Role of the edges in a quasicrystalline Haldane model

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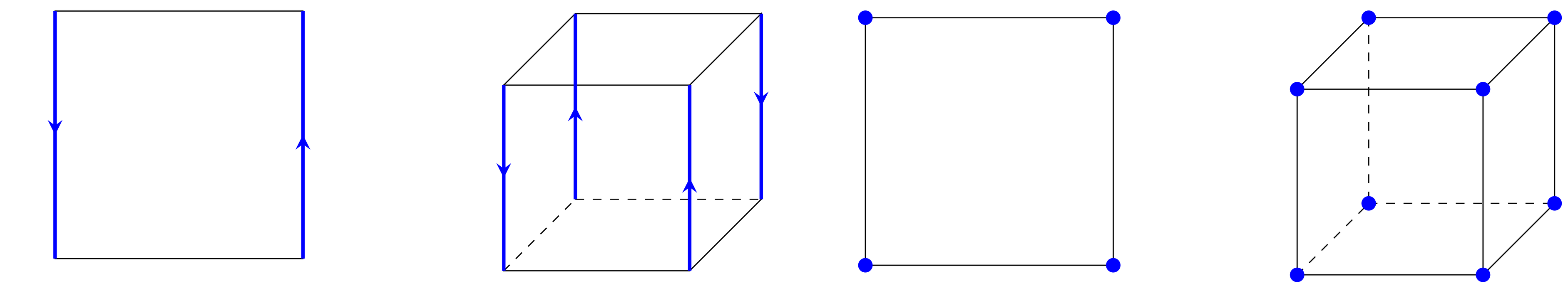
1. Higher order topological insulators

Higher order topological insulators (HOTIs) are systems with non trivial topology, where an insulating d-dimensional bulk coexists with metallic modes that live in a $(d-n)$-dimensional space located at the boundary of the system. Differently from ordinary topological insulators, in HOTIs the topological states are protected by a spatial symmetry (*e.g.* rotational).



(First Order) Topological Insulator

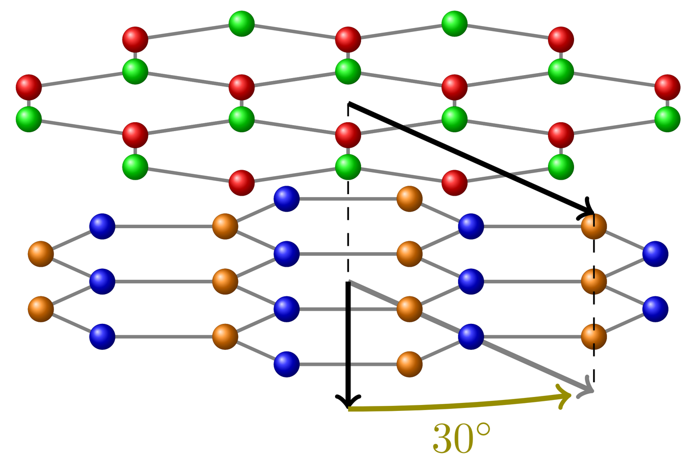
Second Order Topological Insulators

Third Order Topological Insulator

2. A model for a HOTI on the spinless graphene quasicrystal

S. Spurrier and N. Cooper developed a model for a spinless second order topological insulator (SOTI) on a lattice crystallographically equivalent to that of the *graphene quasicrystal* (GQ) [1]

$$H = t\sum_{\langle ij\rangle} c_i^\dagger \tau_0 c_j + \lambda_H \sum_{\langle\langle ij\rangle\rangle} i\nu_{ij} c_i^\dagger \tau_z c_j + \lambda_\perp \sum_{ij} t_{ij}^\perp c_i^\dagger \tau_x c_j .$$



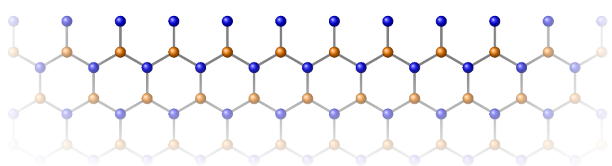
$$t_{ij}^\perp = t^\perp e^{-\frac{|\vec{d}_{ij}|-d_0}{\delta}}$$

This lattice is obtained by superimposing two honeycomb lattices with a relative twist angle of 30°, and possesses a local C_{12} rotational symmetry [2].

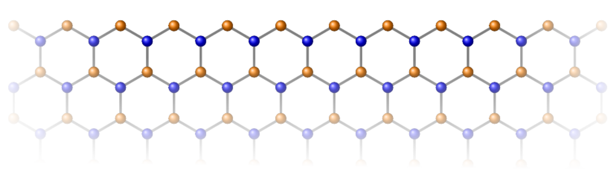
3. Edge dependence of the HOTI phase

We found that the features of the HOTI phase achieved by this model greatly depend on the nature of the finite sample edges. We performed the numerical diagonalization of the model for a finite-size square sample (global C_4 symmetry), both with bearded-armchair edges and with zigzag-armchair edges.

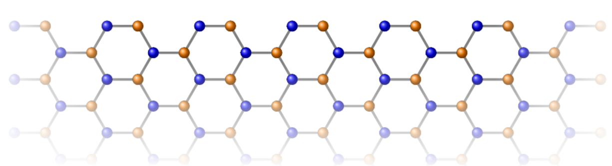
The gap opens at different energies for the two different configurations of the edges. In both cases, four degenerate eigenvalues are found inside the gap: these correspond to 0D states localized on the vertices of the samples (*corner modes*).



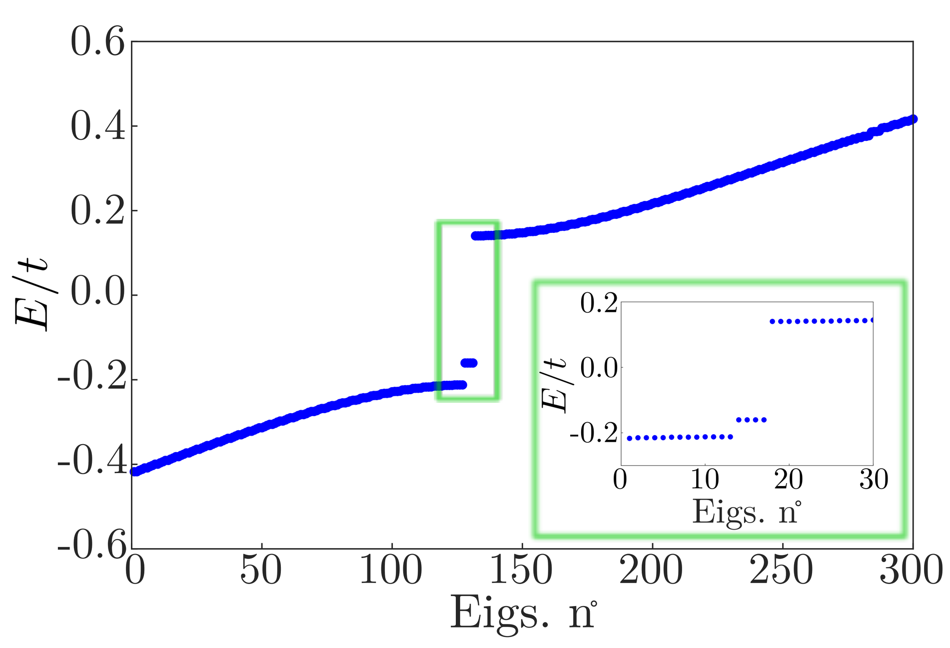
Bearded edge

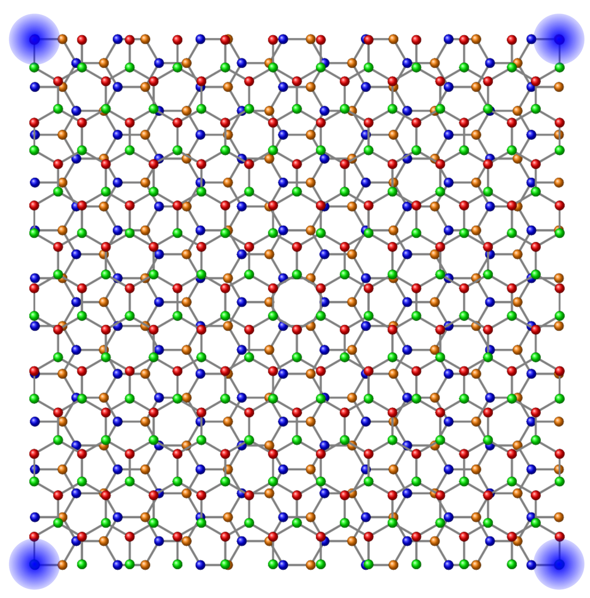


Zigzag edge

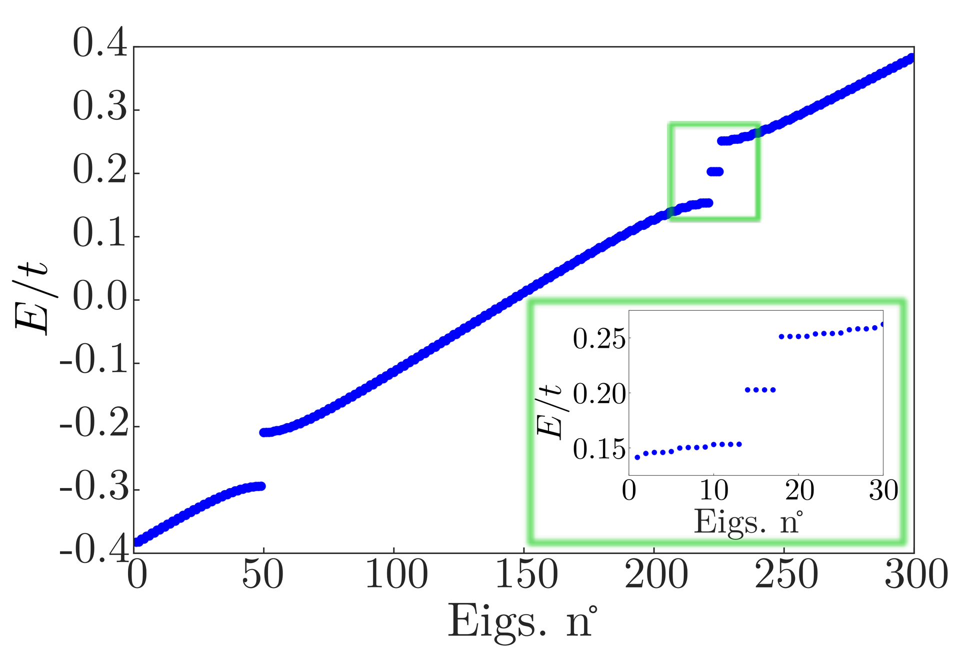


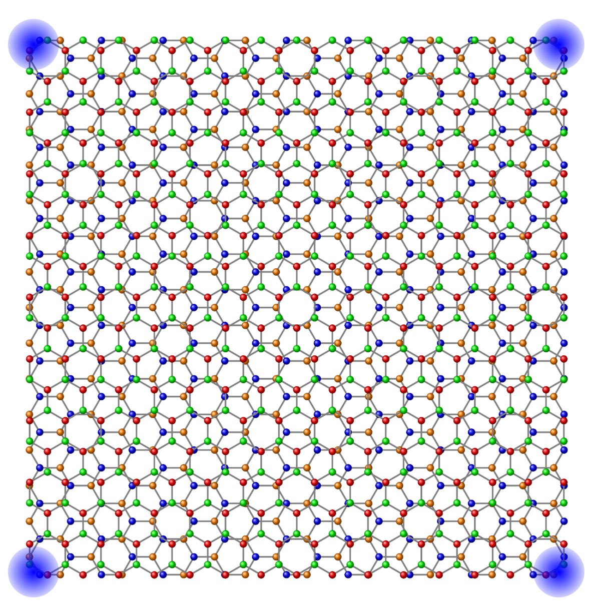
Armchair edge





Eigenvalues for a square sample with bearded-armchair edges.



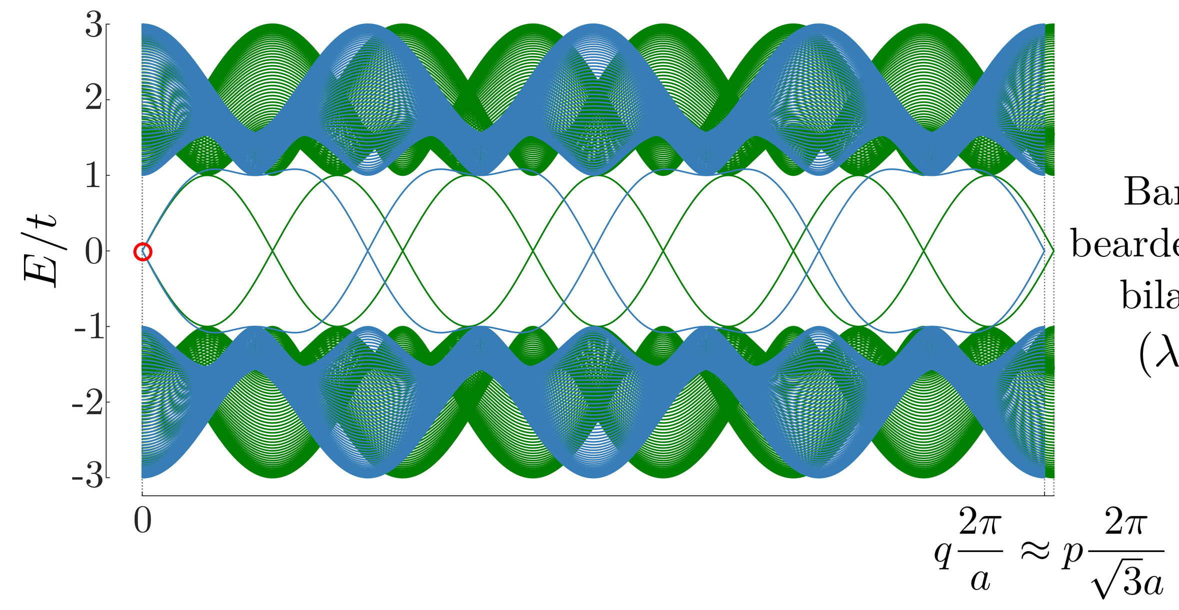


Eigenvalues for a square sample with zigzag-armchair edges.

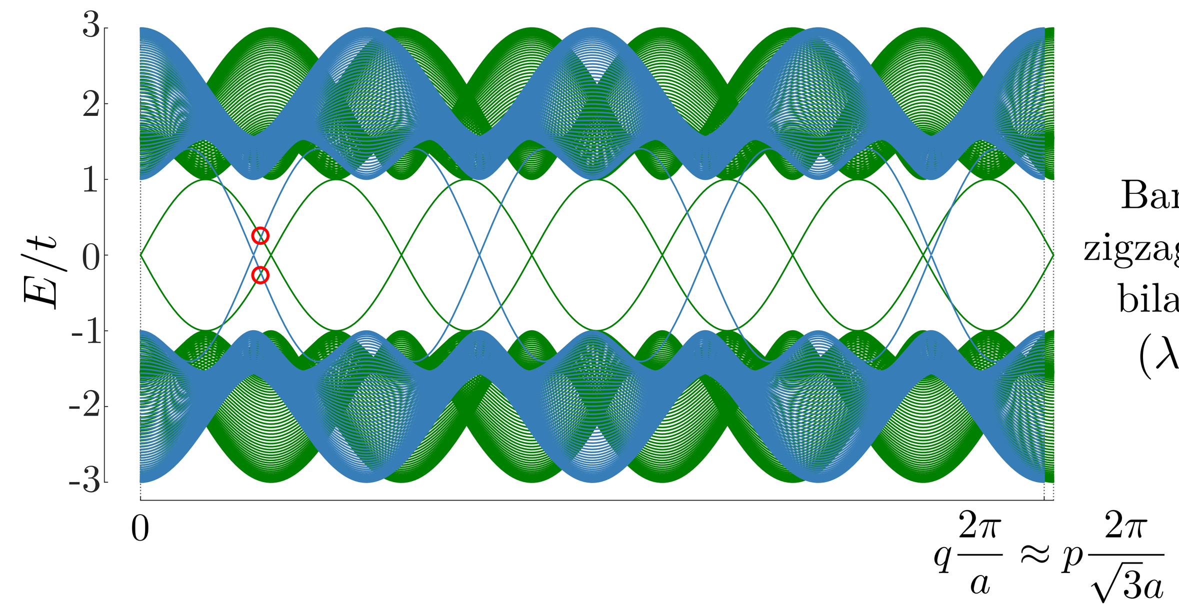
- Why does the low energy spectrum crucially depend on the edge shape?
- How can we predict where the edge spectrum should gap out?

4. The quasiperiodic approximation

To understand why the gap opens at different energies for different configurations of the edges, we started from the approximate edge bands of the uncoupled ($\lambda_\perp = 0$) bilayer system, derived under the *quasiperiodic approximation*.



Bands for a bearded-armchair bilayer strip ($\lambda_\perp = 0$)



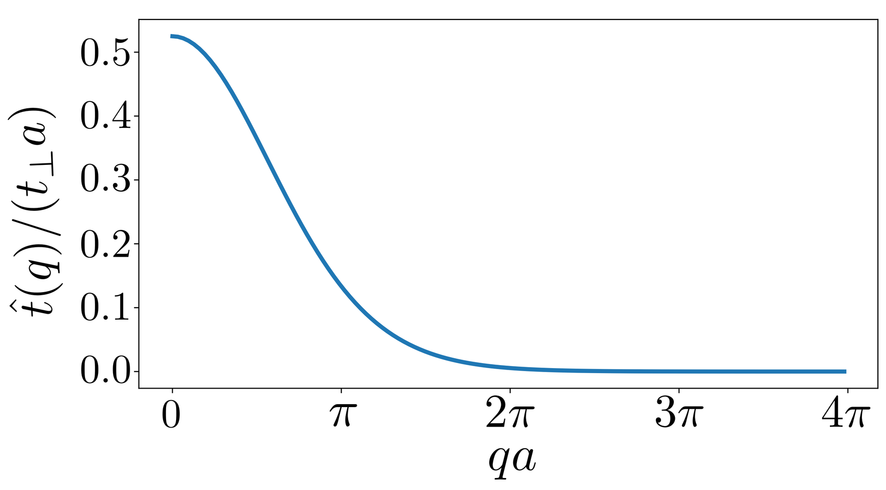
Bands for a zigzag-armchair bilayer strip ($\lambda_\perp = 0$)

The gap opening can only occur at the crossing points between the bands of the two edges!

We studied the effect of the interlayer coupling on the edge bands via an effective low energy theory [3]. The matrix element between the Bloch edge states

$$T^{\alpha\beta}_{k_1,k_2} = \langle \psi^{(1)}_{k_1\alpha} | H_\perp | \psi^{(2)}_{k_2\beta} \rangle ,$$

at the crossing points ($k_1 = k_2 \equiv q$), goes as $\hat{t}(q)$, that is the Fourier transform of the interlayer coupling function along the edge.



Since $\hat{t}(q)$ rapidly decreases with q, the gaps will open at the crossings with the lowest qs!

5. Conclusions

- We have analyzed the edge dependence of the topological modes hosted by a particular SOTI model, built on top of the spinless GQ lattice.
- We have shown that when the structure presents zigzag and armchair edges, the topological modes do not appear at the charge neutrality point. This fact may have consequences on the SOTI phase experimental observability.
- Our analysis suggests the possibility of engineering 1D structures with fractal energy dispersion in systems where the topological states are hosted by armchair and zigzag edges in close proximity to each other.

6. References

[1] S. Spurrier and N. R. Cooper, *Phys. Rev. Research*, 2:033071, Jul 2020.

[2] S. J. Ahn, P. Moon, T.-H. Kim, H.-W. Kim, H.-C. Shin, E. H. Kim, H. W. Cha, S.-J. Kahng, P. Kim, M. Koshino, Y.-W. Son, C.-W. Yang and J. R. Ahn, *Science*, 361(6404):782–786, Jun 2018.

[3] F. Rost, R. Gupta, M. Fleischmann,D. Weckbecker, N. Ray, J. Olivares, M. Vogl, S. Sharma, O. Pankratov and S. Shallcross, *Phys. Rev. B*, 100:035101, Jul 2019.

[4] S. Traverso, M. Sassetti and N. T. Ziani, submitted to *Phys. Rev. B*, Apr 2022.