Moiré Superlattices at Fractional Band Fillings: Particle-Hole Duality and Quantum Geometry

Ahmed Abouelkomsan

AA, Zhao Liu & Emil Bergholtz, Phys. Rev. Lett. **124**, 106803 (2020)

Zhao Liu, **AA** & Emil Bergholtz, Phys. Rev. Lett. **126**, 026801 (2021)

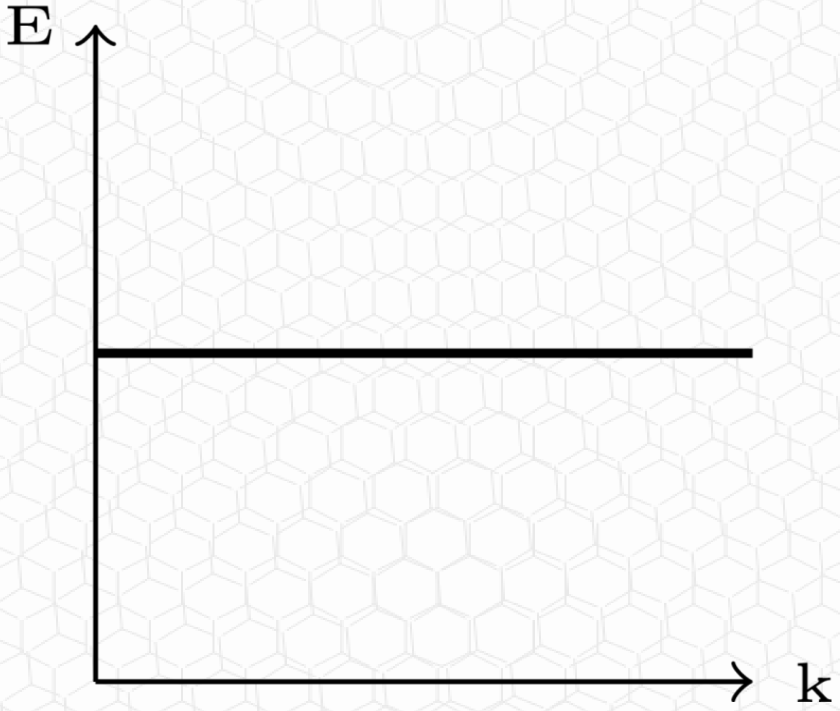
AA, Kang Yang & Emil Bergholtz, arXiv:2202.10467

Knut and Alice Wallenberg Foundation

STOCKHOLMS UNIVERSITET

The Problem

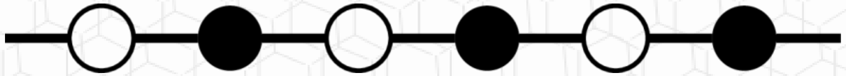
A Flat Band in a Moiré System



The Problem

A Flat Band

Fractional filling



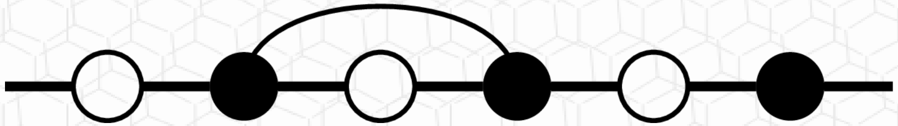
Fractional band filling -> non integer filling per Moiré unit cell

The Problem

A Flat Band

Add interactions at
fractional band fillings

$$V(\mathbf{q}) \sim \frac{1}{\sqrt{\mathbf{q}^2 + \kappa^2}}$$



What is the underlying phase ?

Fractional Chern insulators, Charge density waves, Fermi
Liquids, …. etc. ???

Fractional Chern Insulators, why bother?

- Broadly fractional Chern insulators are lattice analogues of the fractional quantum Hall effect.

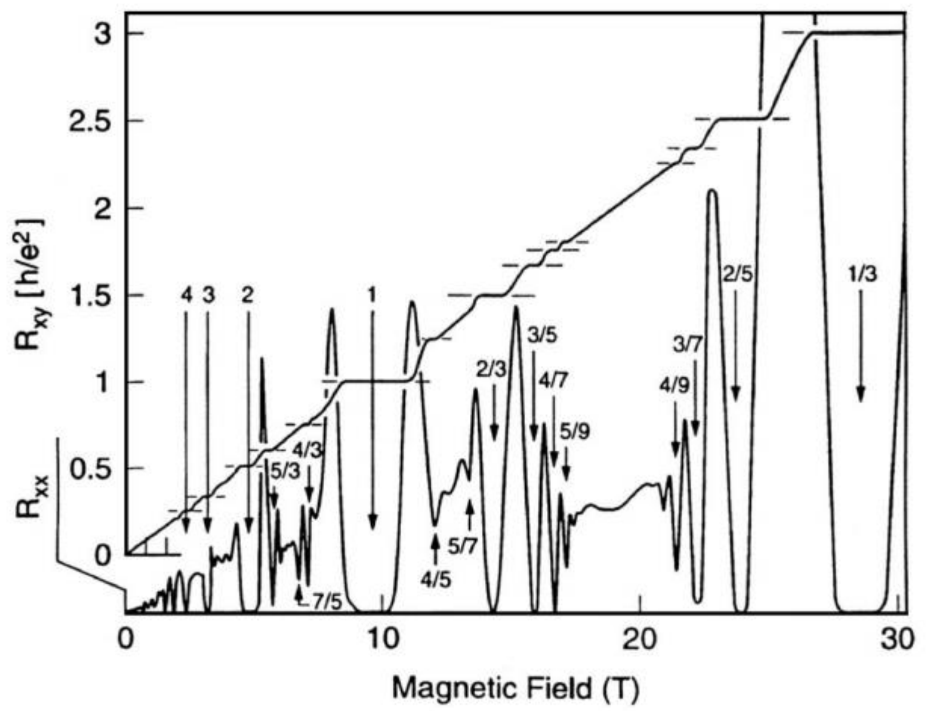


Fig from : Stormer, Physica B **177**, 1-4 (1992)

- Klitzing el at. Phys. Rev. Lett. **45**, 494 (1980)
- Tsui el at. Phys. Rev. Lett. **48**, 1559 (1982)

Fractional Chern Insulators, why bother?

- Overcomes challenges with the conventional FQHE experimental setup!

Very strong magnetic fields

$$|\mathbf{B}| \sim 10 - 30 \text{ Tesla}$$

Extremely low temperatures

$$T \lesssim 1 \text{ Kelvin}$$

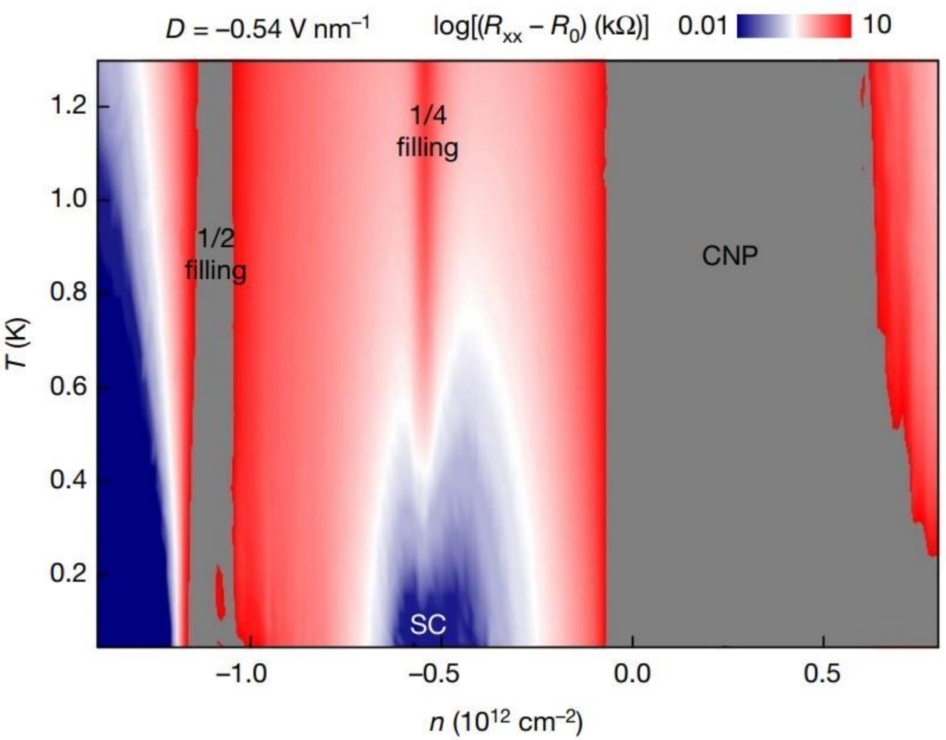
Fractional Chern Insulators, why bother?

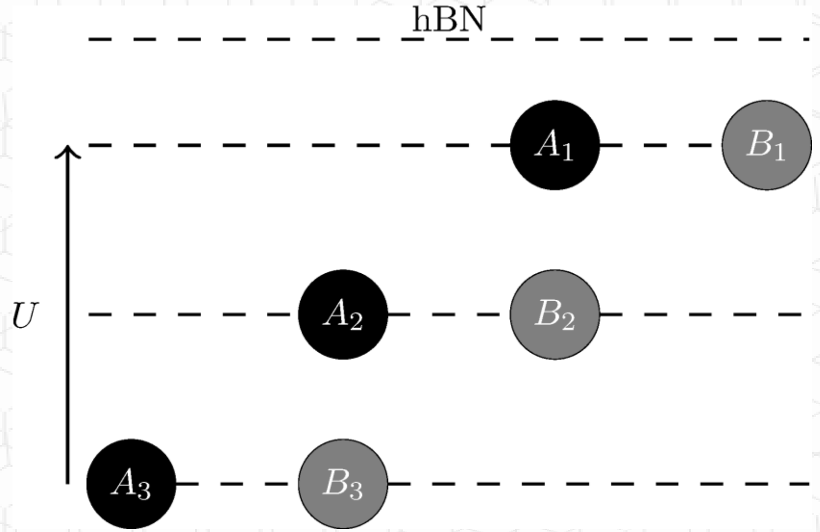
- No magnetic field required!

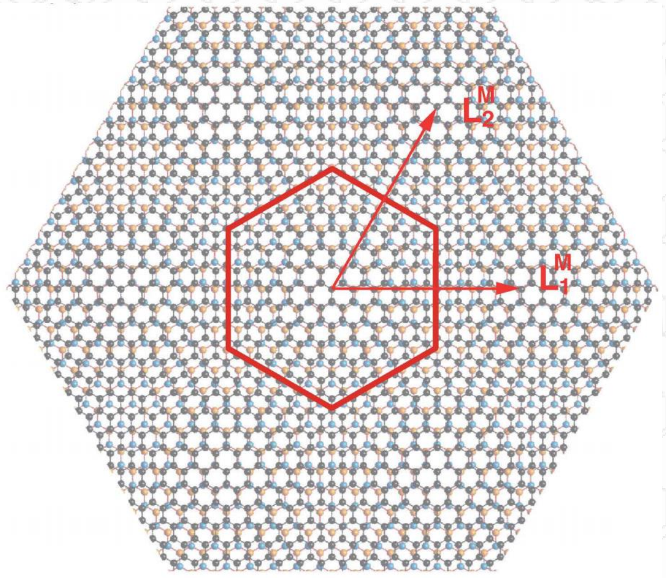
- Interactions on the lattice scale are greater than the magnetic length scale -> Higher energy gap!
- A step towards high temperature topological phases.

- More than FQHE!
- Higher Chern number FCIs are possible -> no mapping to decoupled Landau levels! , PRL, **109**, 186805 (2012)

Trilayer Graphene aligned with Boron Nitride







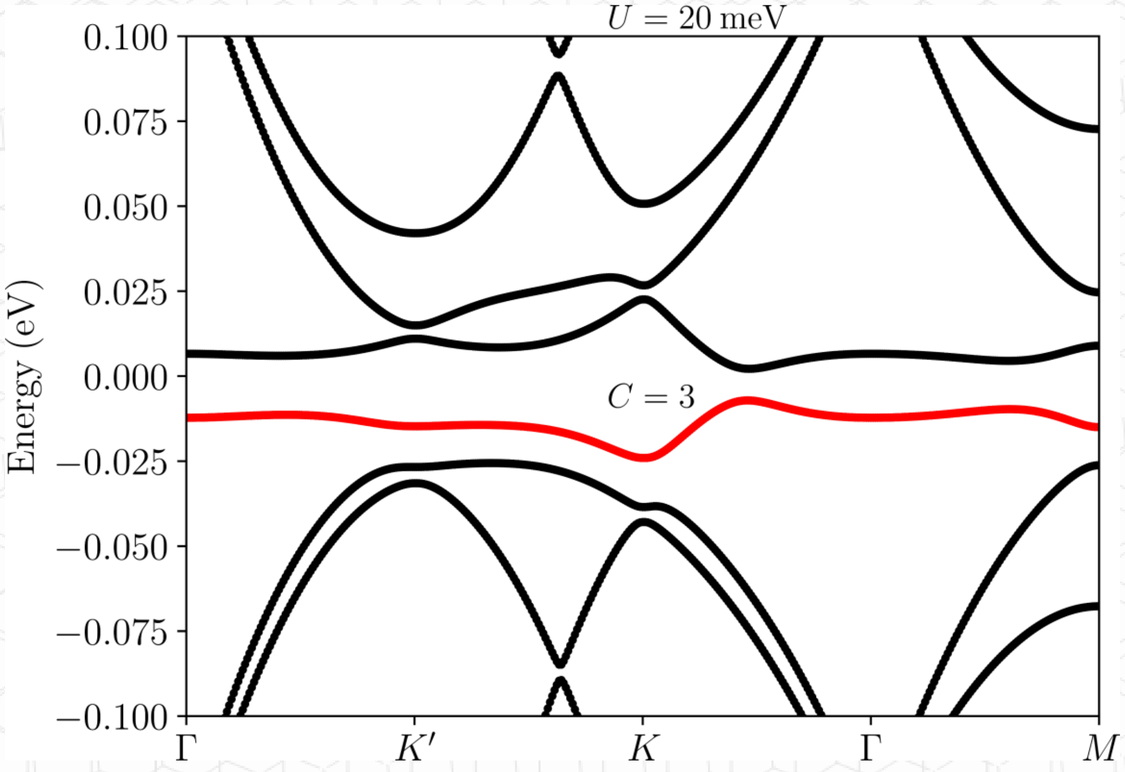
Phys. Rev. B **90**, 155406 (2014)

- Correlated Insulators and superconductors!

  Chen et al. Nature **572**, (2019)

Trilayer Graphene aligned with Boron Nitride

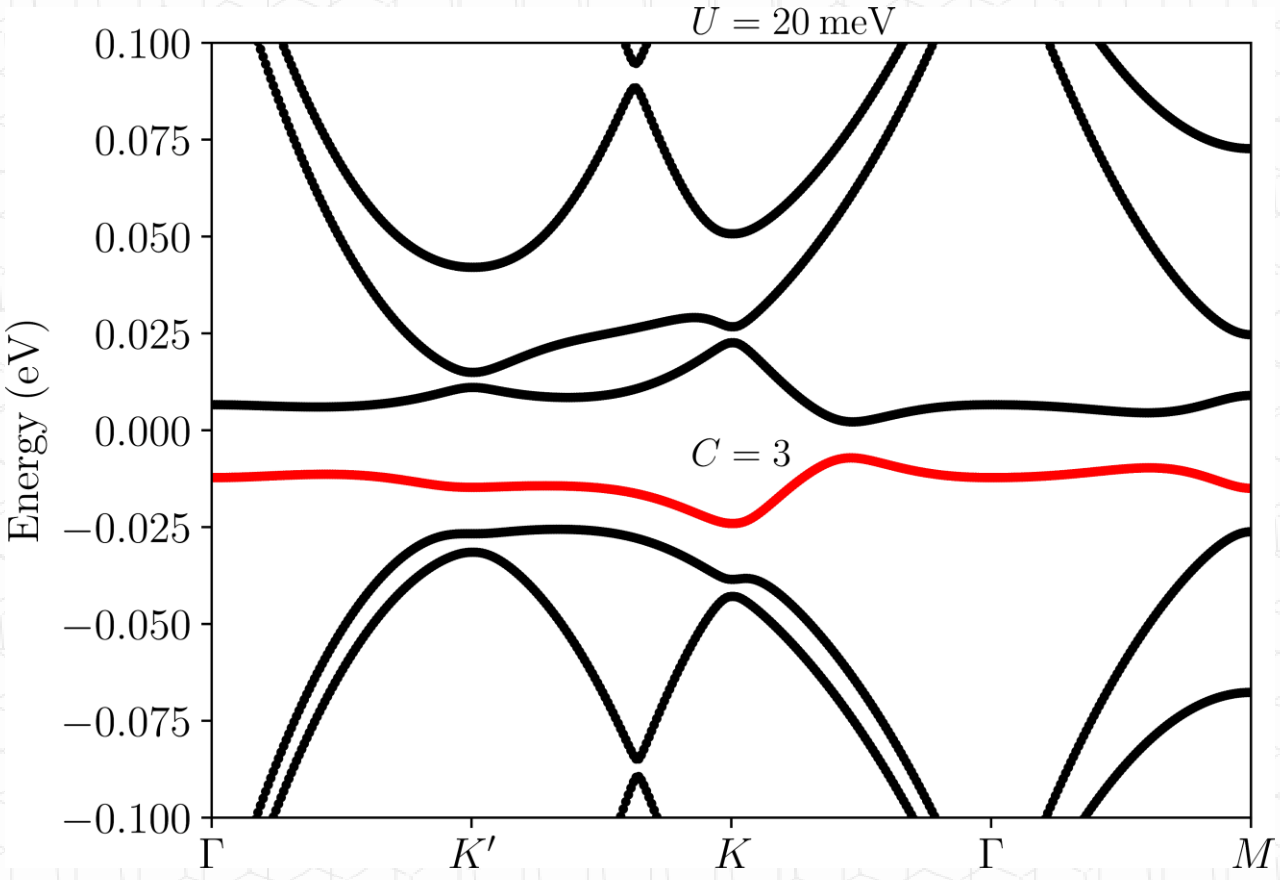
Phys. Rev. Lett. 122, 016401 (2019)



What happens when the red band is fractionally filled?

Trilayer Graphene aligned with Boron Nitride

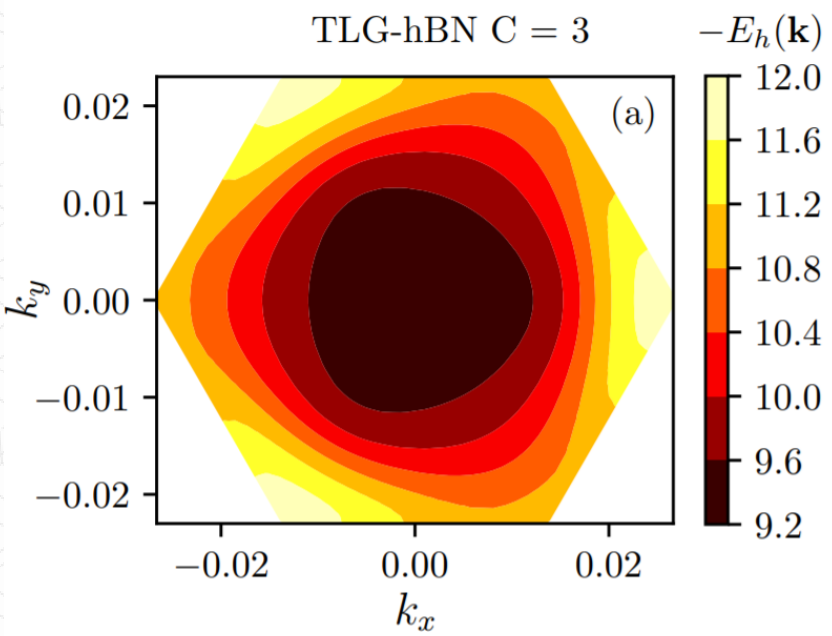
- Are FCI states possible?
- No numerical evidence!
- Why?



Another perspective

$$H_{\text{proj}} = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1} c^\dagger_{\mathbf{k}_2} c_{\mathbf{k}_3} c_{\mathbf{k}_4}$$

$\{c_{\mathbf{k}_\mathbf{i}}\}$ are band operators!



Particle-Hole Transformation, $c_{\mathbf{k}} \rightarrow d^\dagger_{\mathbf{k}}$

$$H_{\text{proj}} \rightarrow \sum_{\mathbf{k}} E_h(\mathbf{k}) d^\dagger_{\mathbf{k}} d_{\mathbf{k}} + \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V^*_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} d^\dagger_{\mathbf{k}_1} d^\dagger_{\mathbf{k}_2} d_{\mathbf{k}_3} d_{\mathbf{k}_4}$$

$$E_h(\mathbf{k}) = \sum_{\mathbf{k}'} \left(V_{\mathbf{k}'\mathbf{k}\mathbf{k}'\mathbf{k}} + V_{\mathbf{k}\mathbf{k}'\mathbf{k}\mathbf{k}'} - V_{\mathbf{k}\mathbf{k}'\mathbf{k}'\mathbf{k}} - V_{\mathbf{k}'\mathbf{k}\mathbf{k}\mathbf{k}'} \right)$$

Dispersive!
for projected interactions

Unlike Landau levels!

Another perspective

$$H_{\text{proj}} = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} c^\dagger_{\mathbf{k}_1} c^\dagger_{\mathbf{k}_2} c_{\mathbf{k}_3} c_{\mathbf{k}_4}$$

Particle-Hole Transformation,

$$H_{\text{proj}} \to \sum_{\mathbf{k}} E_h(\mathbf{k}) d^\dagger_{\mathbf{k}} d_{\mathbf{k}} + \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} V^*_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} d^\dagger_{\mathbf{k}_1} d^\dagger_{\mathbf{k}_2} d_{\mathbf{k}_3} d_{\mathbf{k}_4}$$

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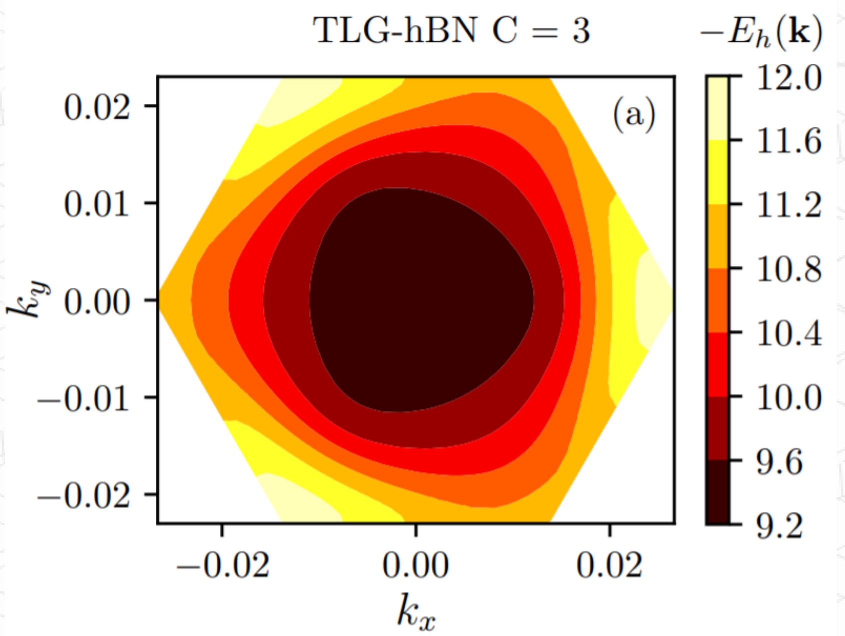


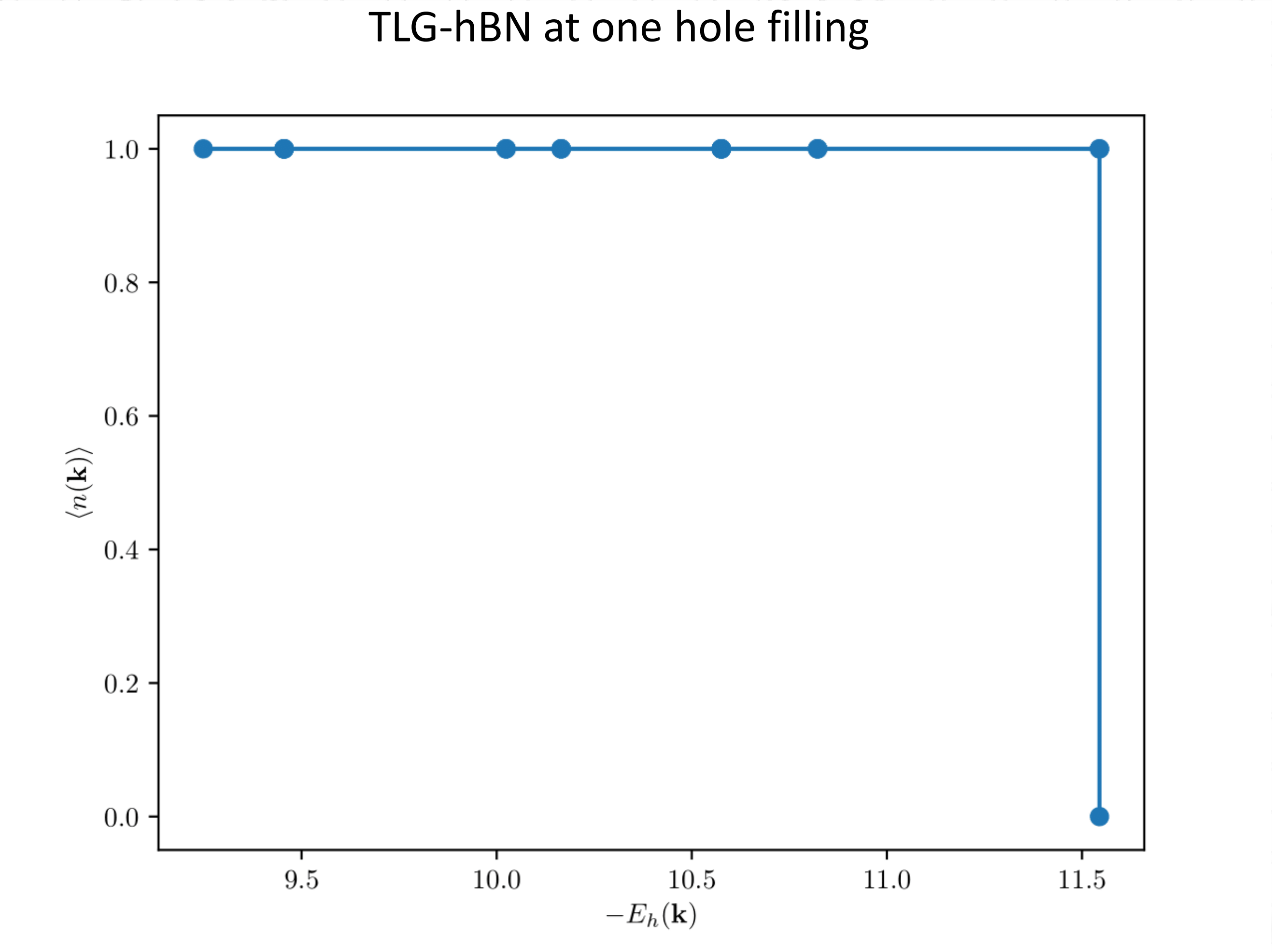
What role does $E_h(\mathbf{k})$ play?

Is there a quantity that $E_h(\mathbf{k})$ correlates with?

Look at the electron occupation in the GS of H_{proj} vs $E_h(\mathbf{k})$

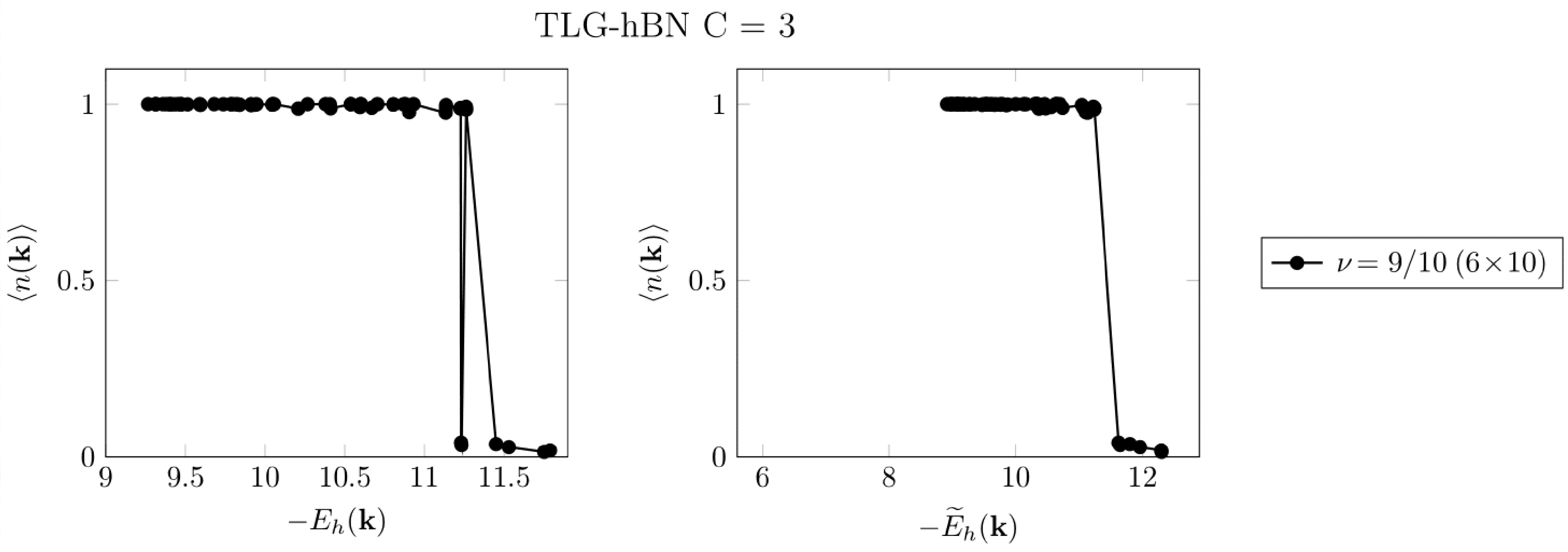
$$\frac{W_h(\text{Hole dispersion bandwidth})}{W(\text{Flatband bandwidth})} \sim 5$$

Trivial example:



$\langle n(\mathbf{k})\rangle = \langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}}\rangle$ is electron occupation in the many-body ground state obtained from exact diagonalization

Emergent Fermi Liquids in TLG-hBN



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$\widetilde{E}_h(\mathbf{k})$ is the renormalized single-hole energy after Hartree-Fock mean field of the interaction

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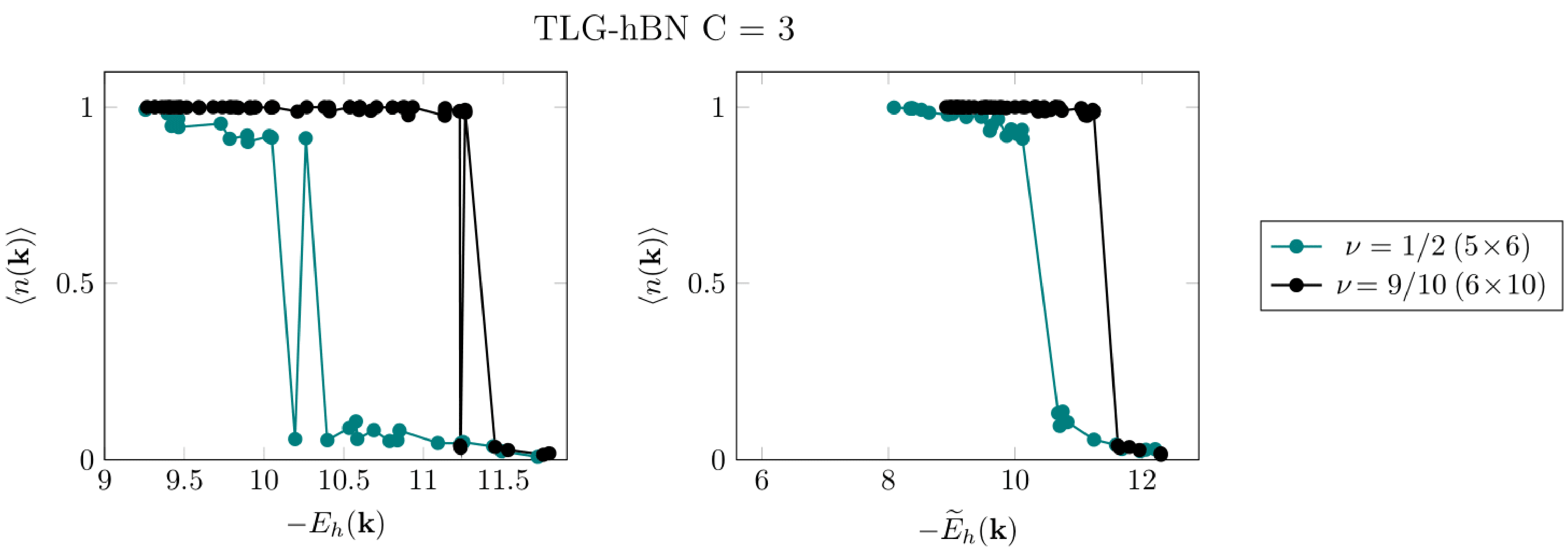

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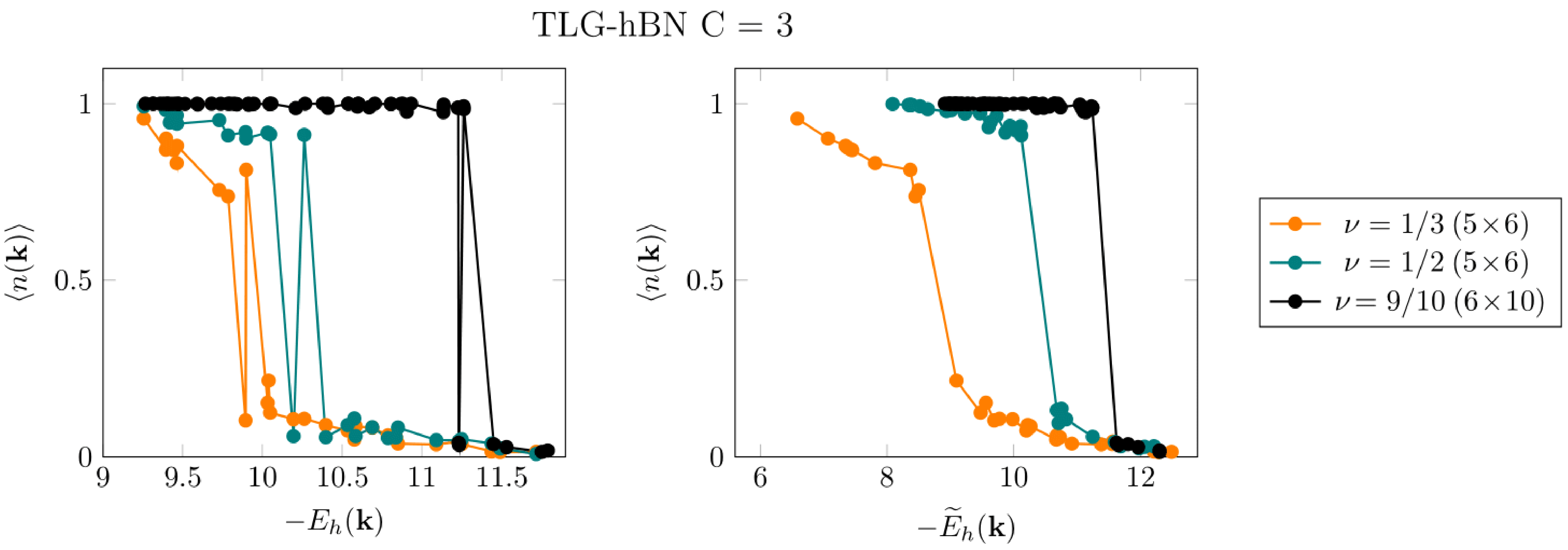

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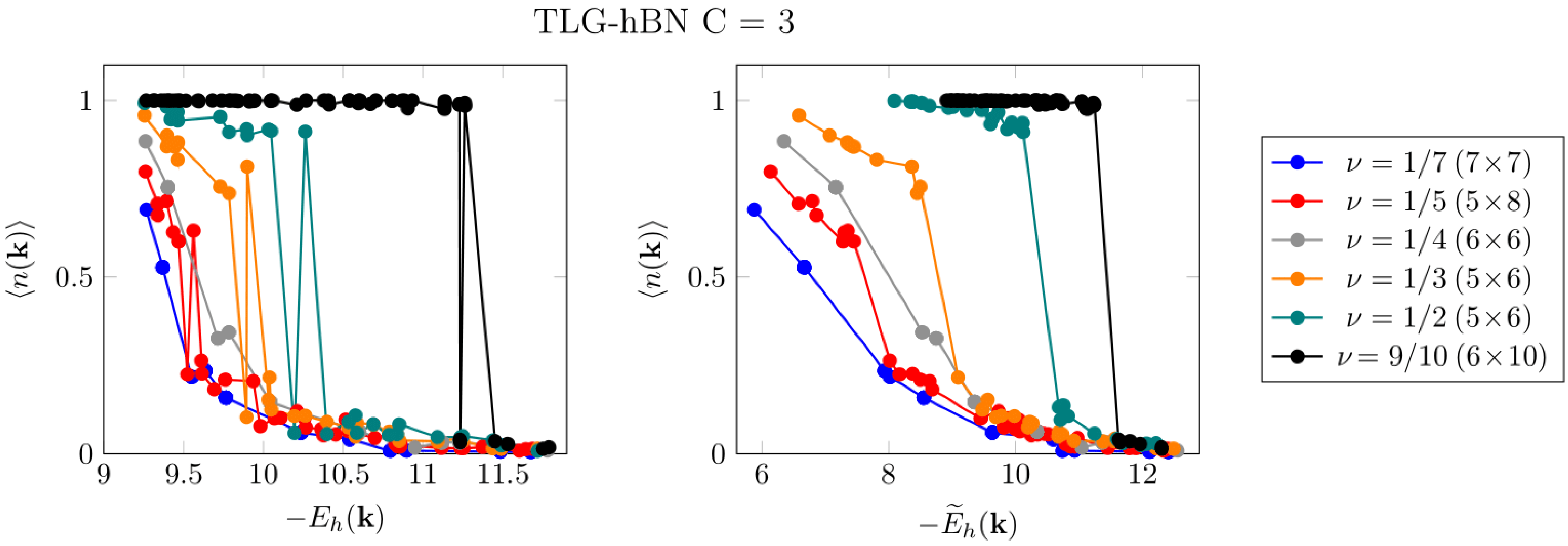

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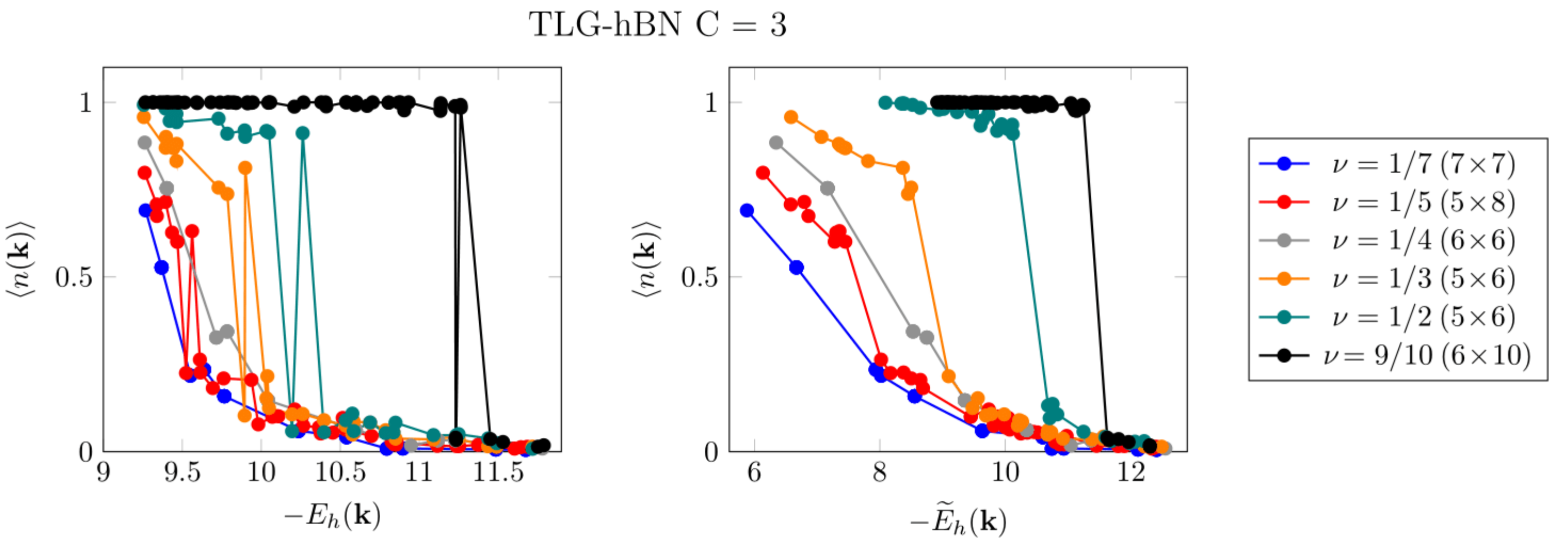
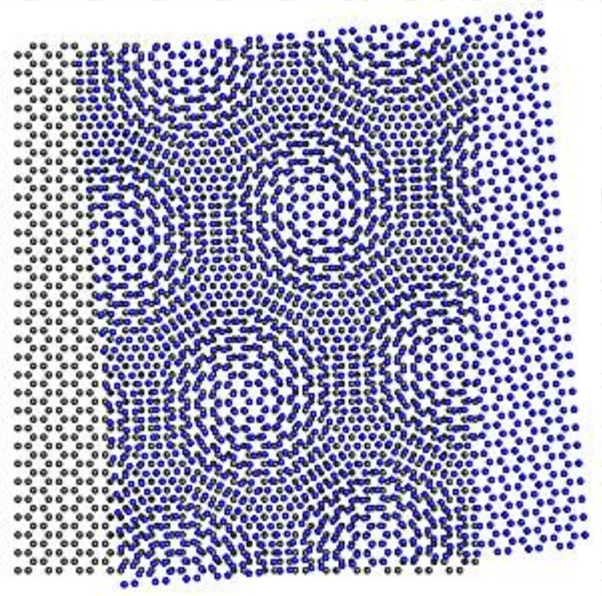
What happened here?



- The problem is weakly interacting in terms of holes!
- Emergent Fermi Liquids from an initial strongly interacting problem.
- The hole dispersion dictates the underlying physics.
- Guiding principle : Electrons prefer to occupy states with the largest hole-energy.

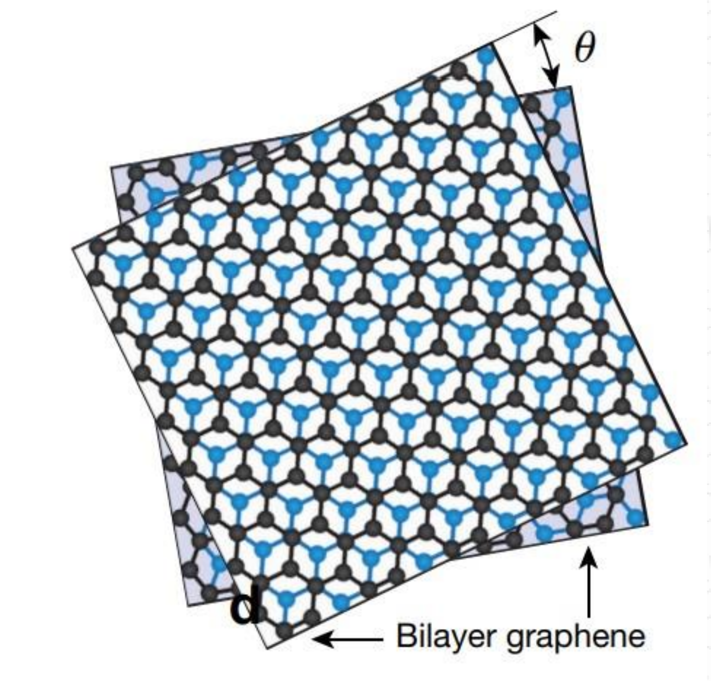
FCIs in other Moiré systems

Twisted Bilayer Graphene



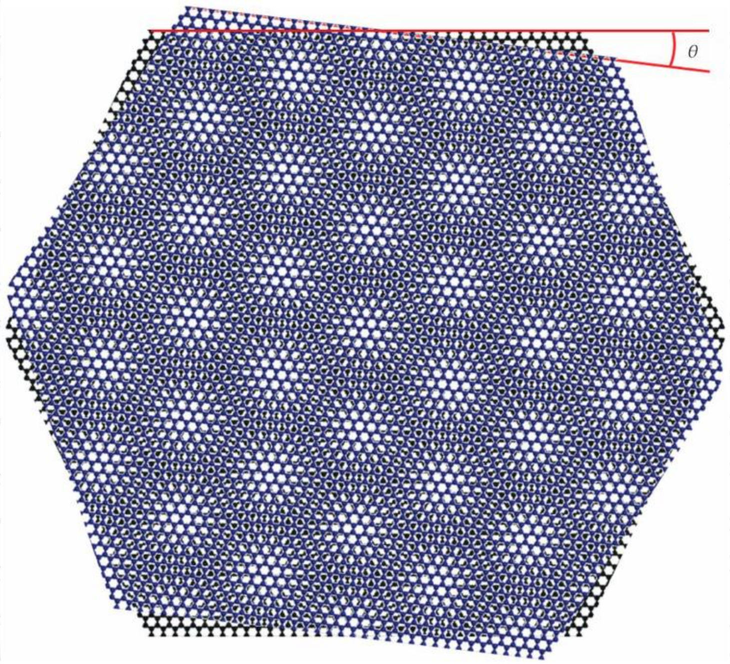
Credit : NIST

Twisted Double Bilayer Graphene



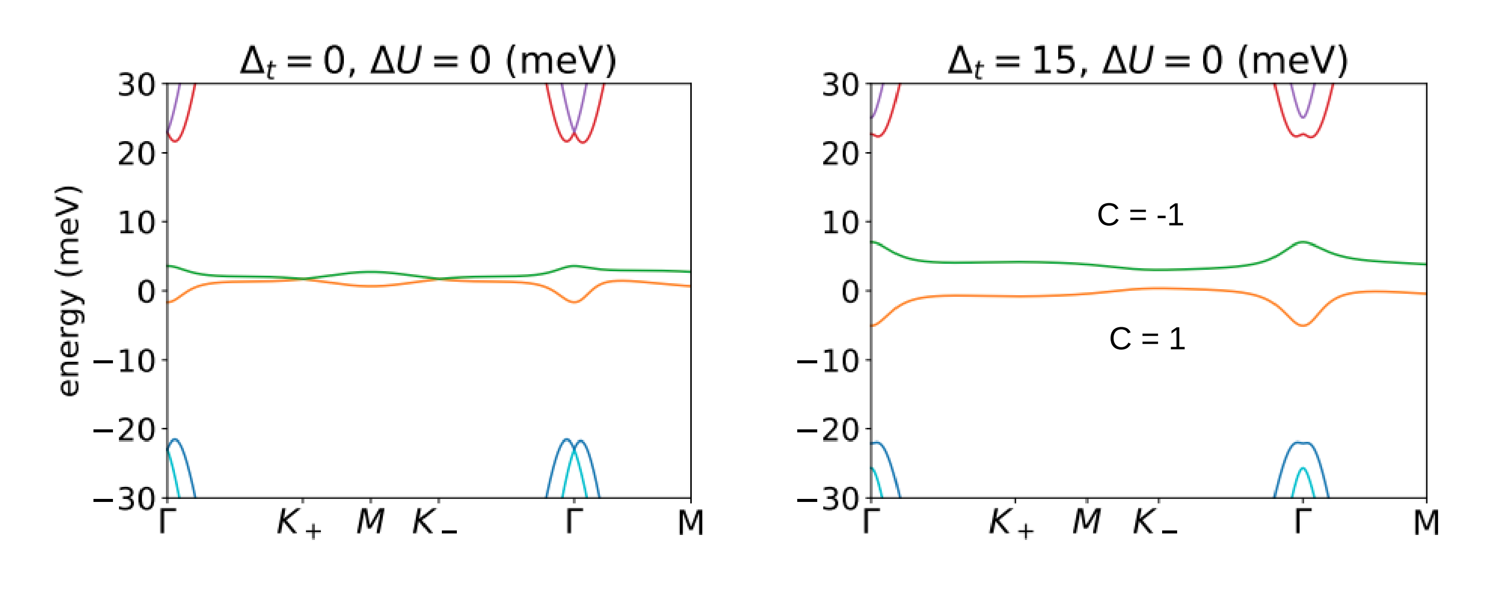
Credit : Liu et al, *Nature* **583**, (2020)

Twisted Bilayer Graphene aligned with Boron Nitride



Courtesy of ICFO/ Xiaobo Lu.

No alignment with BN $\theta \approx 1.05^\circ$ Aligned with BN

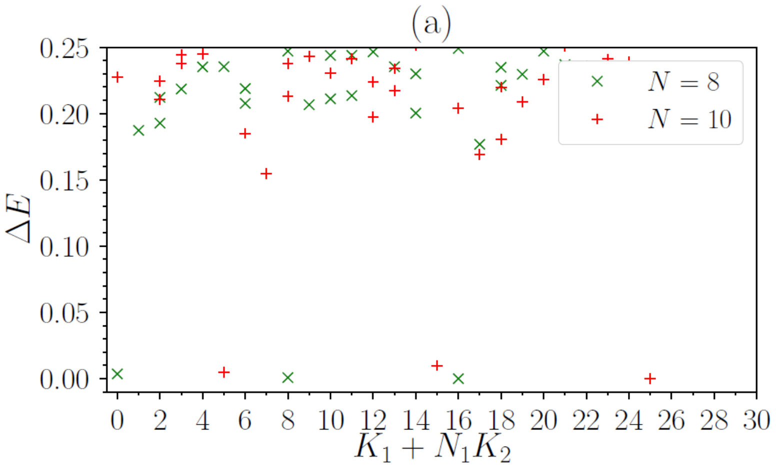


arXiv:1908.00986

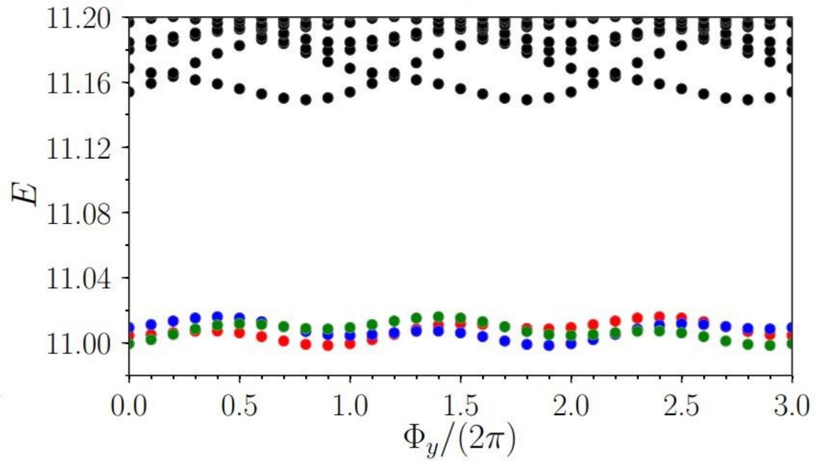
Yes, FCIs, finally!

In twisted bilayer graphene aligned with boron nitride — but only **at slightly weaker inter-layer tunnelling than in current experiments**…

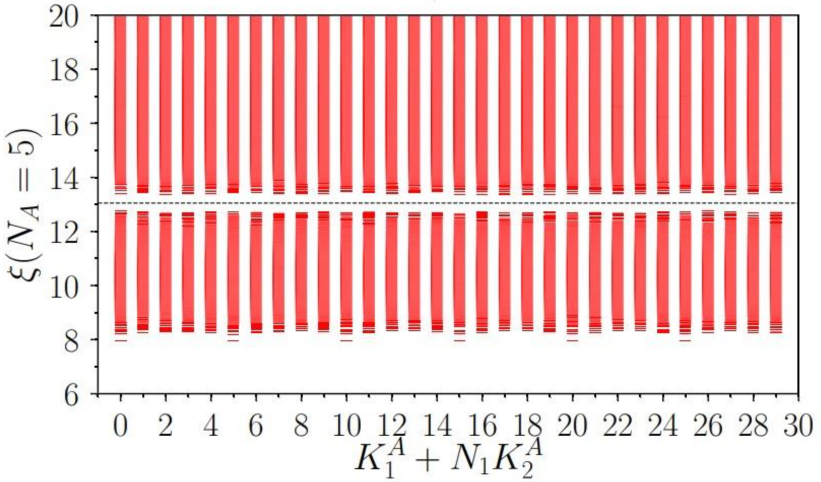
Ground state degeneracy on a torus



Spectral flow of ground states



Particle entanglement spectrum



- Laughlin like state at filling $\nu = 1/3$

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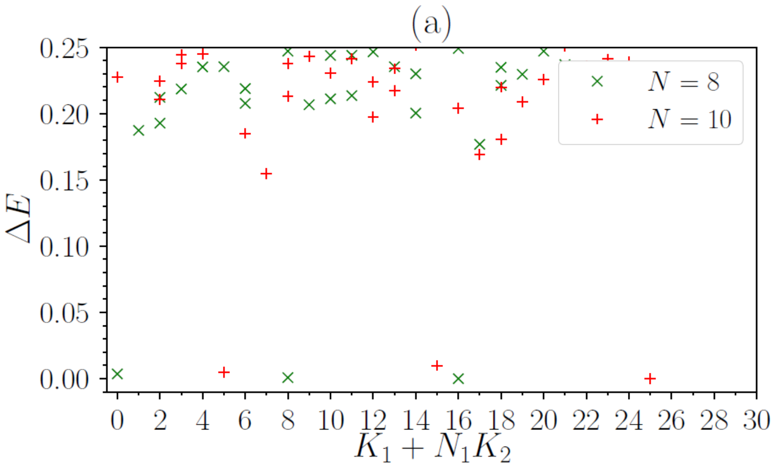

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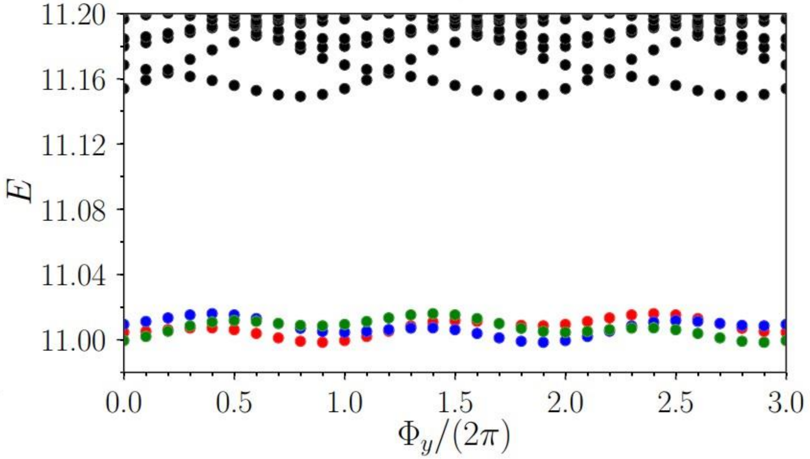
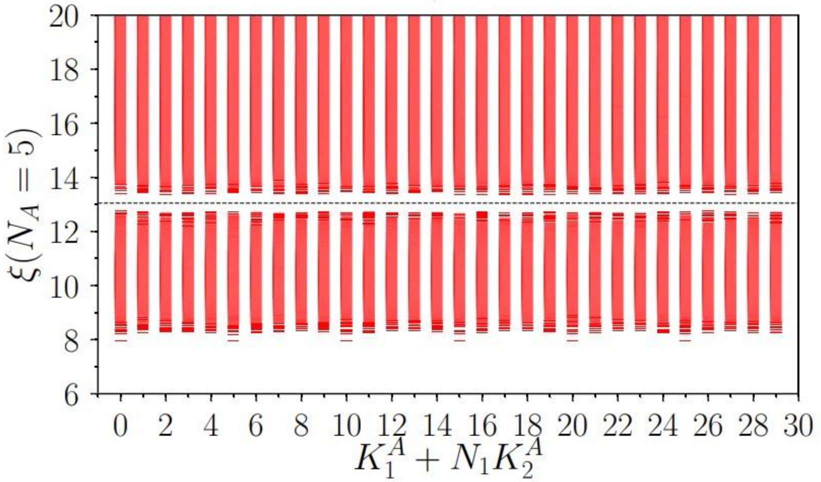


Particle entanglement spectrum



- Corroborated by other works

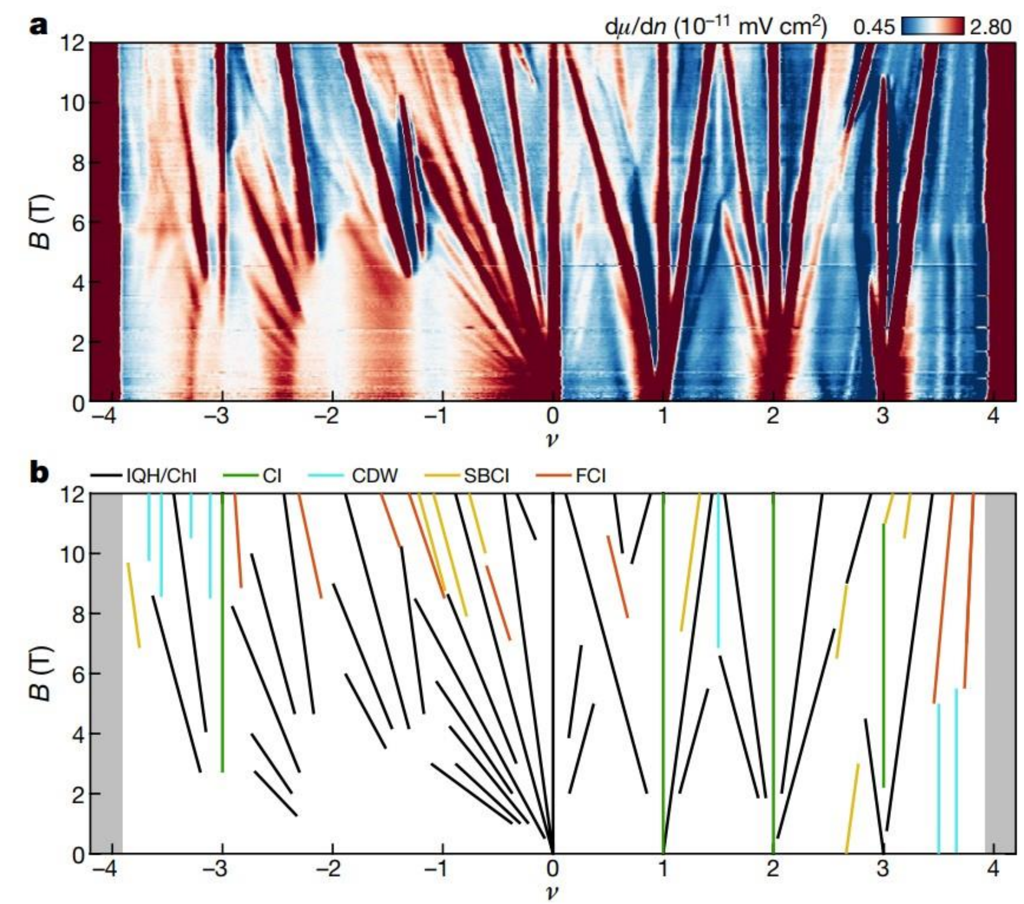
- Spin polarization confirmed by Repellin and Senthil, Phys. Rev. Research 2, 023238 (2020)
- Analytical understanding in the “chiral limit” by Ledwith et. al., Phys. Rev. Research 2, 023237 (2020)

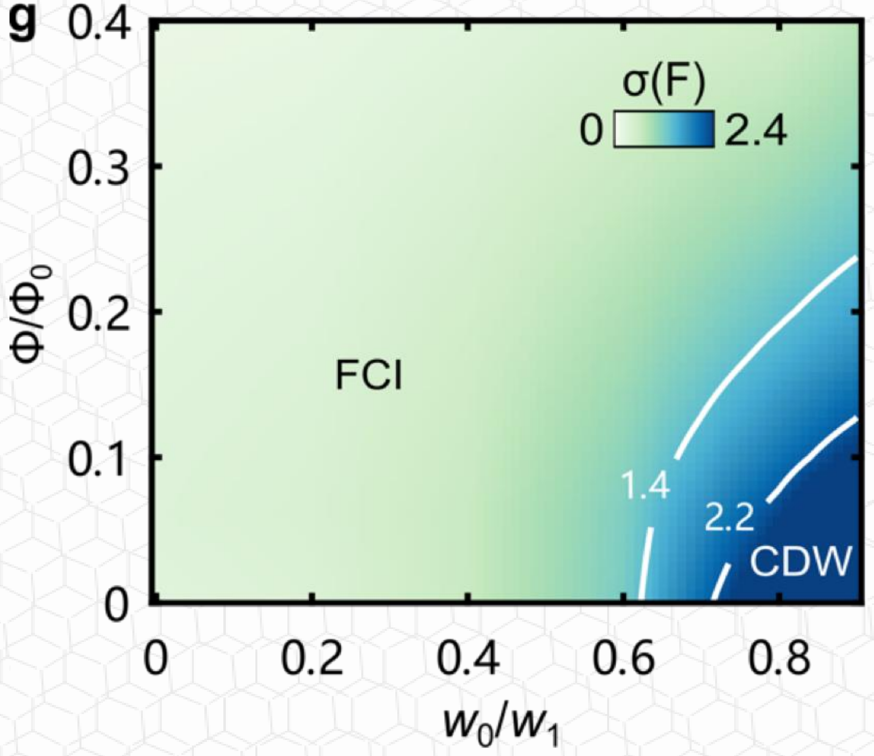
Article | Open Access | Published: 15 December 2021

Fractional Chern insulators in magic-angle twisted bilayer graphene

Yonglong Xie, Andrew T. Pierce, Jeong Min Park, Daniel E. Parker, Eslam Khalaf, Patrick Ledwith, Yuan Cao, Seung Hwan Lee, Shaowen Chen, Patrick R. Forrester, Kenji Watanabe, Takashi Taniguchi, Ashvin Vishwanath, Pablo Jarillo-Herrero & Amir Yacoby

Nature **600**, 439–443 (2021) | Cite this article

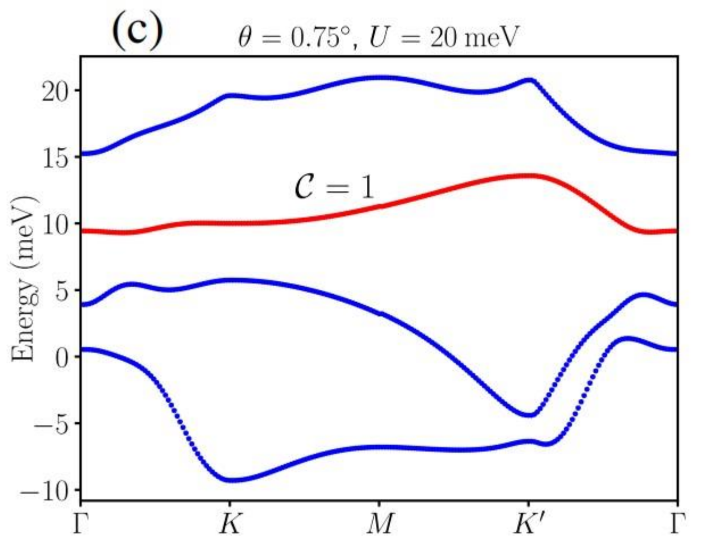


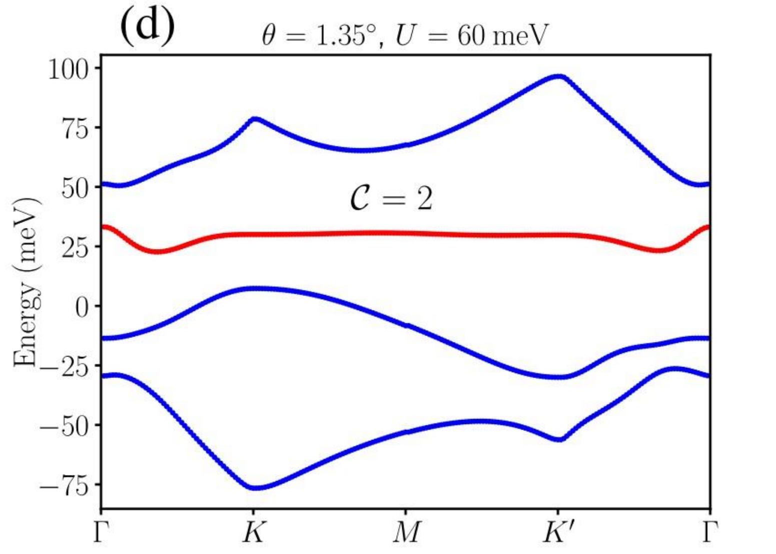


- Weak field (5 Tesla), similar effect as changing inter-layer tunnelling

$$\nu = t\frac{\phi}{\phi_0} + s$$

A series of FCIs in tunable TDBG





- Spin-polarized Laughlin like state at $\nu = 1/3$
- Laughlin state particle-hole conjugate at $\nu = 2/3$
- Spin-singlet Halperin 332 state at $\nu = 2/5$
- Possibly Halperin 332 particle-hole conjugate at $\nu = 3/5$

- Spin-polarized FCI at $\nu = 1/3$ in $C = 2$ band!
- It could be thought of as a weakly interacting state of composite fermions with negative flux attachment!

Full details in Phys. Rev. Lett. **126**, 026801

Quantum Geometry : Another look at the competition

$$\eta_{\mu\nu}(\mathbf{k}) = g_{\mu\nu}(\mathbf{k}) + \frac{i}{2}\epsilon_{\mu\nu}F(\mathbf{k})$$

Quantum Geometric Tensor

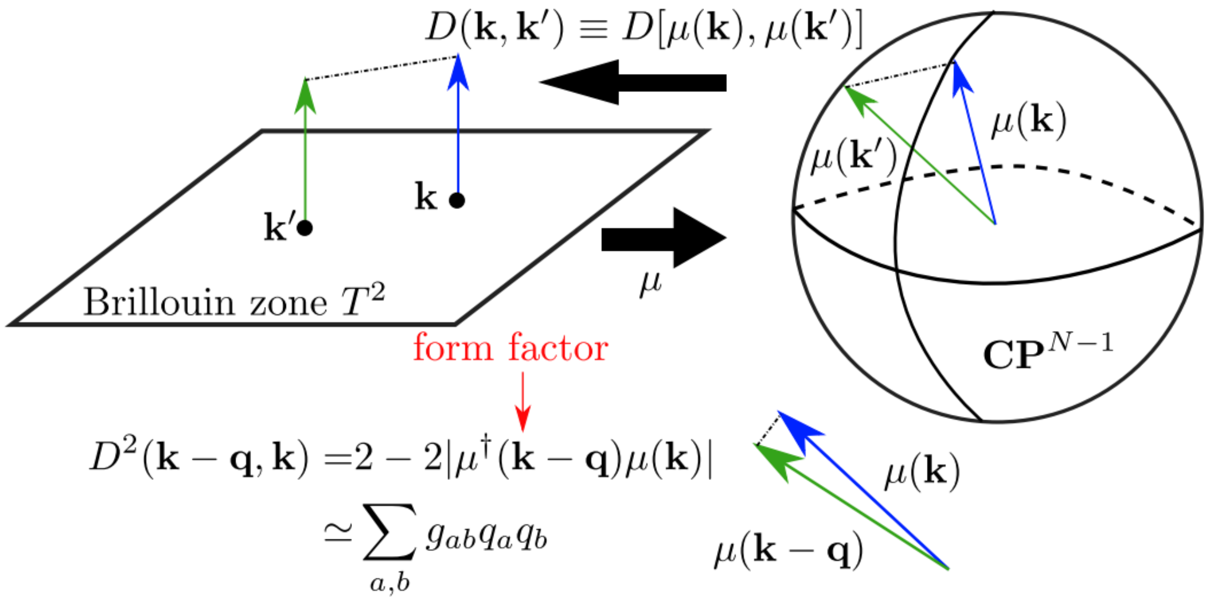
"Fubini-Study Metric"

Berry Curvature

arXiv:2202.10467

Fubini-Study metric

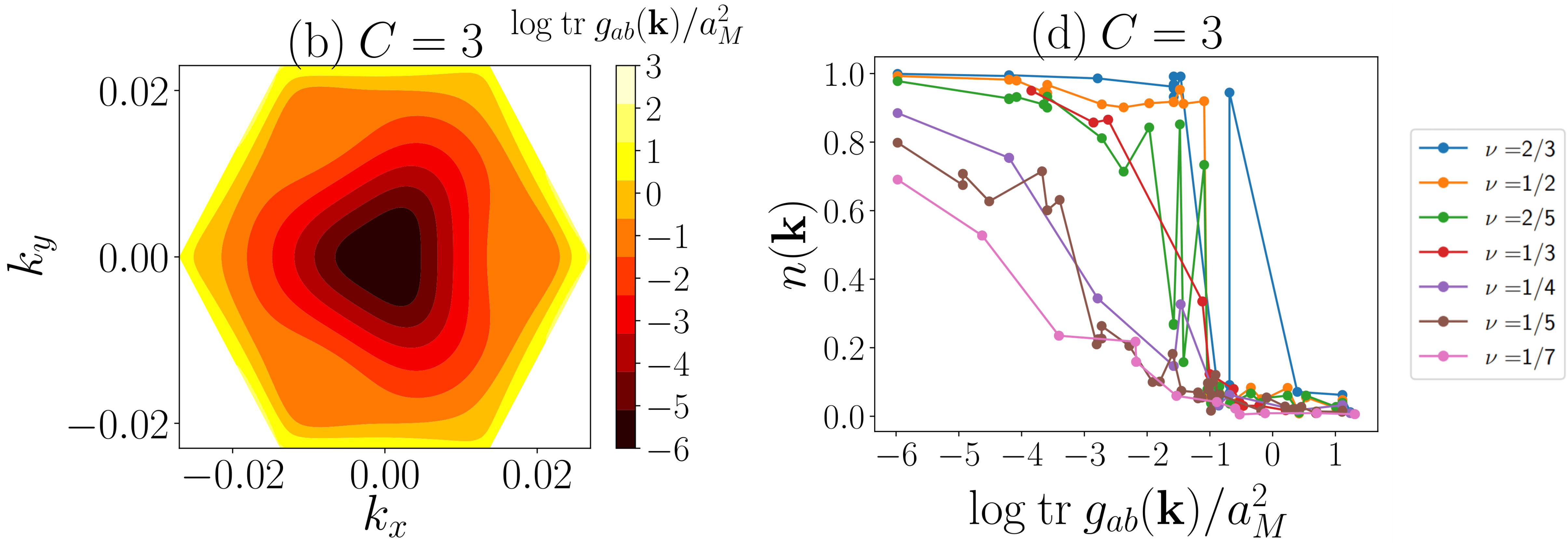
- Natural interpretation in terms of the distance between Bloch states



- It appears in the hole dispersion $E_h(\mathbf{k}) = \sum_{\mathbf{q}} V(\mathbf{q}) |\mu^\dagger(\mathbf{k}-\mathbf{q})\mu(\mathbf{k})|^2 \approx \sum_{\mathbf{q}} V(\mathbf{q})(1 - q_a q_b g_{ab}(\mathbf{k}))$
- It appears also in the Fock term, $E_k(\mathbf{k}) \approx \sum_{\mathbf{q}} V(\mathbf{q}) |\mu^\dagger(\mathbf{k}-\mathbf{q})\mu(\mathbf{k})|^2 \langle c^\dagger_{\mathbf{k}+\mathbf{q}} c_{\mathbf{k}+\mathbf{q}} \rangle \approx \sum_{\mathbf{q}} V(\mathbf{q})(1 - q_a q_b g_{ab}(\mathbf{k})) \langle c^\dagger_{\mathbf{k}+\mathbf{q}} c_{\mathbf{k}+\mathbf{q}} \rangle$

- How does the metric $g_{ab}(\mathbf{k})$ affect the electron occupation $\langle n(\mathbf{k}) \rangle = \langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}} \rangle$?

Let's look again



- <u>Guiding principle</u> :

Electrons tend to occupy states with the highest hole-energy. ⟺ Electrons tend to occupy states with the lowest Fubini-Study metric trace!

Take home message

- Moiré systems are promising platforms for fractional Chern insulators.
- The particle-hole asymmetry of interactions in a single band has dramatic consequences.
- The Fubini-Study metric is a relevant quantity to the low energy physics of Moiré materials.