

***Electronic phase transitions in charge-neutral
graphene multilayers:
Does graphene ever become graphite
upon increasing thickness?***

Alberto Morpurgo



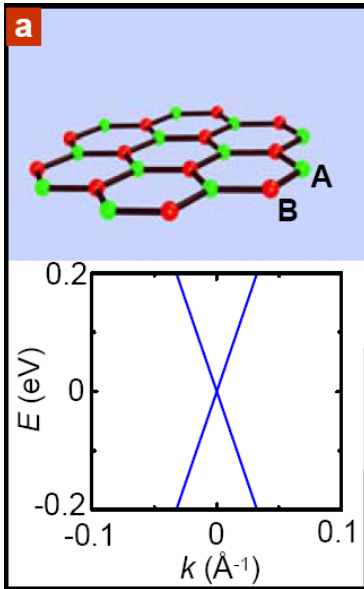
UNIVERSITÉ
DE GENÈVE

Work done by:

DongKeun Ki, YoungWoo Nam, David Soler Delgado, Anna Grushina

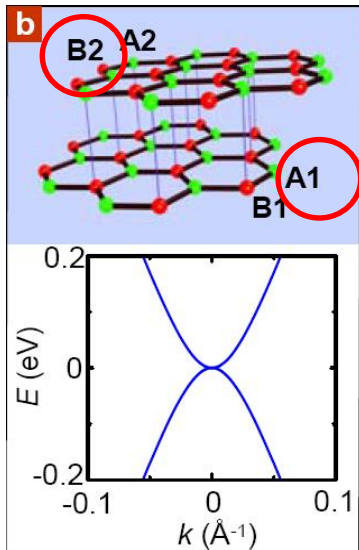
Single & bilayer graphene

In the absence of interactions



Monolayer = 1LG

$$H_{1L} = \begin{pmatrix} 0 & \hbar v_F (k_x - i k_y) \\ \hbar v_F (k_x + i k_y) & 0 \end{pmatrix}$$

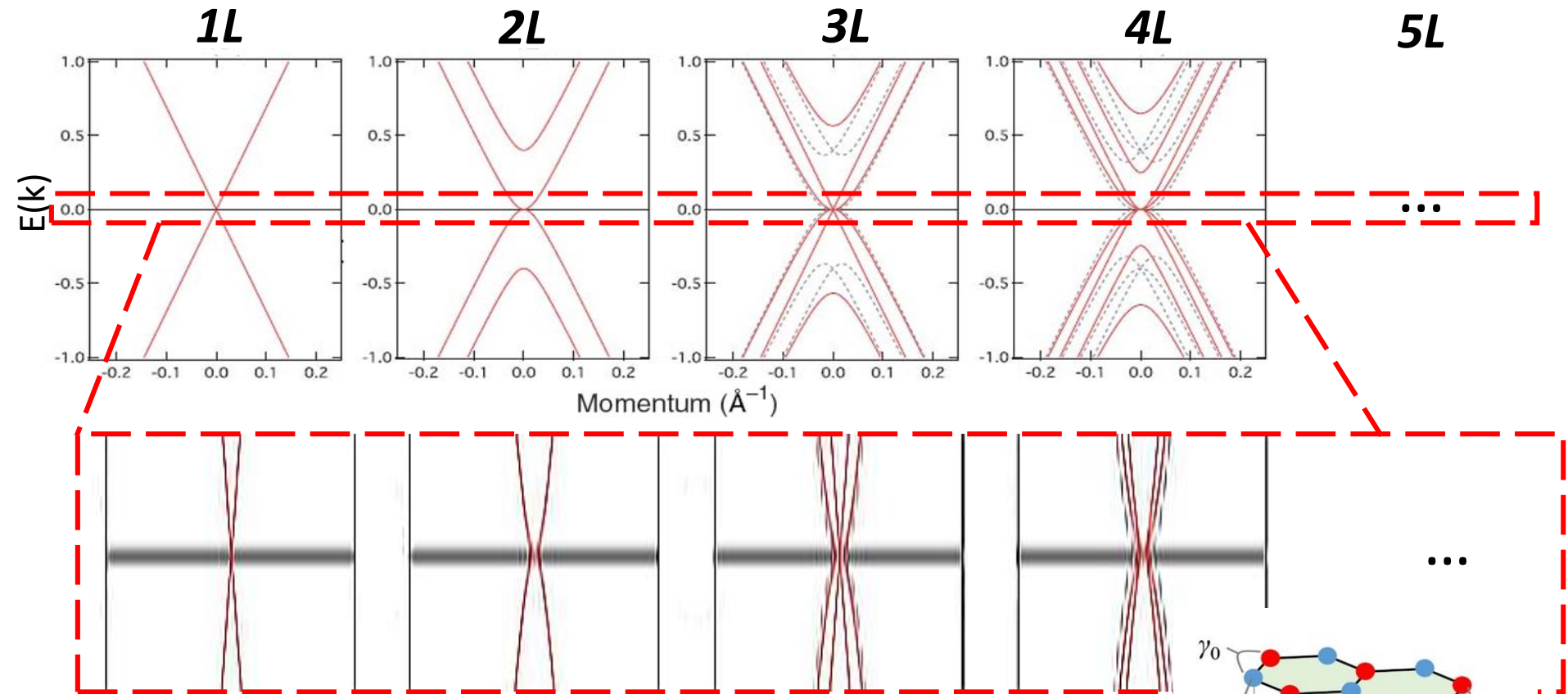


Bilayer = 2LG

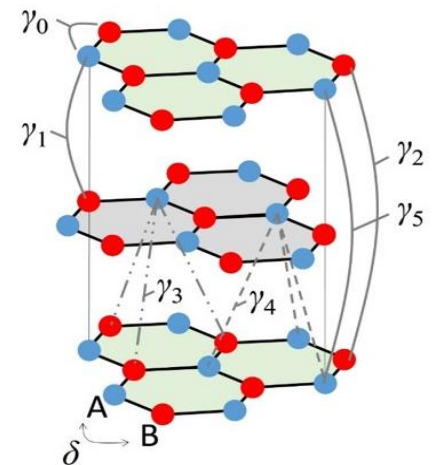
$$H_{2L} = \begin{pmatrix} 0 & \frac{\hbar^2}{2m^*} (k_x - i k_y)^2 \\ \frac{\hbar^2}{2m^*} (k_x + i k_y)^2 & 0 \end{pmatrix}$$

Compare kinetic energy to interactions

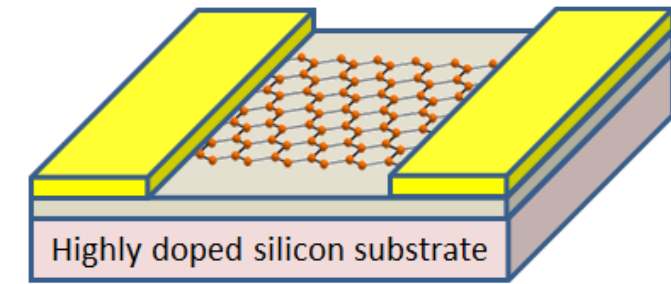
Search for interaction effects? zoom in on $E=0$ as close as possible



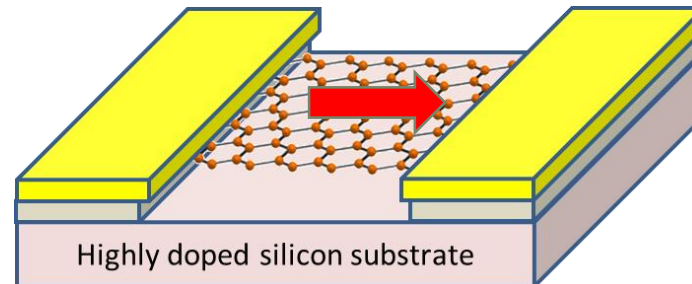
*Today we will look at what happens for
Bernal stacked multilayers up to 8LG*



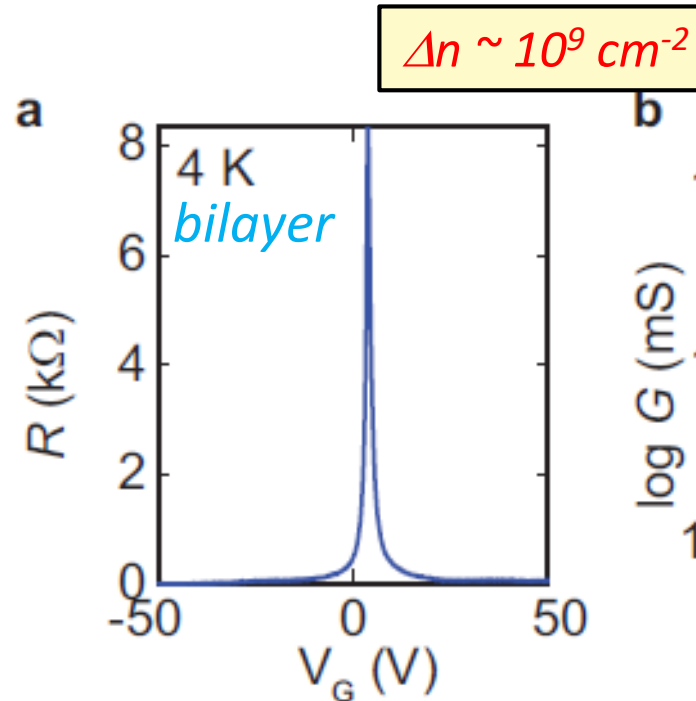
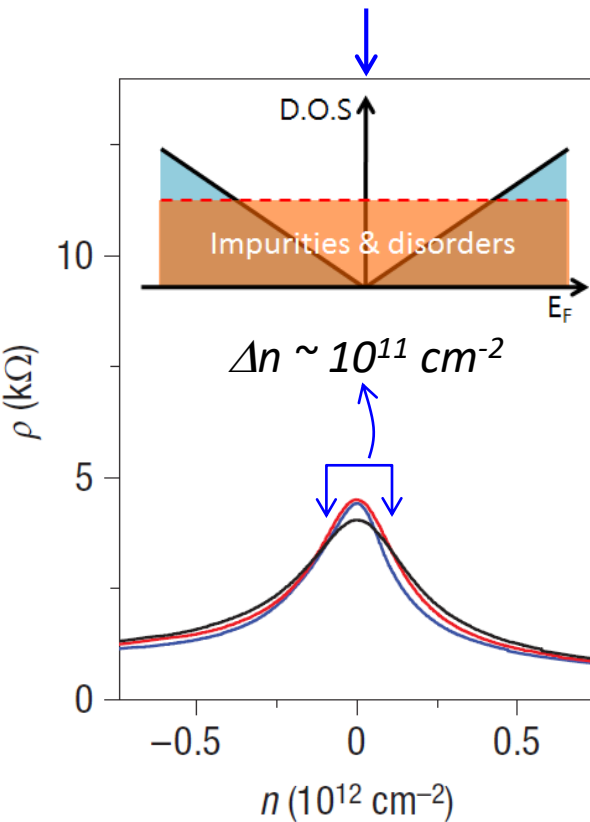
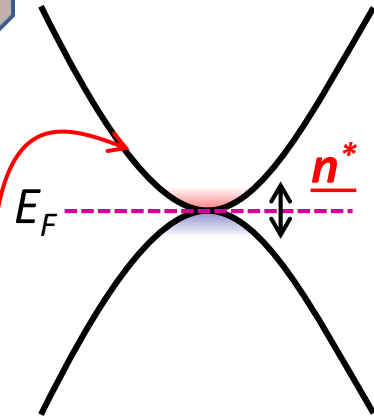
Approaching the Dirac point in Suspended graphene



On substrate



Suspended



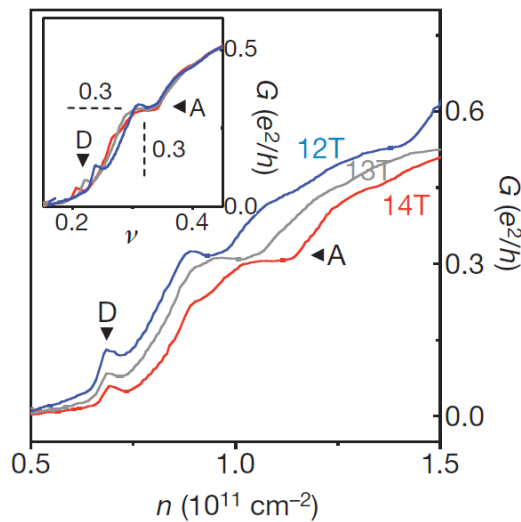
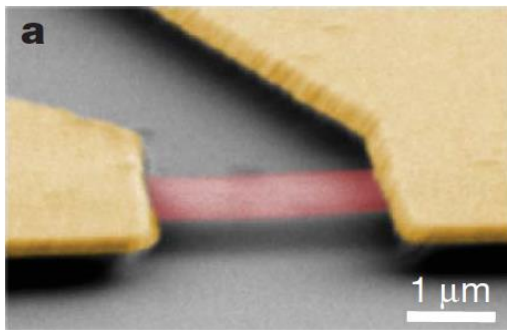
Δn approaching 10^8 cm^{-2} for $T < 1\text{K}$

Early days of suspended Graphene

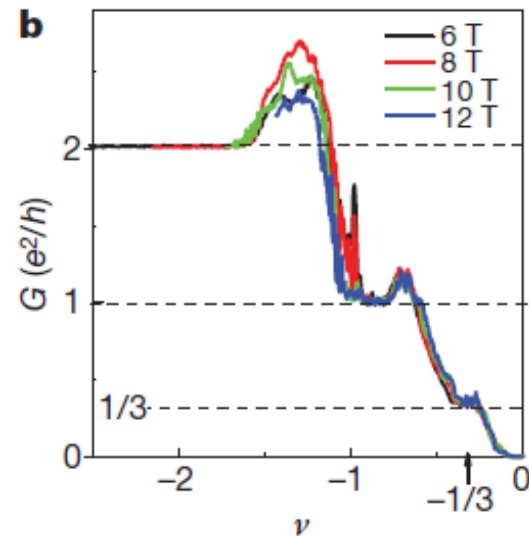
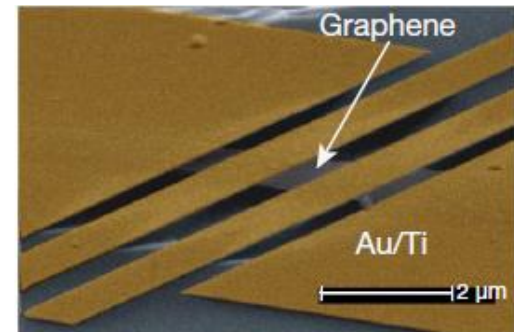
Under magnetic field

FQHE at $\nu = 1/3$ in monolayer graphene

Kim's group

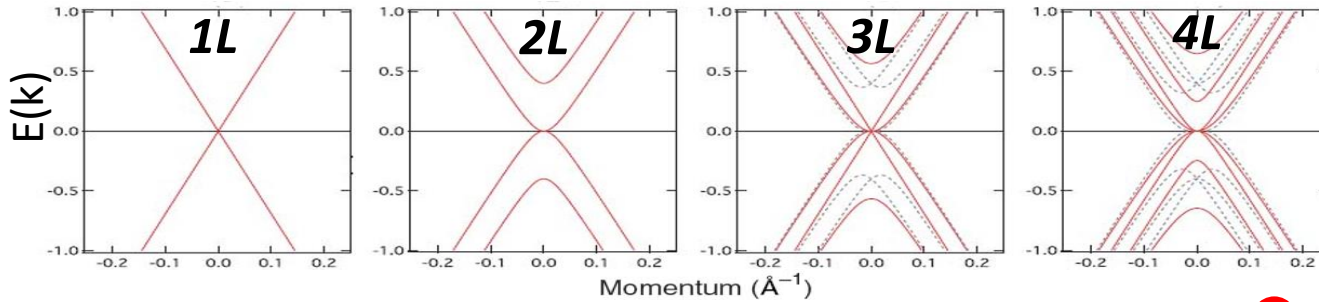


Andrei's group



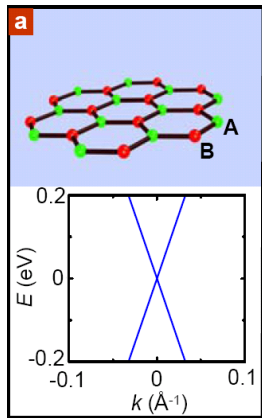
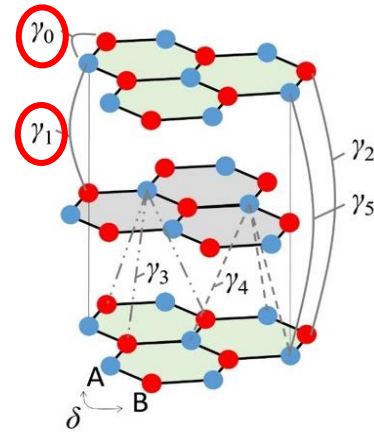
Nature 2009

“Ideal” single-particle mono & bilayer at $B=0$



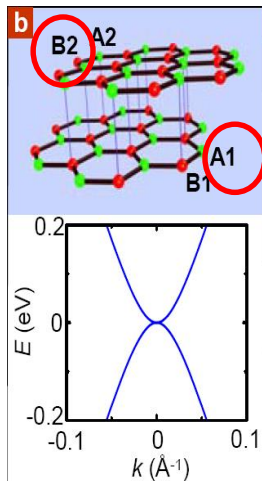
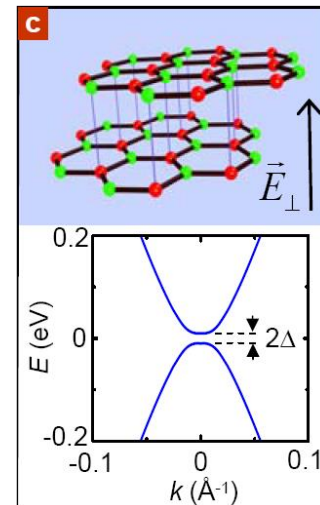
Monolayer = 1LG

$$H_{1L} = \begin{pmatrix} 0 & \hbar v_F (k_x - i k_y) \\ \hbar v_F (k_x + i k_y) & 0 \end{pmatrix}$$



Bilayer = 2LG

$$H_{2L} = \begin{pmatrix} \Delta & \frac{\hbar^2}{2m^*} (k_x - i k_y)^2 \\ \frac{\hbar^2}{2m^*} (k_x + i k_y)^2 & -\Delta \end{pmatrix}$$

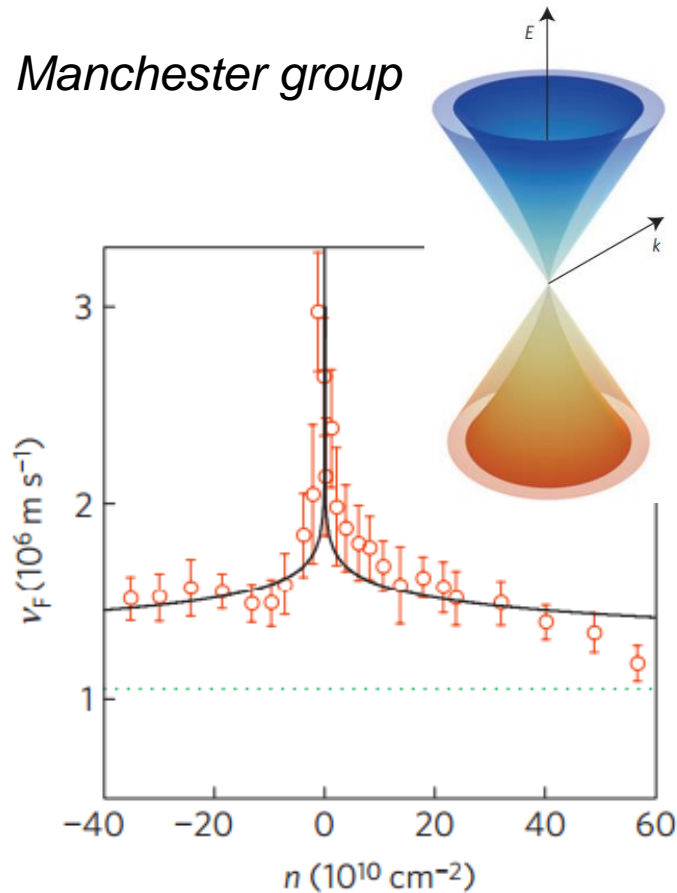


Interaction effects at $B=0$

Monolayer

Renormalization of Fermi velocity

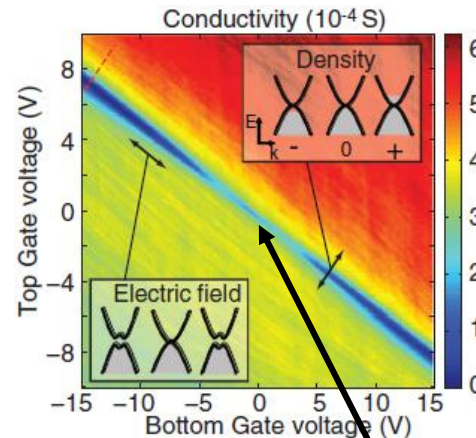
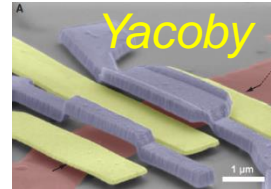
Manchester group



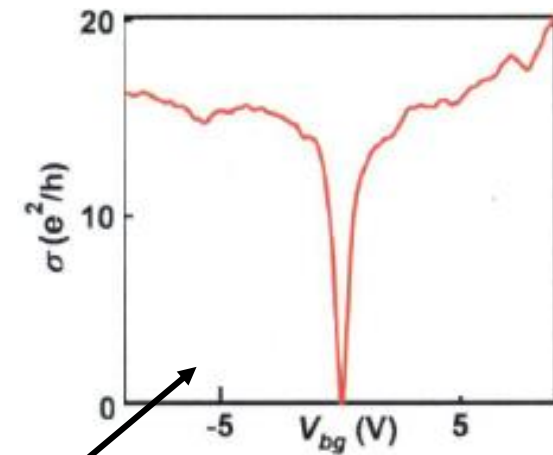
Nat. Phys. 2011

Bilayer

Insulating bilayer @ charge neutrality



Science 2011



PNAS 2012

Suppressed conductance
at charge neutrality

All the action happens for $n < 10^{10} \text{ cm}^{-2}$

Symmetry broken gapped state in bilayers

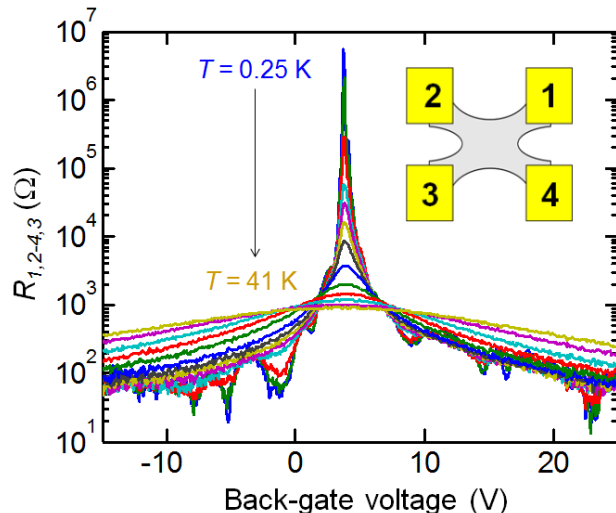
What does it really mean?



Phase transition

*a gap opens because of
e-e interactions:*

*The bilayer becomes
an insulator*



$$H_{2L} = \frac{\hbar^2}{2m^*} \begin{pmatrix} \Delta_{s,v} & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & -\Delta_{s,v} \end{pmatrix}$$

- $\Delta \neq 0$ occurs spontaneously due to e-e interactions
- Broken symmetry state with Δ as order parameter
- Sign of Δ depends on valley and spin
- Exchange energy: in general there is no electric field between the two layers

...and then:

*virtually all the world started working on graphene on hBN
and forgot about suspended graphene....*

...BUT...

...Graphene on hBN is NOT Graphene...

&

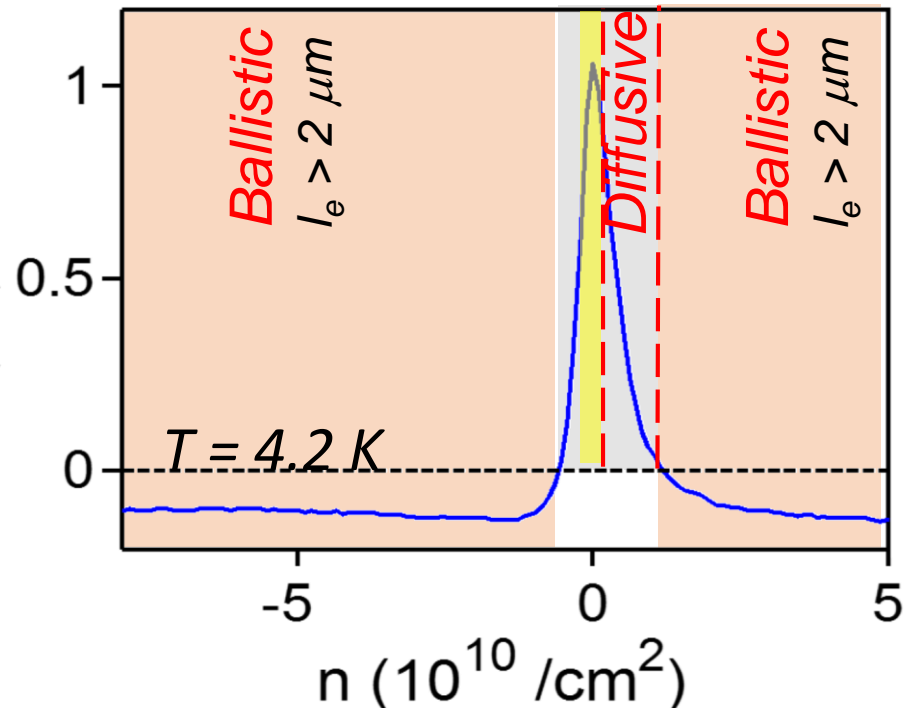
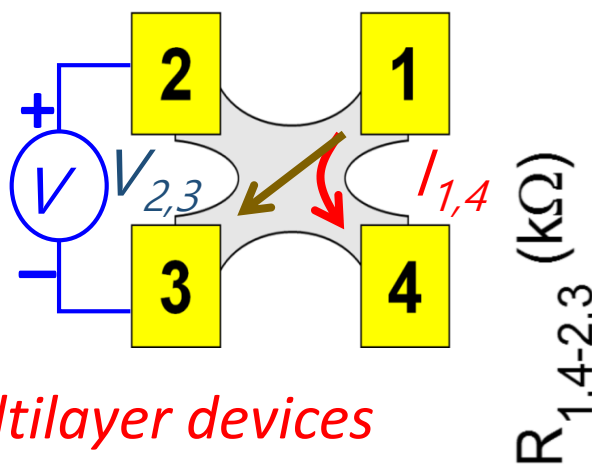
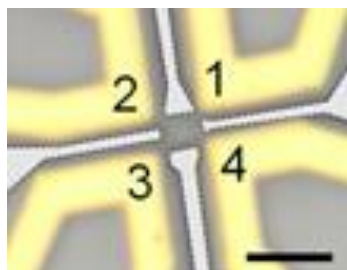
*...Graphene on any substrate does not reach
the quality of suspended Graphene!*

So:

We continued working on suspended graphene!

“Multi” devices: Multi-terminal and/or Multi-layer devices

Bilayer Graphene

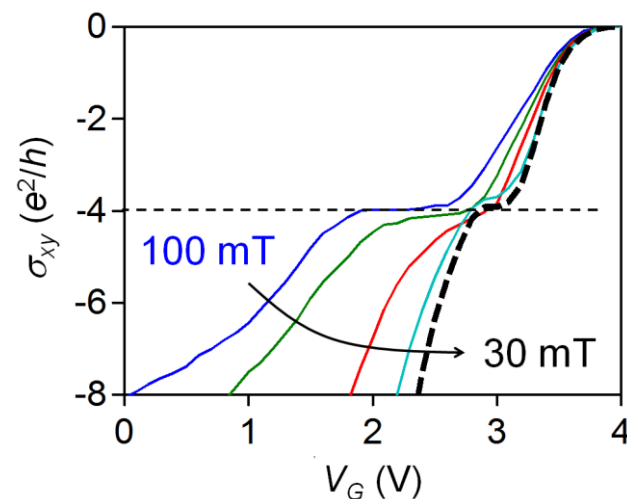


*Suspended multilayer devices
of very high quality*

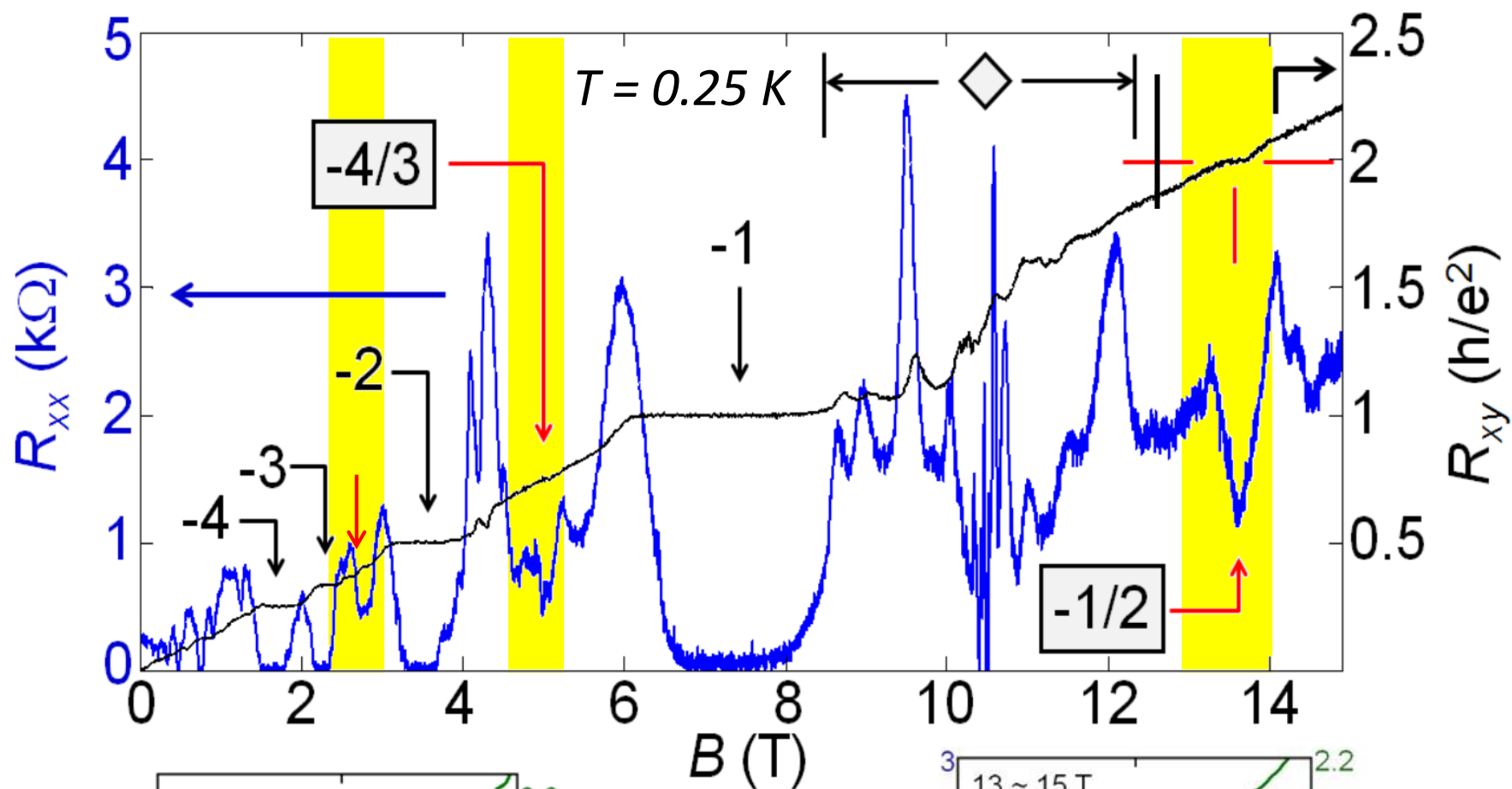
- Ballistic transport
For $n > 10^{10} \text{ cm}^{-2}$

- Quantum Hall plateaus
starting from 300 Gauss

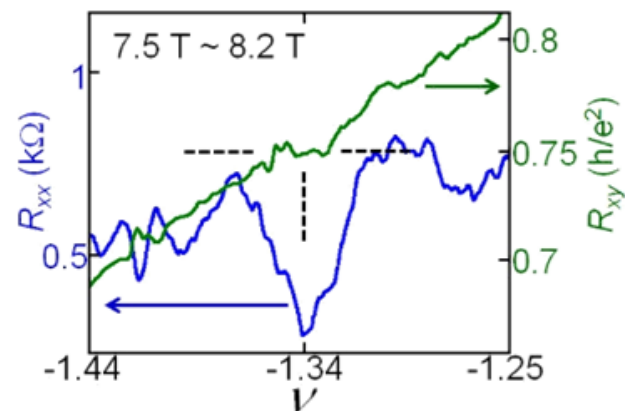
- Observed
even-denominator FQHE



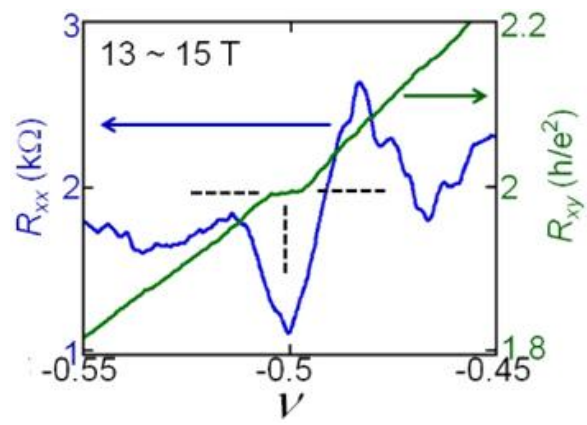
Even denominator FQHE: first time not in GaAs-2DEGs



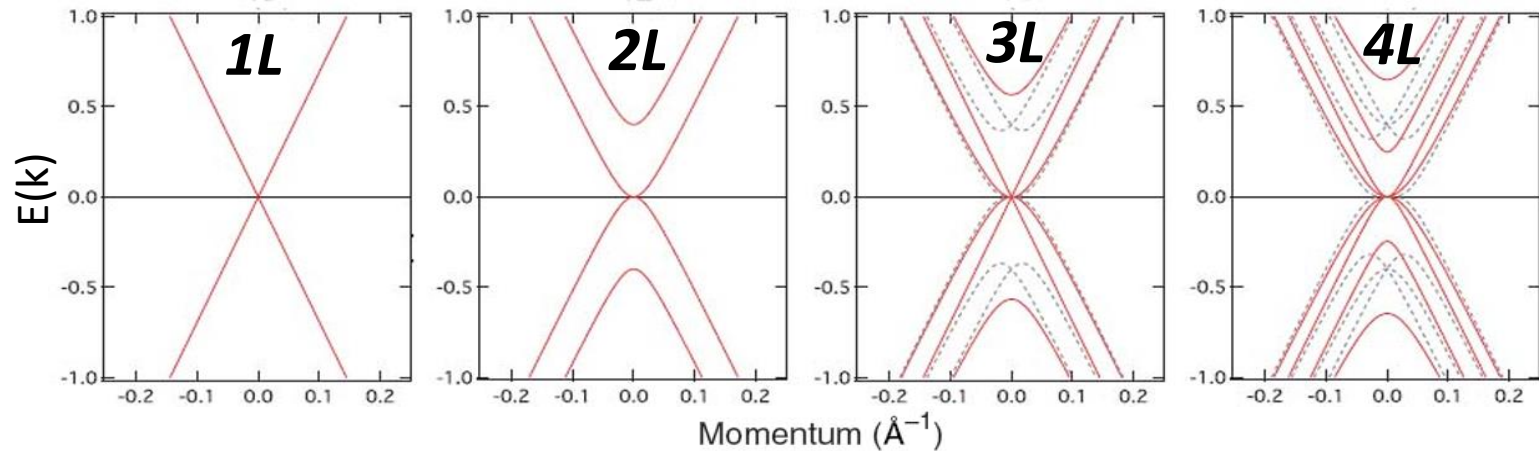
$\nu = -4/3$



$\nu = -1/2$



Identifying Multilayers from Low-B integer QHE



Dirac Band Landau Levels

$$\varepsilon_N = \pm v_F \sqrt{2e\hbar BN}$$

$E=0$ level: **4 x Degenerate**

Bilayer Band Landau Levels

$$\varepsilon_N = \pm \hbar \omega_c \sqrt{N(N-1)}$$

$E=0$ level: **8 x Degenerate**

Shared by valence and conduction band

Ex. Bernal trilayer:

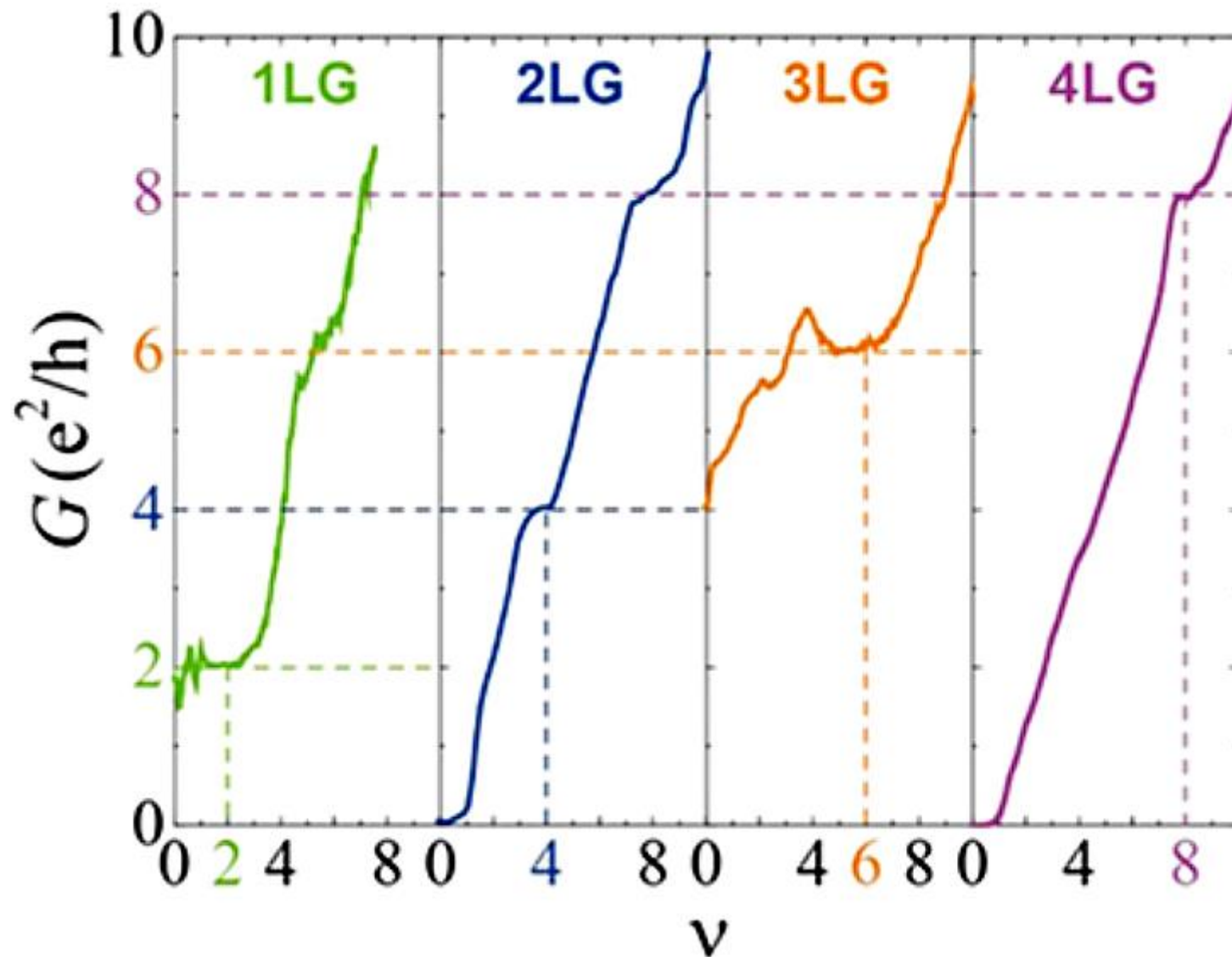
1 x Dirac + 1 x Bilayer bands ----- $E=0$ level: **12 x Degenerate**

First low-B plateaus appearing at filling factor:

$\nu = -6$ (valence band) & $\nu = +6$ (conduction band)

It works -- but often requires multiterminal devices

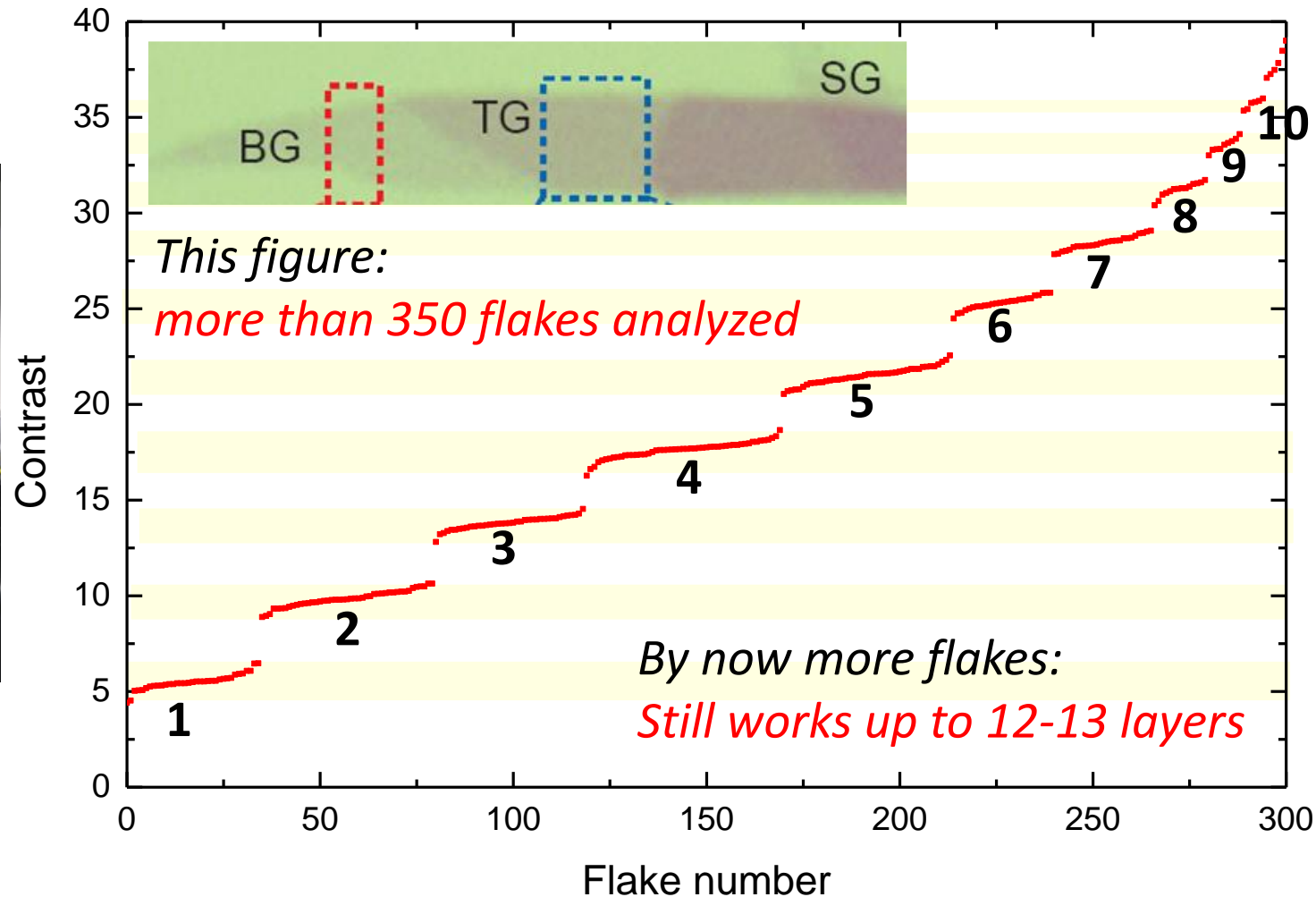
At low B look for the first plateau appearing



Successfully identified up to 8LG so far

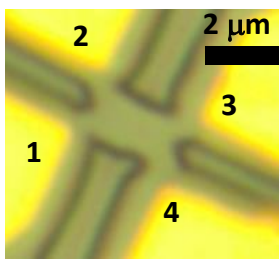
Thickness determination from careful contrast analysis

Graphene multilayers on Si/SiO₂

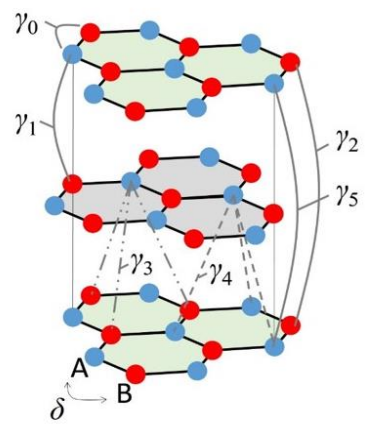
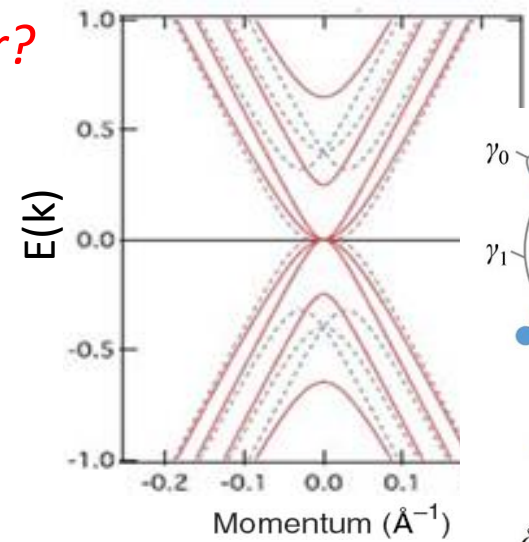
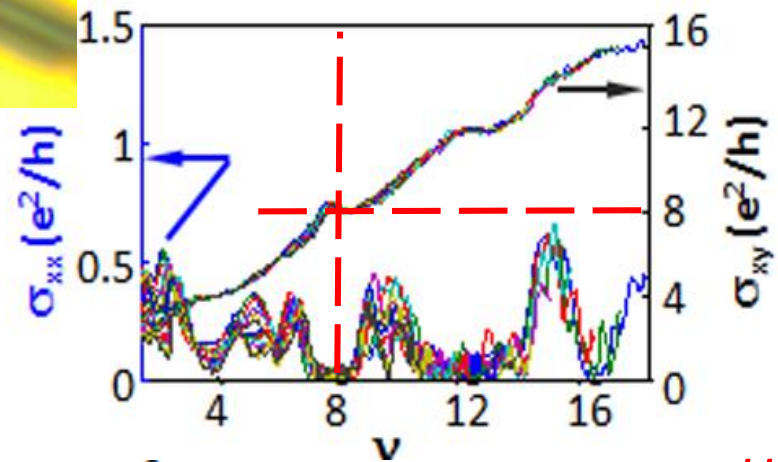


Tell you thickness but does not tell you stacking:
if natural graphite is used to exfoliate Bernal predominates

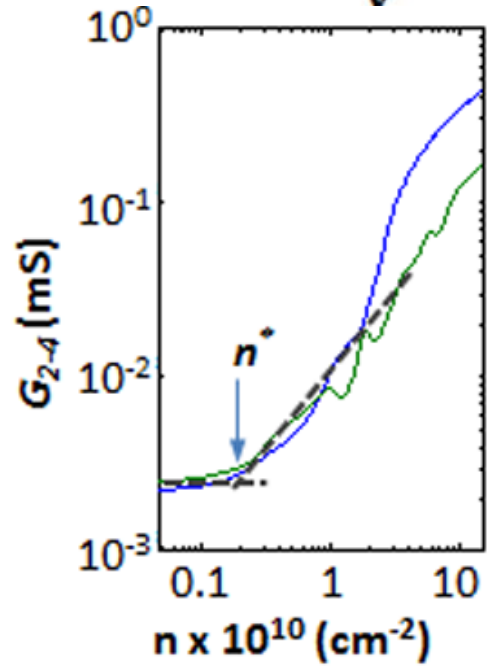
OK...But is there anything interesting in thicker multilayers?



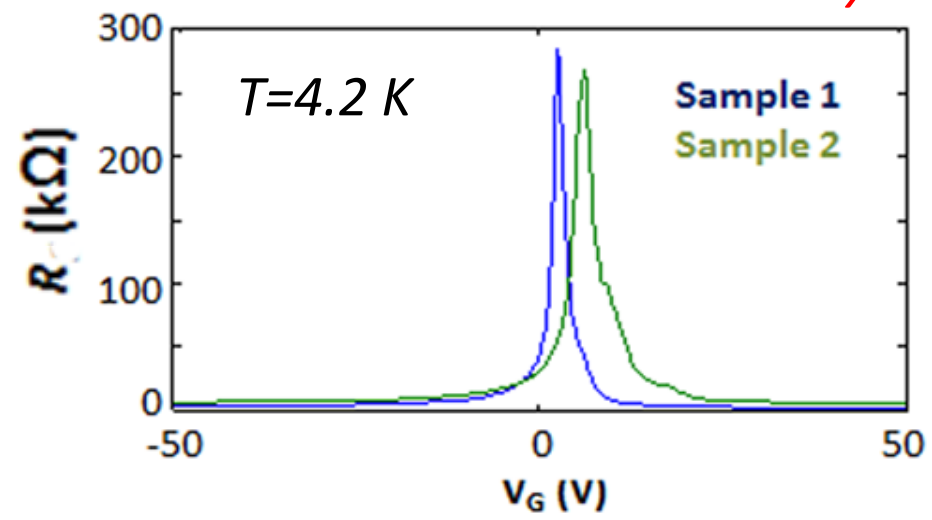
4LG: - Zero-gap semiconductor?
- Semi-metal?



Very narrow Dirac peak
($n^* \sim 3 \times 10^9 \text{ cm}^{-2}$ at 4.2 K)



Highly resistive @ T=4.2 K:
50 x more resistive than bilayers

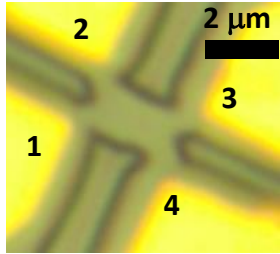


...Yes: there is!

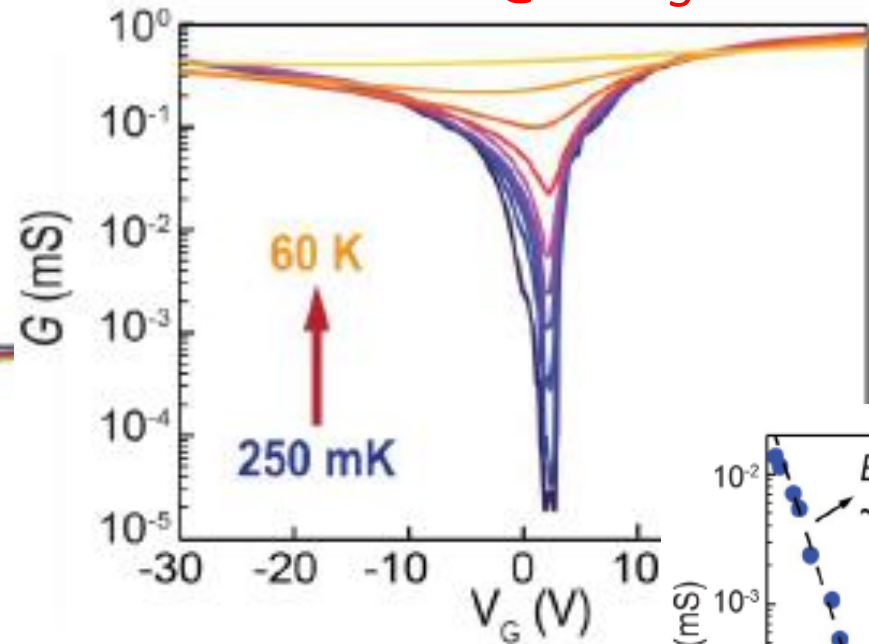
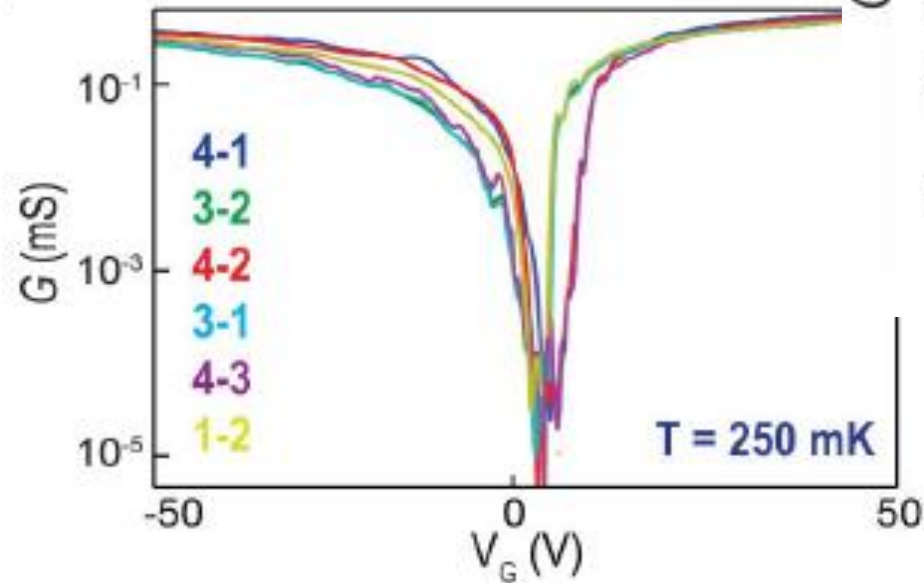
Bernal 4LG are insulators --- more insulating than bilayers

Thermally activated conductance

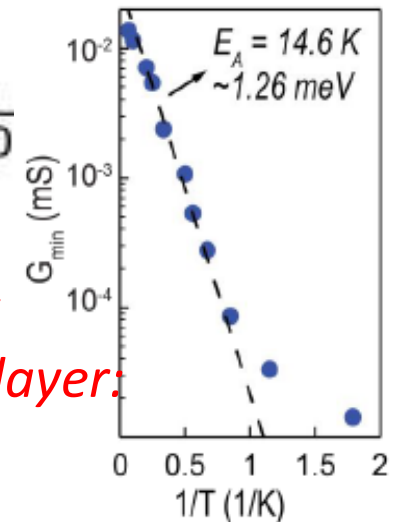
@ charge neutrality



Uniform:
seen in all pair of contacts



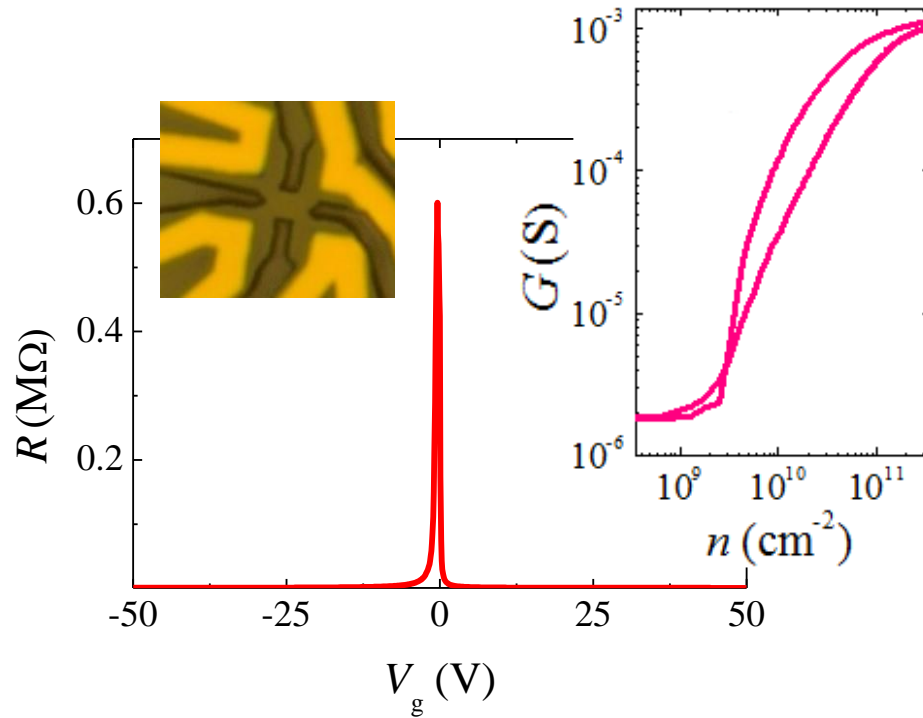
Activation energy
Comparable to bilayer:
....strange



Increasing thickness makes the behavior and deviate more from that of graphite....is this a coincidence?

Let's check Bernal-stacked 6LG

- *Very sharp peak: $\delta n \sim 2 \cdot 10^9 \text{ cm}^{-2}$;*

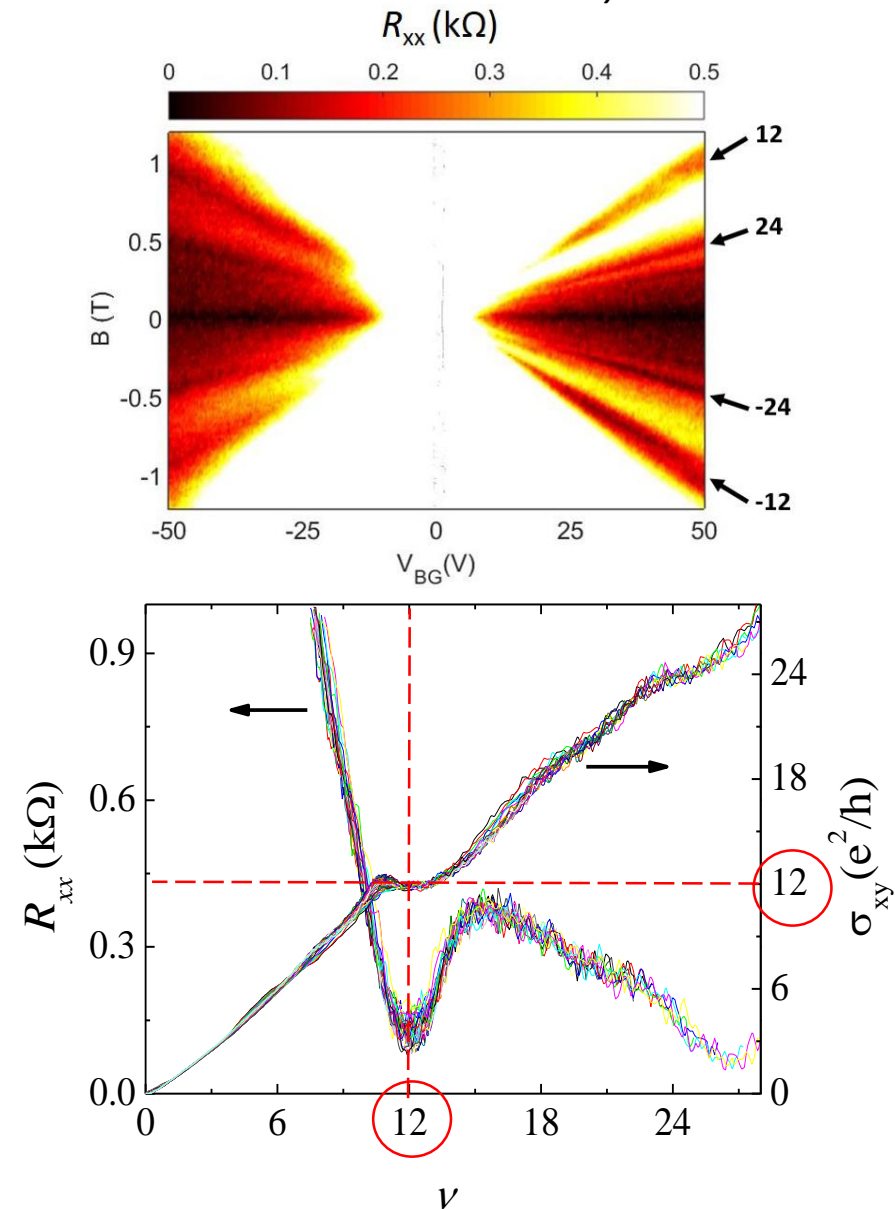


High resistance:

@ charge neutrality and $T = 4.2 \text{ K}$
 $R = 0.6 \text{ M}\Omega$

- *At low B*

Dominant plateau in σ_{xy} at $12 e^2/h$

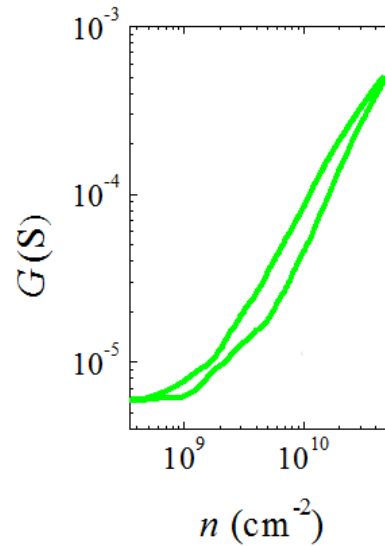
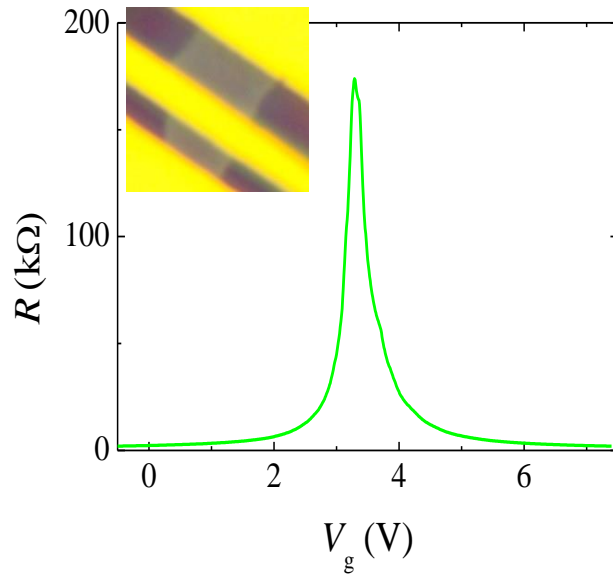


6L Graphene also has high resistance at CNP

It does not stop...Bernal-stacked 8LG

In this case only 2-terminal device

- Again: very sharp peak, $\delta n \sim 2 \cdot 10^9 \text{ cm}^{-2}$;



High resistance:

@charge neutrality & $T = 4.2$ K

$R = 0.2 \text{ M}\Omega$

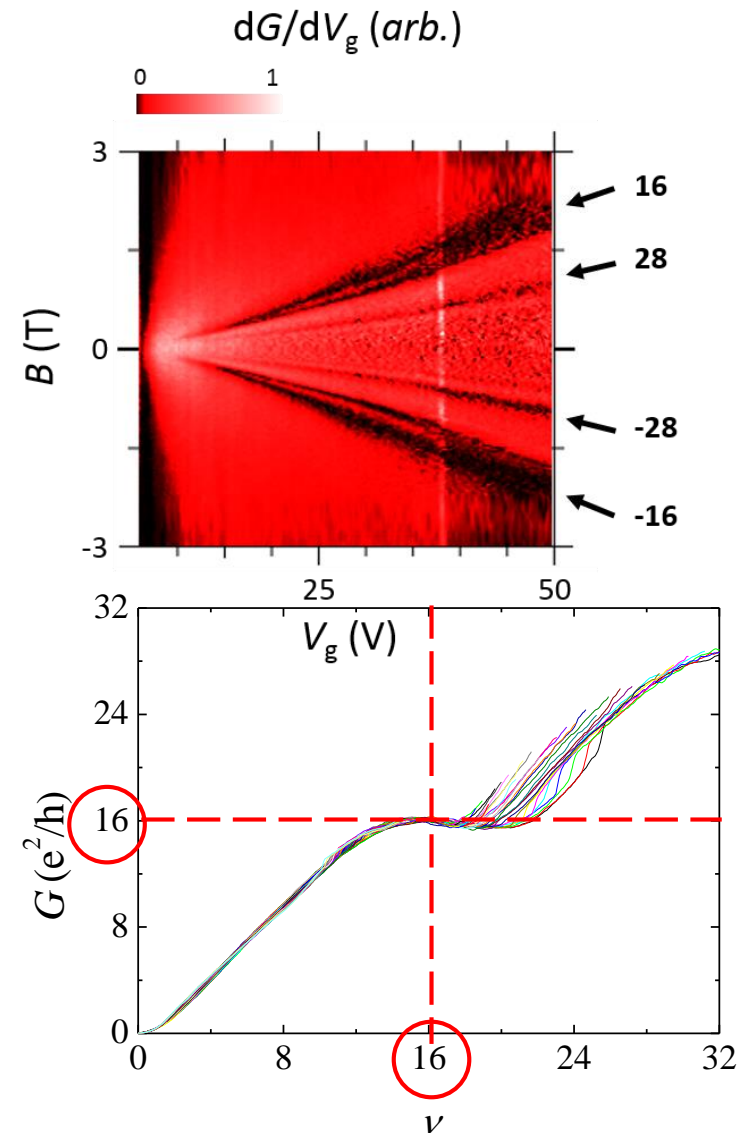
Square resistance $\sim 350 \text{ k}\Omega$

Same as in 4LG and 6LG

At low B

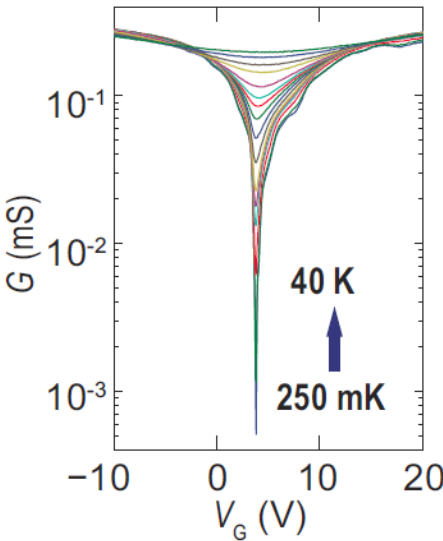
Dominant plateau

σ_{xy} at $16 e^2/h$

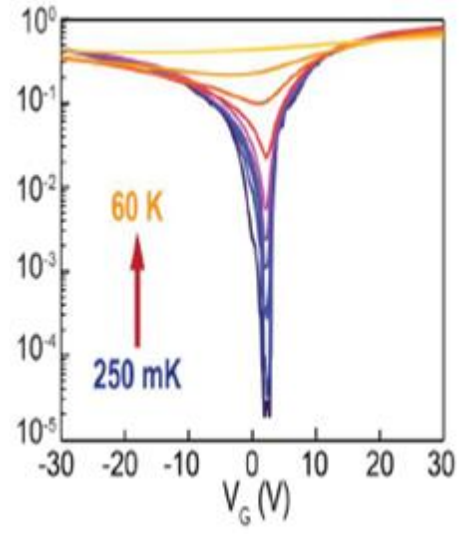


Resistance temperature dependence

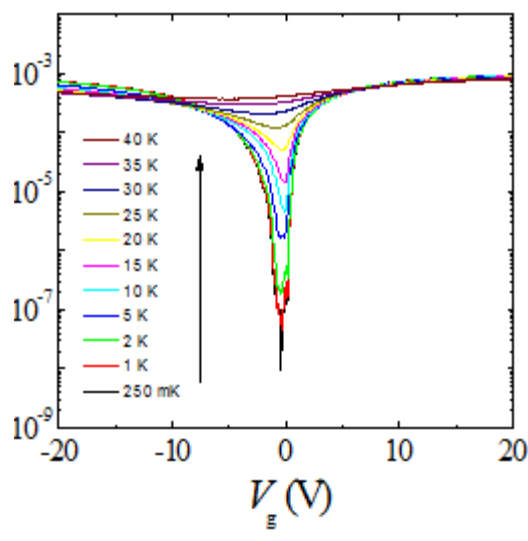
2LG



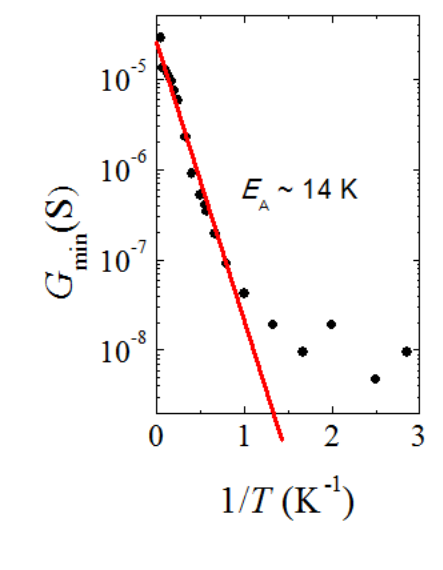
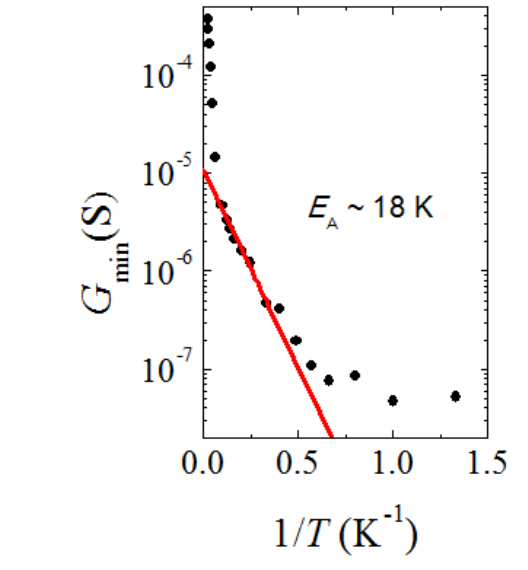
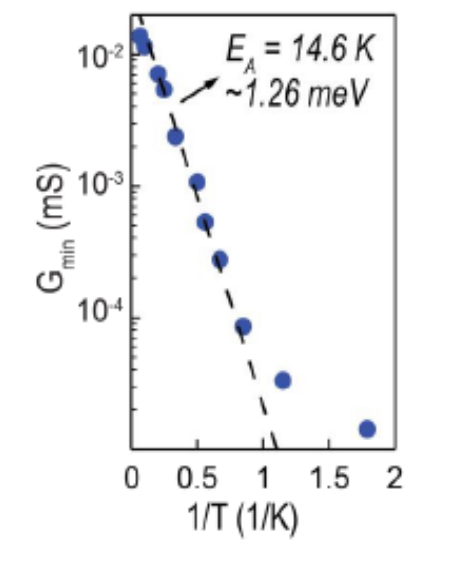
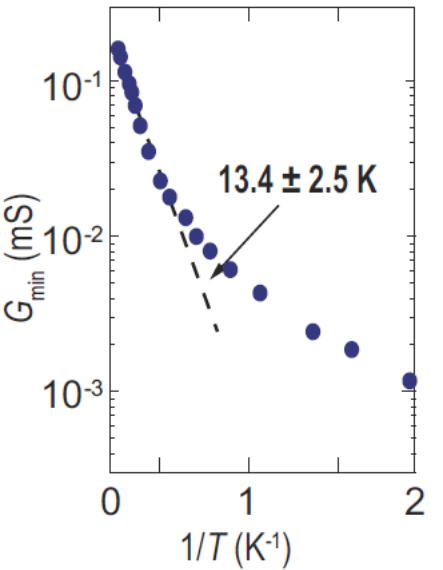
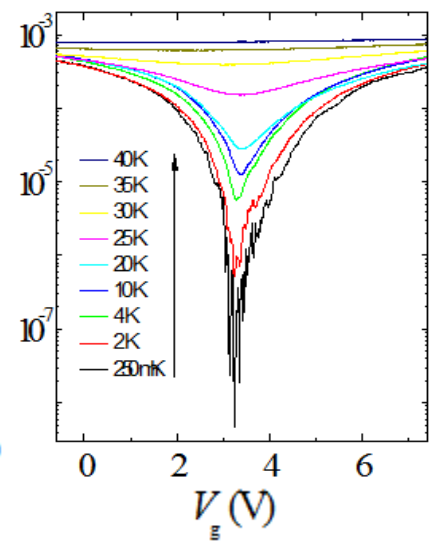
4LG



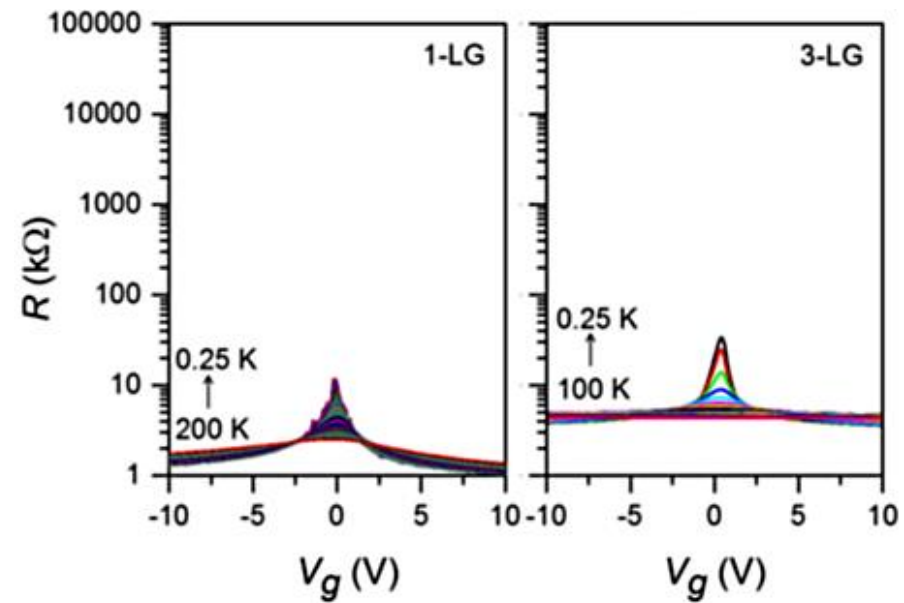
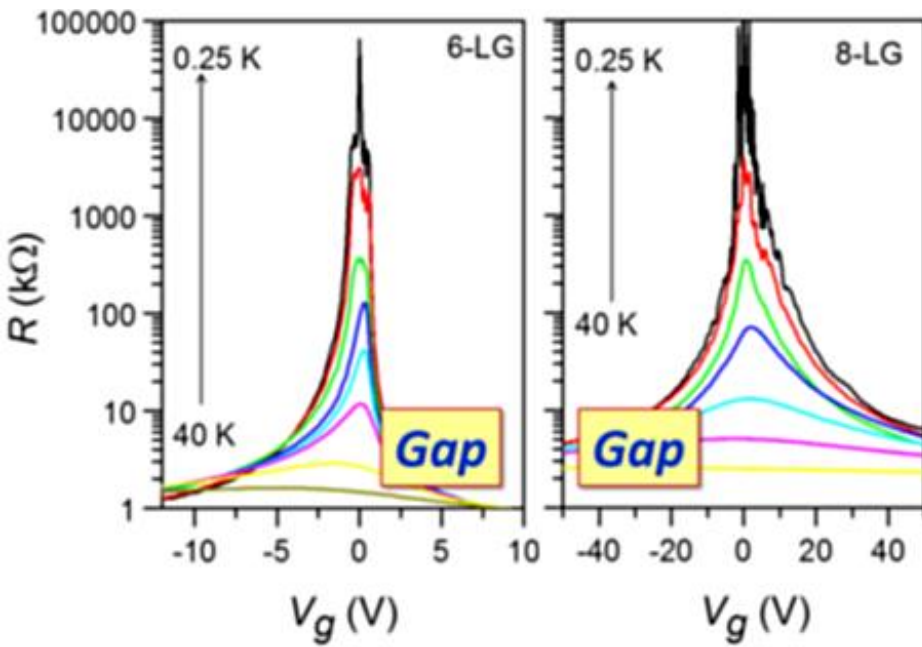
6LG



8LG



What about odd multilayers? Even-Odd !



We are onto something...but what?

Let's not forget: Even-Odd !

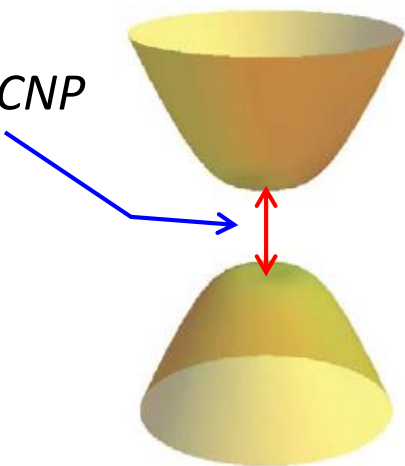
Δ — a *self-consistent mean-field potential* explains insulating bilayer

— $+\Delta$

— $-\Delta$

$H = \begin{pmatrix} +\Delta & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & -\Delta \end{pmatrix}$

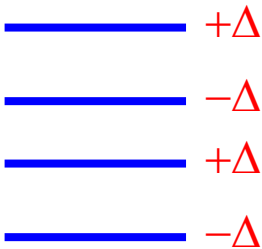
opens a gap at CNP



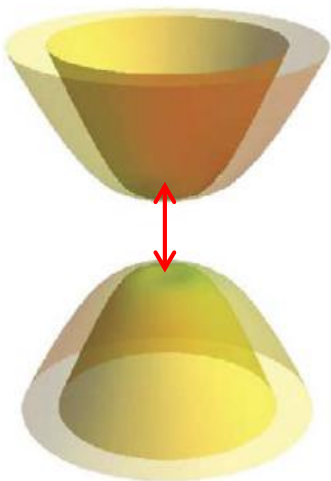
Let's generalize this idea to thicker multilayers:

Δ — a self-consistent mean-field *staggered potential*

Ex.: 4-layer graphene

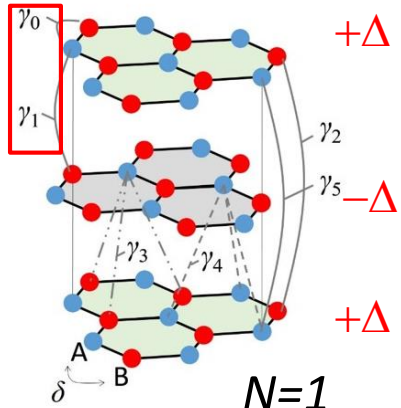


Staggered potential:
opens a gap @ CNP in 4LG

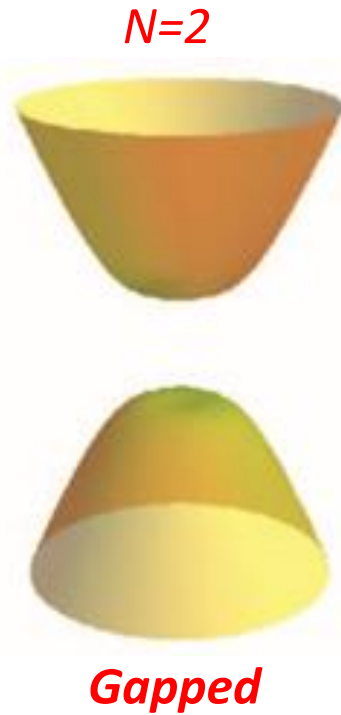
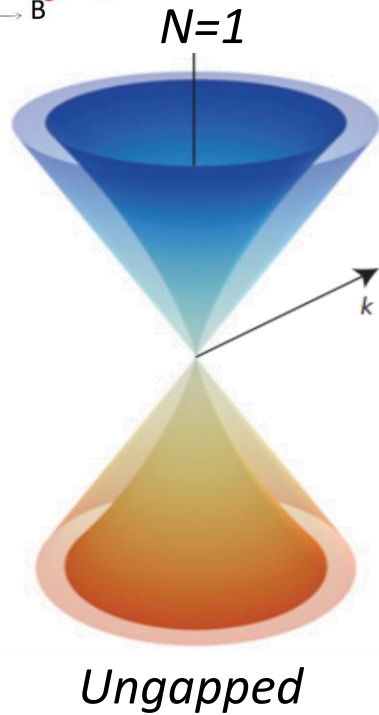


Nat. Commun. **6** 6419 (2015)
2D Mater. **3**, 045014 (2016)

Staggered potential: it works for all thicknesses!



- All even multilayers:
fully gapped
- All odd multilayers:
parabolic bands gap out, but Dirac band doesn't



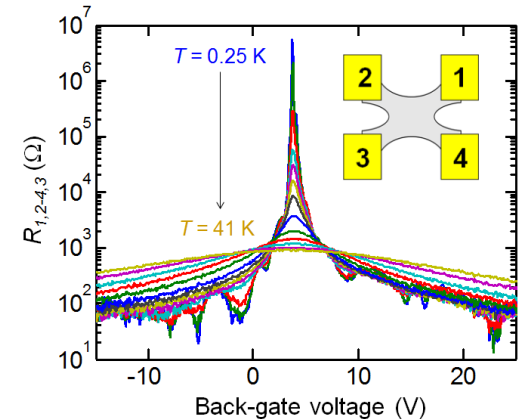
Are we done ? Not really

- Do we have any *direct evidence* that *e-e interactions* are doing anything in *odd layers*? **No !**

- *Mean-field staggered potential*:

There must be a *phase transition* at which the staggered potential (which is the order parameter) appears. What kind of phase transition?

- *Quantum phase transition* at $T=0$ as a function of n ?
- *Finite temperature* phase transition at T_c ? What is T_c ?



- *How large is the staggered potential*? Is it the same for all thicknesses?
- *At which thickness* do we *recover* the behavior of *graphite*?

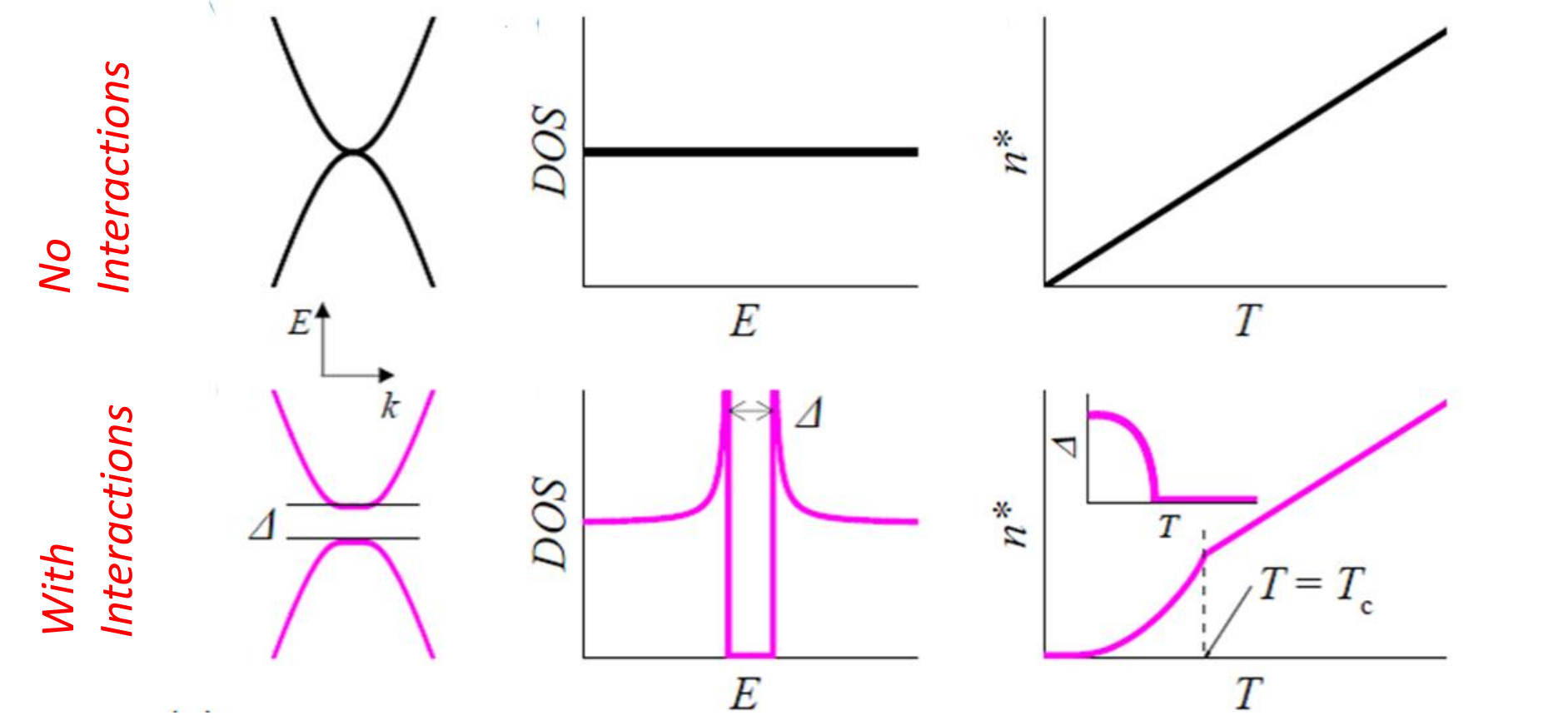
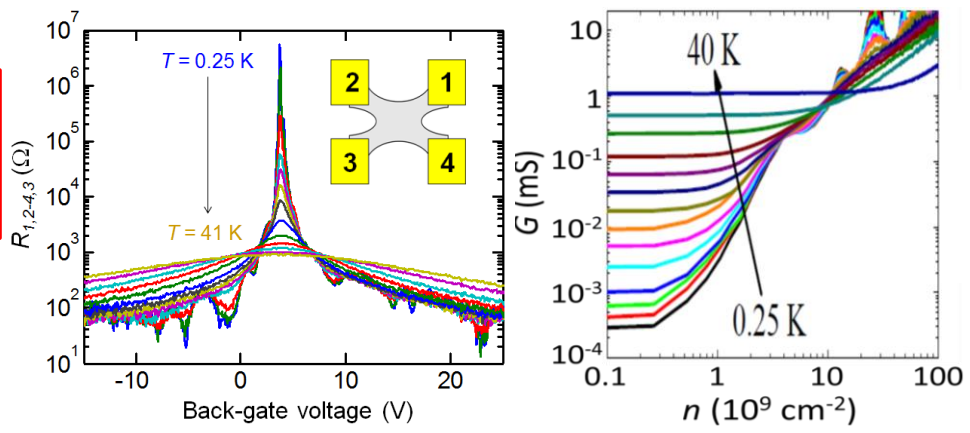
Probing the density of states from the width of the Dirac peak

Beyond $R(T)$ measurements

$$n^*(T) = \int_0^\infty g(E,T) \times f(E,T) dE$$

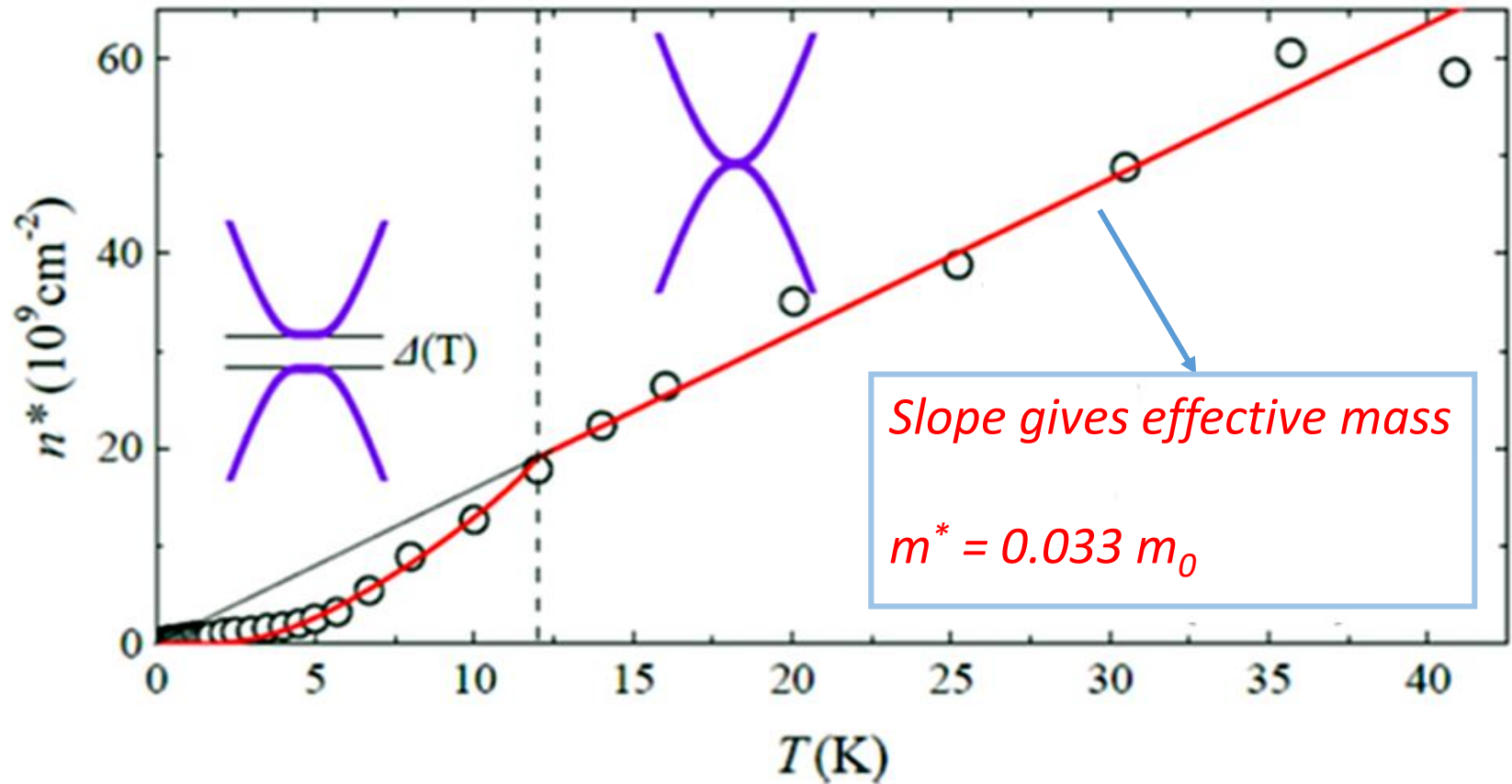
Density of states

Ex.: Bilayer Graphene



It works!

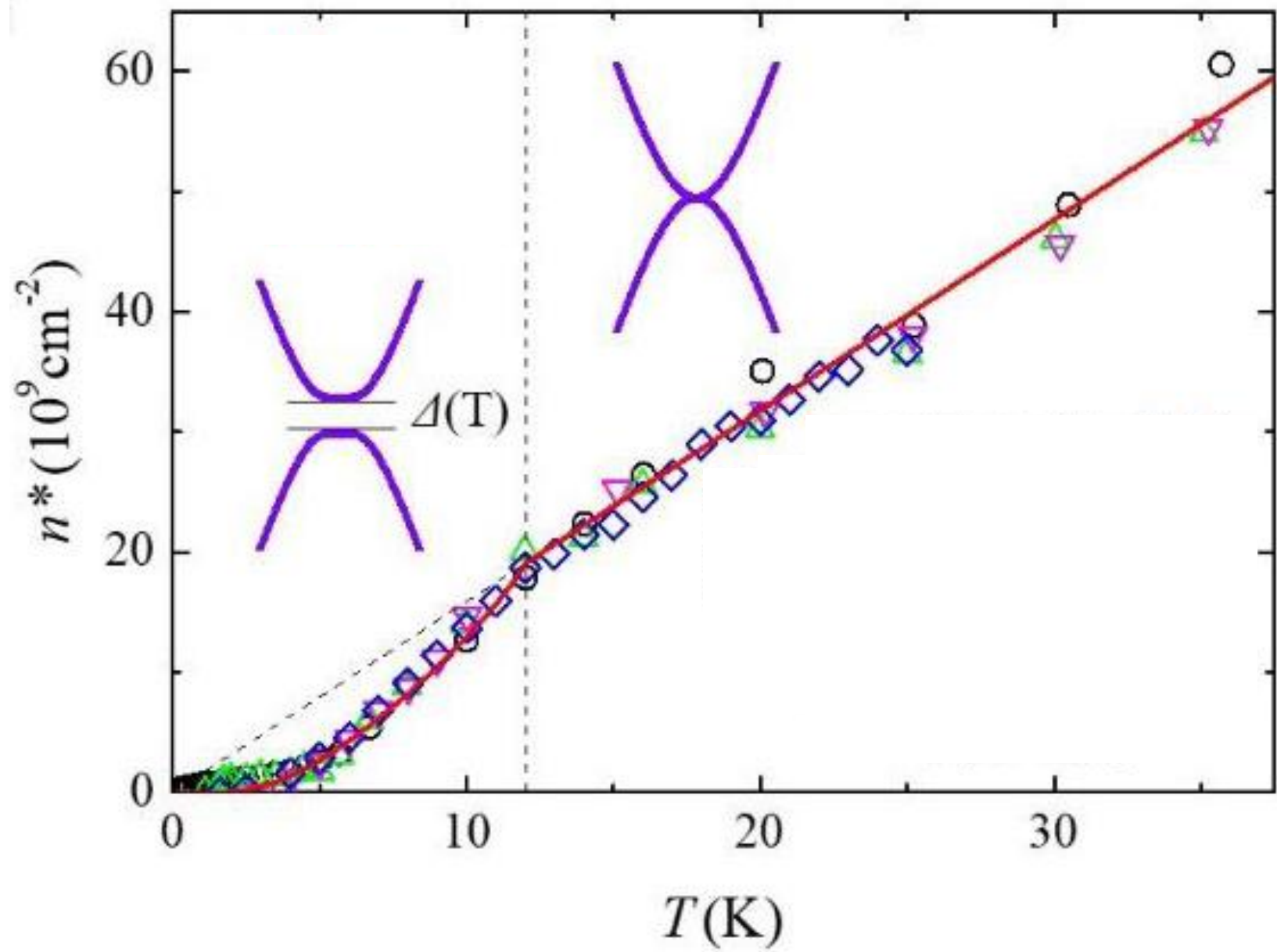
Second-order phase transition at finite T : Δ goes to zero at $T_c=12$ K



*Mean-field T -dependence
of the gap $\Delta(T)$*

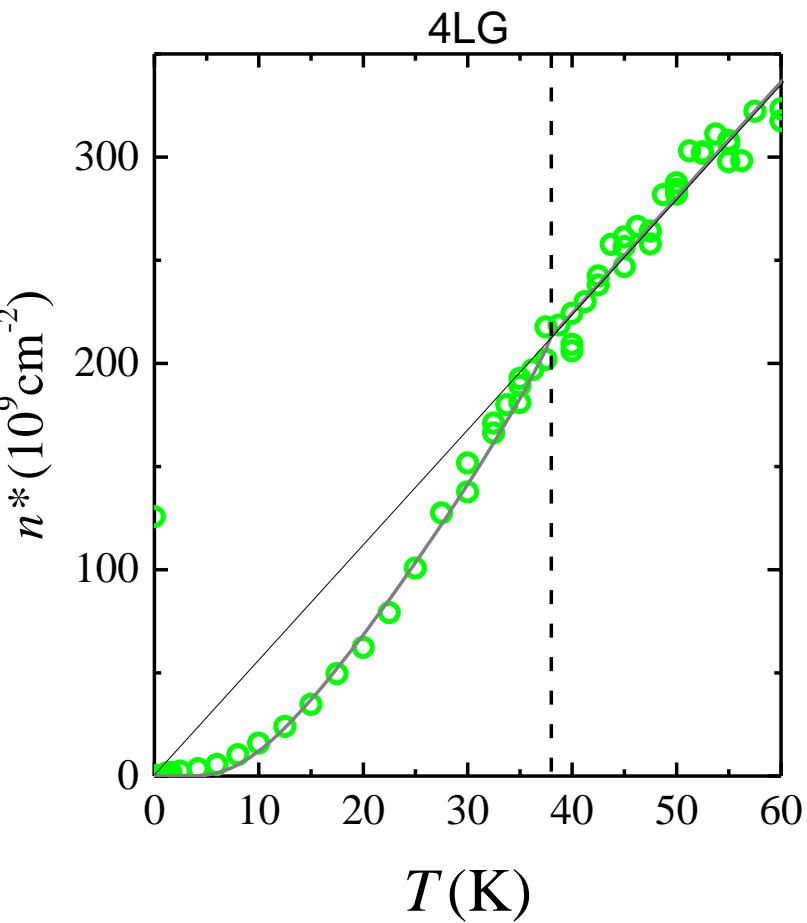
$$\Delta_{T < T_c} = \Delta_0 \tanh \left(1.74 \sqrt{\frac{T_c}{T} - 1} \right)$$
$$\Delta_{T > T_c} = 0$$

Very Reproducible

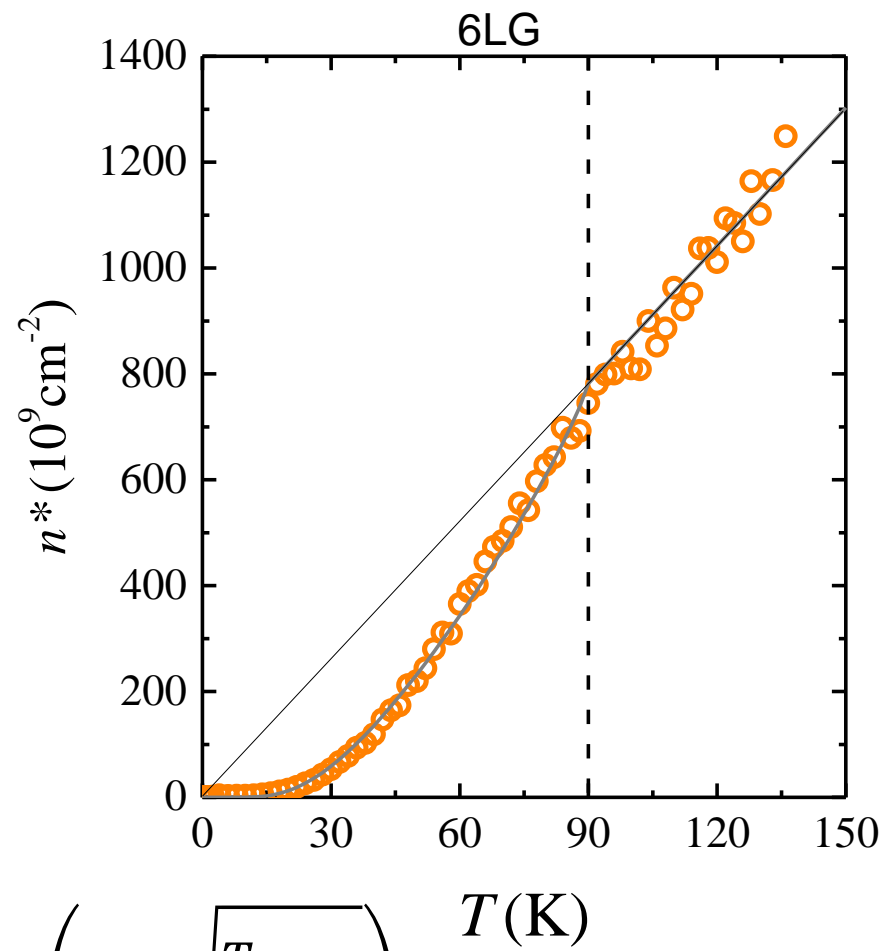


Not only for bilayer, also for 4LG, 6LG,...

And look at the values of T_c ...



4LG
 $T_c = 38 \text{ K}$
 $\Delta(0) = 60 \text{ K}$

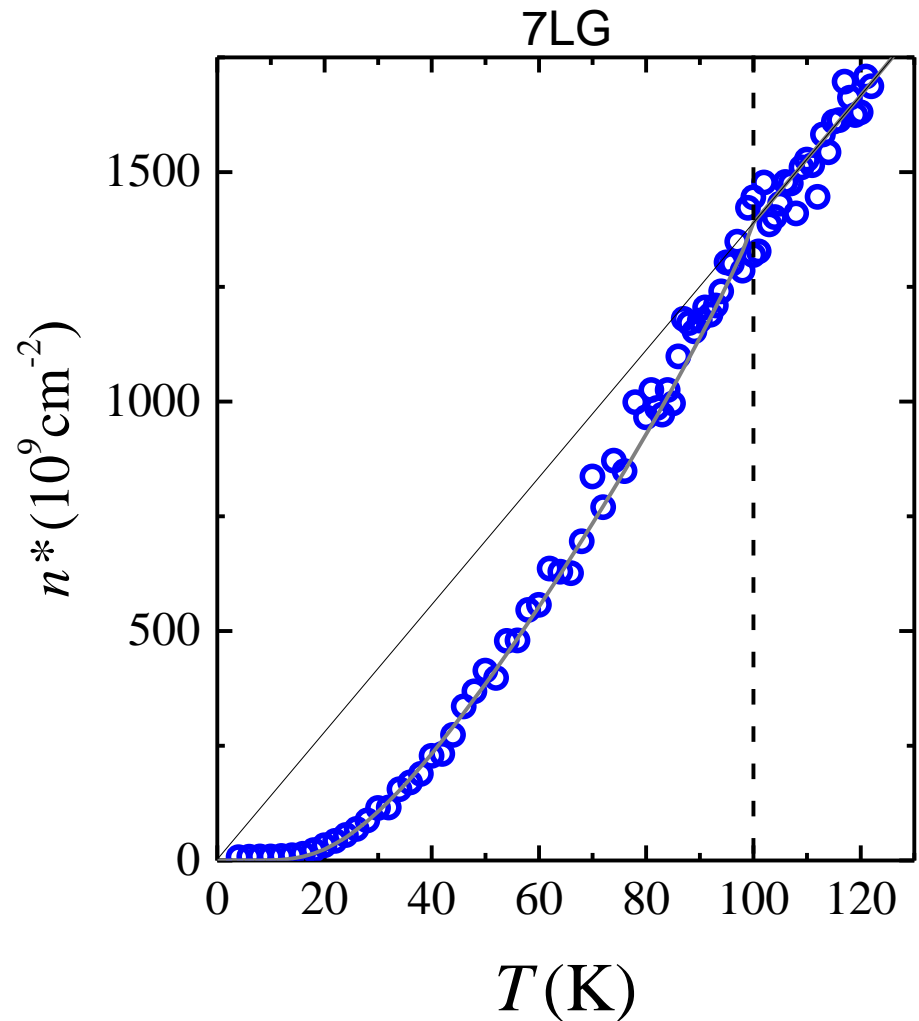
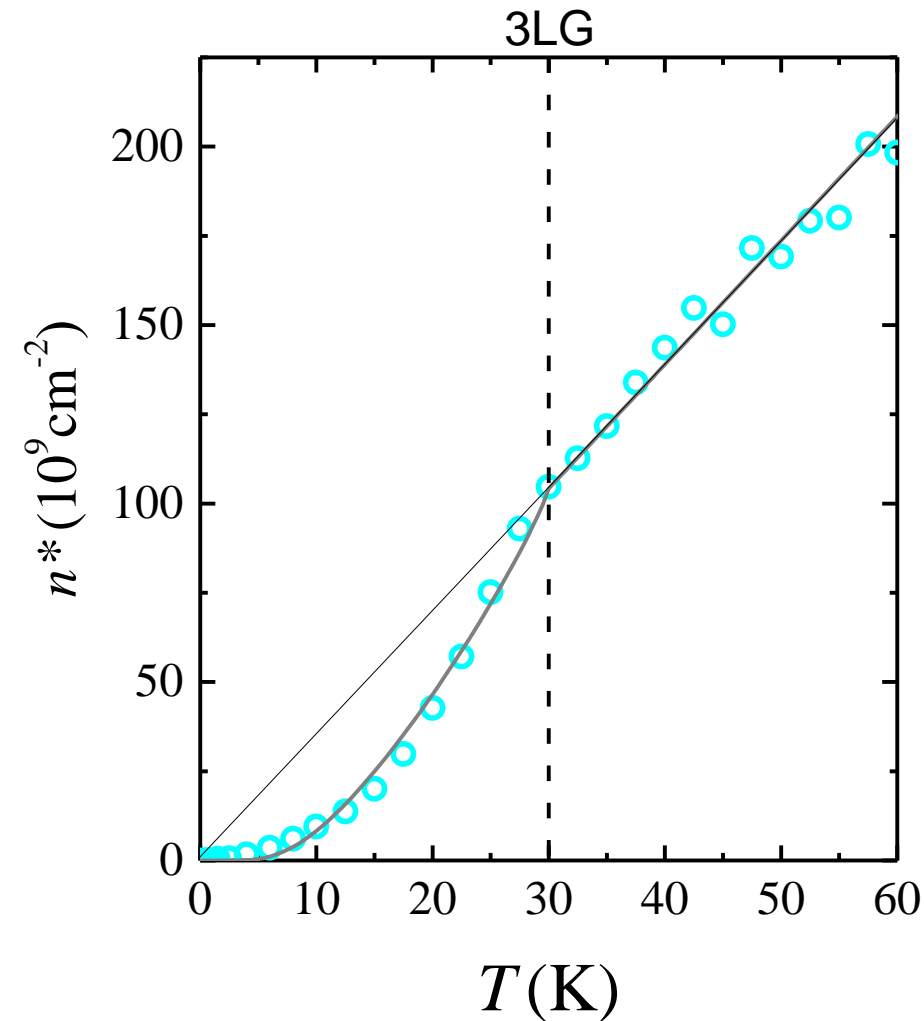


6LG
 $T_c = 90 \text{ K}$
 $\Delta(0) = 150 \text{ K}$

$$\Delta_{T < T_c} = \Delta_0 \tanh \left(1.74 \sqrt{\frac{T_c}{T} - 1} \right)$$
$$\Delta_{T > T_c} = 0$$

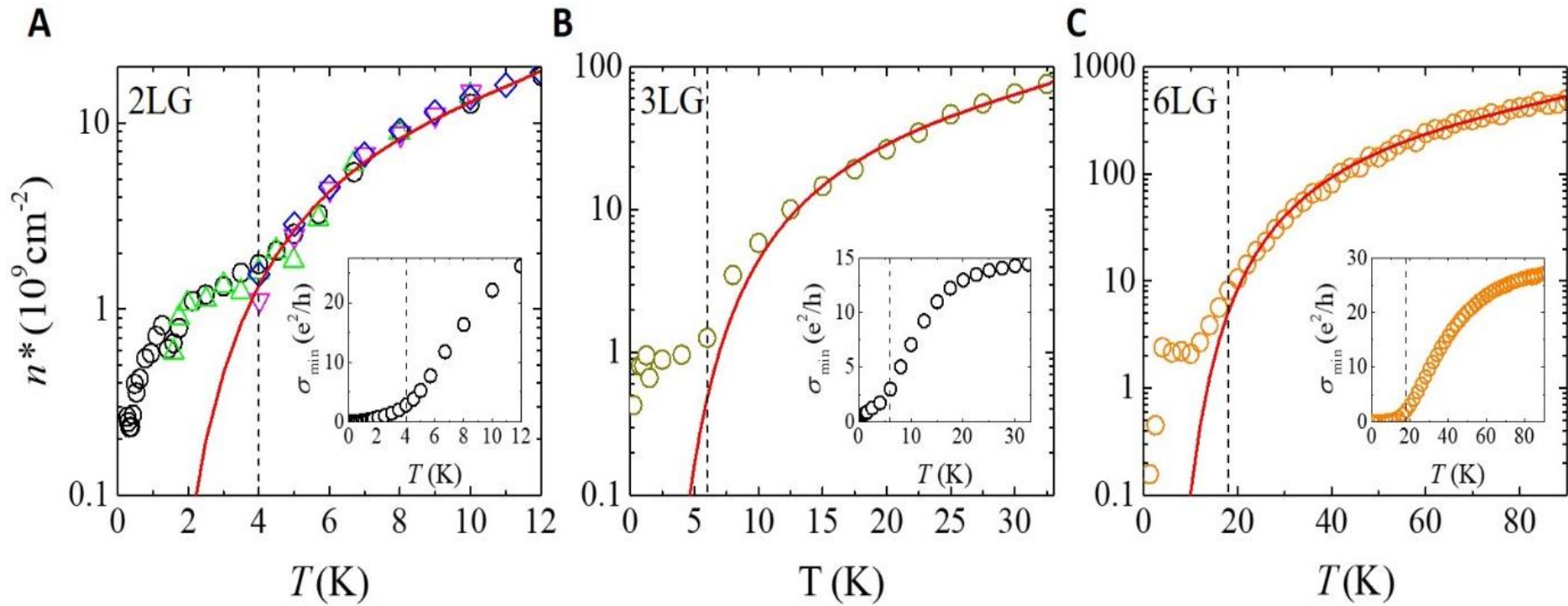
...and it works for odd layers as well: 3LG, 5LG, 7LG

*In this density range the DOS of the quadratic bands
is $\sim 100 \times$ the DOS of the Dirac band*



**Direct demonstration of e-e interaction
effects in odd Bernal-stacked multilayers**

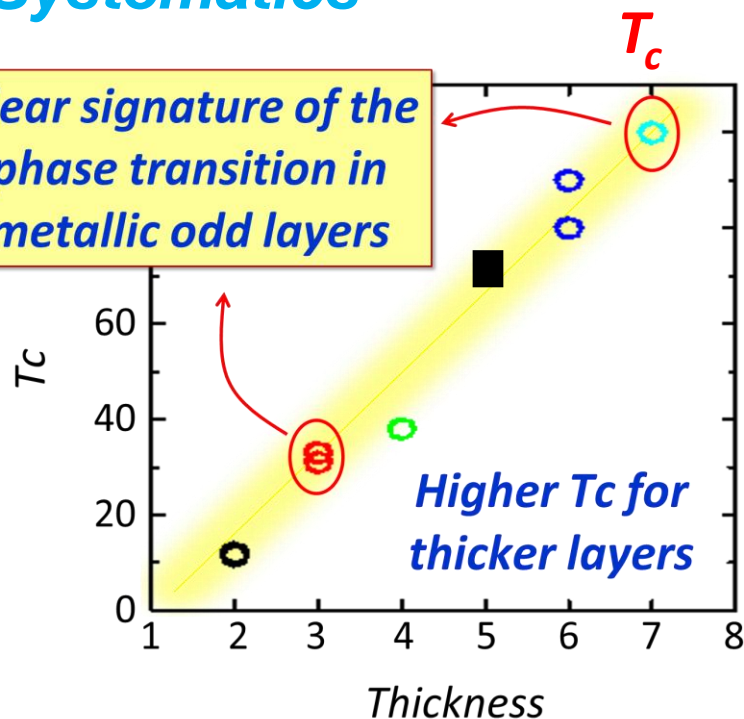
Looks good also on log scale



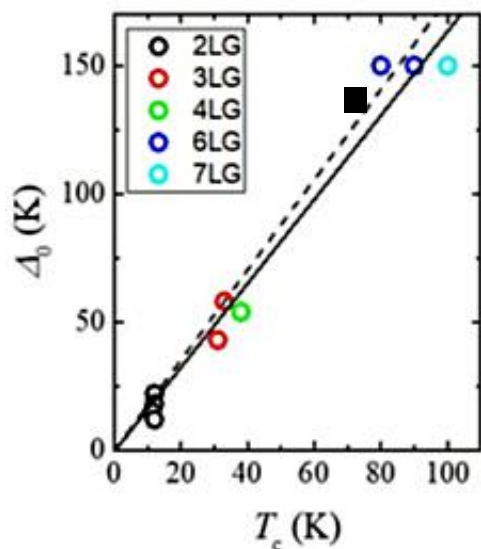
Methods break when $G < e^2/h$

Systematics

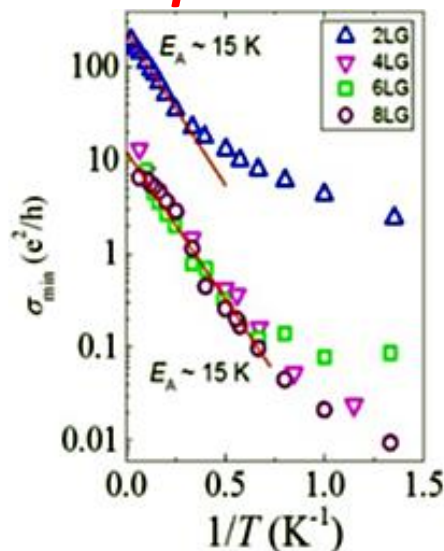
Clear signature of the phase transition in metallic odd layers



Gap

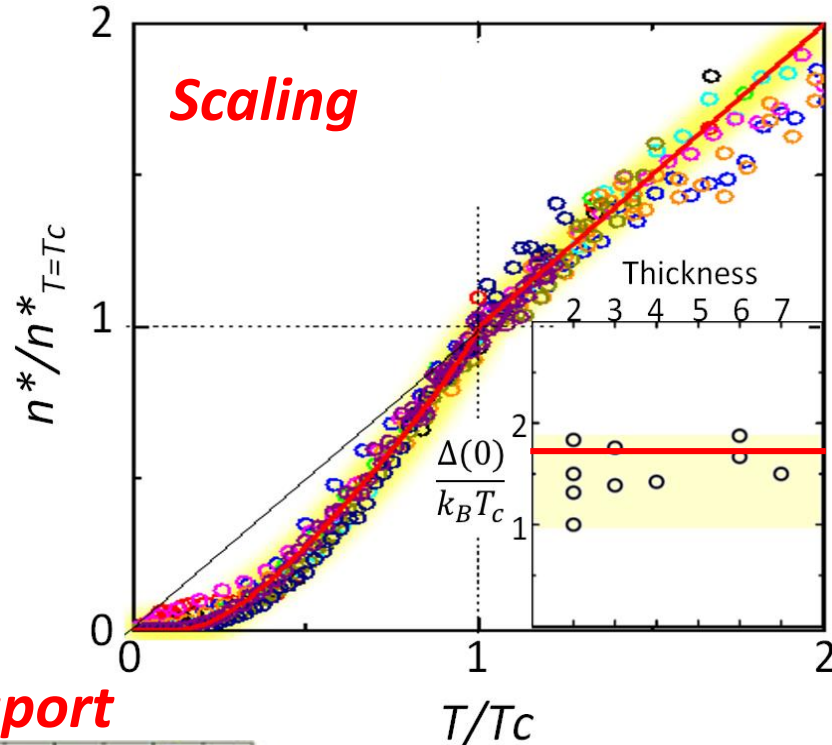


Transport

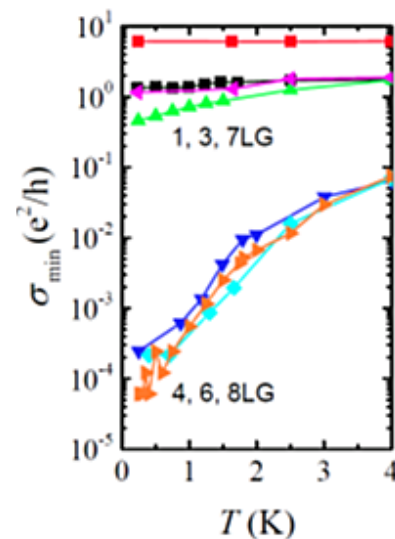


Activation energy is NOT the gap (edge transport?)

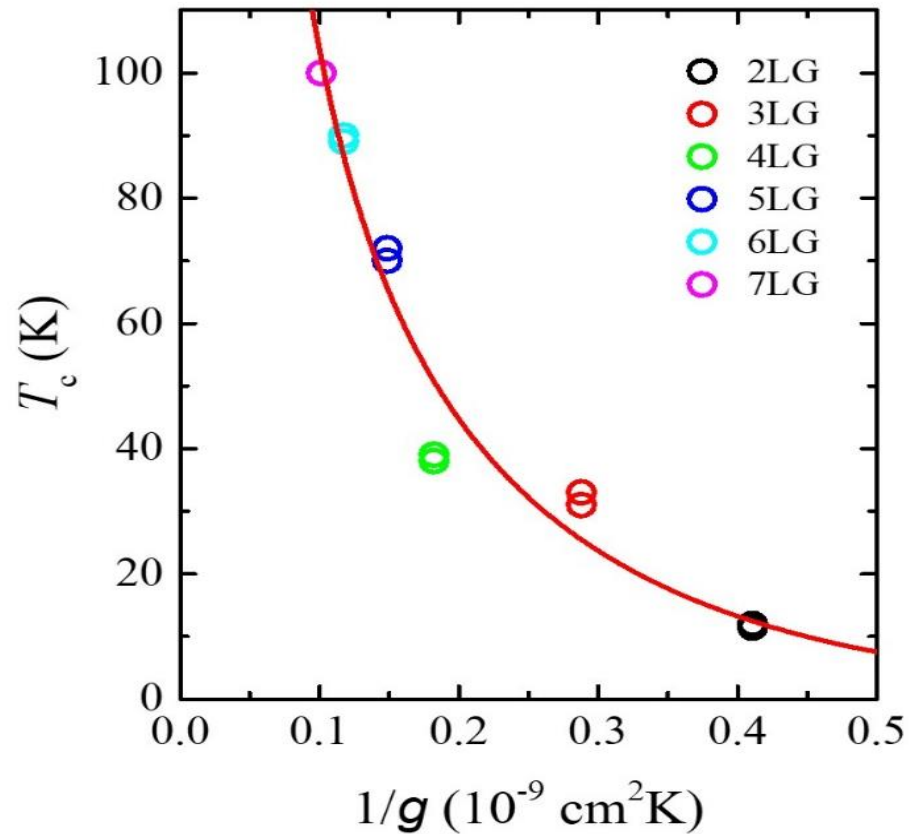
Scaling



Even-odd



*Can we understand why T_c increases
with increasing thickness?*

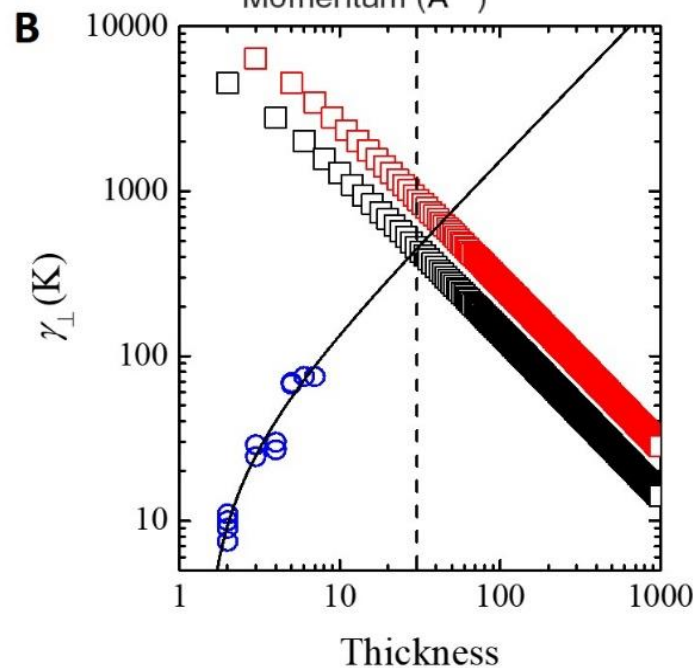
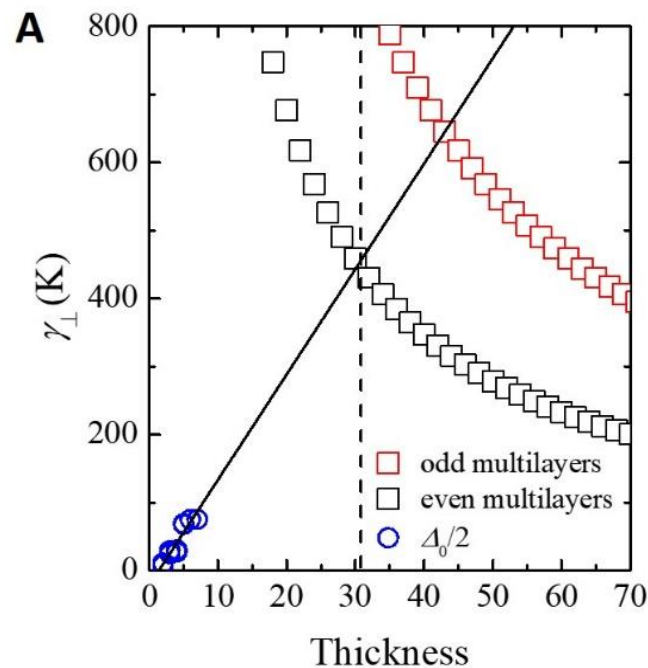
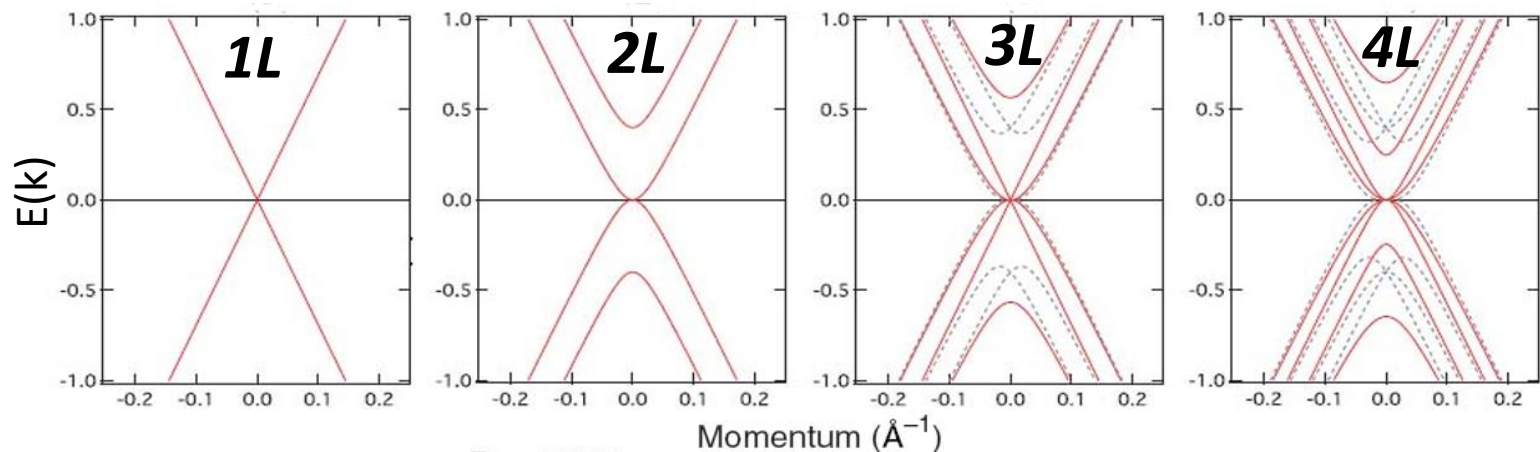


$$k_B T_c = \frac{E_{\text{cut-off}}}{2 \sinh[1/(gV)]}$$

Maybe, but can it be justified...?

When will it stop? Don't know, but...

If Δ gets too large, can't use low-energy Hamiltonian anymore



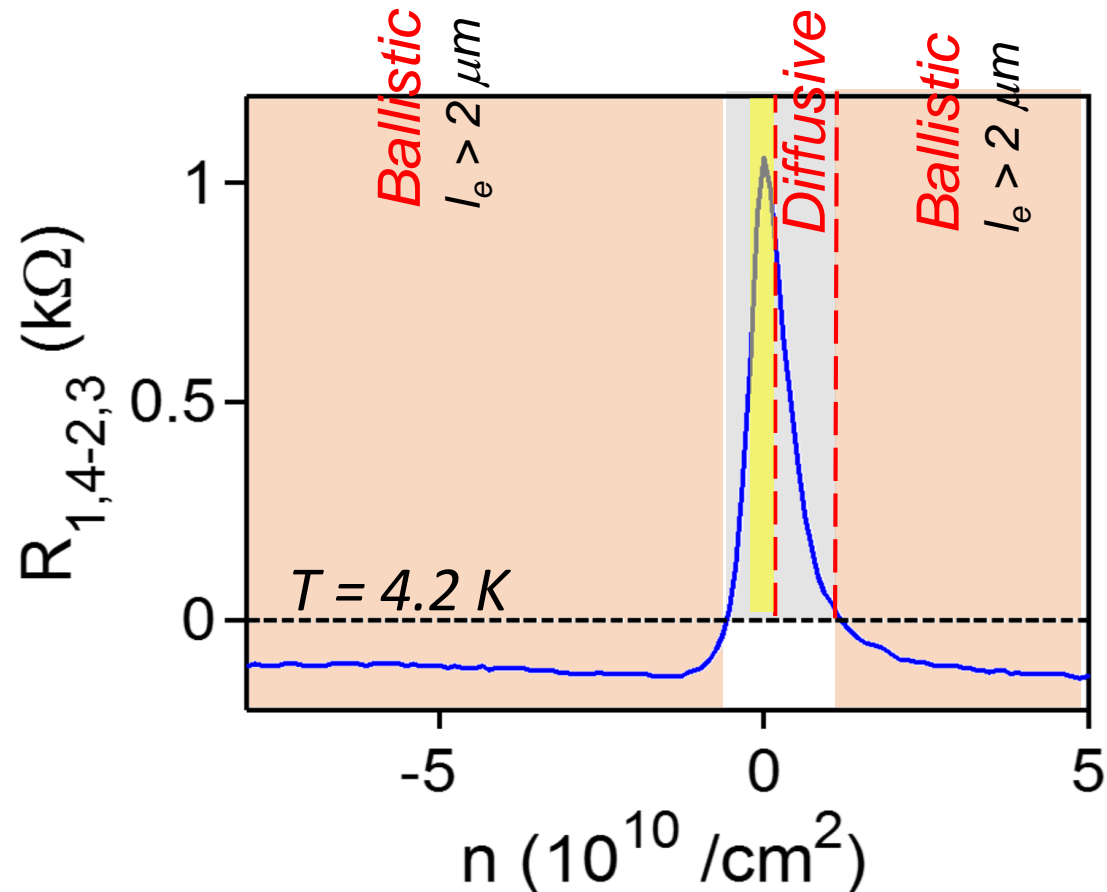
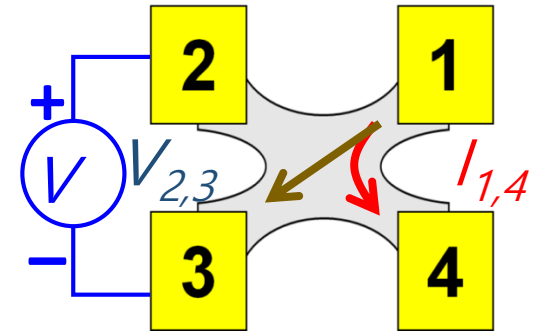
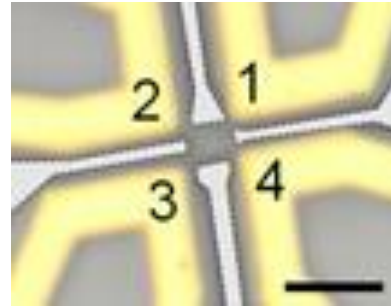
At around 20 LG Δ will hit the “high energy bands”
New scenario certainly needed past that thickness

***Ballistic transport limited by e-h collisions
in charge neutral suspended bilayer graphene***

What limits ballistic transport in suspended bilayers?

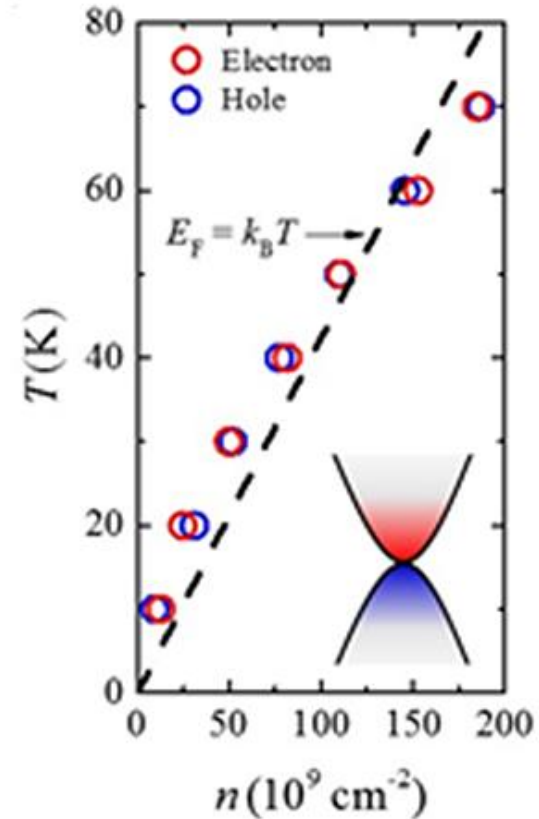
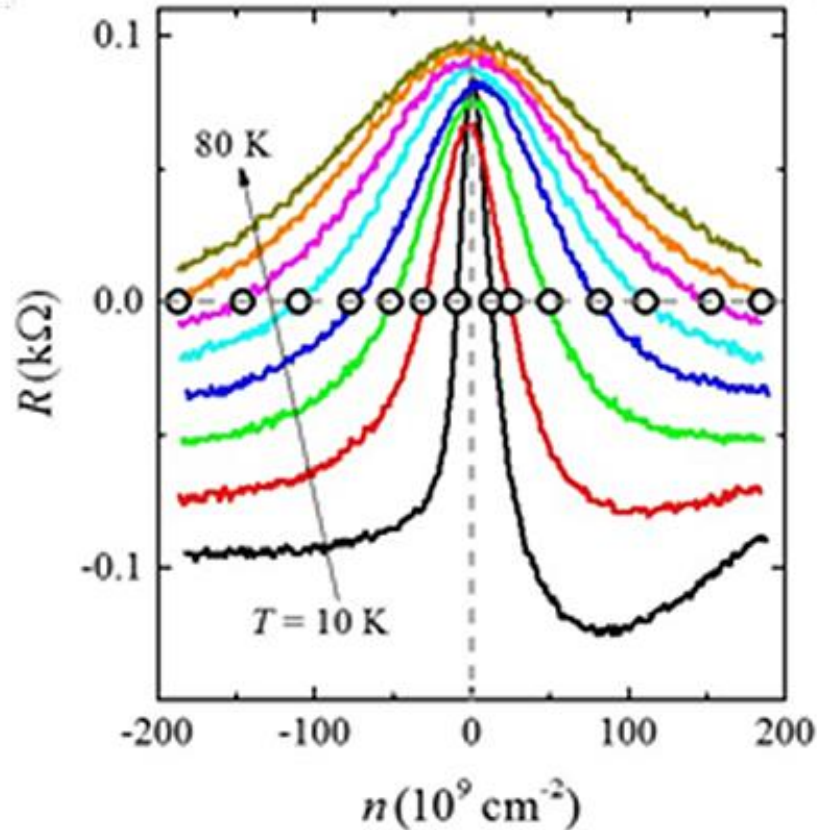
Suspended bilayer devices of very high quality

- Charge fluctuations $< 10^9 \text{ cm}^{-2}$
- Quantum Hall plateaus starting from 300 Gauss
- Observed even-denominator FQHE



Strongly T & n dependent scattering mechanism

*Onset of negative resistance (=ballistic transport)
depends strongly on temperature and carrier density*

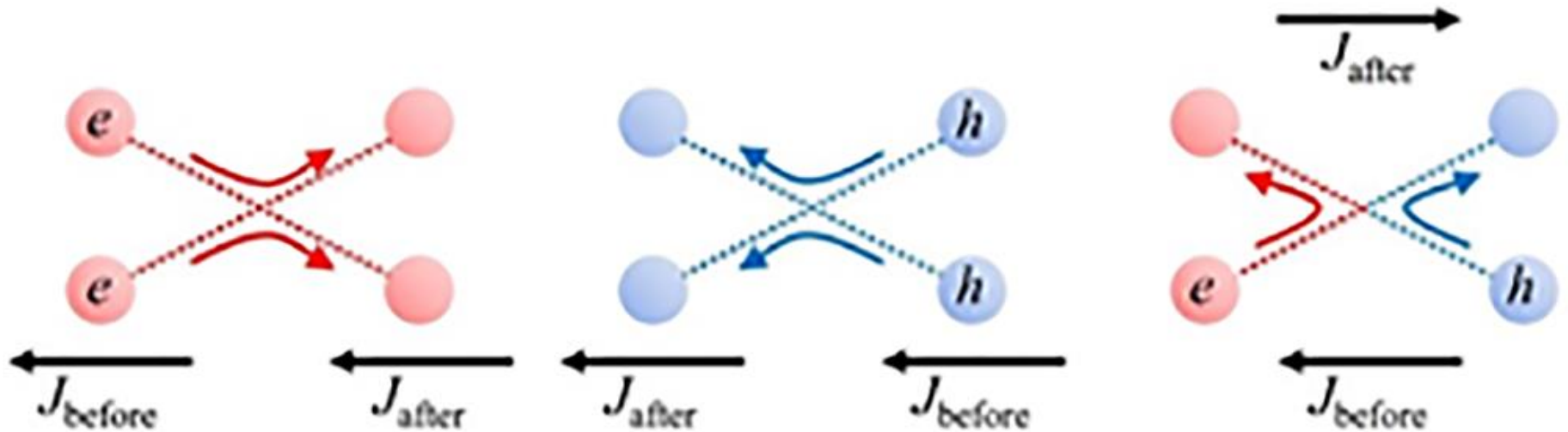


- Onset of negative resistance transport at $E_F \sim kT$
- Ballistic transport for $E_F > kT$

Hint for the role of e-h collisions

How does it work?

*Coulomb scattering always conserves momentum
(we do not consider umklapp processes)*



*But in the presence of multiple bands momentum conservation
does not imply velocity conservation*

Electron-hole scattering “dissipates” current

Can e-h collisions explain transport in the diffusive regime?

At charge neutrality ($= kT > E_F$) we have electron and holes

$$(1) \quad \sigma = n_e e \mu_e + n_h e \mu_h \quad \mu_{e/h} = \frac{e \tau_{e/h}}{m}$$

Assume: *e-h collision determine velocity relaxation*

$$(2) \quad \frac{1}{\tau_e} = \Gamma \frac{n_h}{n_e + n_h} \quad \frac{1}{\tau_h} = \Gamma \frac{n_e}{n_e + n_h} \quad \Gamma = C \frac{kT}{\hbar} \quad C \sim 1$$

We obtain:

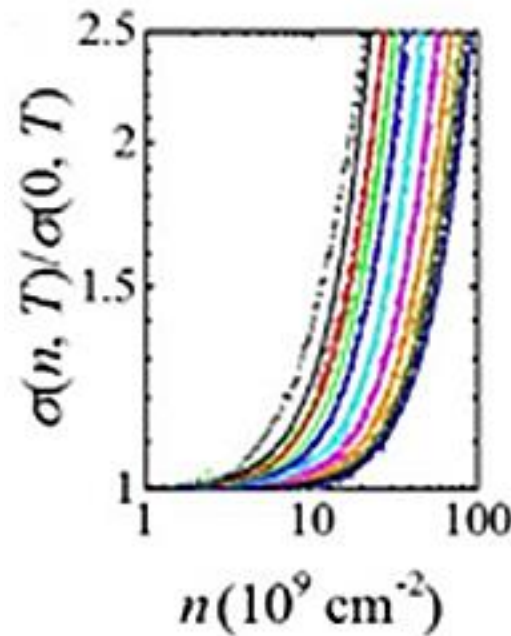
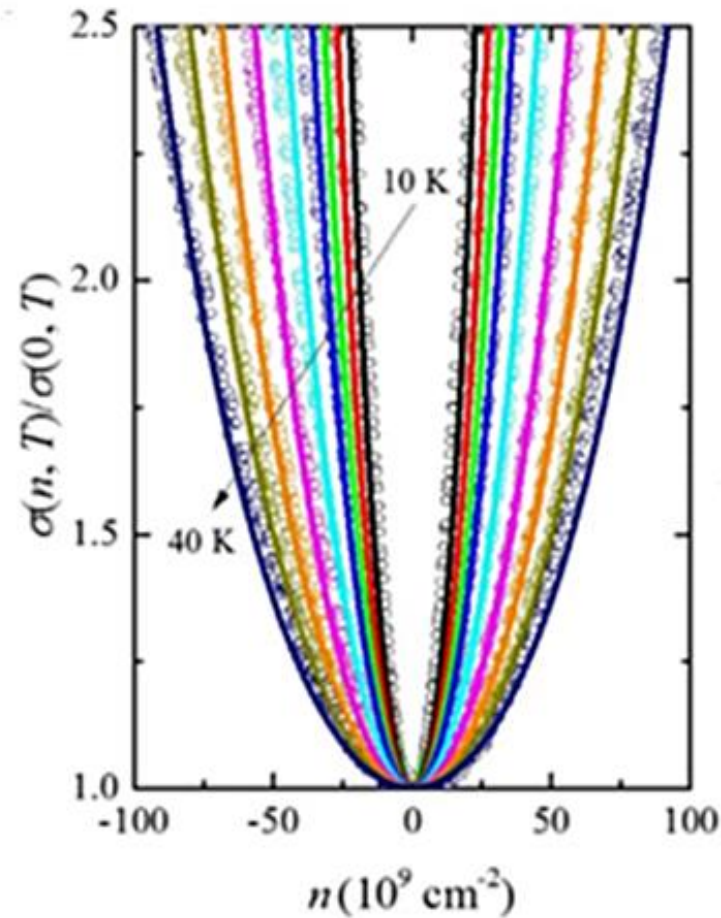
$$(1) + (2) \quad \sigma(n, T) = C^{-1} \frac{\hbar}{kT} \frac{e^2}{m^*} (n_e + n_h) \frac{n_e^2 + n_h^2}{n_e n_h}$$

$$\frac{\sigma(n, T)}{\sigma(0, T)} = \frac{\pi \hbar^2}{8kT m^* \ln(2)} \frac{(n_e + n_h)(n_e^2 + n_h^2)}{n_e n_h}$$

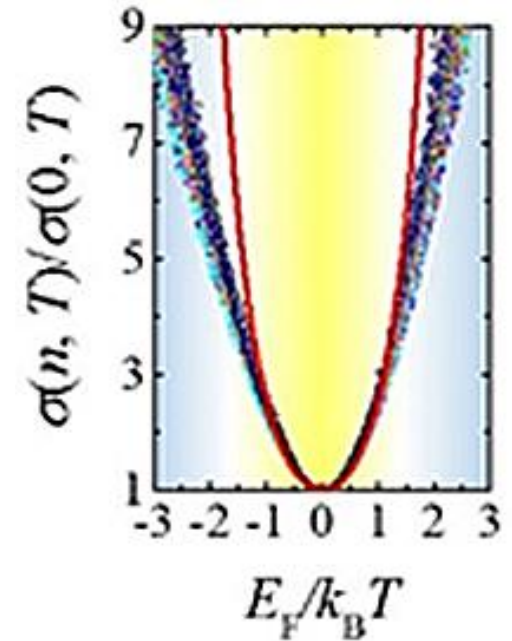
$$m^* = 0.033 m_o$$

No free parameters: either it reproduces the data or it does not

Perfect agreement with no free parameters **when $kT > E_F$**



Excellent agreement:
**over 2 orders of
magnitude in n**



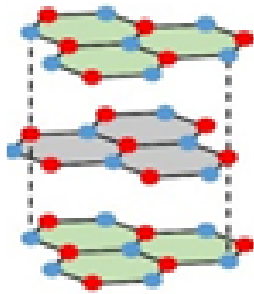
Normalized
conductivity:
**Near CNP it only
depends on E_F/kT**

Observed in 4 out of 4 samples investigated in detail

- **between 10 and 100 K,**
- **with $m^* = 0.031\text{-}0.034 m_0$**

It also works on Bernal-stacked trilayer graphene

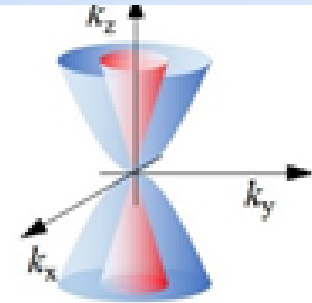
Bernal stacked
trilayer



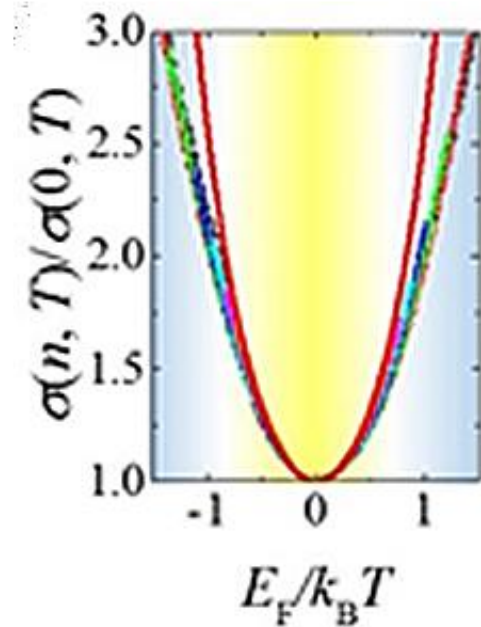
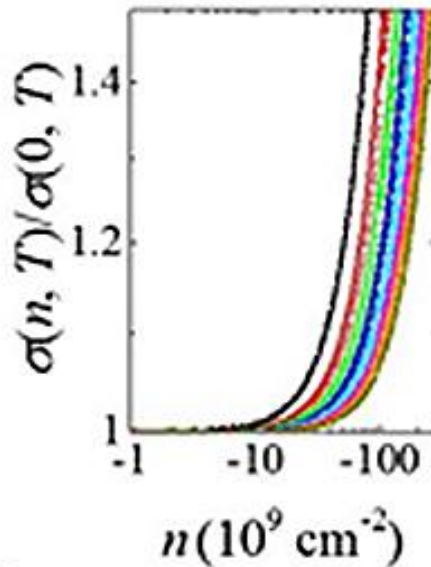
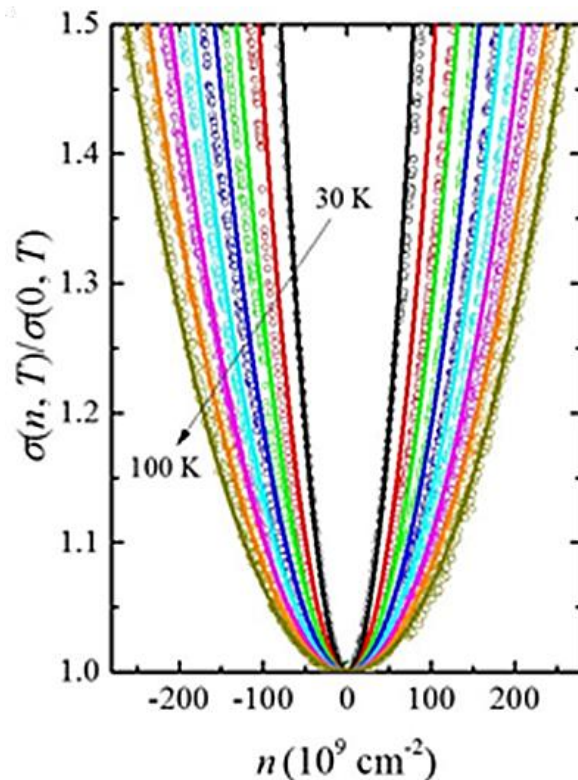
Bands:

1 x linear

1 x quadratic



- For $E \sim kT > E_F$ quadratic band DOS $> 100 \times$ linear band DOS
- Quadratic band dominates transport



Perfect agreement with $m^* = 0.06 m_0$
(expected $m^* = 0.05 m_0$)

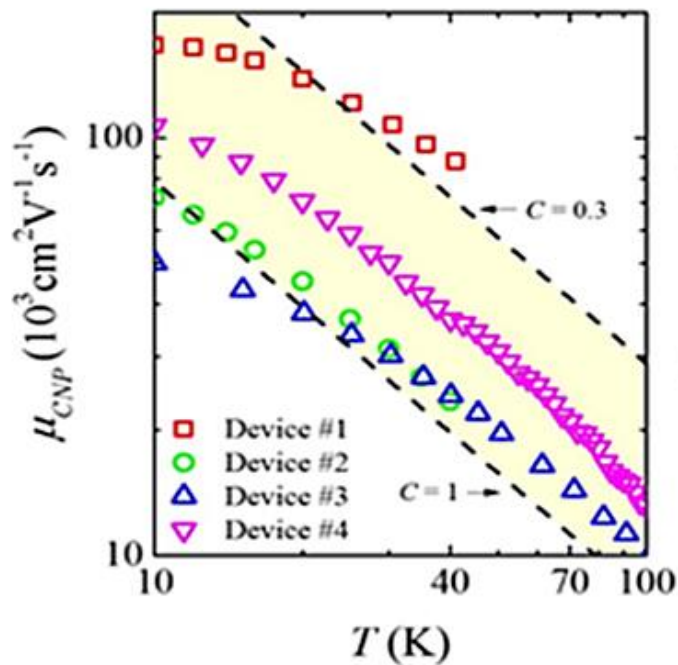
Also quantitative agreement with no free parameters

$$\sigma(n, T) = C^{-1} \frac{\hbar}{kT} \frac{e^2}{m^*} (n_e + n_h) \frac{n_e^2 + n_h^2}{n_e n_h} \quad C \sim 1$$

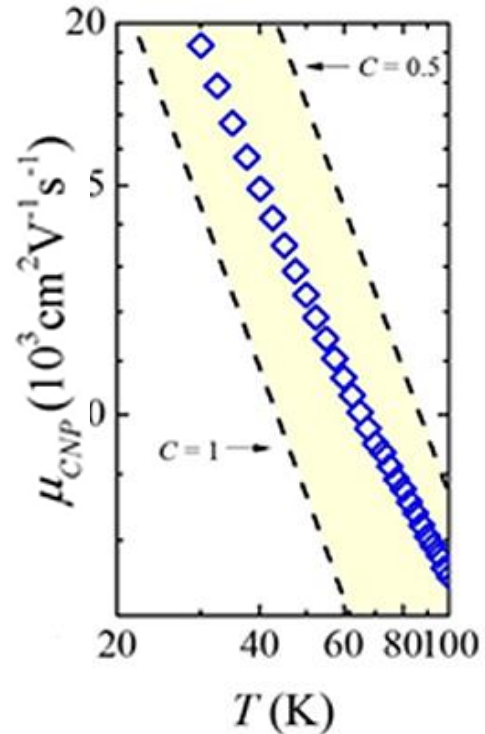
T-dependent mobility at CNP

$$\mu(T) = \frac{\sigma(n = 0, T)}{n_e(n = 0, T) + n_h(n = 0, T)}$$

Bilayer



Trilayer

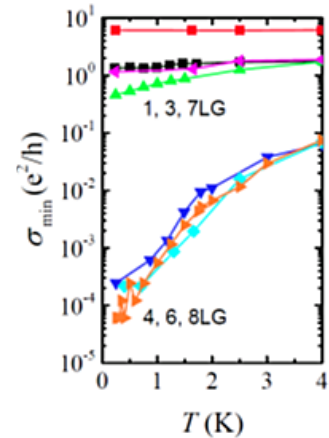


- *Quantitative agreement within a factor of 2-3 or better*
- *Expected due to indetermination on C and precise geometry*

Conclusions

Unexpected insulating state @ CNP in all even Bernal multilayers

Even-odd effect: at low T all odd multilayers remain conducting with conductivity of 1 Dirac band



*2nd order phase transition gapping quadratic bands
In all layers, with T_c increasing with thickness*

e-h scattering limits ballistic motion at CNP and cause diffusive transport

Inelastic scattering rate @ CNP is $\Gamma = C \frac{kT}{\hbar}$

