READOUT OF PARAFERMIONIC STATES BY TRANSPORT MEASUREMENTS

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Introduction

In the pursuit for realizing topologically protected quantum computation, experimental signatures of the existence of parafermionic zero-energy modes (parafermions for short) are highly desired as they enter as building blocks in several proposals. We show here that it is possible to detect the shared degree of freedom between two such modes by charge transport measurements. To this end, we present two complementary hybrid systems composed of a fractional quantum Hall (FQH) liquid and a superconductor (SC).

Parafermions

We consider $\mathbb{Z}_6$ parafermions:

$$\alpha^6 = \mathbb{1}\,, \qquad \alpha^\dagger = \alpha^{-1}\,.$$

A pair of these localized topological zero-energy modes fulfil the commutation relations:

$$\alpha_2\alpha_1 = e^{-i\frac{\pi}{3}}\,\alpha_1\alpha_2\,, \qquad \alpha_2^\dagger\alpha_1 = e^{i\frac{\pi}{3}}\,\alpha_1\alpha_2^\dagger\,.$$

Together they define a 6-fold degree of freedom that can be characterized by the parity operator: $e^{-i\frac{\pi}{6}}\,\alpha_2^\dagger\alpha_1 = e^{-i\pi q/3}$, where the eigenvalues of q can be $0,\ldots,5$. In the presence of a weak coupling between α_1 and α_2, they acquire a finite energy:

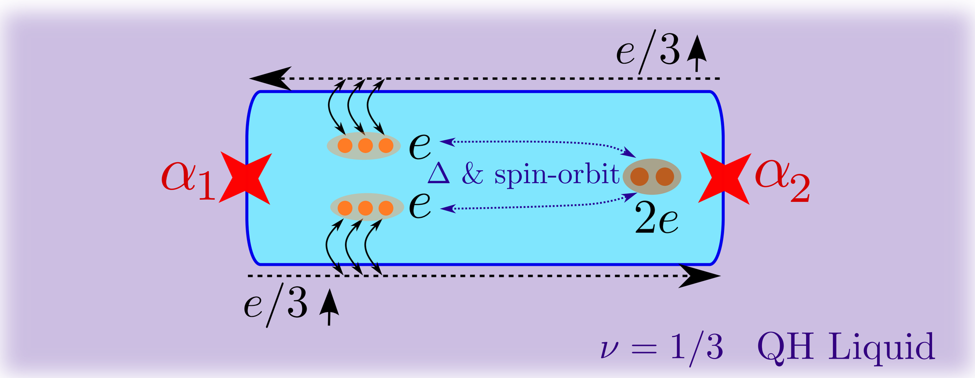
$$H_{\rm pf} = -\varepsilon\, e^{-i(\pi/6+\phi)}\,\alpha_2^\dagger\alpha_1 + \text{H.c.} = -2\varepsilon\cos\left(\pi q/3+\phi\right)\,,$$

where ε indicates the overlap of the two modes and ϕ is an arbitrary phase.

Notice that the operators $\alpha_i^3 = (\alpha_i^\dagger)^3 \equiv \gamma_i$ anticommute and define two effective localized Majorana zero-energy modes.

Physical realization

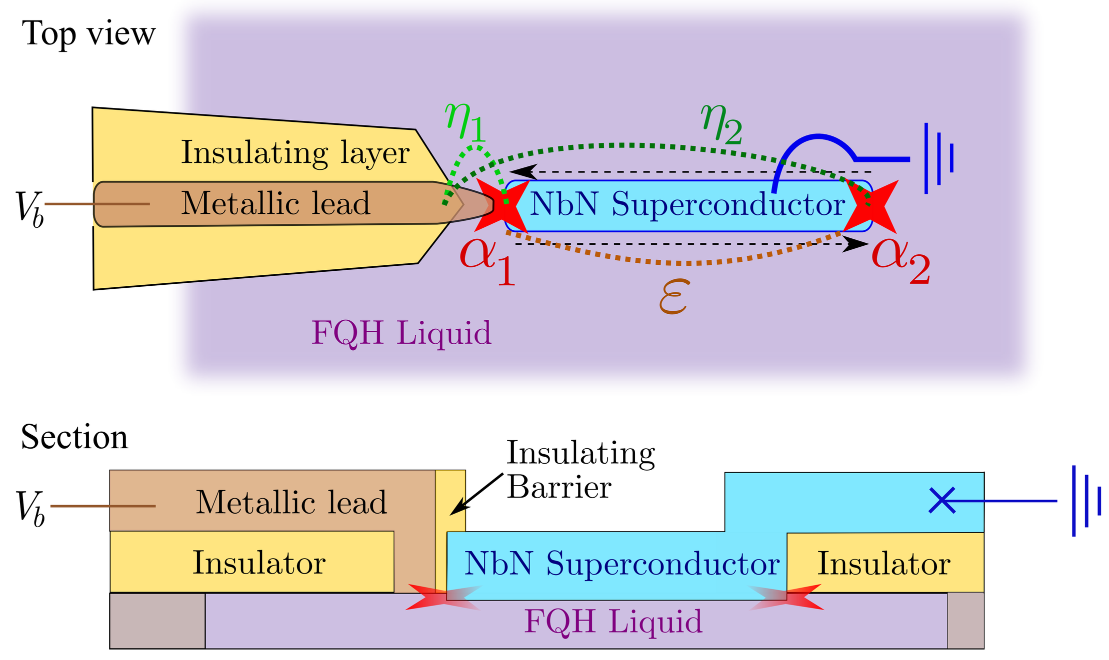
Parafermions are generated when counterpropagating edge modes of a FQH liquid are coupled through an induced superconducting pairing Δ. Recent experiments [2] have realized this by etching a thin trench in a FQH system at filling $\nu = 1/3$ and depositing a "finger" of NbN therein. The SC induces a gap $\Delta_{e/3}$ in the edge modes and the composite system now has a 6-fold groundstate degeneracy. These can be distinguished by the fractional charge $eq/3$ in the system, such that the charge is defined $\bmod\ 2e$. At each end of the SC there is a localized $\mathbb{Z}_6$ parafermion that we label by $\alpha_{1,2}$.



The parafermions can be tunnel coupled to external normal (N) electrodes through the property $\alpha_i^3 = \gamma_i$.

Grounded N-SC device

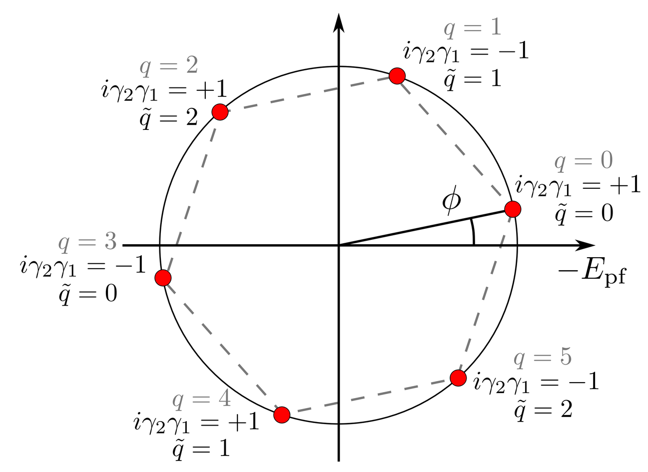
In this setup the SC is grounded and the parafermions coupled to a metallic lead.



The tunnel coupling with the lead changes the system charge by $\pm e$, and hence the topologically protected quantity is $e\tilde{q}/3 \equiv qe/3 \mod e$. With the mapping:

$$e^{i\pi q/3} = i\gamma_2\gamma_1\, e^{i\pi 4\tilde{q}/3}$$

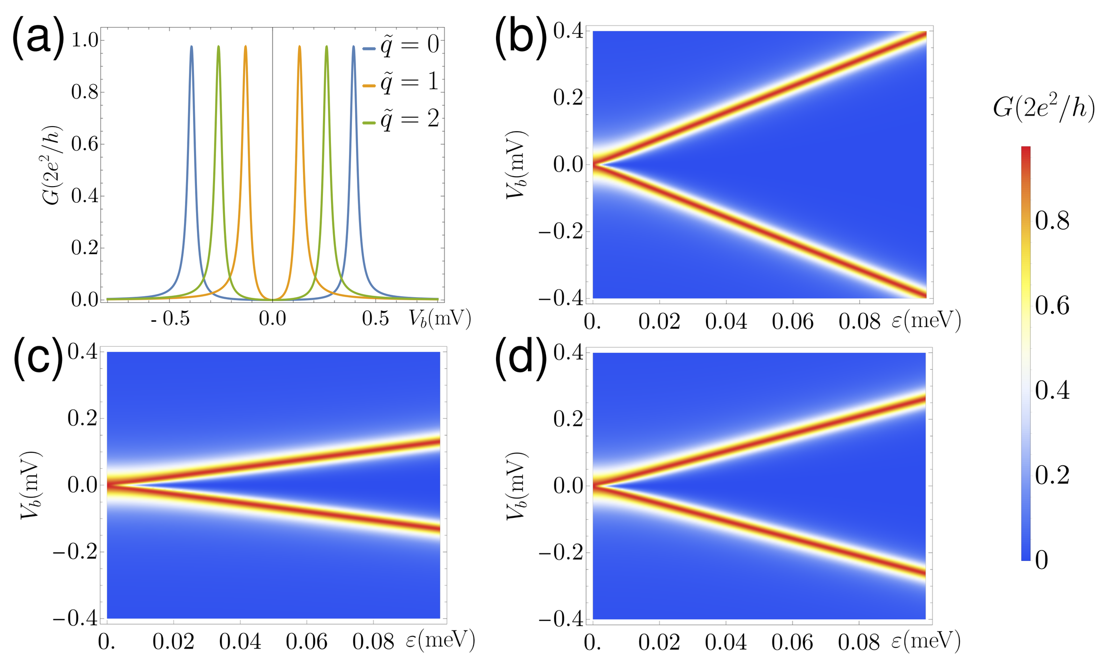
we can translate $|q\rangle \to |p,\tilde{q}\rangle$, where $p = i\gamma_2\gamma_1$ is the parity of two Majoranas.



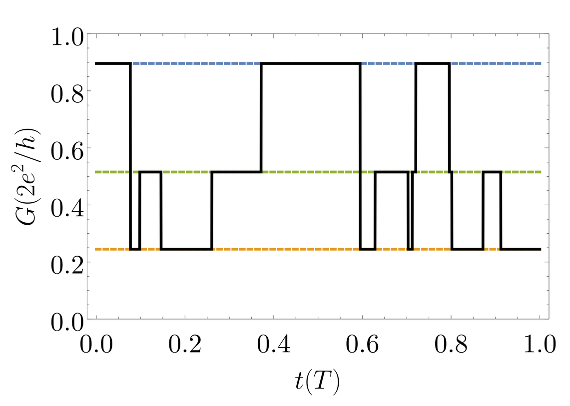
The parafermion coupling then becomes:

$$H_{\rm pf} = -i2\varepsilon\gamma_2\gamma_1\cos(4\pi\tilde{q}/3+\phi)\,,$$

and the conductance between the N lead and the grounded SC electrode can be derived through the corresponding two-Majorana setup for each value of $\tilde{q}$. Here we show it as a function of ε and bias voltage, V_b, for $\tilde{q} = 0, 1, 2$, (b,c,d respectively), and at fixed $\varepsilon = 0.1$meV (a).

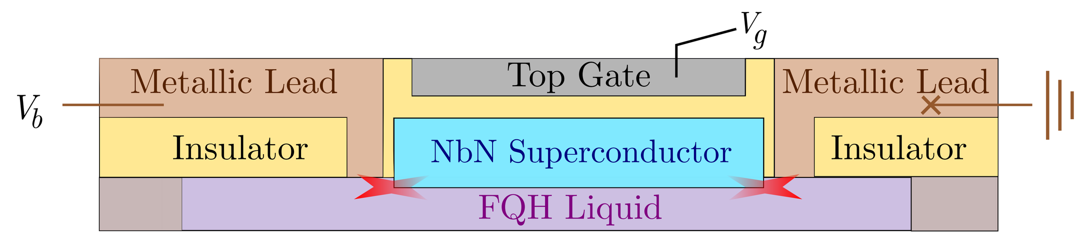


Decoherence can affect $\tilde{q}$ for example through poisoning with fractional quasiparticles from the FQH liquid. If the poisoning rate is slow compared with the current measurement time, the conductance at suitable V_b can show a three-state telegraph noise, unique for fractional subgap states.



Coulomb blockaded device

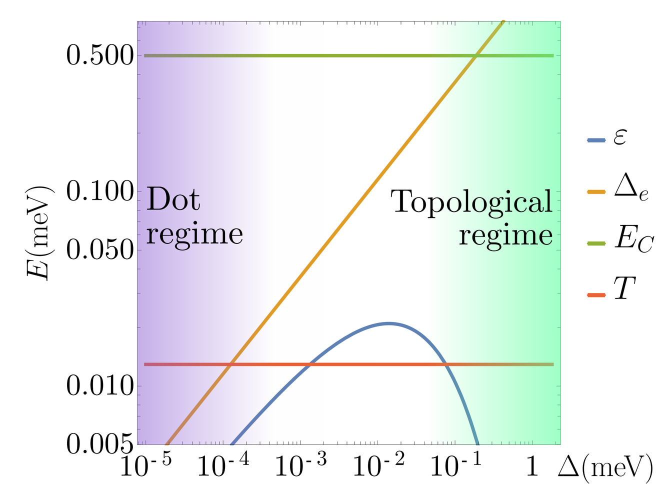
We consider here the SC to be a floating island with strong charging energy E_C. The induced charge can be varied by a top gate voltage V_g.



The model Hamiltonian is:

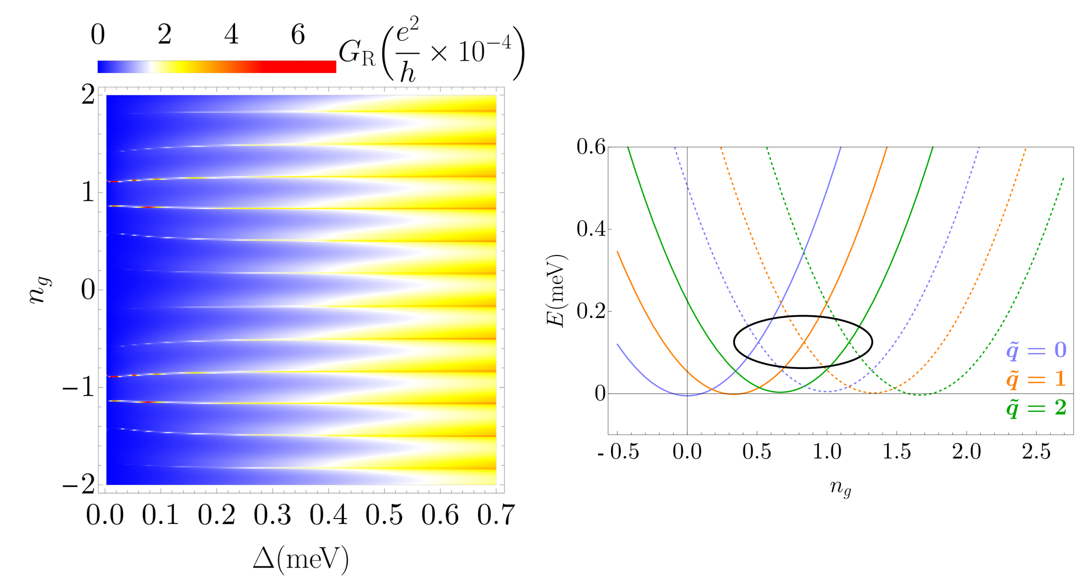
$$H_{\rm SC}(N_{\rm C}, N_e, N', q, n_g) = H_{\rm pf} + N_e\Delta_e + N'\Delta_{e/3} + E_C\left(2N_{\rm C} + N_e + N'/3 + q/3 - n_g\right)^2\,.$$

$N_{\rm C}$ refers to the number of Cooper pairs on the island and $N_e(N')$ to quasielectron(fractional quasiparticle) excitations in the device. We compare here, for varying induced pairing Δ, the typical energy scales E_C, temperature T, ε, and the energy gap of a quasielectron in the paired FQH edge modes, Δ_e.



To model the transport across the system, we consider **resonant tunnelling** of electrons mediated by the parafermions (transitions between q and $q+3$) and **single-electron incoherent sequential tunnelling** across gapped states (transitions between $N_e = 0$ and $N_e = 1$). Fractional quasiparticle poisoning is captured by considering a thermal equilibrium distribution of states $|N_{\rm C}, N_e, N', q\rangle$.

For realistic parameters the zero-bias conductance contribution from resonant tunnelling, $G_{\rm R}$, is:



A sixfold pattern with $2e$-periodicity is observed for $\Delta \lesssim 0.3$meV and evolves into an $e/3$-periodic pattern with increasing Δ (vanishing ε). This signals the onset of the topological phase with strongly localized parafermions.

Conclusions

- Parafermions in FHQ-SC devices can be investigated by electron transport spectroscopy and the groundstates distinguished by current readout.
- For weak quasiparticle poisoning a three-state telegraph noise, unique to fractional states, is expected.

References

[1] Ida E. Nielsen, Karsten Flensberg, Reinhold Egger, and Michele Burrello. Readout of parafermionic states by transport measurements. *arXiv:2109.02300.*

[2] Önder Gül, Yuval Ronen, and Si Young Lee et al. Andreev reflection in the fractional quantum Hall state. *arXiv:2009.07836.*

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