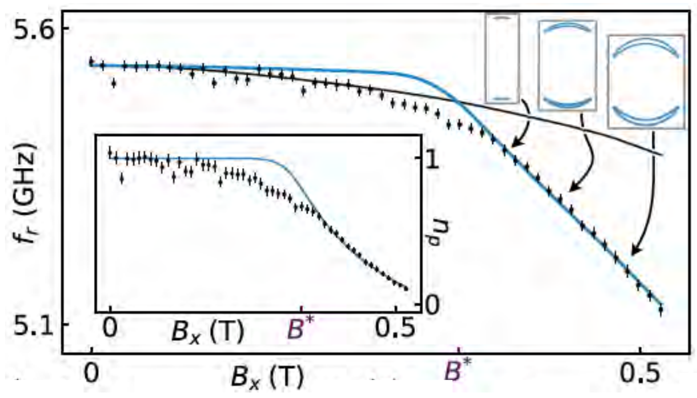
should have an effect on kinetic inductance

S. Serafim et al. arXiv:2311.09347 (2023)

First experimental evidence





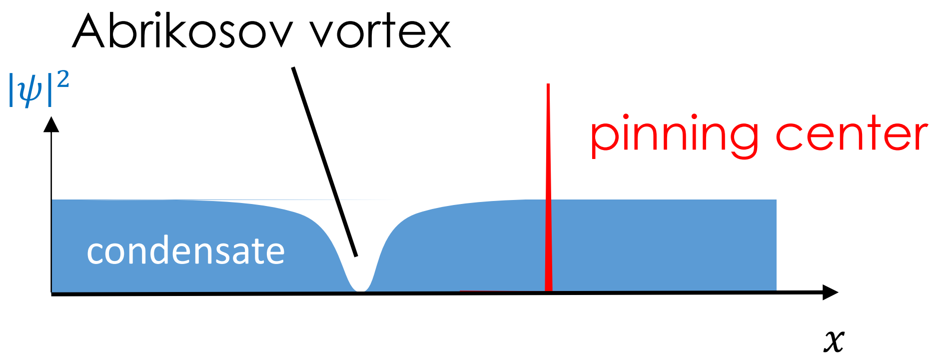


D. Phan, et al., Phys. Rev. Lett. **128**, 107701 (2022)

open questions:

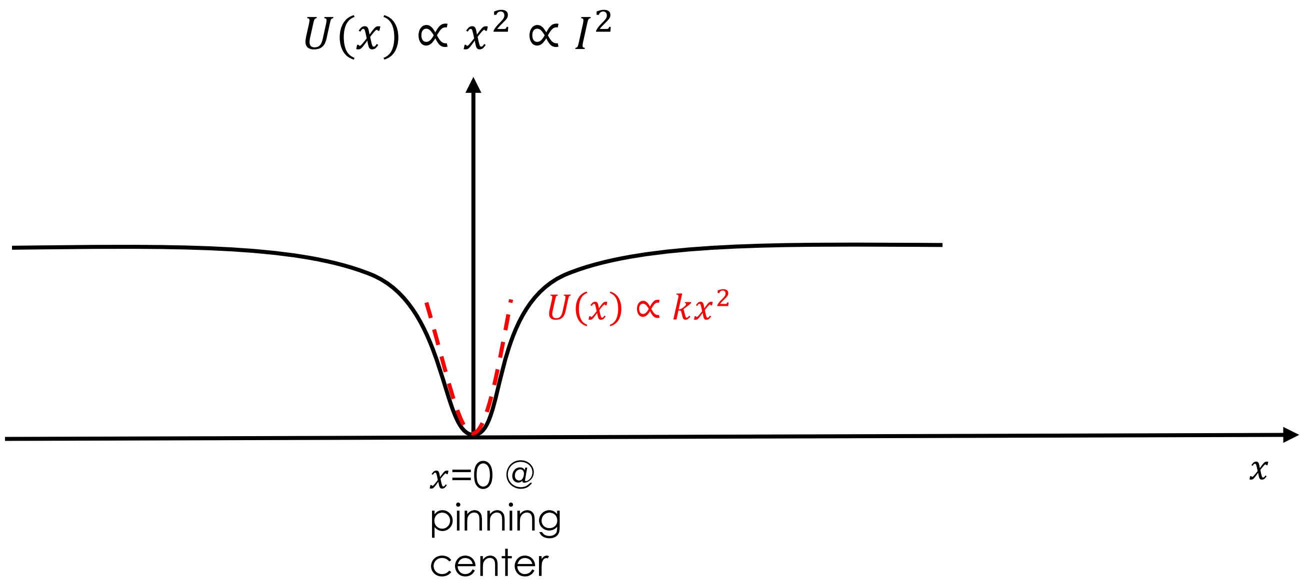
- effects of pinned vortices from misalignment?
- orientation of suppression of L_{kin} in xy-plane opposite to expectations from Bogoliubov Fermi surfaces?
- anisotropy of g-factor?
- direct evidence for triplet component?

Vortex inductance



Pinning potential is convolution of defect potential (delta-like) and order parameter profile:

$$U(x) \propto |\psi|^2$$



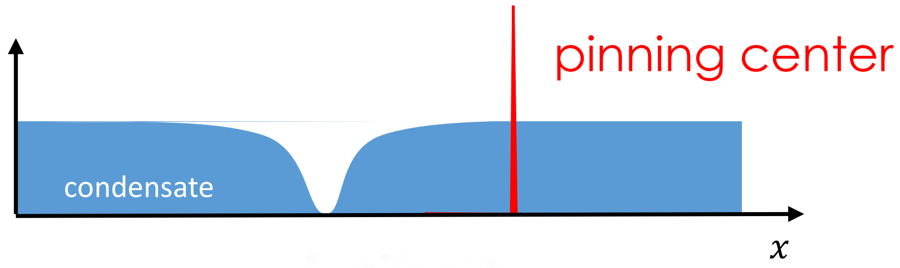
Q: What is the voltage response of a vortex to a low frequency AC current drive?

A: At low frequency, a purely imaginary impedance arises

Pedagogical review: M. Golosovsky et al., *Supercond. Sci. Tech*. **9**, 1 (1996)

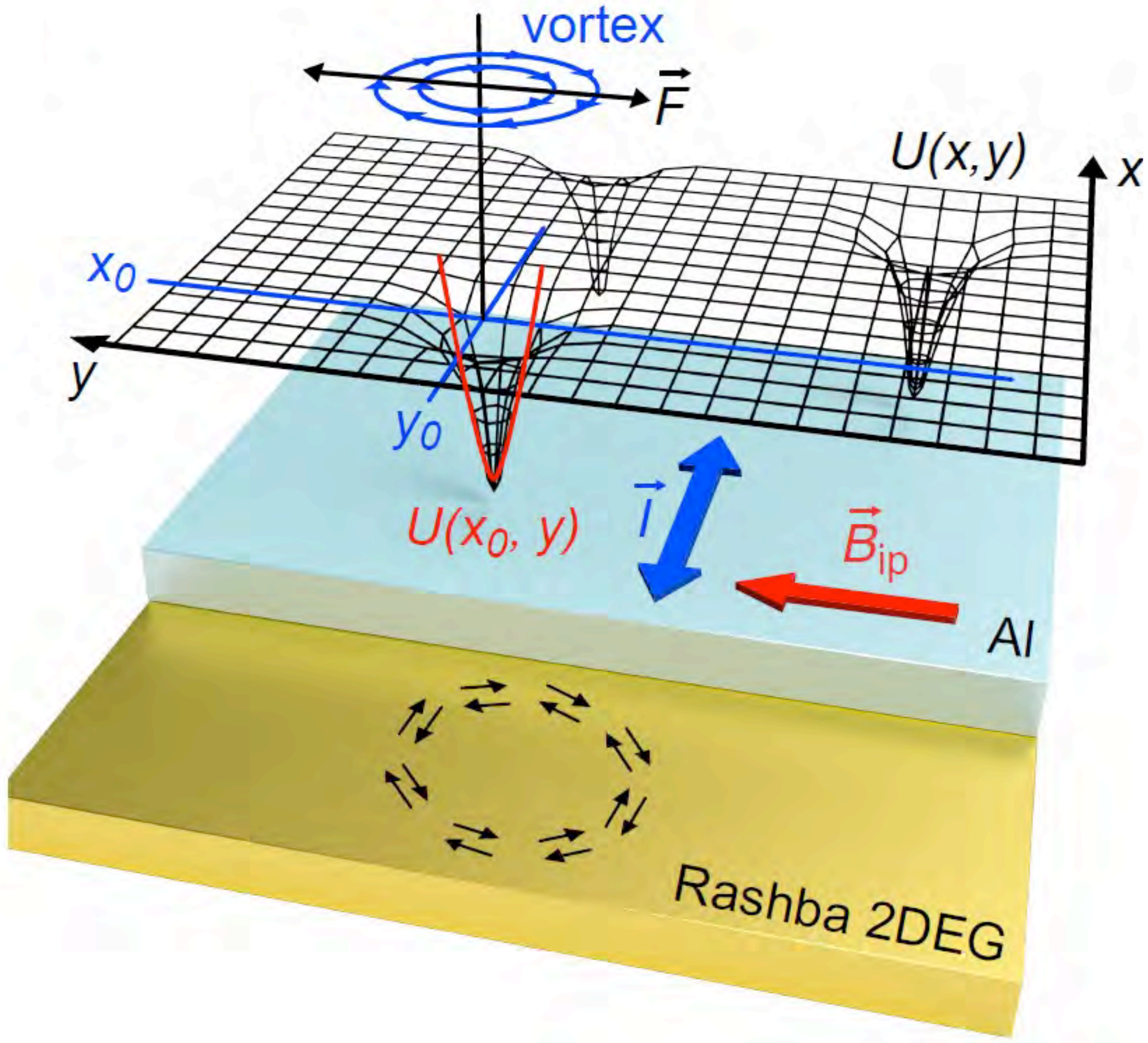
Our experiment

- Vortex inductance linear in B_z
- L_v increases when $|\psi|^2$ decreases

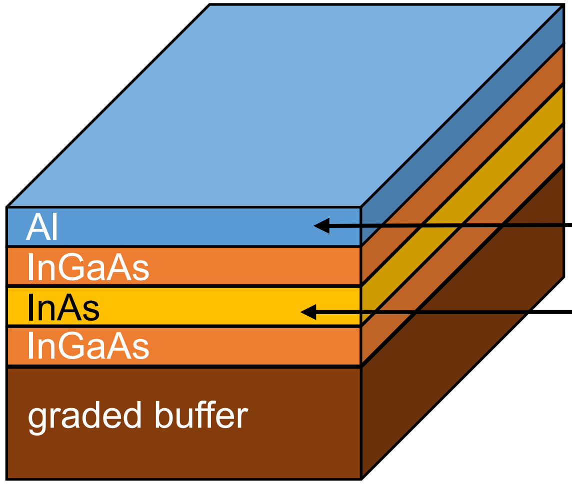


$$L_v = N_\square \frac{B_z \Phi_0}{k}$$

Vortex inductance measures (inverse) curvature of pinning potential



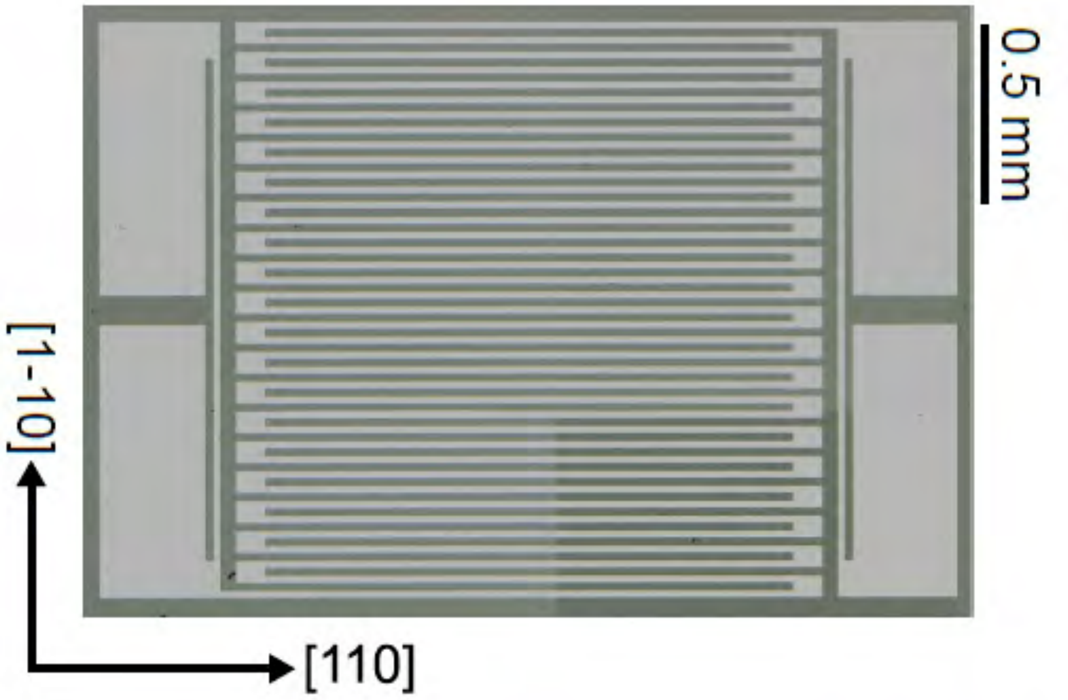
Our experiment

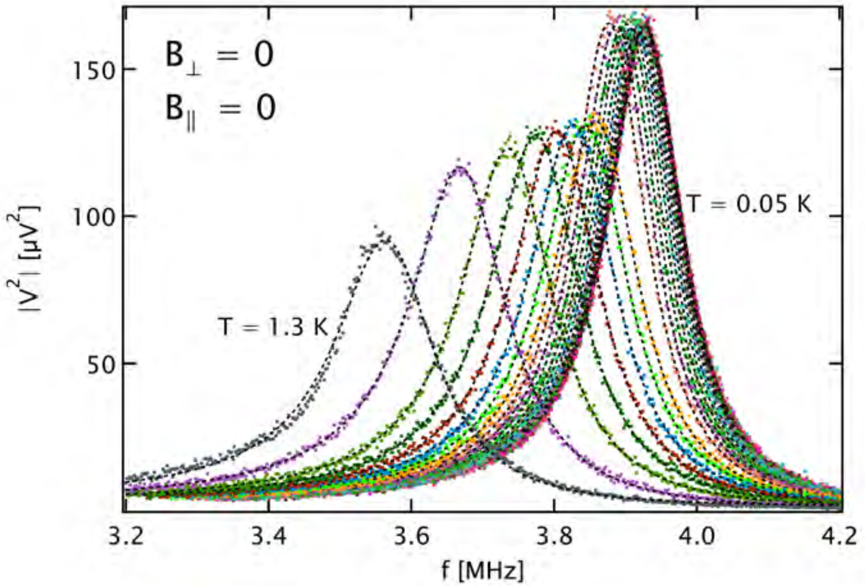


material by Manfra group

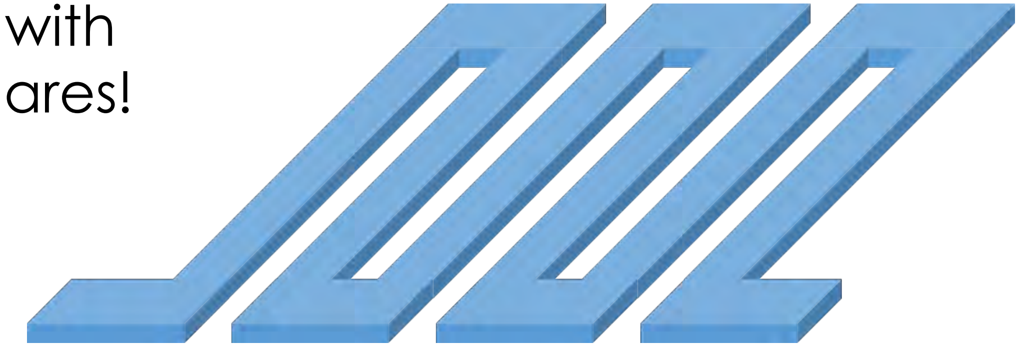
Epitaxial Al, gap Δ

2DEG, ind. gap Δ*





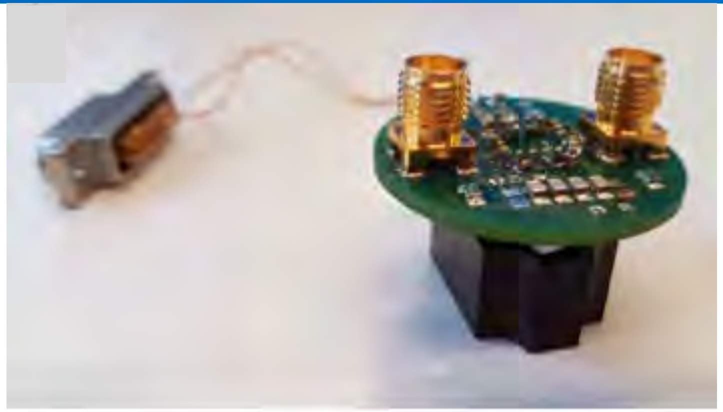
superconducting meander with >3000 squares!

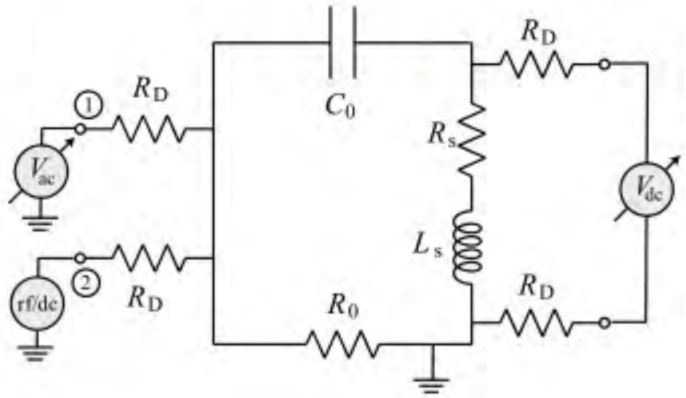


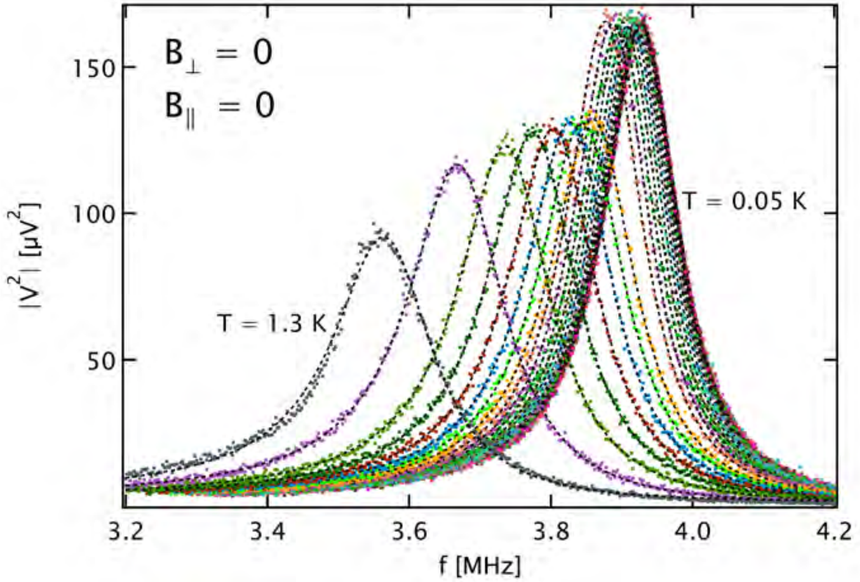
Inductance measurements

Lumped element resonator : superconducting strip series with Cu-coil and thin film capacitor

- decrease in resonance frequency f_0 corresponds to increase in kinetic inductance of sample
- decrease in internal Q-factor Q_i corresponds to increase in sample resistance
- J_s is very sensitive to magnetic field







measure superfluid stiffness via kinetic inductance $L_\blacksquare$:

$$J_s = \frac{\Phi_0^2}{16\pi^2 k_B L_\blacksquare} = \frac{\hbar^2}{4e^2 k_B} \cdot \frac{d}{\mu_0 \lambda^2(T)}$$

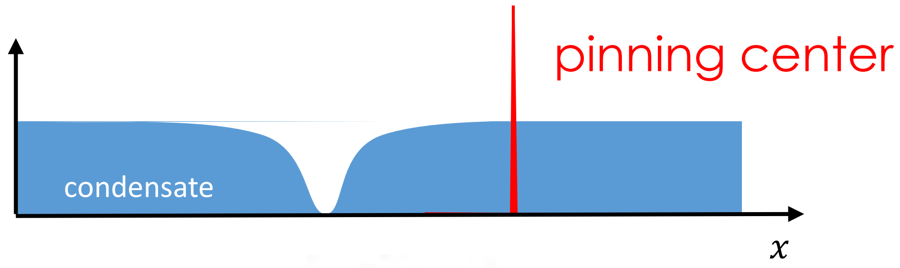
Dirty limit BCS: $J_s(T=0) = \dfrac{\Phi_0^2 \Delta(0)}{4\pi k_B \hbar R_N}$

idea based on:

Meservey & Tedrow, J. Appl. Phys., **40**, 2028 (1969)

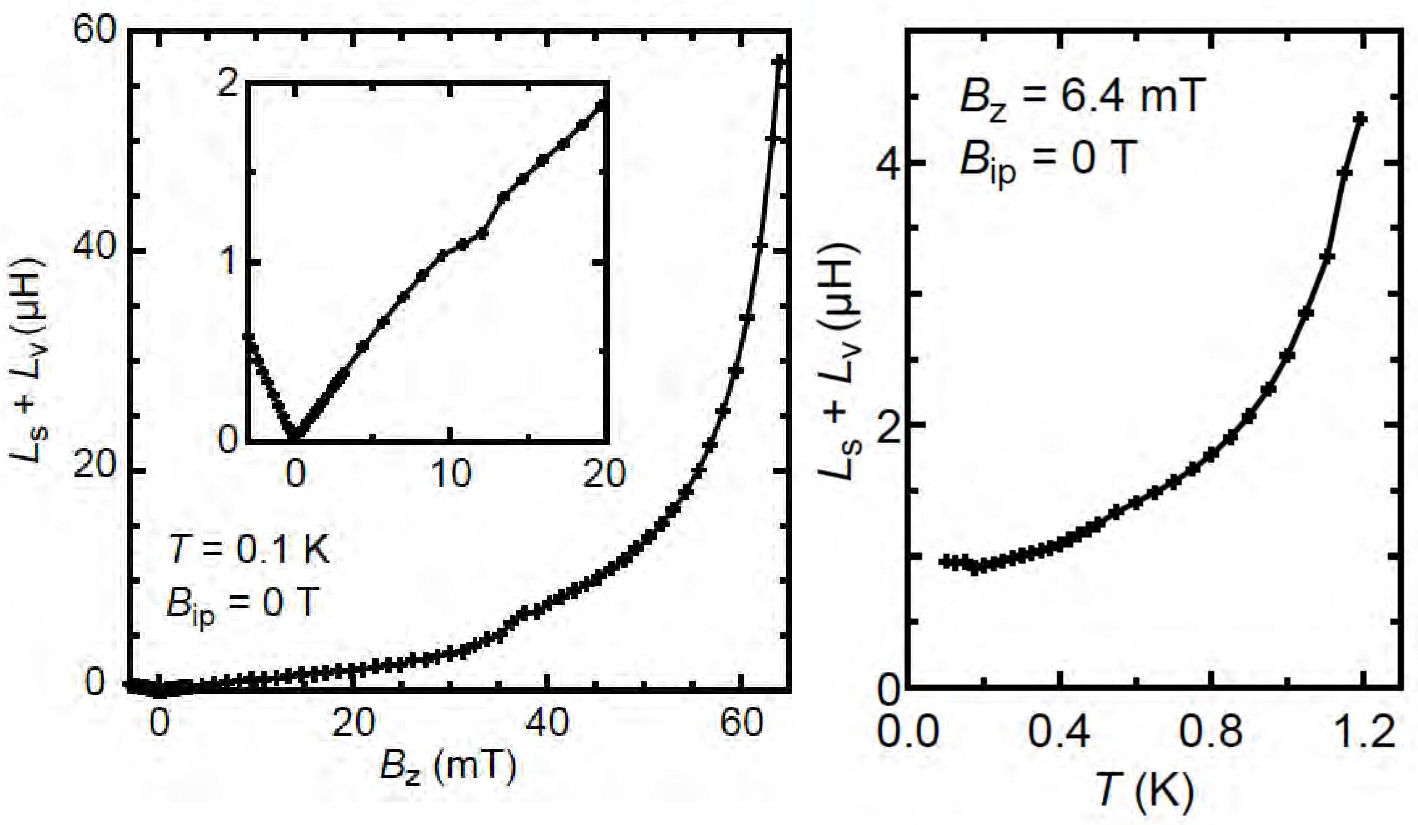
First characterization: everything behaves as it should

- Vortex inductance linear in B_z
- L_v increases when $|\psi|^2$ decreases



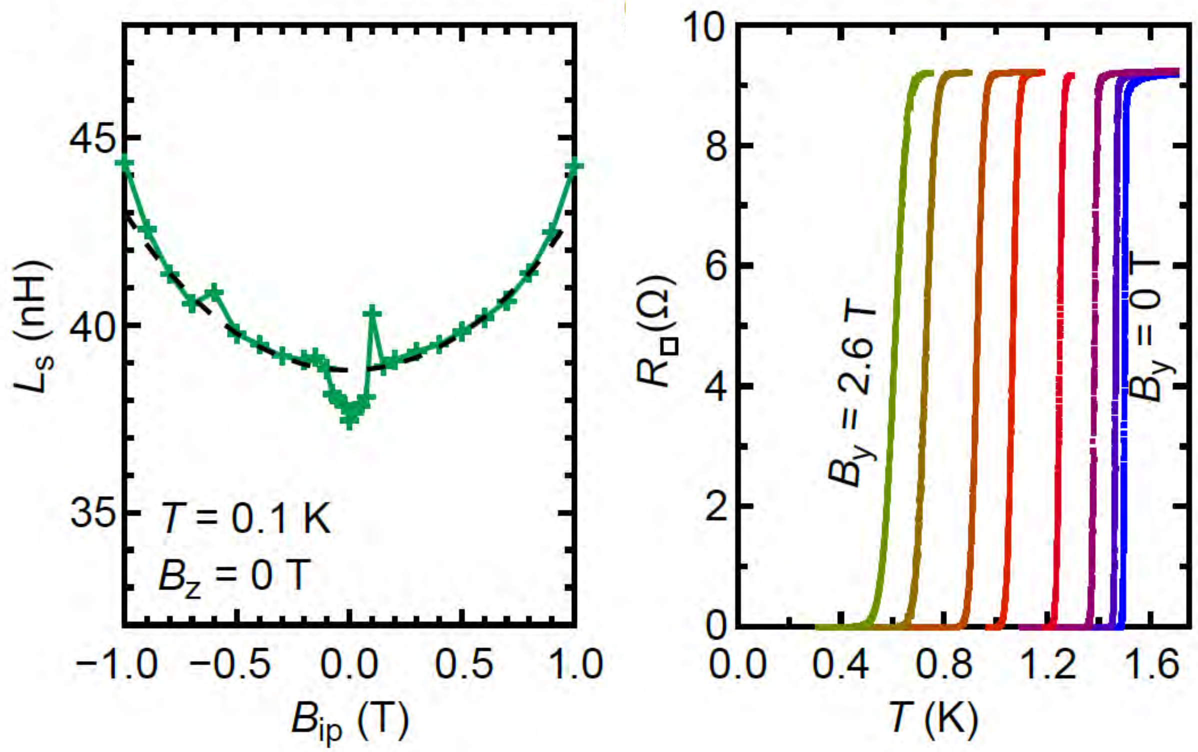
$$L_v = N_\Box \frac{B_z \Phi_0}{k}$$

Vortex inductance measures (inverse) curvature of pinning potential



First characterization: everything behaves as it should

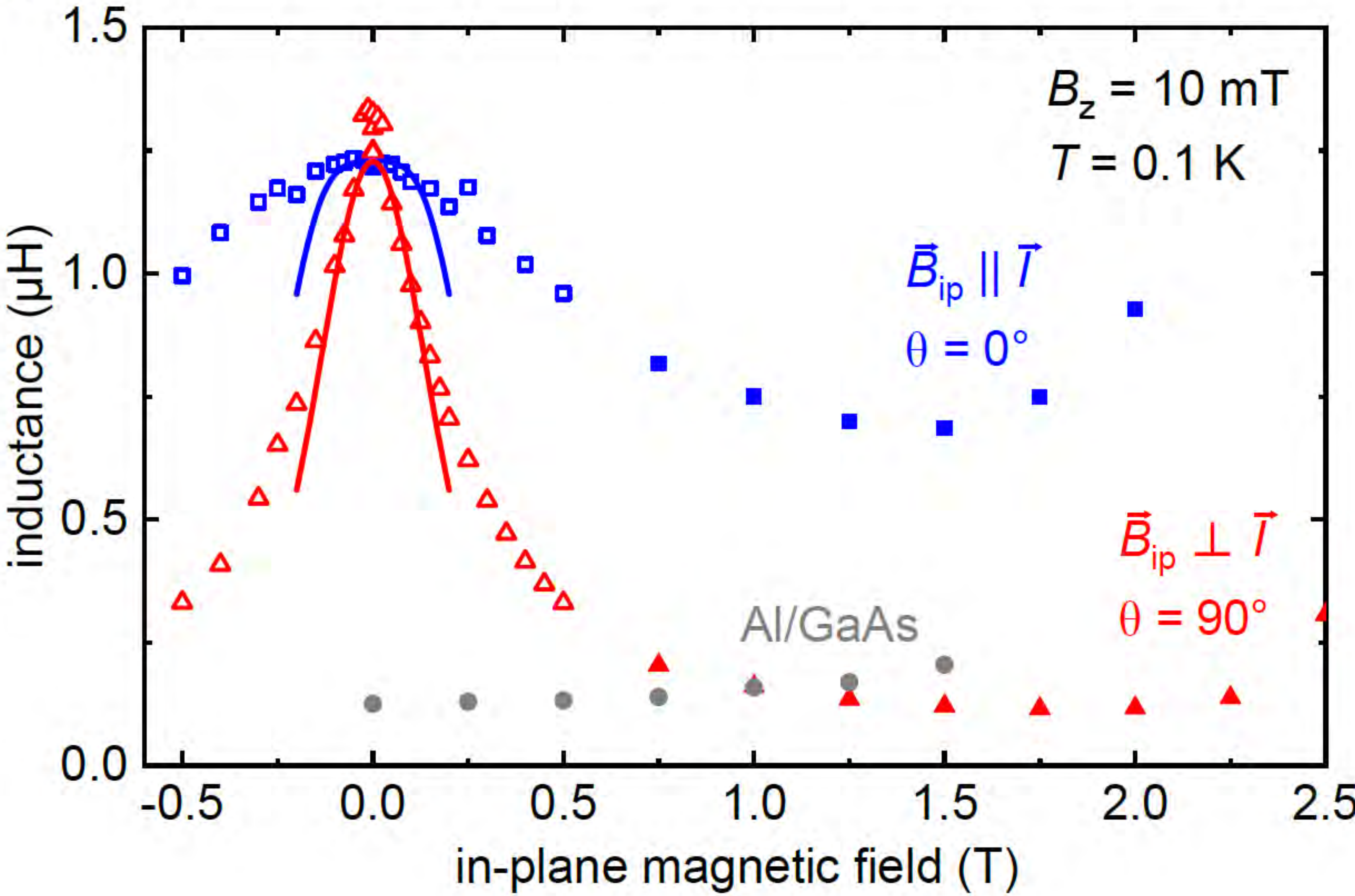
- Kinetic inductance increases with B_{ip}
- L_{kin} increases if $|\psi|^2$ decreases
- T_c decreases if $|\psi|^2$ decreases



Inductance suppression = pinning enhancement

Main experimental result

- L_v decreases with B_{ip}
- The decrease is anisotropic
- Al on GaAs (low SOI) behaves regularly



L. Fuchs et al.,
Phys. Rev. X **12**, 041020 (2022)

The Ginzburg-Landau way: Lifshitz Invariants

An effective way to describe SOI effects on the condensate:
GL free energy + Lifshitz invariant

E. M. Lifshitz,
On the theory of the second order phase-transitions I + II
JETP **11**, 255 and 269 (1941)

D. Kochan

$$F[\psi,\mathbf{A}] = a(T)|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4m}(\mathbf{D}\psi)^* \cdot \mathbf{D}\psi + \frac{\mathbf{B}^2}{2\mu_0} + F_L[\psi,\mathbf{A}] \equiv F_{GL}[\psi,\mathbf{A}] + F_L[\psi,\mathbf{A}],$$

$$F_L[\psi,\mathbf{A}] = -\frac{1}{2}\kappa(\mathbf{n}\times\mathbf{B})\cdot\mathbf{Y}_\psi \equiv -\frac{1}{2}\kappa(\mathbf{n}\times\mathbf{B})\cdot\left[(\psi)^*\mathbf{D}\psi + \psi(\mathbf{D}\psi)^*\right],$$

$$\mathbf{D} = \frac{\hbar}{i}\nabla - 2e\mathbf{A}$$

V. M. Edelstein, J. Physics: Cond. Matter **8**, 339 (1996)
M. Smidman *et al.*, Rep. Prog. Phys. 80 036501 (2017)
Denis Kochan *et al.*, arXiv:2303.11975 (2023)

reflects Rashba-SOI at GL level !

Effect of SOI on Abrikosov vortices

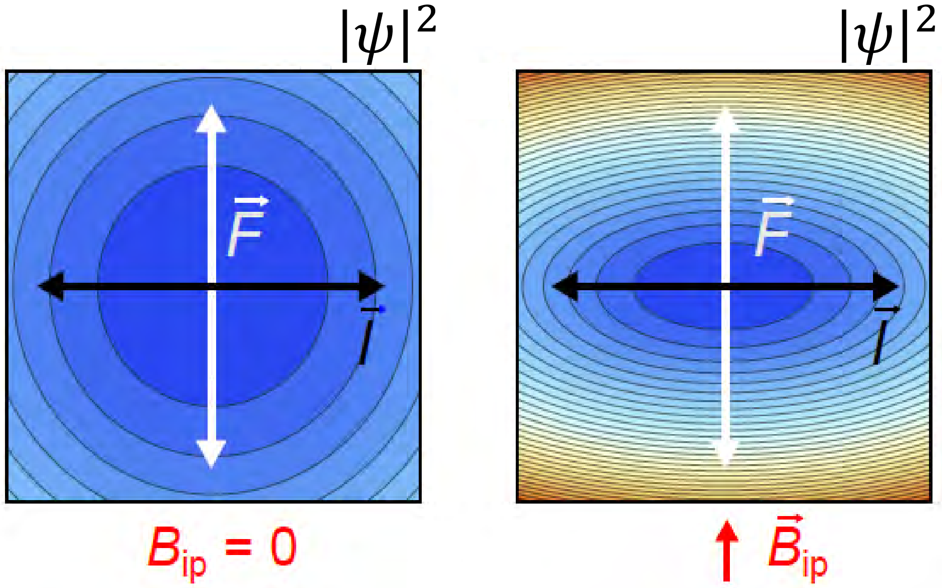
Signatures of SOI at the Ginzburg-Landau level ?

Theory:

- Minimize GL-free energy F with Lifshitz invariant term + in-plane field
- Solve GL-equations in 2 dimensions

Surprising result:

- an in-plane field squeezes the vortex cores in an anisotropic fashion



color scales identical

Inductance suppression = pinning enhancement

Main experimental result

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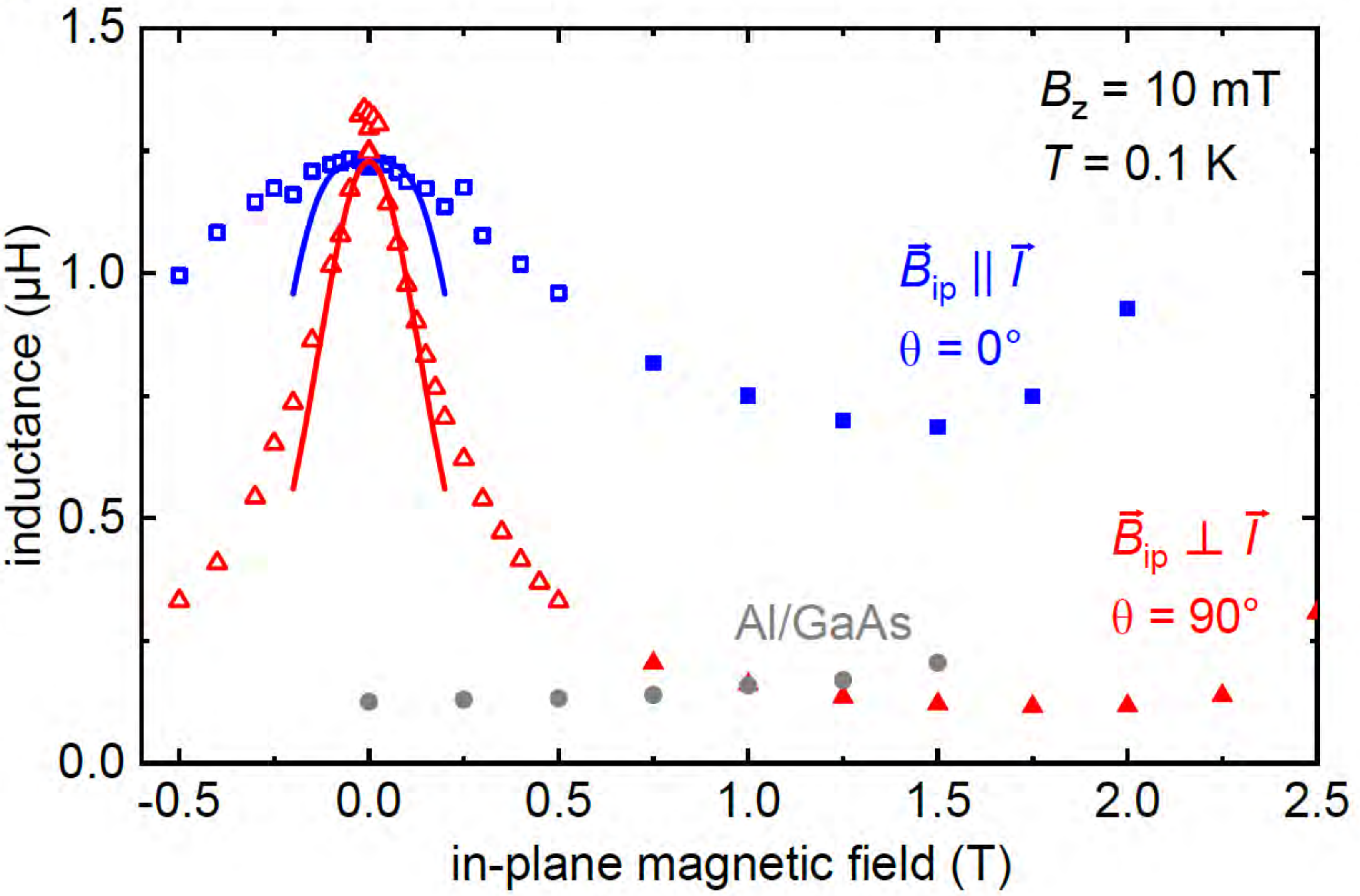
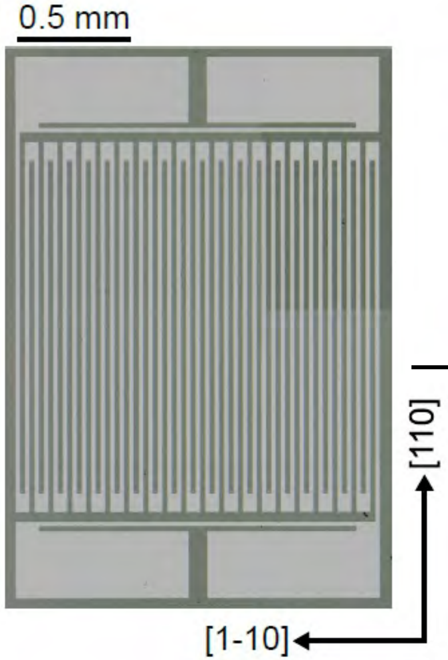
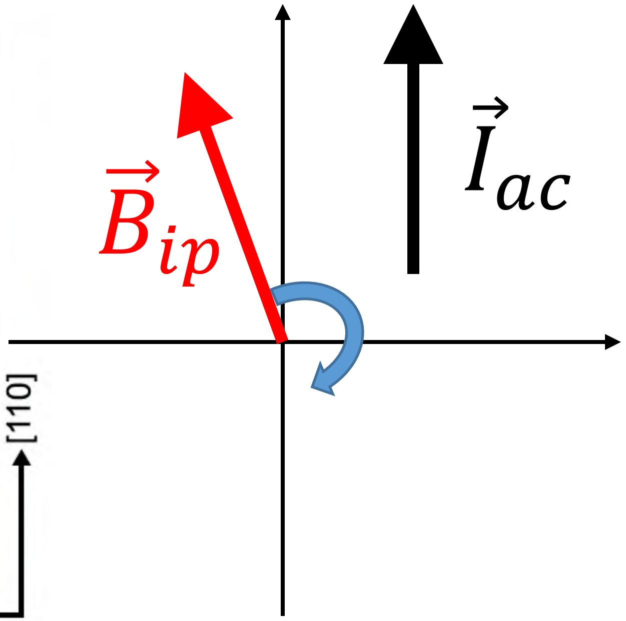
- The decrease is anisotropic
- Al on GaAs (low SOI) behaves regularly

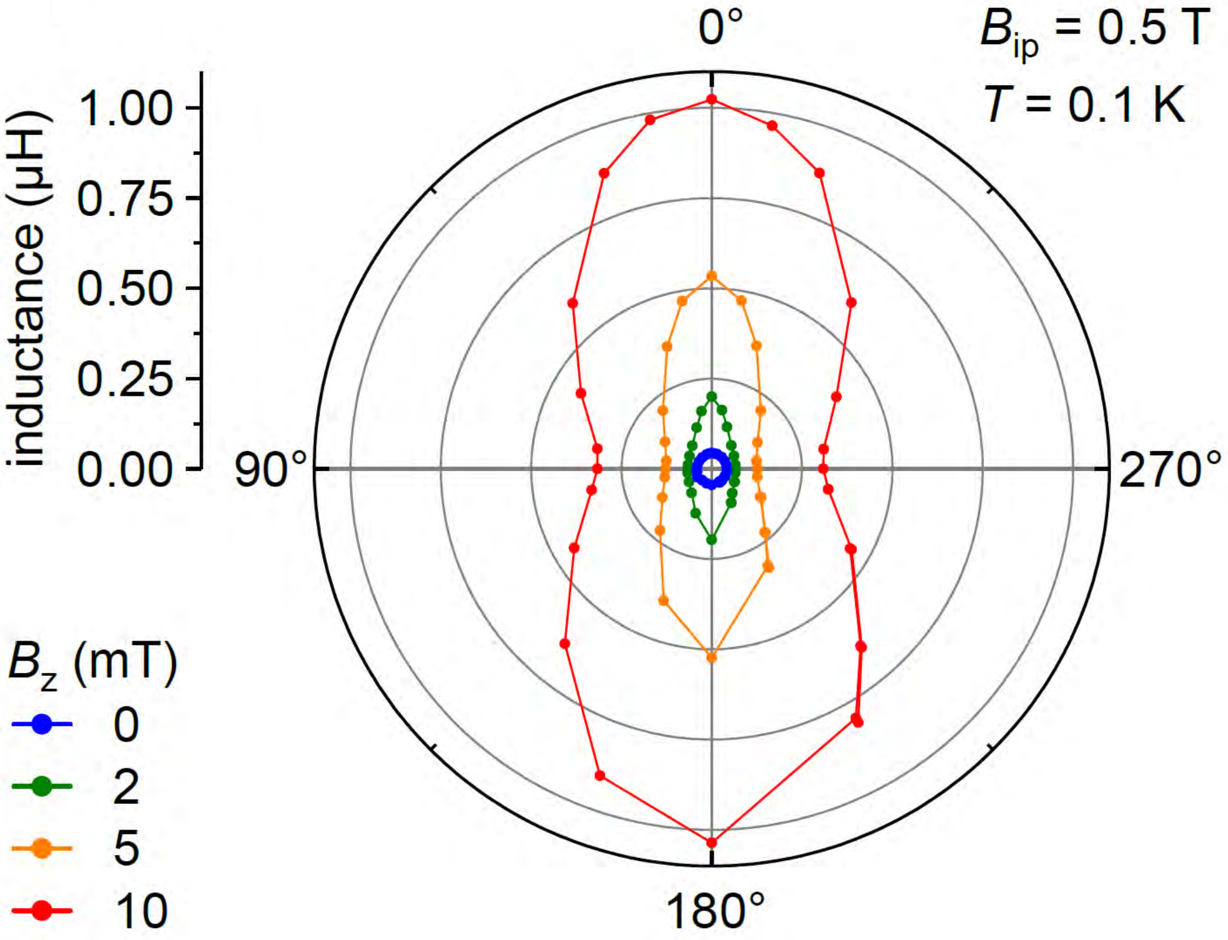


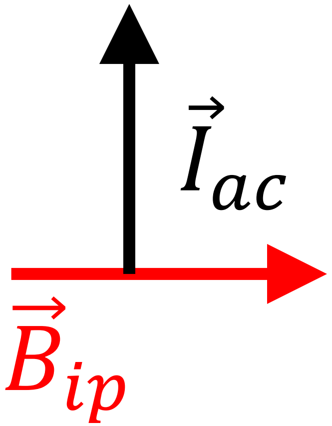
L. Fuchs et al.,
Phys. Rev. X **12**, 041020 (2022)

Anisotropy of the effect



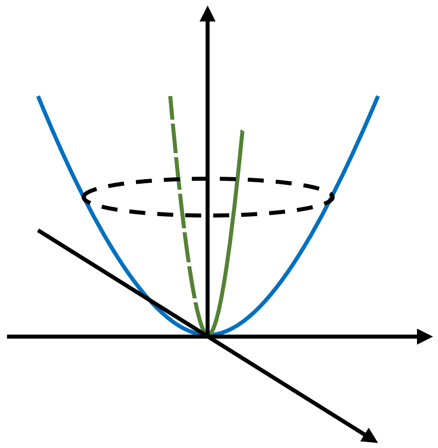






Maximal effect: strongest suppression of L_v,
strongest enhancement of pinning potential

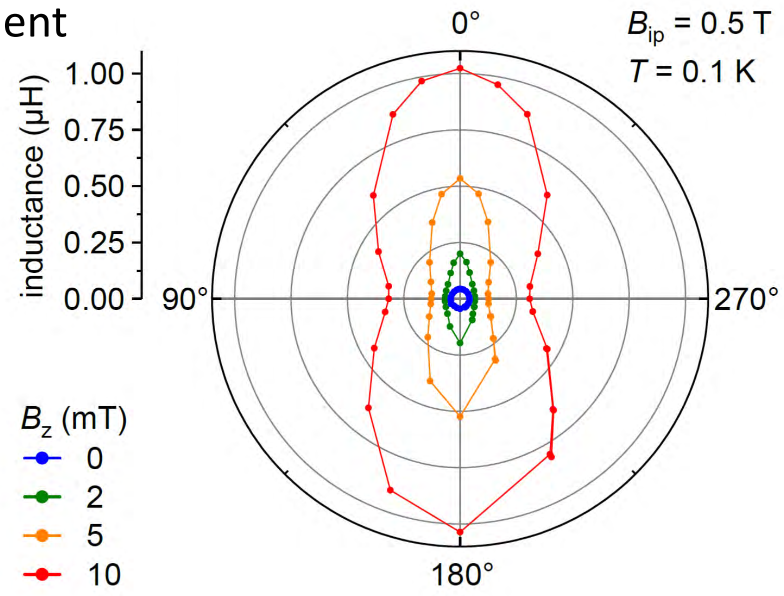
Vortex tomography



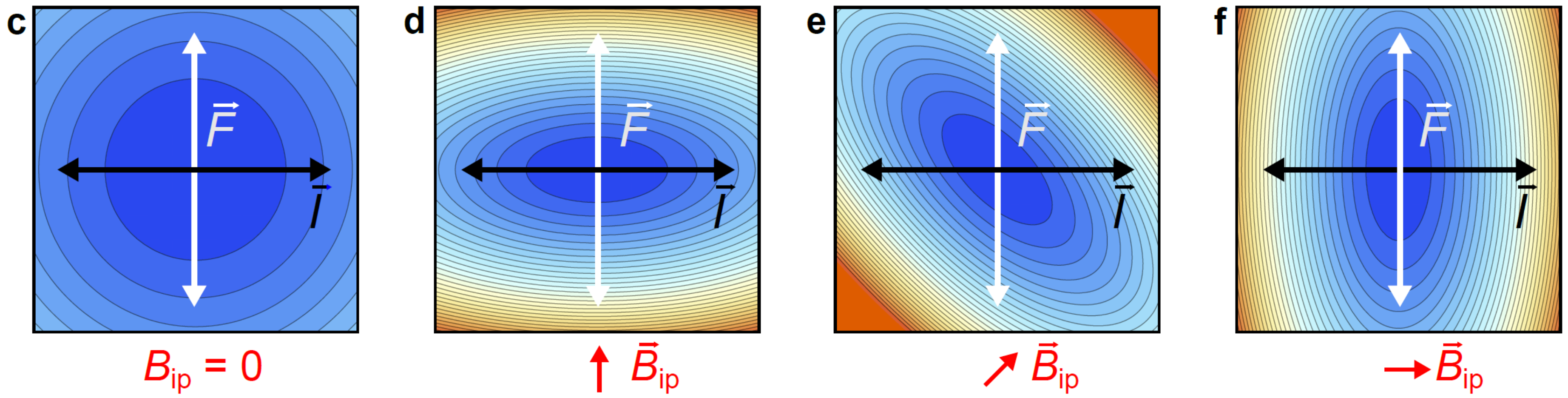
Lower curvature, higher inductance

higher curvature, lower inductance

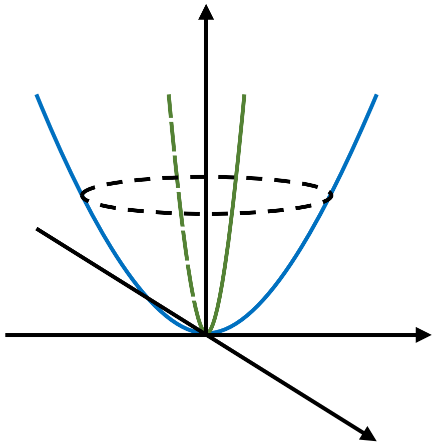
Experiment



Result of calculations:

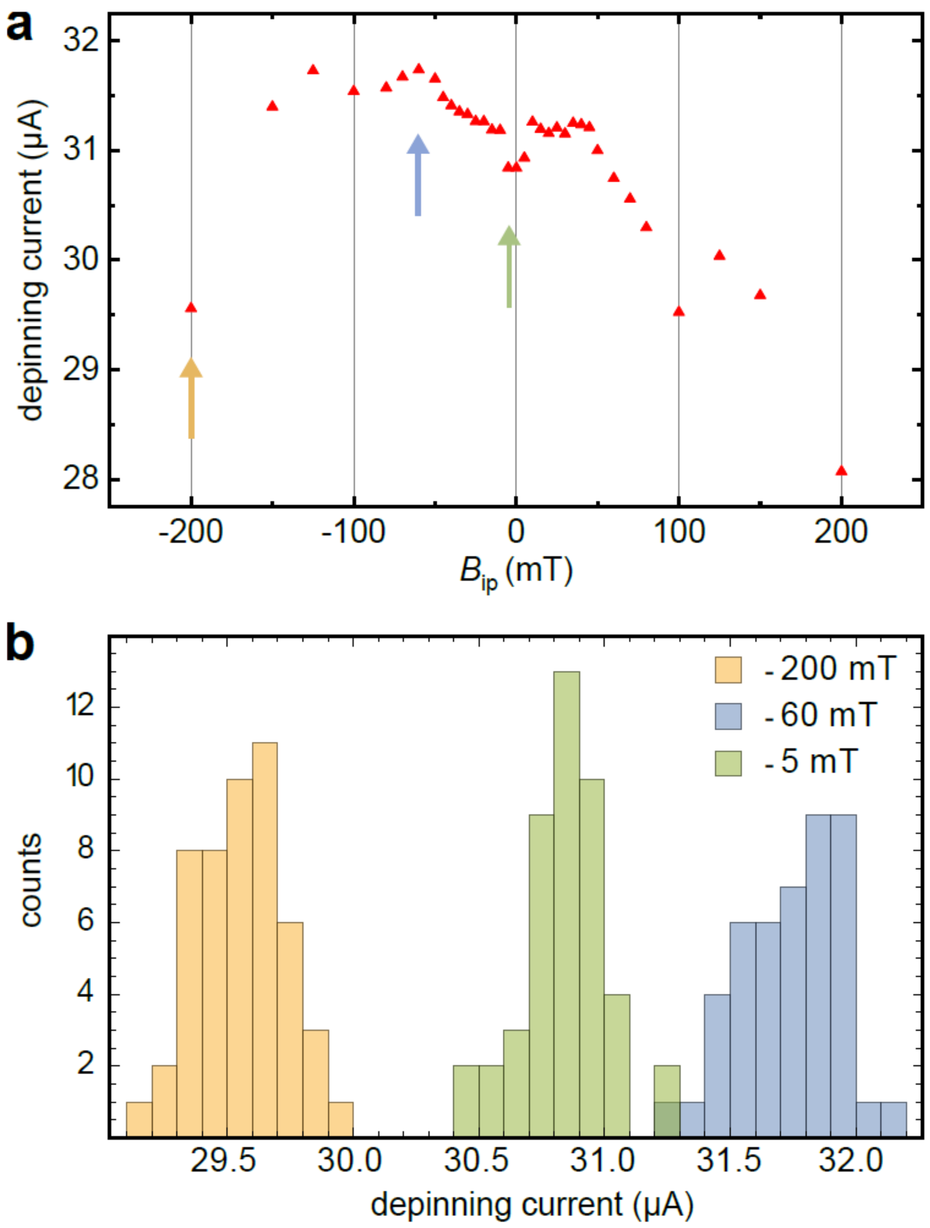


DC transport signatures



Lower slope,
easier to depin

higher slope,
Higher depinning current



Experiments at high current bias are more difficult due to heating instability.

L. Fuchs et al.,
Phys. Rev. X **12**, 041020 (2022)

Conclusions

✓ AC supercurrent diode effect demonstrated in Josephson junctions

✓ $\Delta\varphi_0$ crossover detectable in inductance: minimal model captures the main features

✓ Anisotropic vortex squeezing in in-plane field