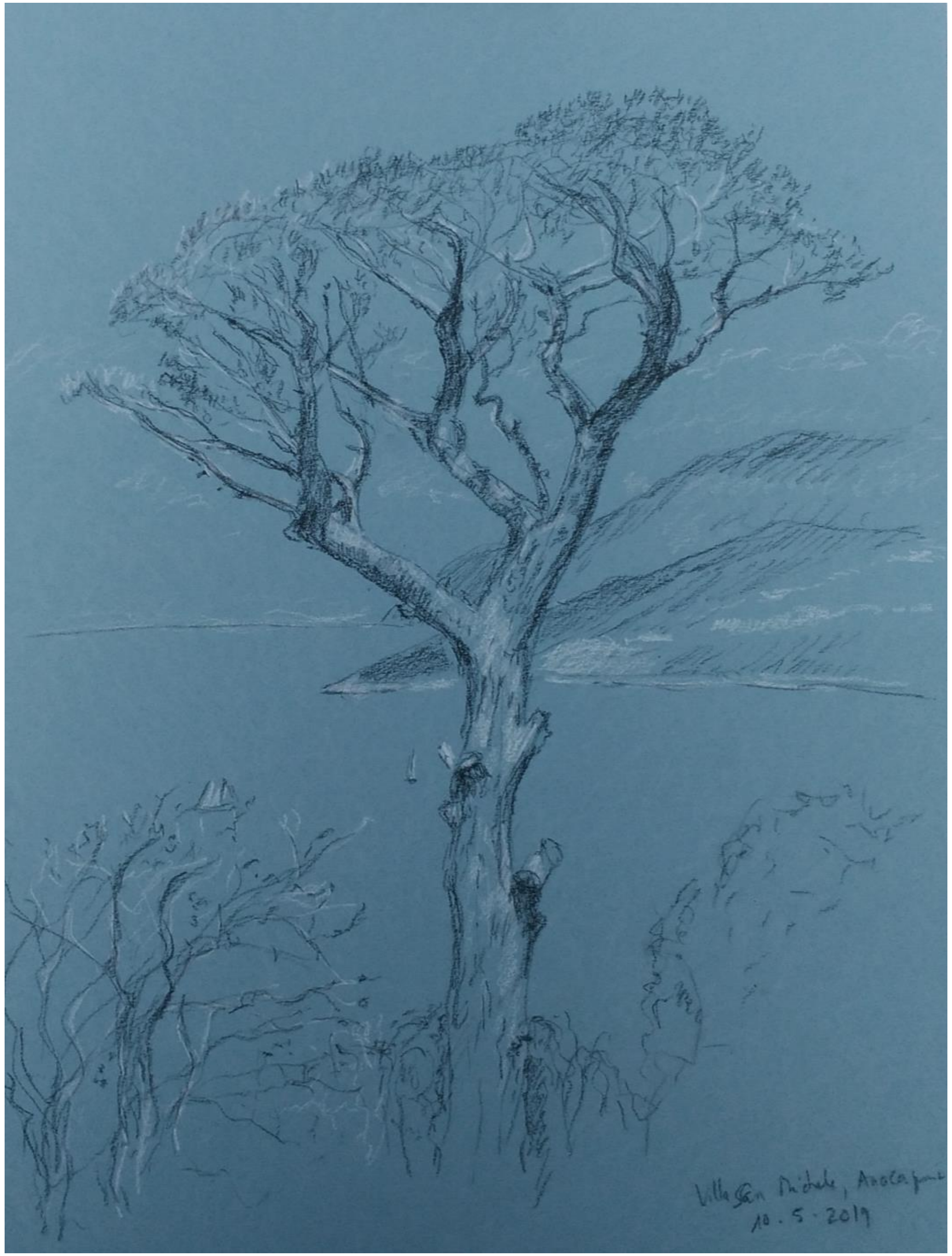
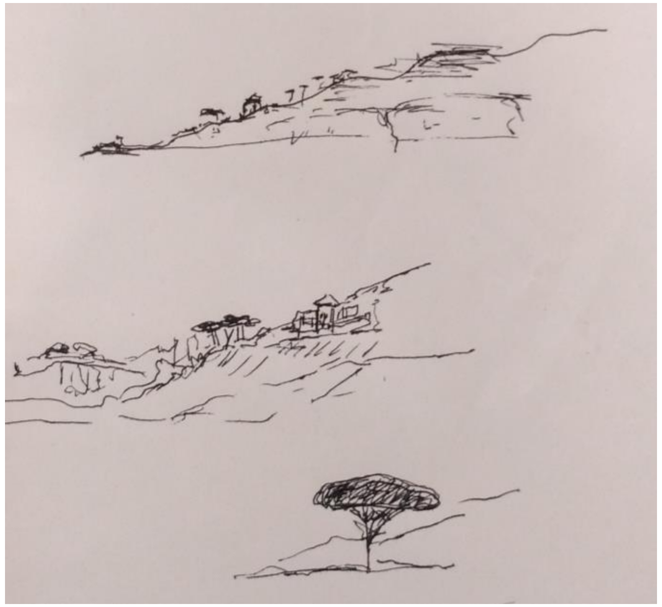
Sophie Guéron

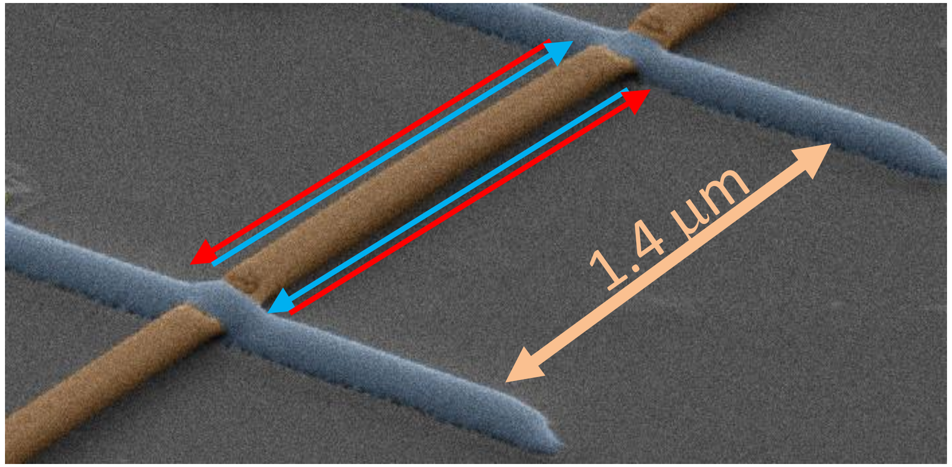
Capri Spring School on Transport in Nanostructures 2019

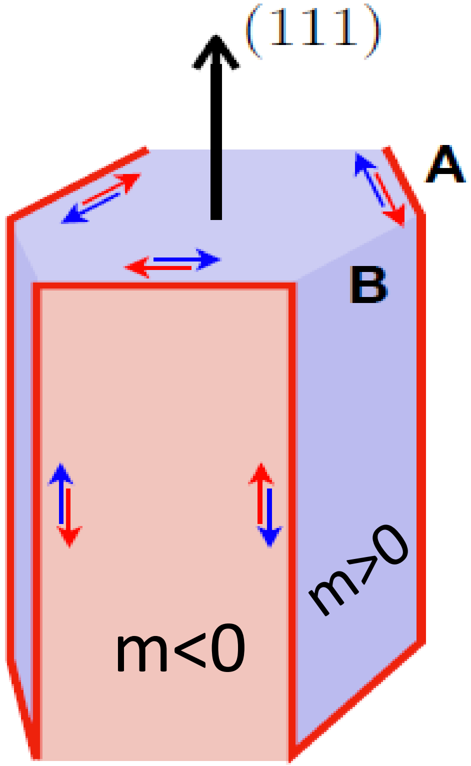




Bismuth, a "real life" Second Order Topological Insulator ?
Mesoscopic physics lends a helping hand to topological quantum chemistry

Anil Murani, B. Dassonneville, A. Kasumov, R. Delagrange, S. Sengupta, C. Li, F. Brisset, F. Fortuna,
A. Chepelianskii, R. Deblock, M. Ferrier, H. Bouchiat, S. Guéron (Orsay, France)
K. Napolskii, D. Koshkodaev, G. Tsirlina, Y. Kasumov, I. Khodos (Moscow and Chernogolovka)
F. Schindler, Z. Wang, M. Vergniory, A., B. A. Bernevig, T. Neupert (Zurich, San Sebastian, Princeton)





Murani et al, Phys. Rev. Lett (2019)
Schindler et al, Nature Physics (2018)
Murani et al, Nature Comm. (2017)
Murani et al, Phys.Rev. B **96**, 165415 (2017)

ERC advanced grant, Hélène Bouchiat,
“Revealing 1D ballistic charge and spin currents in second order topological insulators“,
April 2020-April 2025

Mesoscopic Physics group, Orsay

Sophie Guéron

Alik Kasumov

Meydi Ferrier

Sandrine Autier-Laurent

Richard Deblock

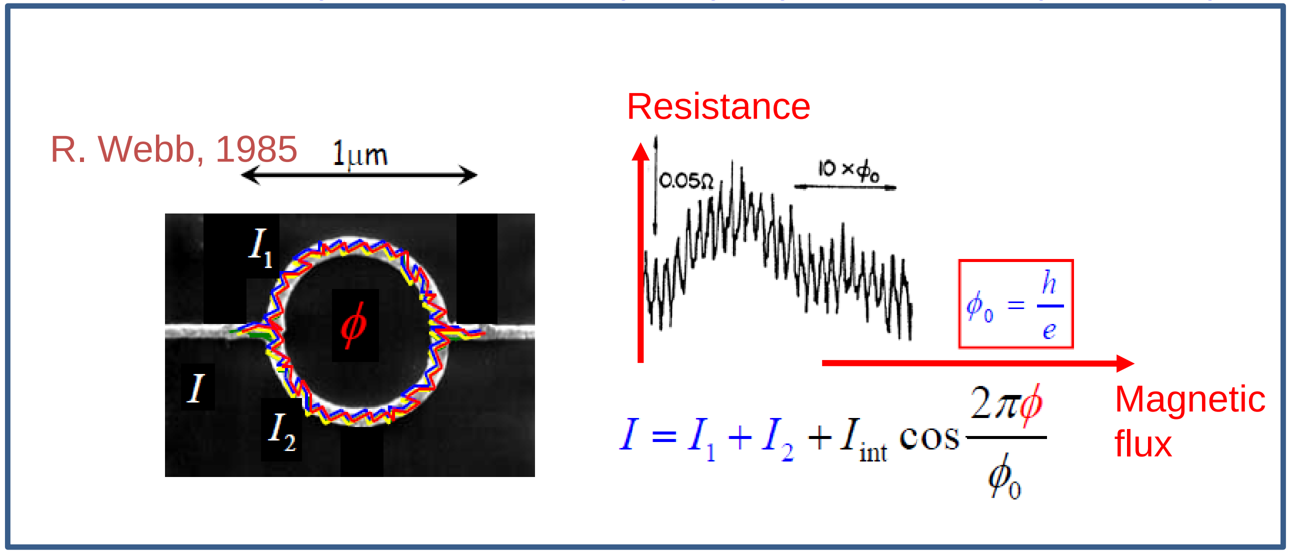
Hélène Bouchiat

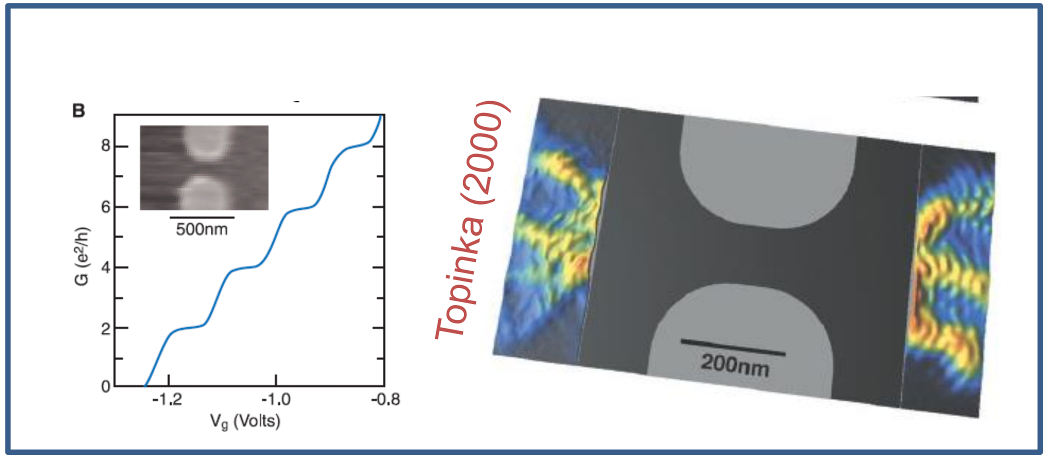
Alexei Chepelianskii

PhD students and Post-docs welcome!

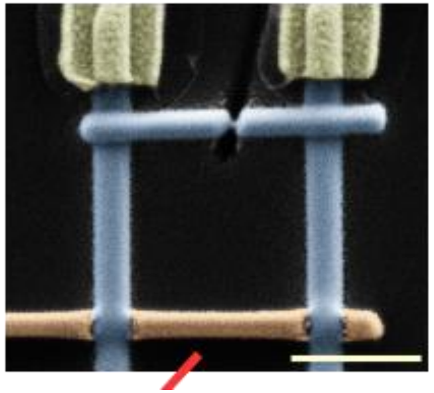
Motivation: Mesoscopic physics to explore topology

- Previously: Mesoscopic physics to explore quantum mechanics in condensed matter





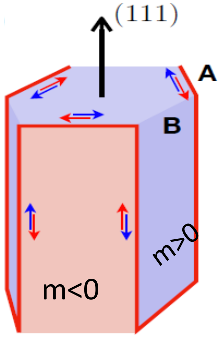
- More recently: Mesoscopic physics to explore different phases/order/coupling in condensed matter: pairing, spin order, spin-orbit…



- What characterizes mesosocopic physics?:
  - individual objects
  - small (micron): phase coherent!
  - contacts! Invasive/tunnel/no contacts, superconducting, magnetic

Bismuth, a "real life" Second Order Topological Insulator ?
Mesoscopic physics lends a helping hand to topological quantum chemistry

1- Bismuth, a Second Order Topological Insulator?

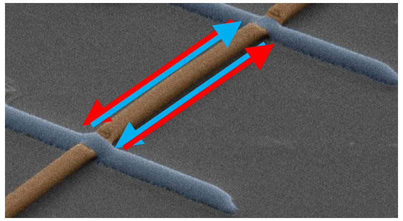


From bulk to surfaces to nanowires

(or why we chose to work with bismuth).

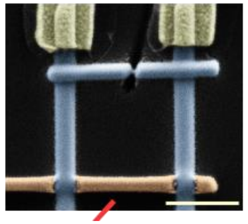
Different experimental techniques probe different parts/aspects of a crystal

2- Induced superconductivity through a Bi nanowire to probe possible SOTI character



a- What is the superconducting proximity effect

b- Critical current displays interference : edge states



c- (dc) Supercurrent versus Phase relation (CPR) to probe transport regime. Also detects effect of spin-orbit.

d- Dynamic response of the system: (ac) susceptibility $\chi=dI/d\varphi$: topologically protected !

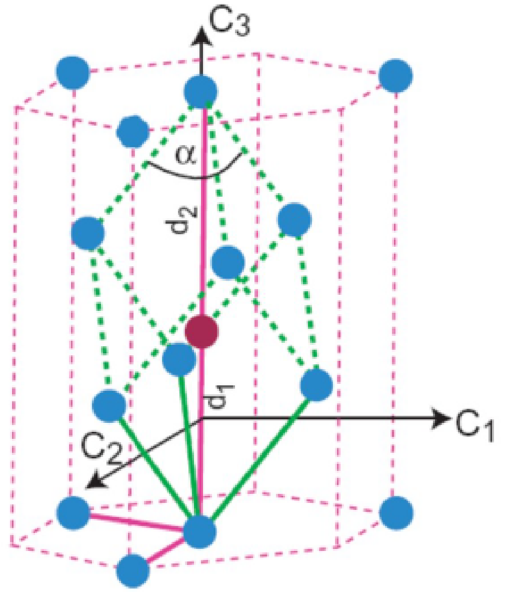
4- Summary of open questions. Other (mesoscopic) probes? Persistent charge/spin currents?

What is bismuth?

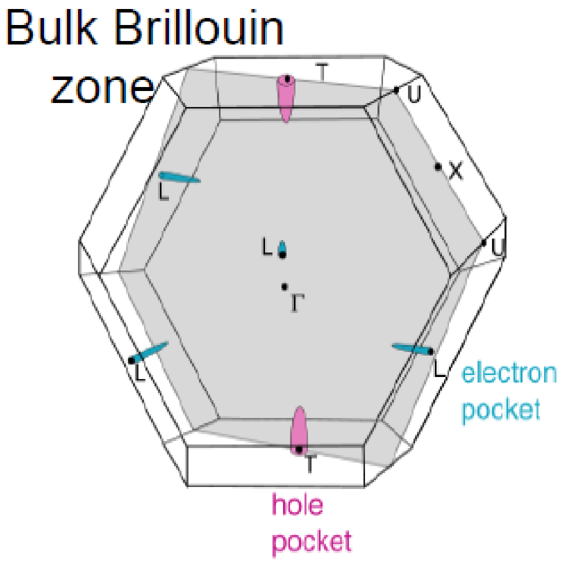
The heaviest non radioactive element

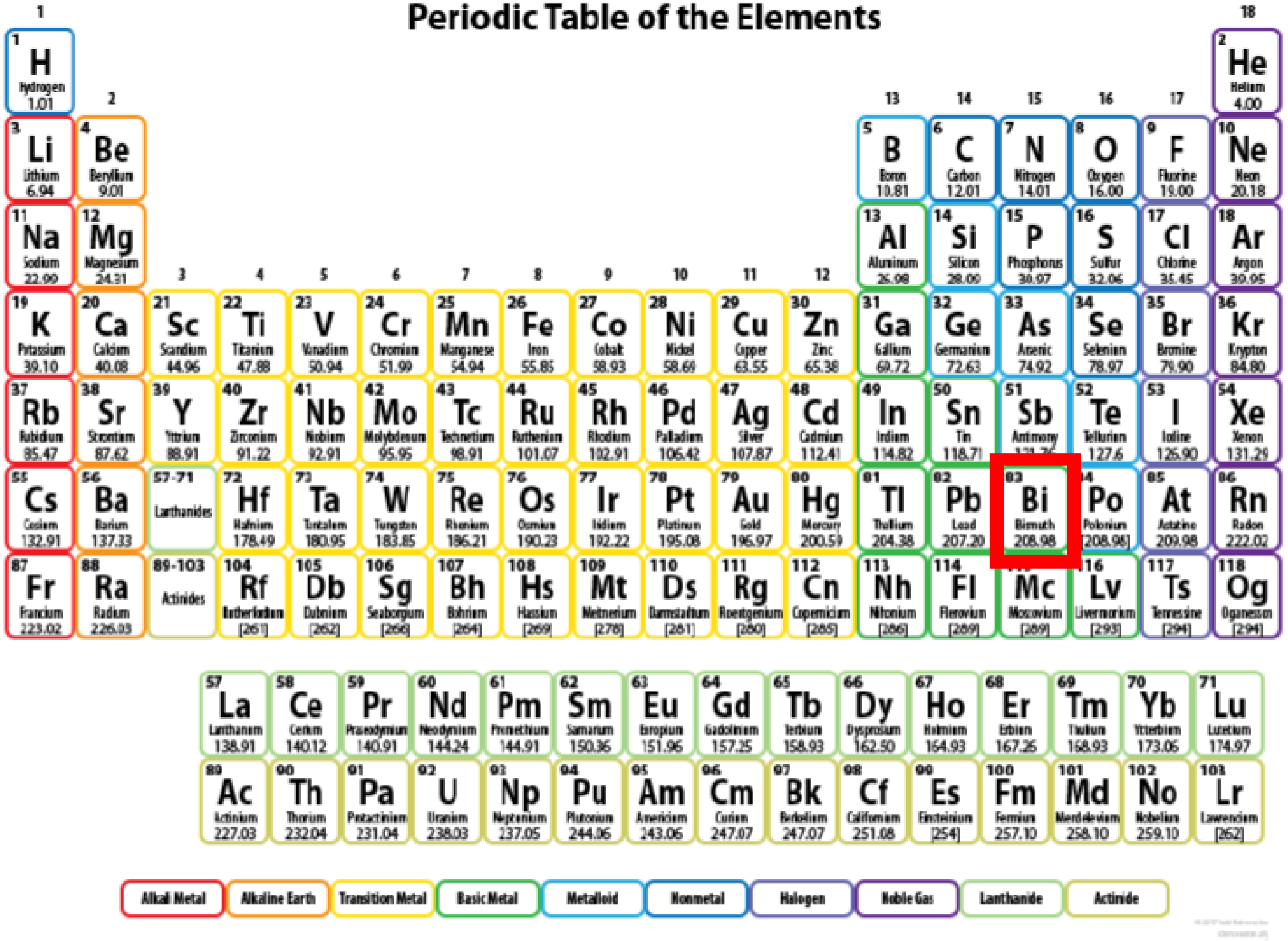
→ Strong spin-orbit interaction

<u>Lattice structure</u>



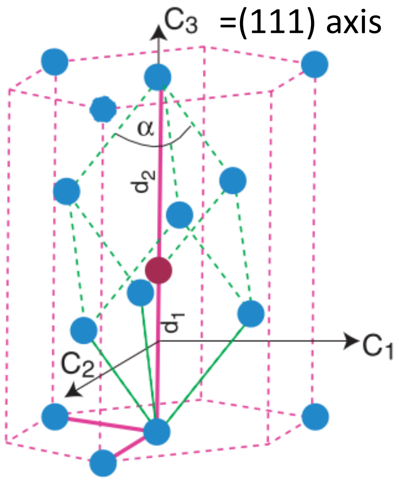
<u>Semimetal</u>





Ph. Hofmann, Prog. Sci. Surf. **81**, 191 (2006).

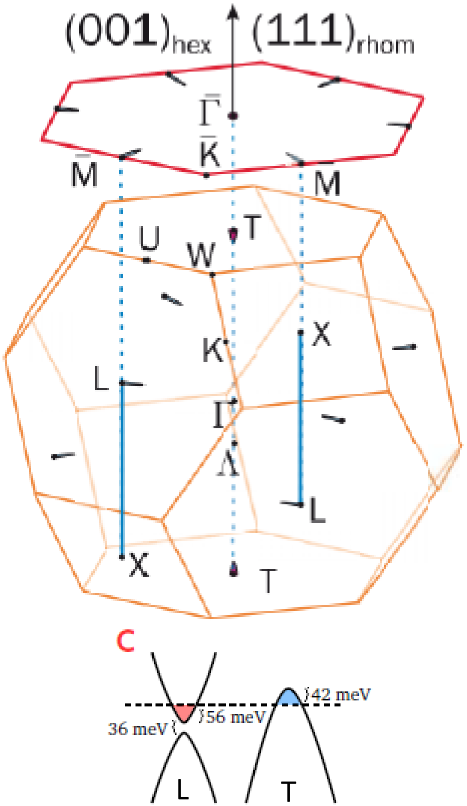
Bismuth as it was known one year ago: bulk and surfaces



Hofmann 2006 review

Z=83: huge spin-orbit

Bulk Brillouin zone

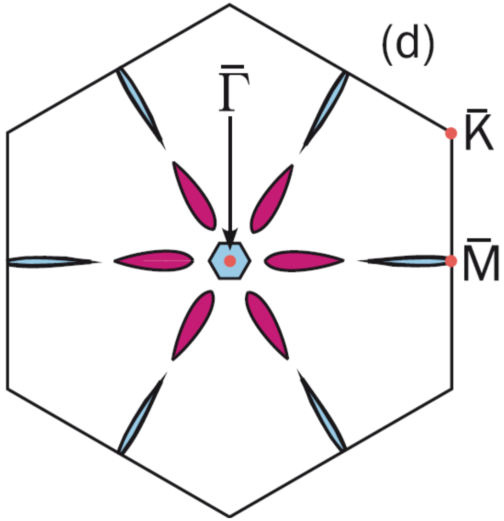


Bulk
semi-metal
$\lambda_F \approx 50$ nm

→No bulk states left in structures smaller than 50 nm

Light electrons, heavy holes
λ_F~50 nm, g ~ up to 1000

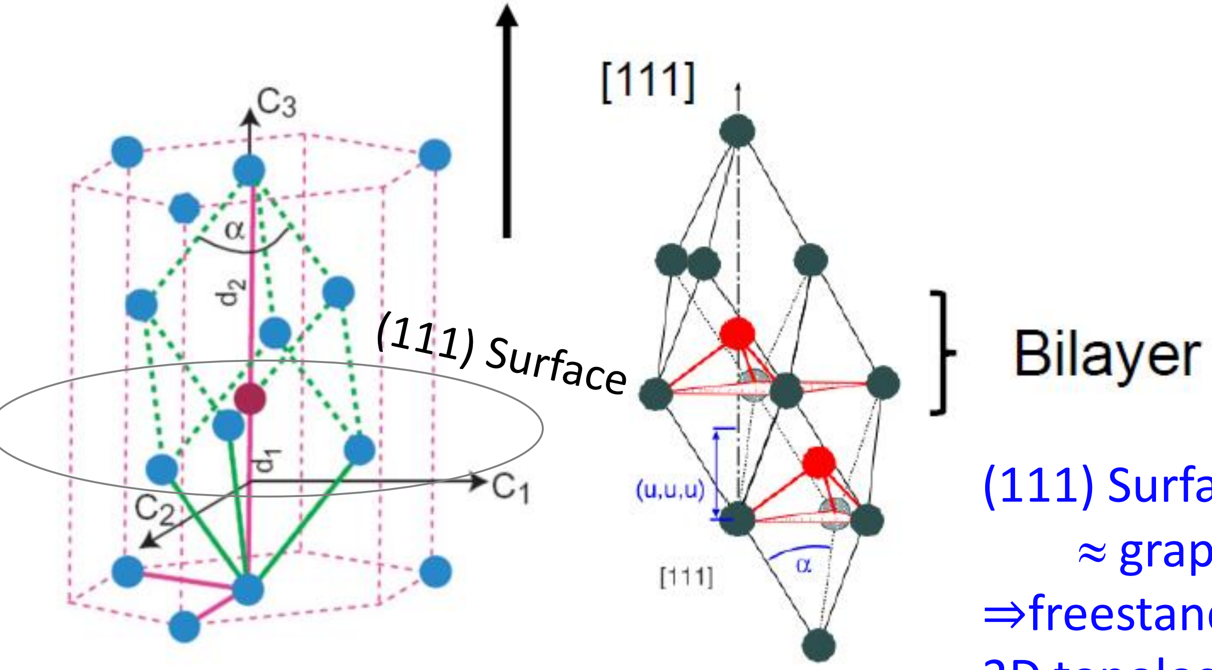
Surface Brillouin zone



Surfaces
$\lambda_F \approx 1$ nm,
spin-split,
E_{SO} ~ E_F ~ 100 meV, g_{eff}: 1~ 100

Topological properties?

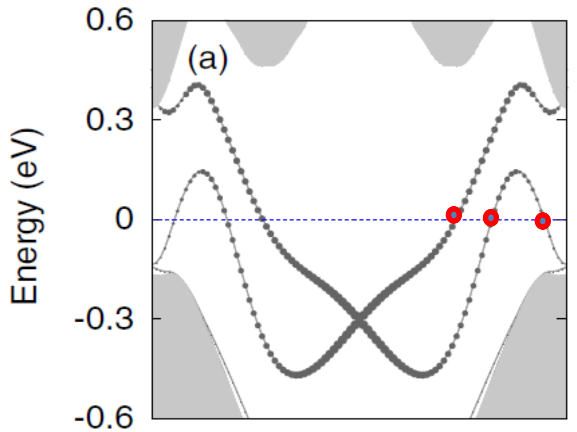
(111) Bi bilayers predicted to be 2D topological insulators in 2006



(111) Surface= buckled honeycomb
≈ graphene with huge spin-orbit!
⇒freestanding bilayer is predicted
2D topological insulator
With 3 Quantum Spin Hall (helical) edge states

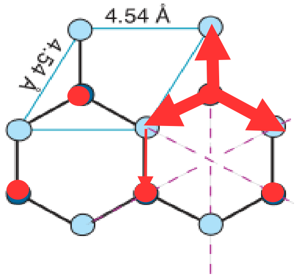
Murakami, 2006

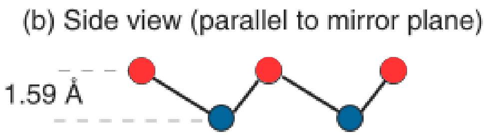
Liu & Allen, 1991



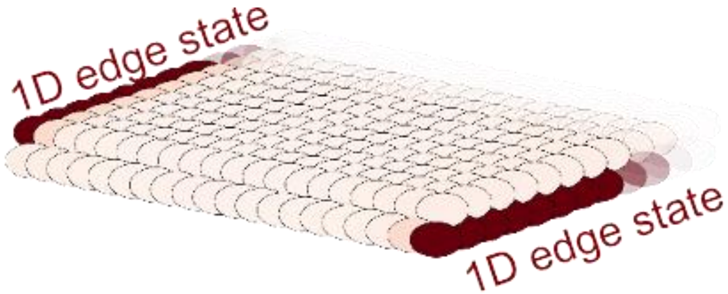
Yeom 2016

Free standing (111 bilayer)
Spin-polarized 1D edge states





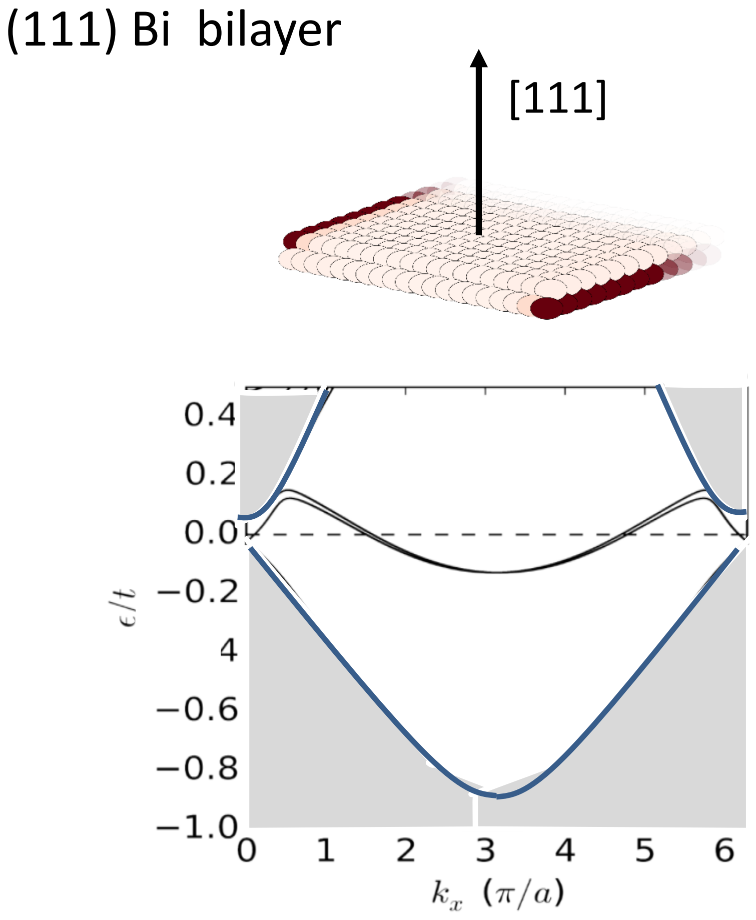
Hofmann 2006 review

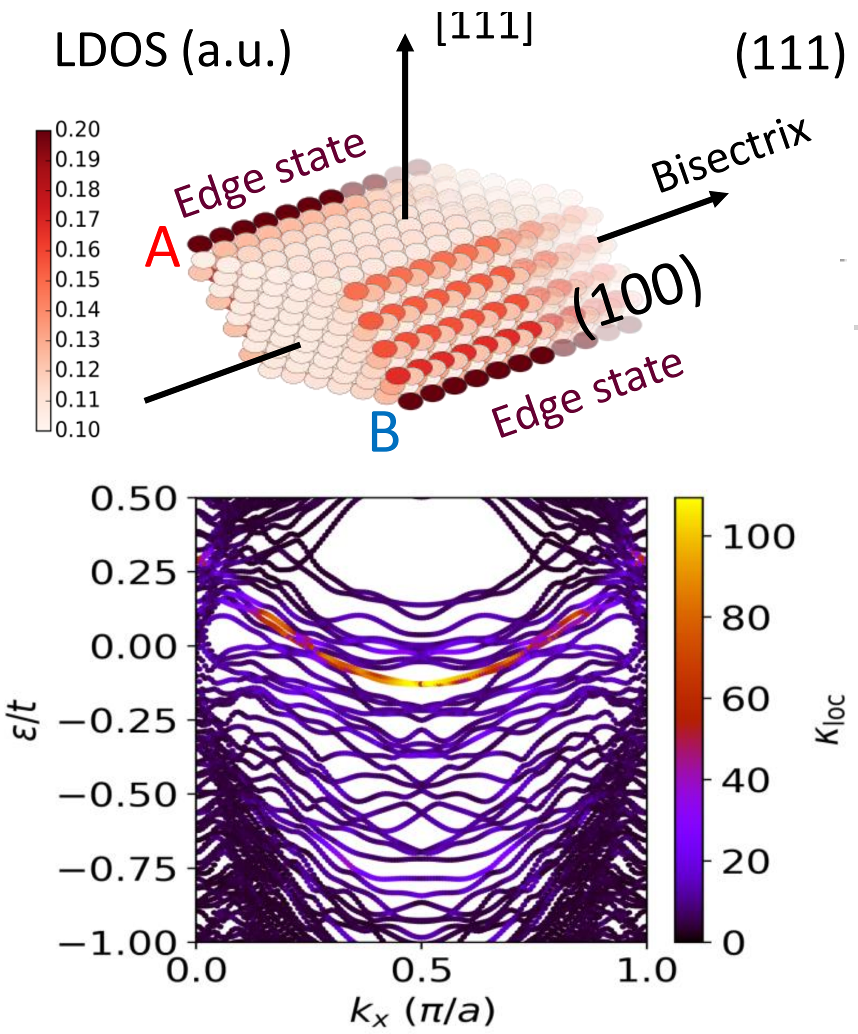


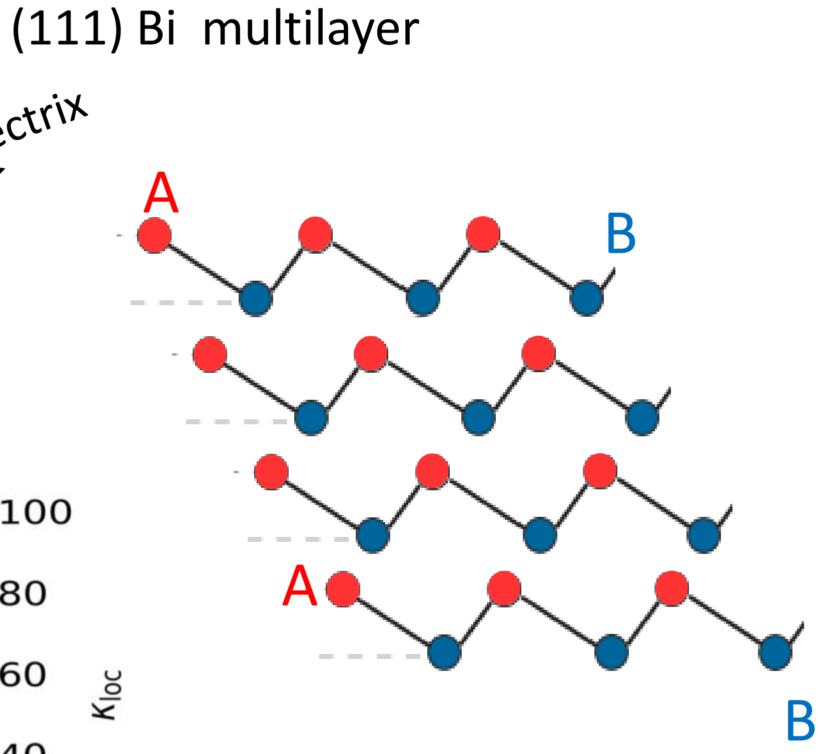
We chose to work with monocrystalline nanowires with (111) surfaces

What about a stack of (111) bilayers?

Tight binding simulation of bilayer and (small) nanowire (Anil Murani)





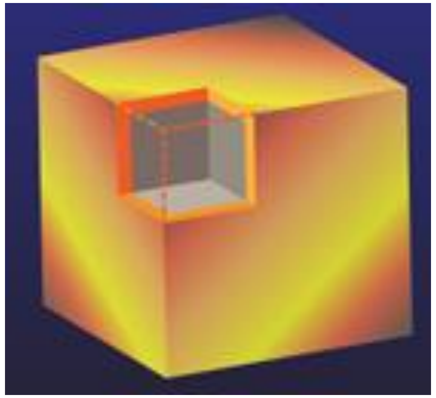


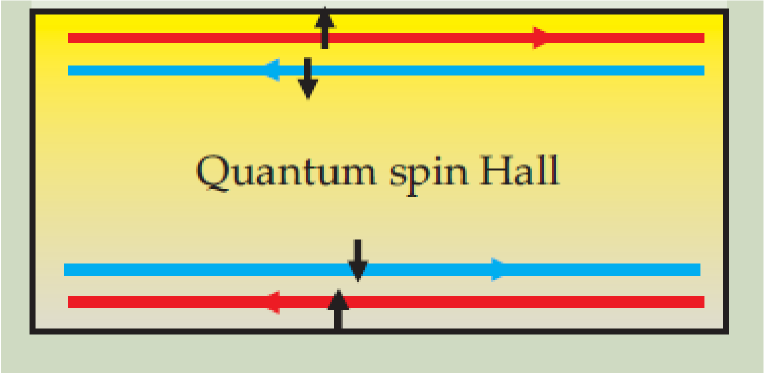
1D edge states seem to persist in 3D wires (at sharp angles of nanowires), but many other states as well…

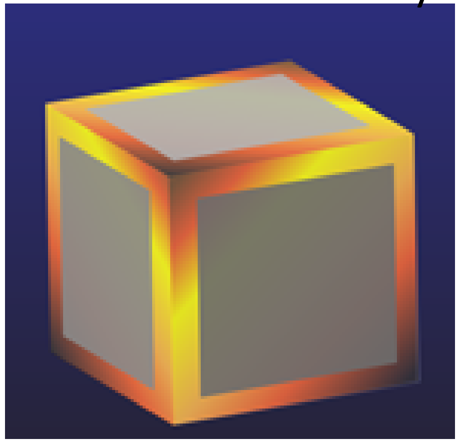
Does theory confirm this? Does experiment confirm this?

Higher order Topological Insulators

From Physics







3D topological insulator

3D insulating bulk

2D Conducting surfaces

2D topological insulator

2D insulating bulk

1D conducting « helical » edges

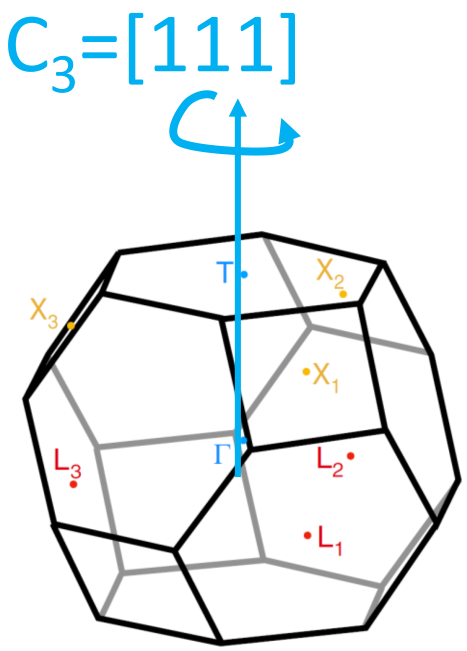
Second Order Topological Insulator

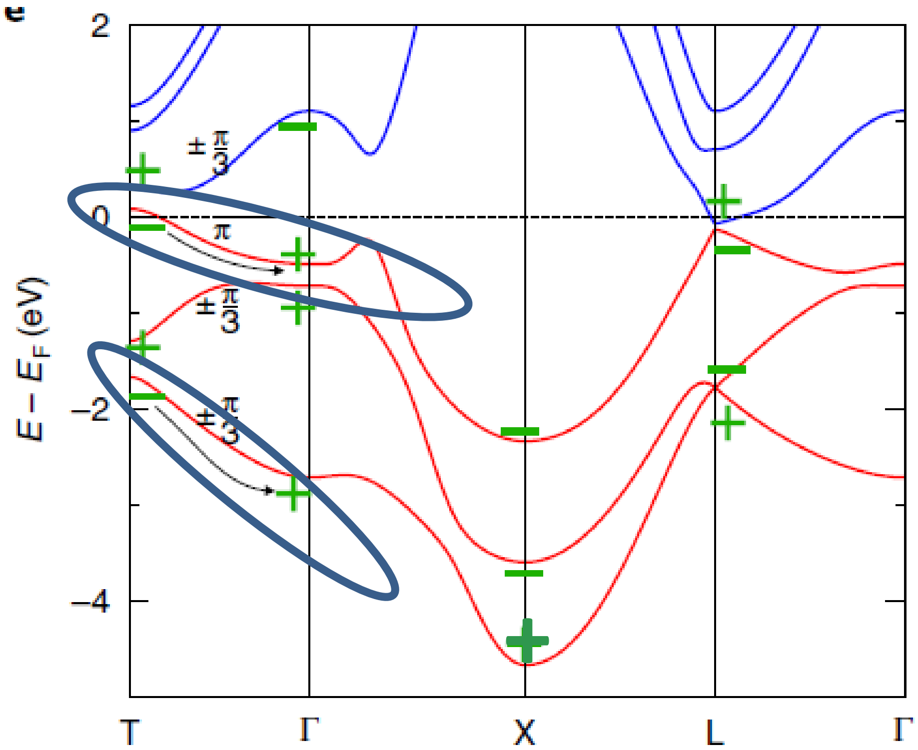
3D insulating bulk

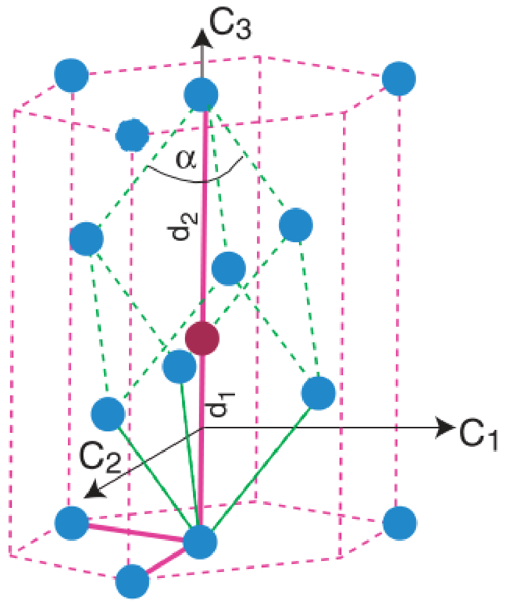
2D insulating surfaces

1D conducting helical « hinges »

Last year: Bulk Bismuth may not be topologically trivial!







Symmetries of Bi:
-Time Reversal
-Inversion
-C3 rotation: defines two subspaces with eigenvalues $e^{i\pi}$, $e^{\pm i\pi/3}$

Fu-Kane topological index: ν=1 (trivial) or -1 (topological)

$\nu=\nu_T\nu_\Gamma\nu_X\nu_L$= 1: bulk Bi is not a 3DTI

$\nu^{(\pi)}=-1=\nu^{(\pm i\pi/3)}$: each subspace is a topological insulator ("C3-graded double band inversion at T point")

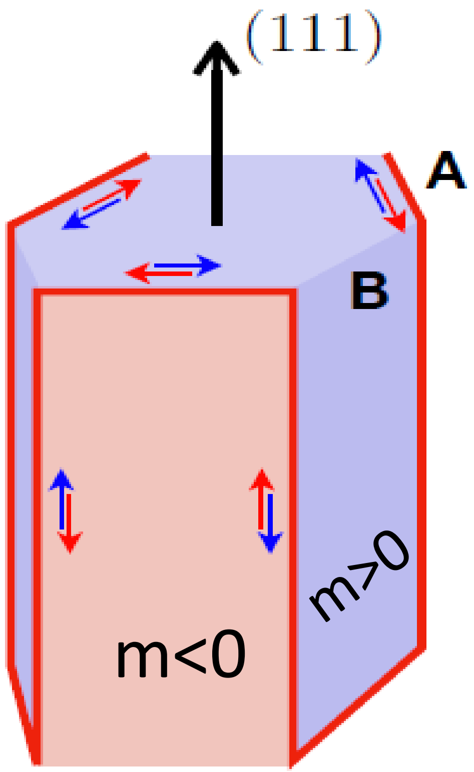
Bi is a superposition of two TI

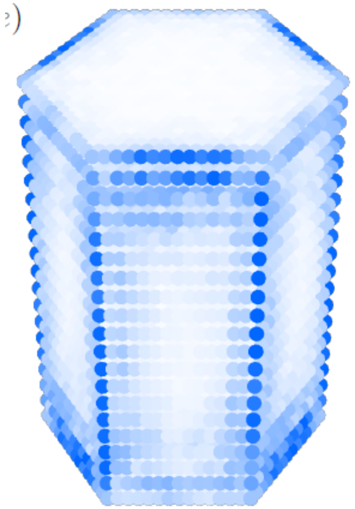
Bulk Bismuth is a Second Order TI (Schindler et al, Nat. Phys. 2018)

Bismuth, a Second Order Topological Insulator?

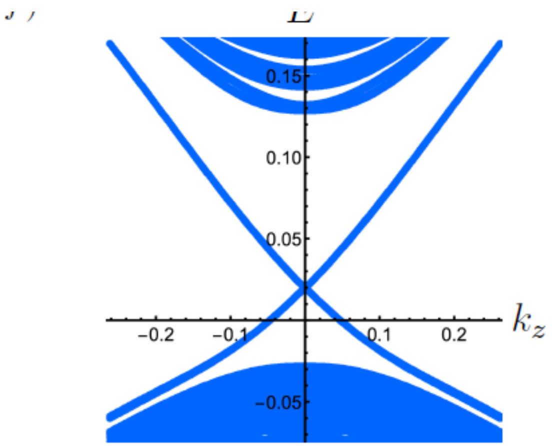
Bi ~ superposition of two topological insulators in 2 independent subspaces

⇒ Topologically protected Kramers pairs of gapless modes at “hinges”





Helical edge state that winds around the cristal



Is bismuth really like that?

(Schindler et al, Nat. Phys. 2018)

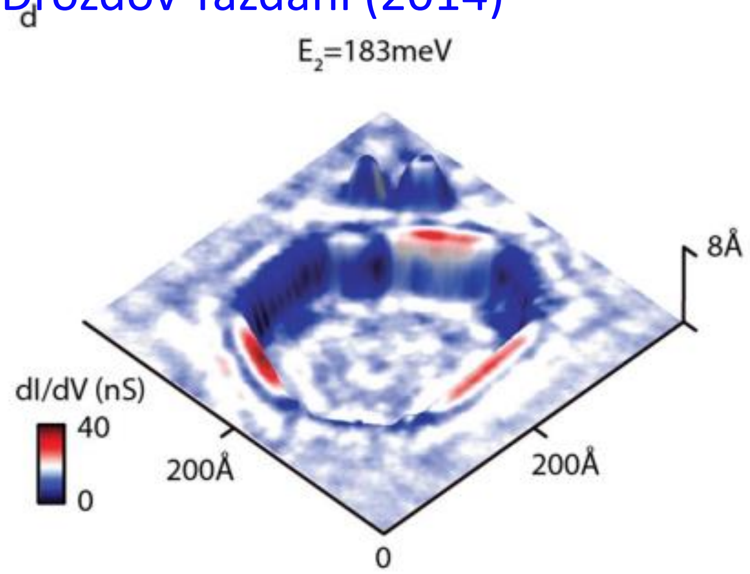
1D edge states observed by STM! (decoupled from bulk Bi)

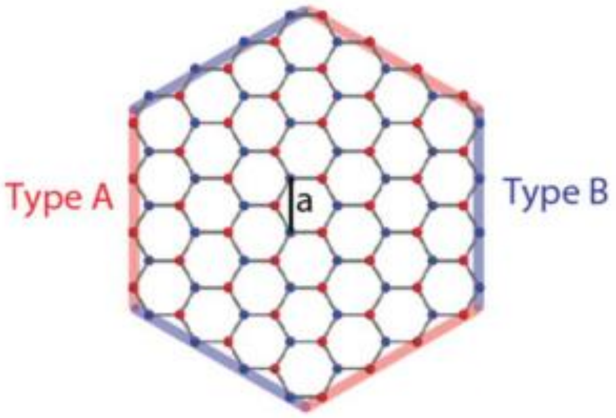
Bilayer pits in (111) bulk Bi crystals

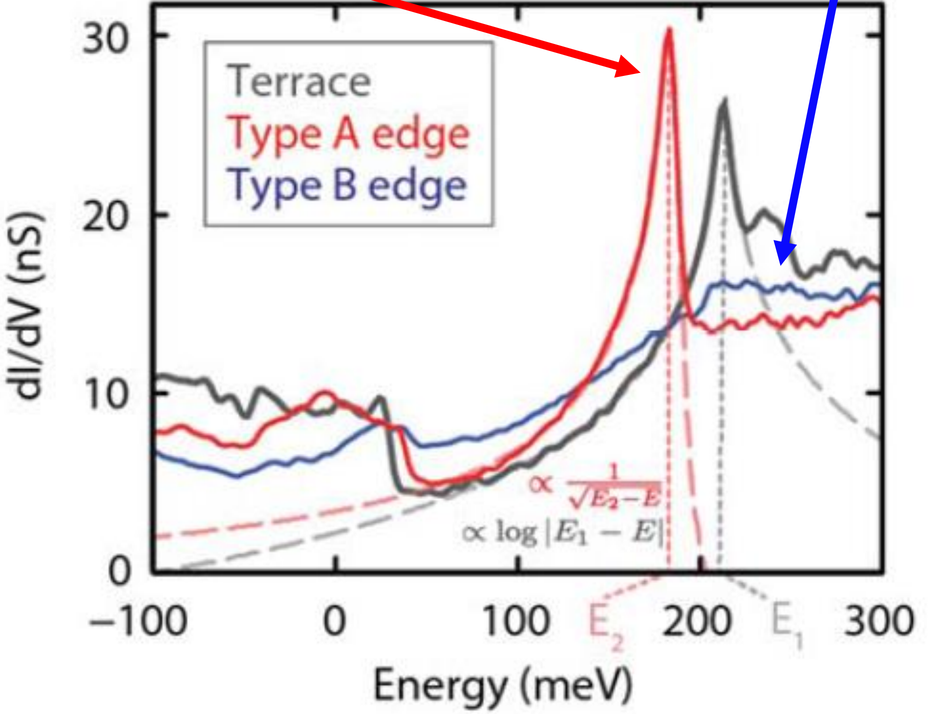
Drozdov Yazdani (2014)

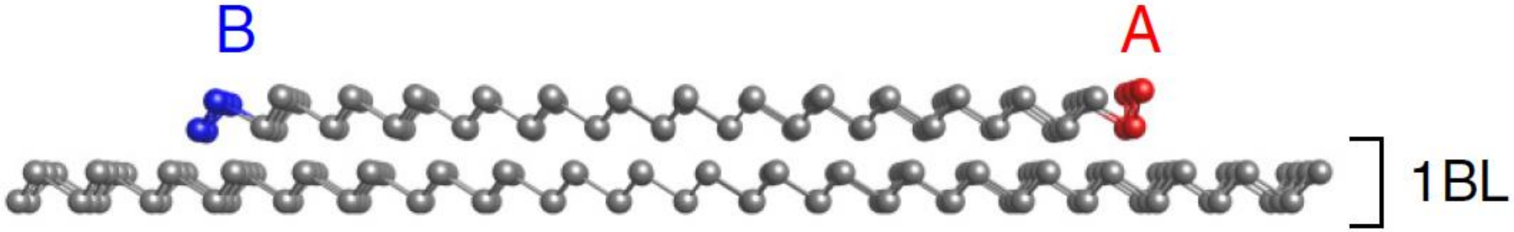
1D-like Van Hove singularity on type A edge

No van Hove on type B edge





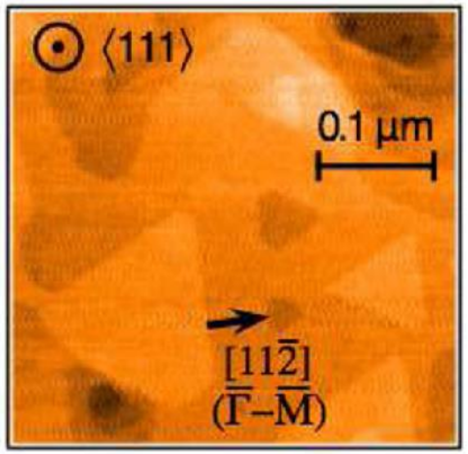


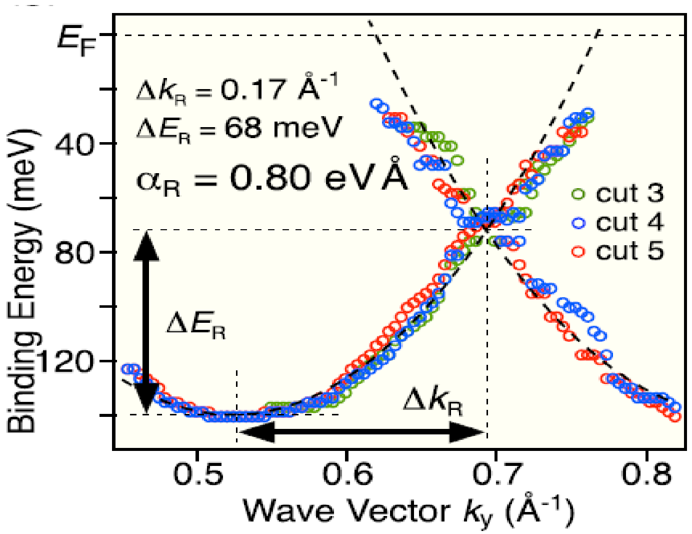


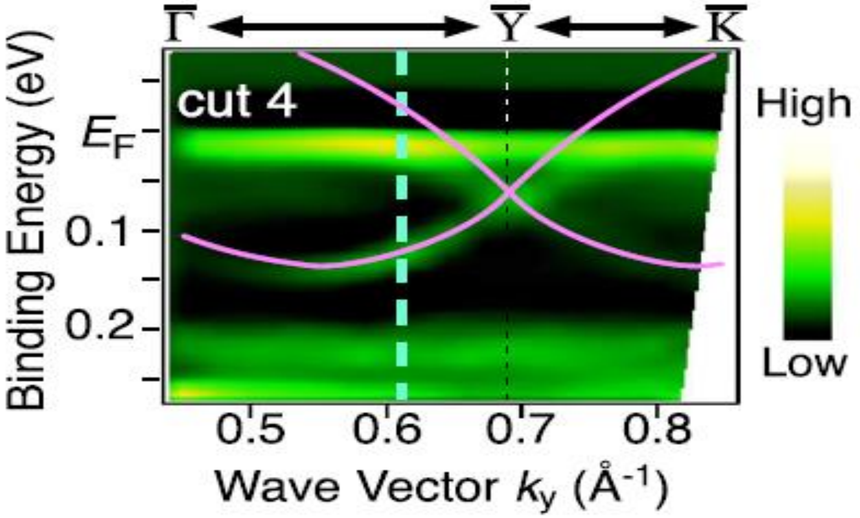
- Only A-type edges show 1D features
- Suppressed backscattering

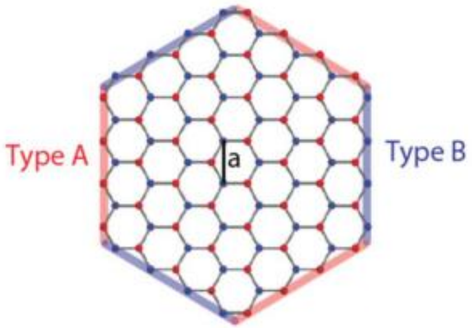
Photoemission on many (111) few layer Bi

Takayama PRL 2015: mostly triangular crystals, less than 15 nm thick





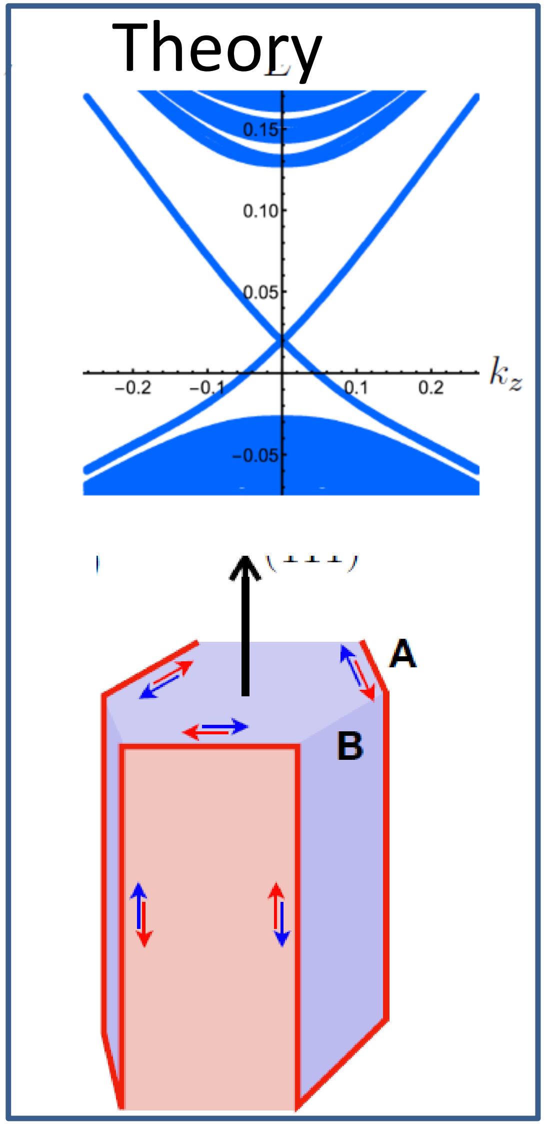


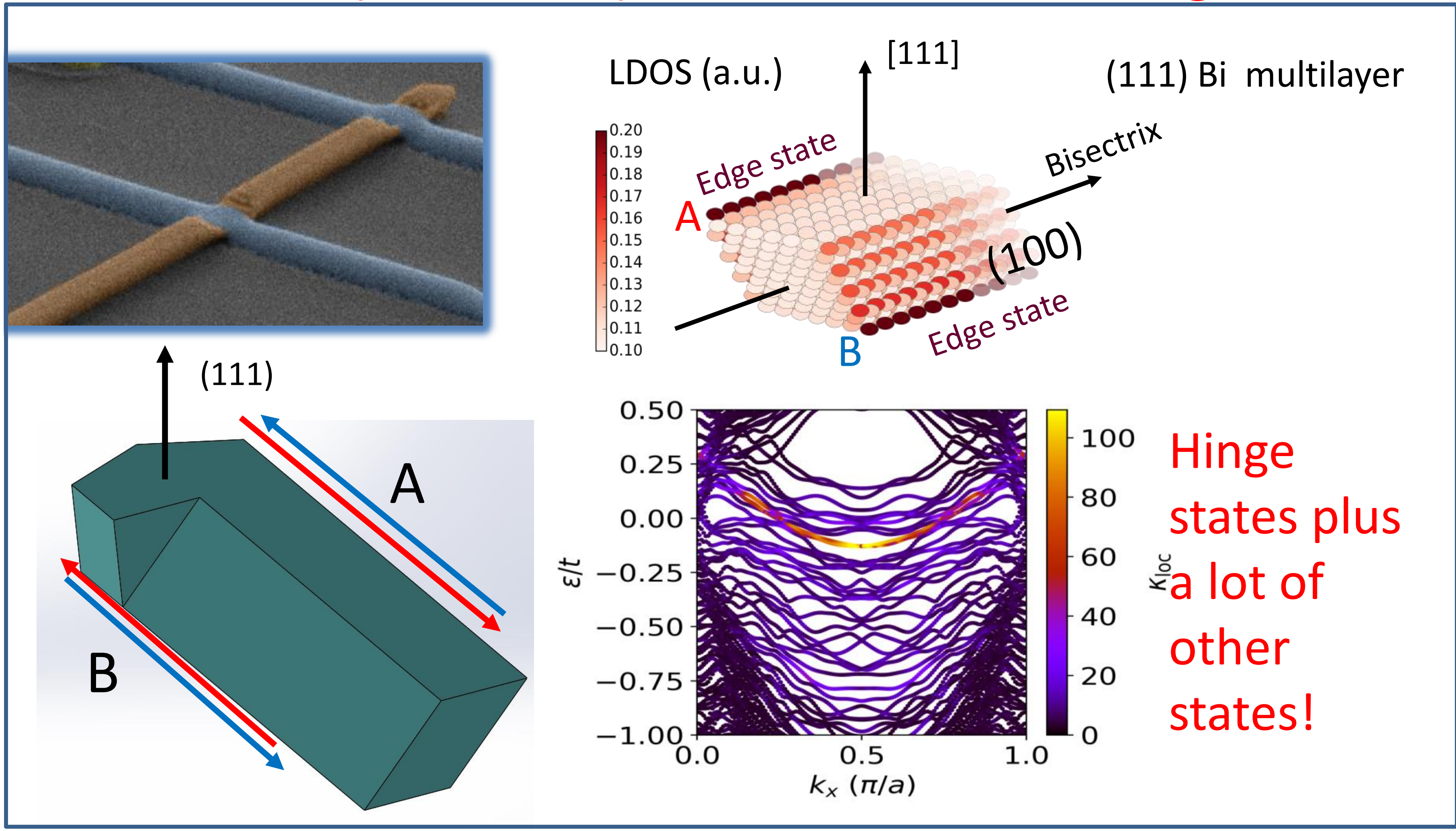


Photoemission detects spin-split surfaces and 1D edge states

… but maybe not topological

Bismuth Nanowires IRL (in real life): bulk surfaces and edges?





Hinge states plus a lot of other states!

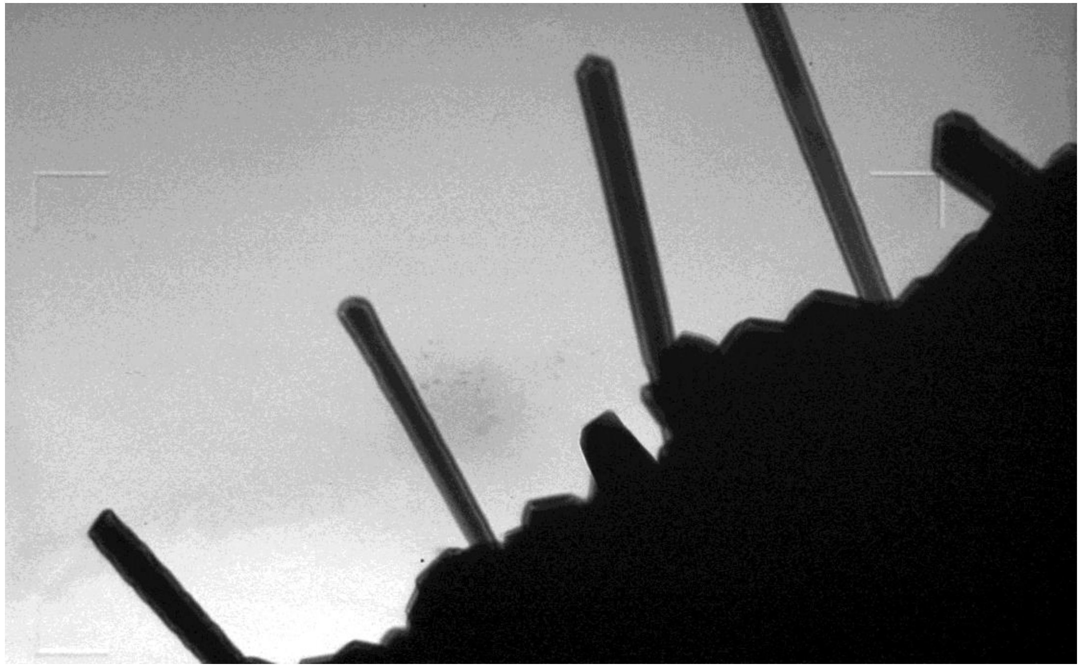
Can the ballisticity of the few topological hinge states emerge?

Mesoscopic physics!

Our samples Monocrystalline Bismuth nanowires

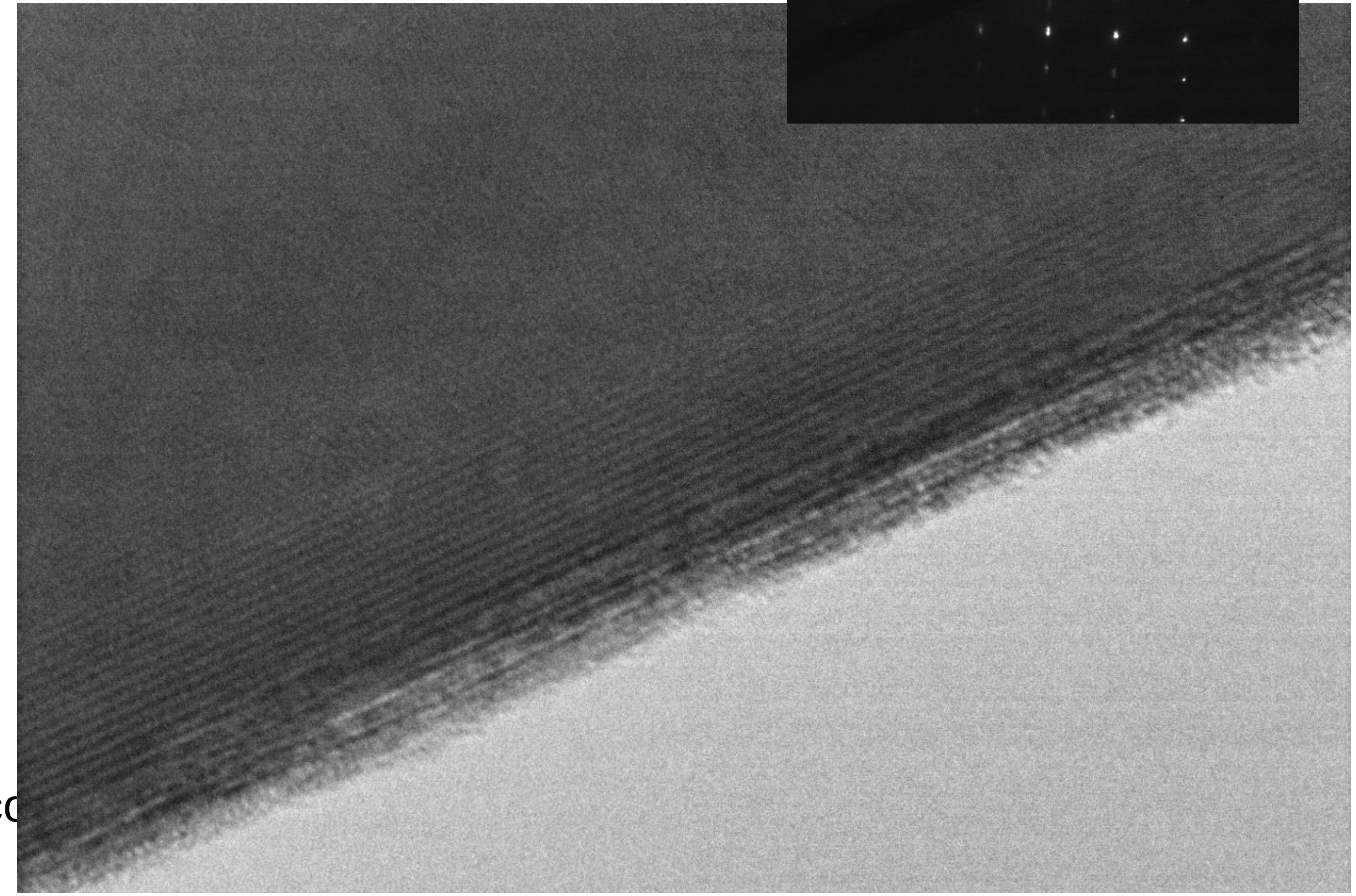
High quality single crystals
Sputtering, buffer layer of Fe or V
(A. Kasumov)

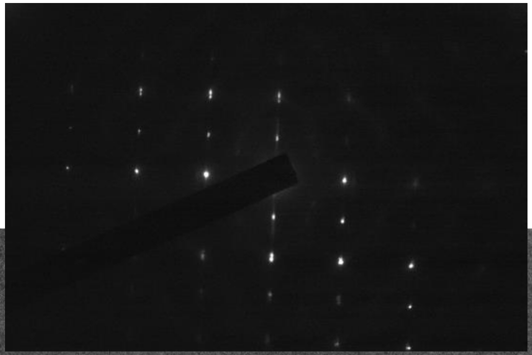
Diameter ~100 nm



Low magnification, Transmission Electron Microsco

High resolution TEM

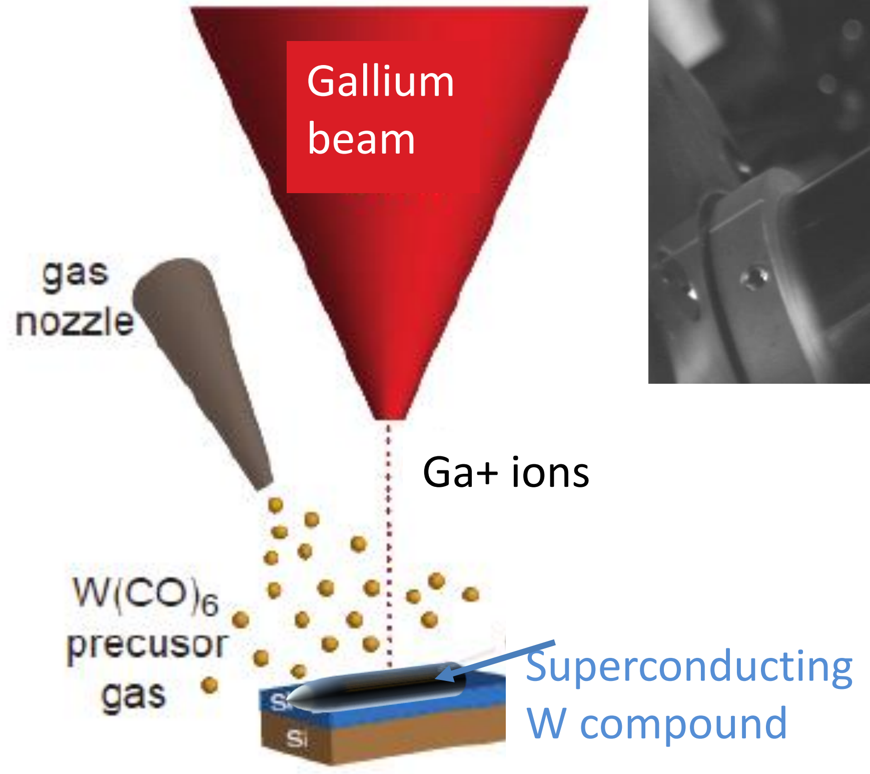


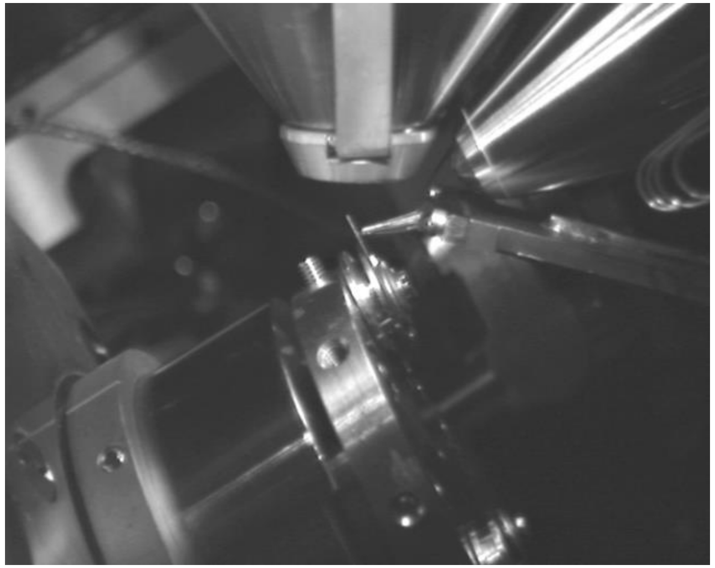


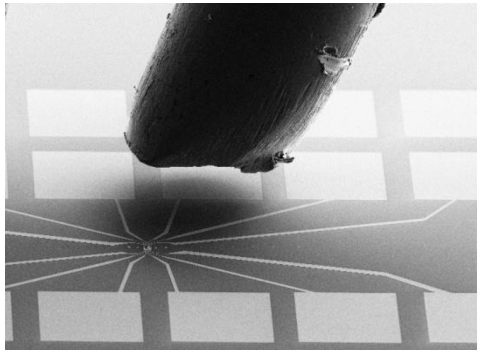
High quality monocrystals

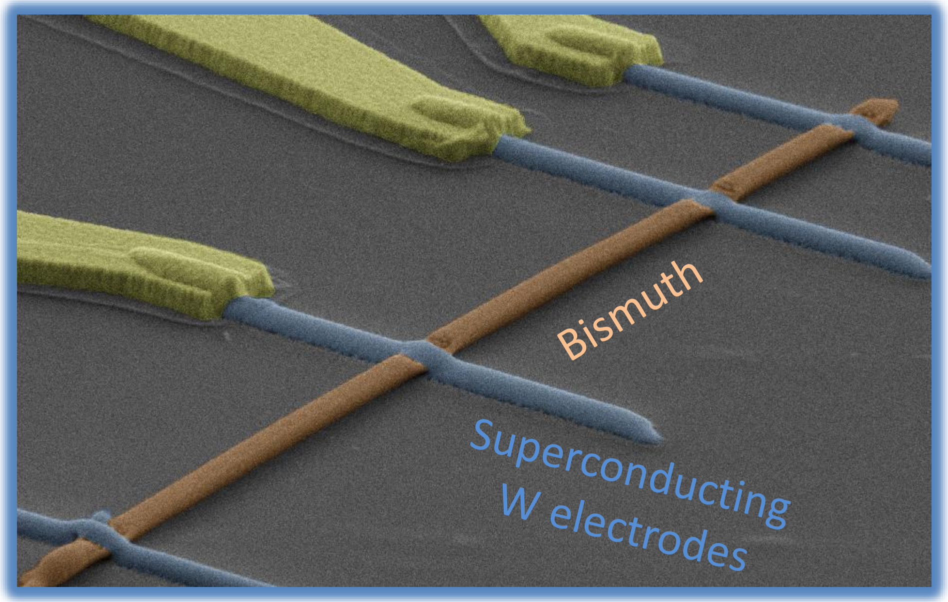
Connect selected nanowires with Superconducting contacts by focused-ion-beam-assisted deposition

Kasumov 2005









Superconducting electrodes:

C and Ga-doped amorphous tungsten

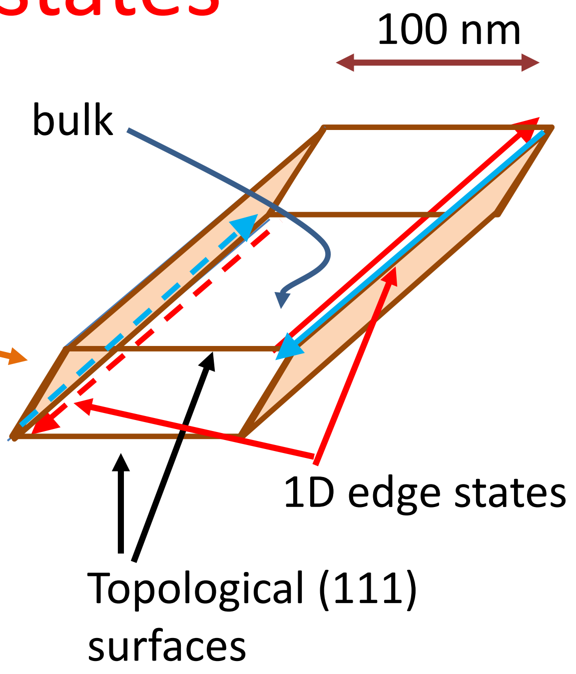
200 nm thick and wide

Great superconducting properties: T_c~4 K, Δ~0.8 meV, H_c~12 Tesla!

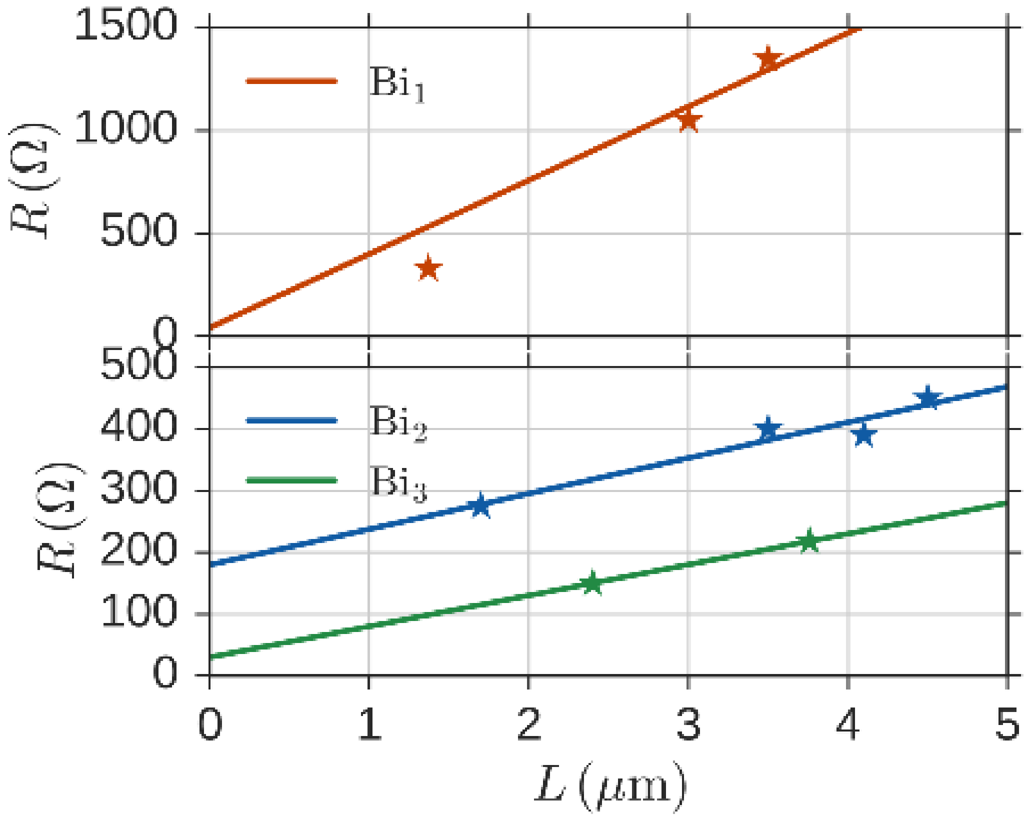
Normal transport doesn't show any ballistic states

Diffusive Surface states carry most of the normal current

Bulk $\lambda_F \simeq 50\,\mathrm{nm}$
Surface $\lambda_F \simeq 5\,\mathrm{nm}$
} Roughly 50 times more surface states than bulk states

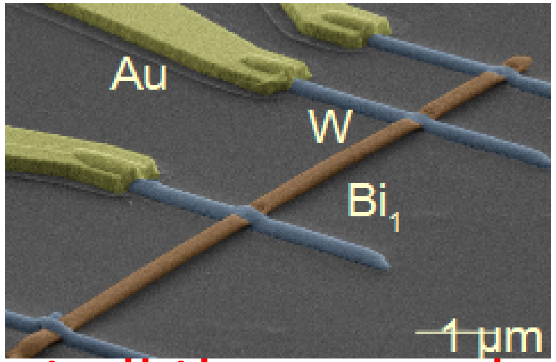


Normal state resistance of nanowire



$$R(L) = R_c + \frac{R_Q}{M}\frac{L}{l_e}$$

Thus $l_e \lesssim 200\,\mathrm{nm}$



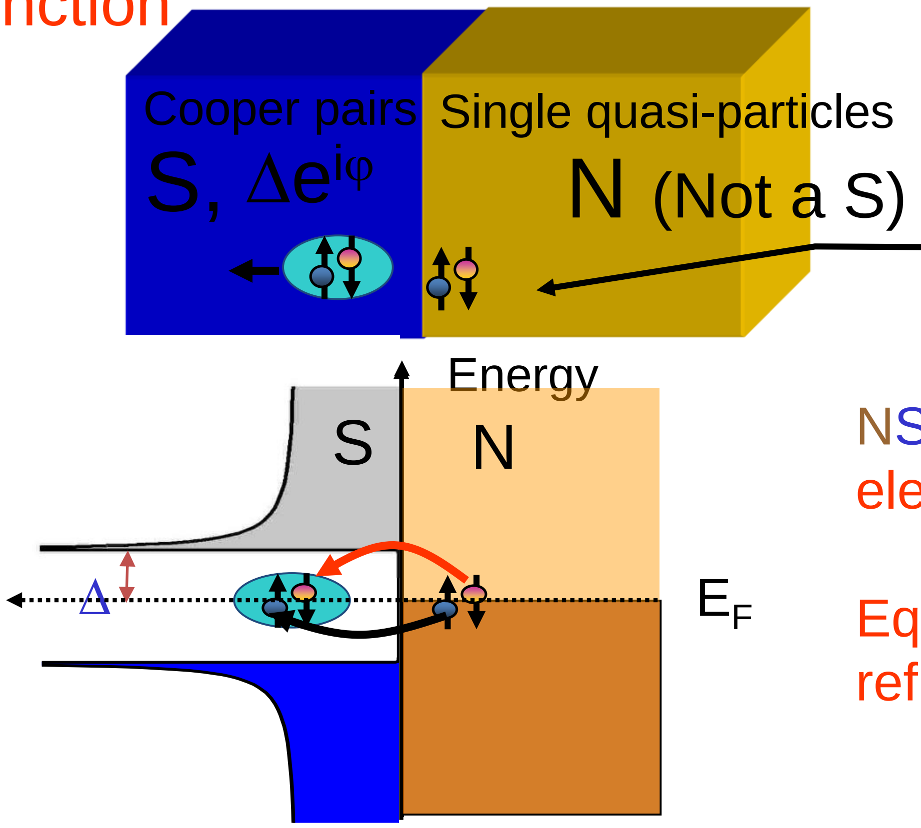
Diffusive surfaces states carry almost all the normal current

$\Rightarrow$ Turn to supercurrent to enhance visibility of ballistic states

How does the superconducting proximity effect help ?

Induced superconductivity

NS junction

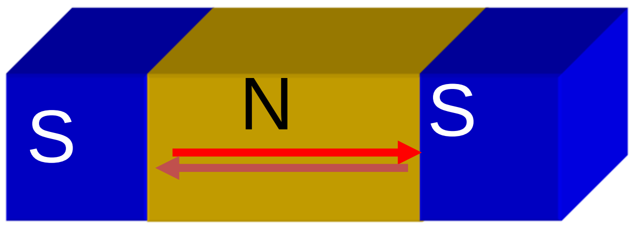


Time reversed quasi-particles are injected

NS current : two (time-reversed) electrons pass from N to S

Equivalent to an electron Andreev reflected into a hole

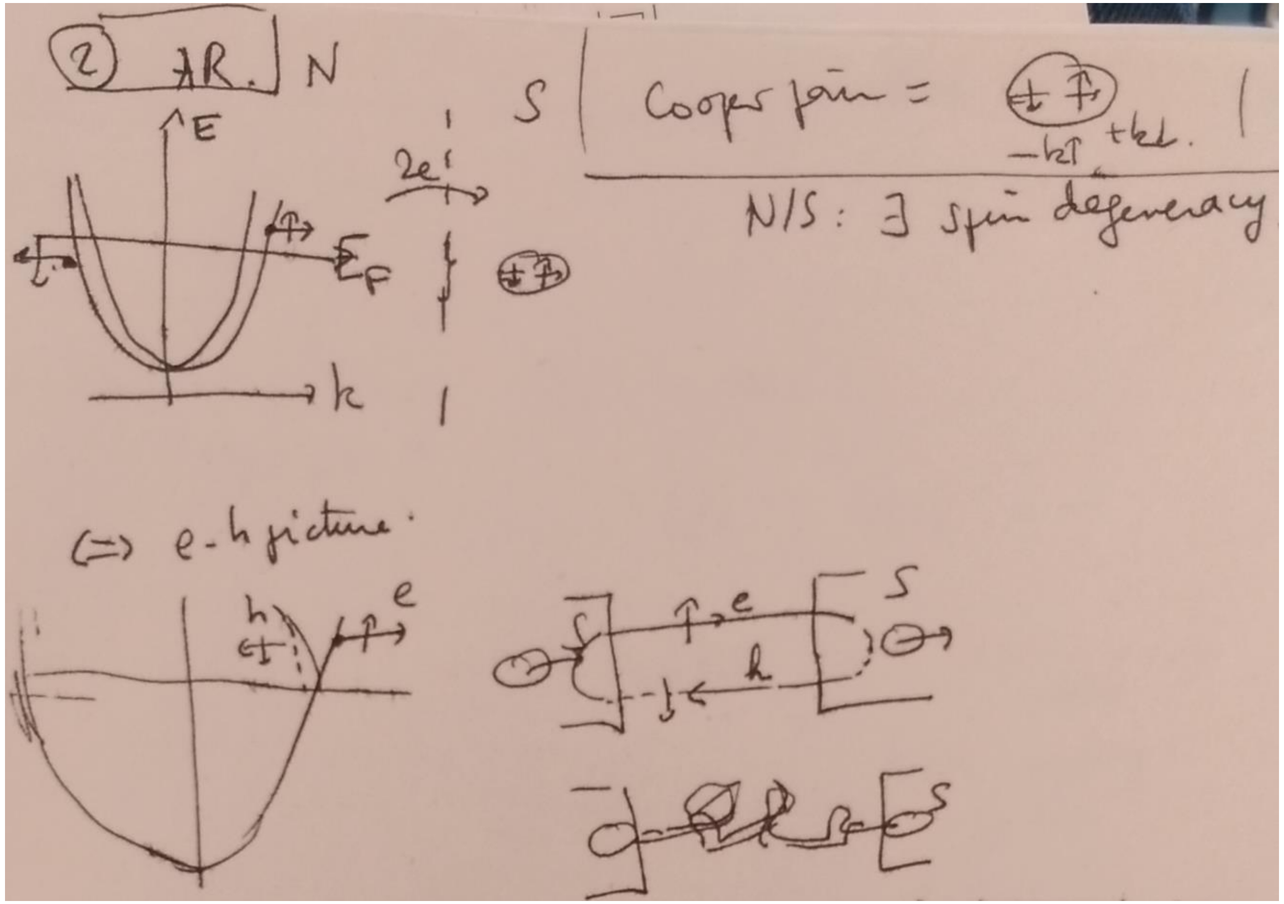
SNS junction

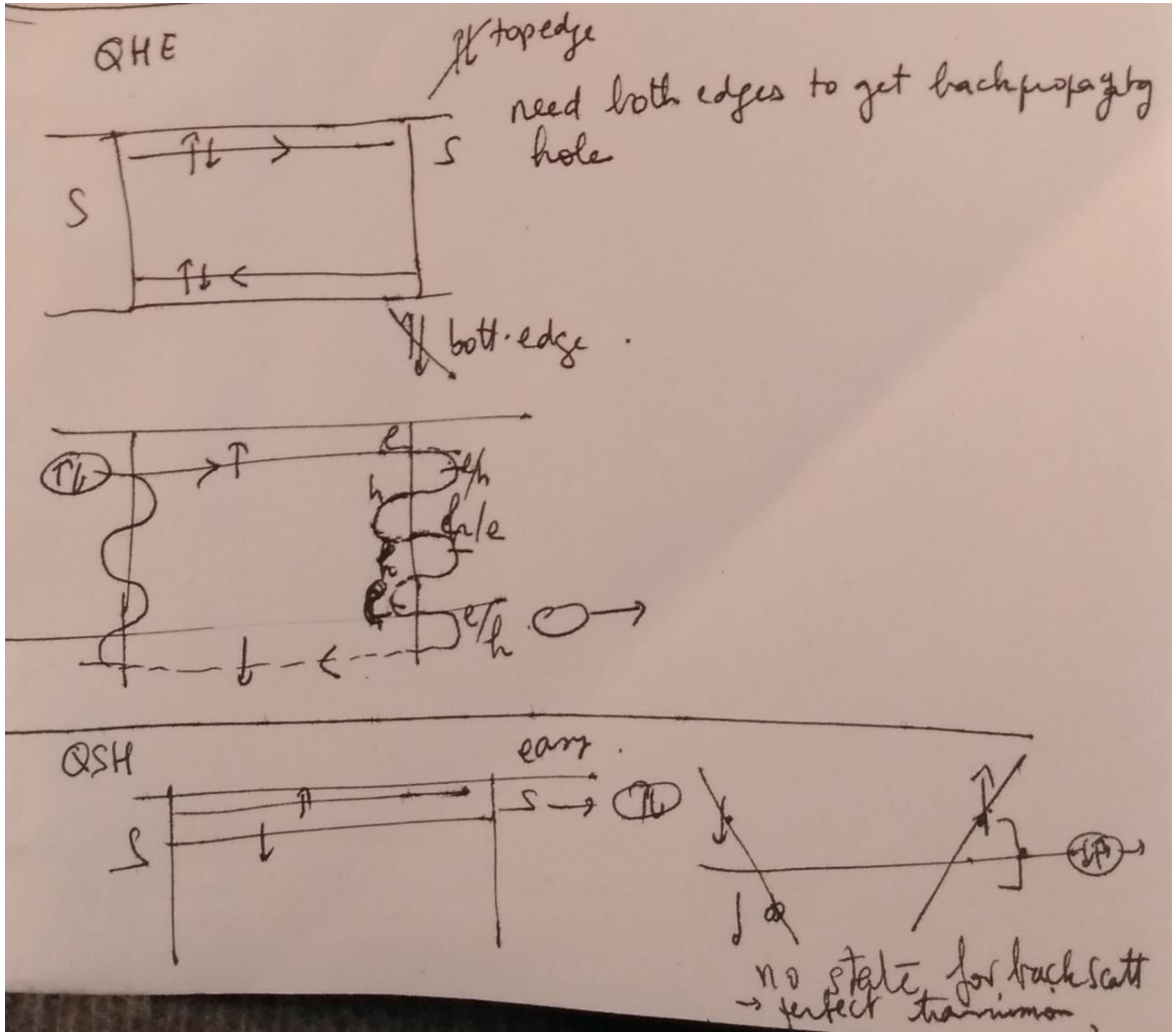


A supercurrent flows (zero resistance!) if N is quantum coherent ($L<L_\varphi$).

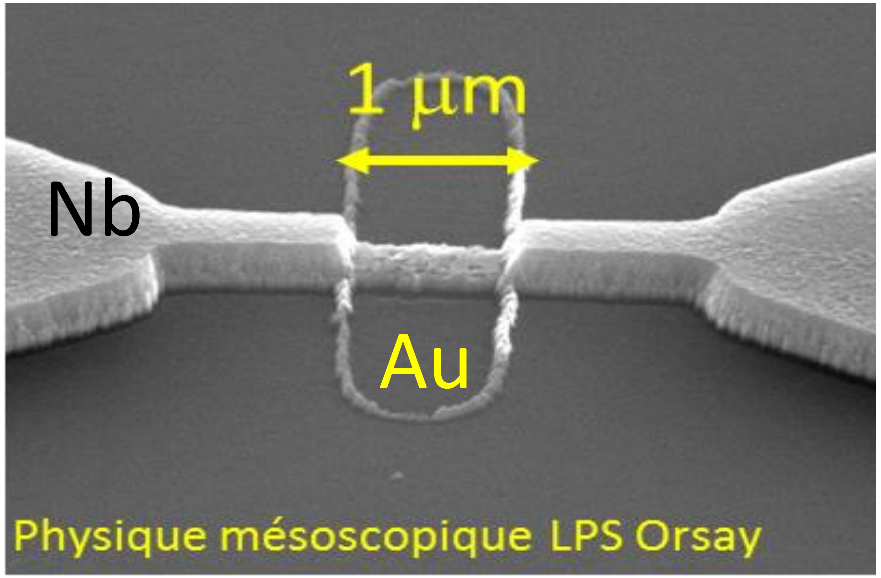
No change of the quantum state of electrons during propagation

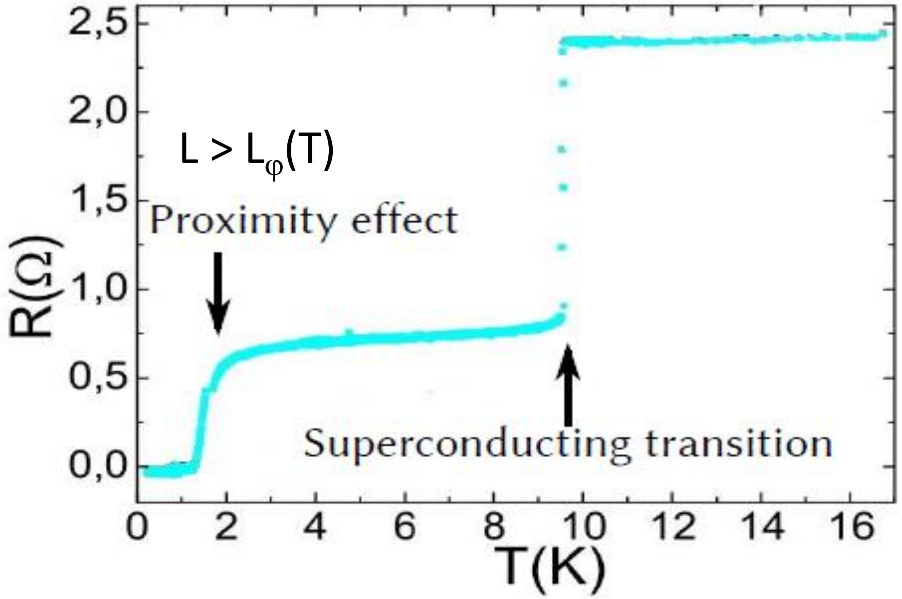
Interesting AR situations

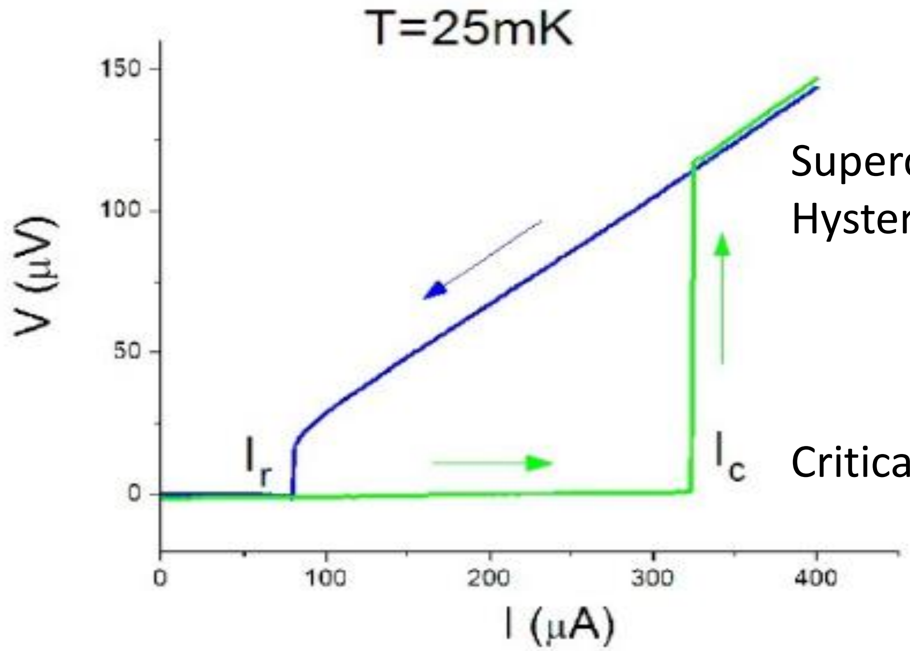




Conventional superconducting proximity effect







Supercurrent
Hysteretical I(V)

Critical current

How much supercurrent can flow?

Where does it flow?

Maximum supercurrent depends on junction length

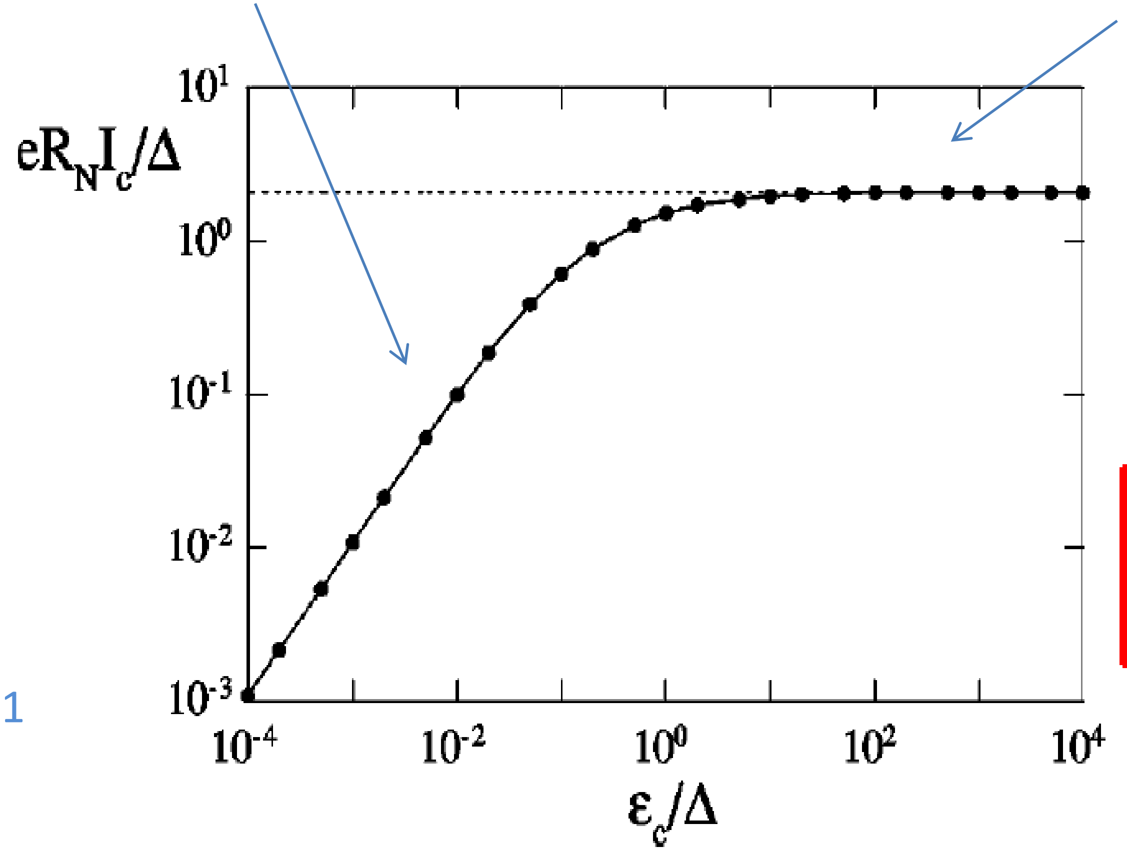
$R_N I_c = \min(\Delta, E_{Thouless})$

Long junction limit $L \gg \xi_S$

$$eR_N I_c(T=0) = 10.82 E_{\mathrm{Th}} = 3.2\Delta_g .$$

Short junction limit $L \ll \xi_S$

$$eR_N I_c \simeq 1.326\pi \Delta/2 \simeq 2.07\Delta .$$

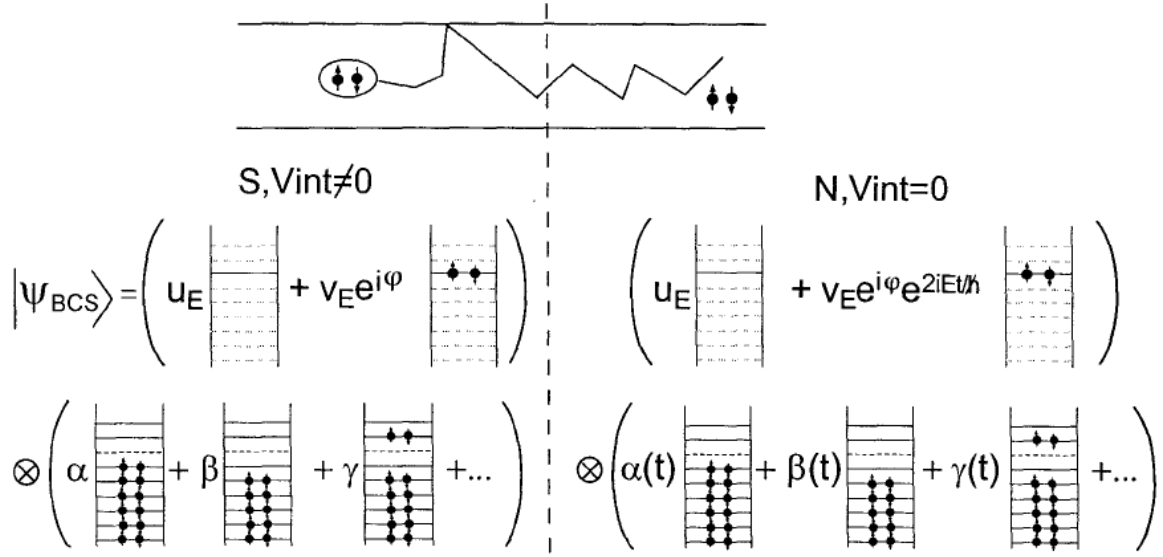


$$E_{Th} = \frac{\hbar D}{L^2} = \frac{\hbar}{\tau_D}$$

$$\Delta = \frac{\hbar D}{\xi_S{}^2}$$

Dubos Wilhem PRB 2001

Simplified picture of why diffusion time determines energy scale of proximity effect (in long junctions)



Dephasing $\exp\left(-i2Et/\hbar\right)$: 2π after $t=h/E \sim hD/L^2$, with spread.

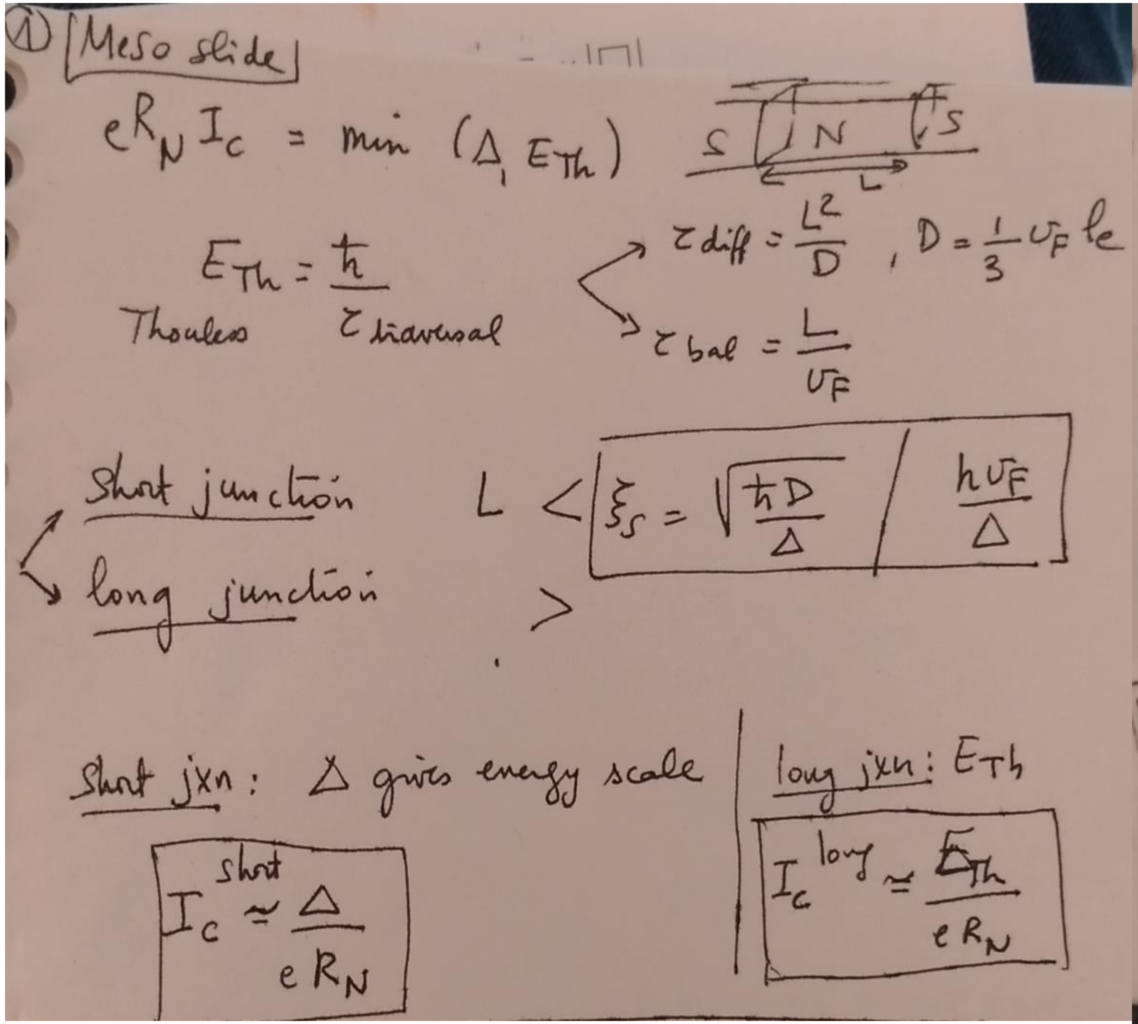
Pair correlation of energy E can propagate in N a distance

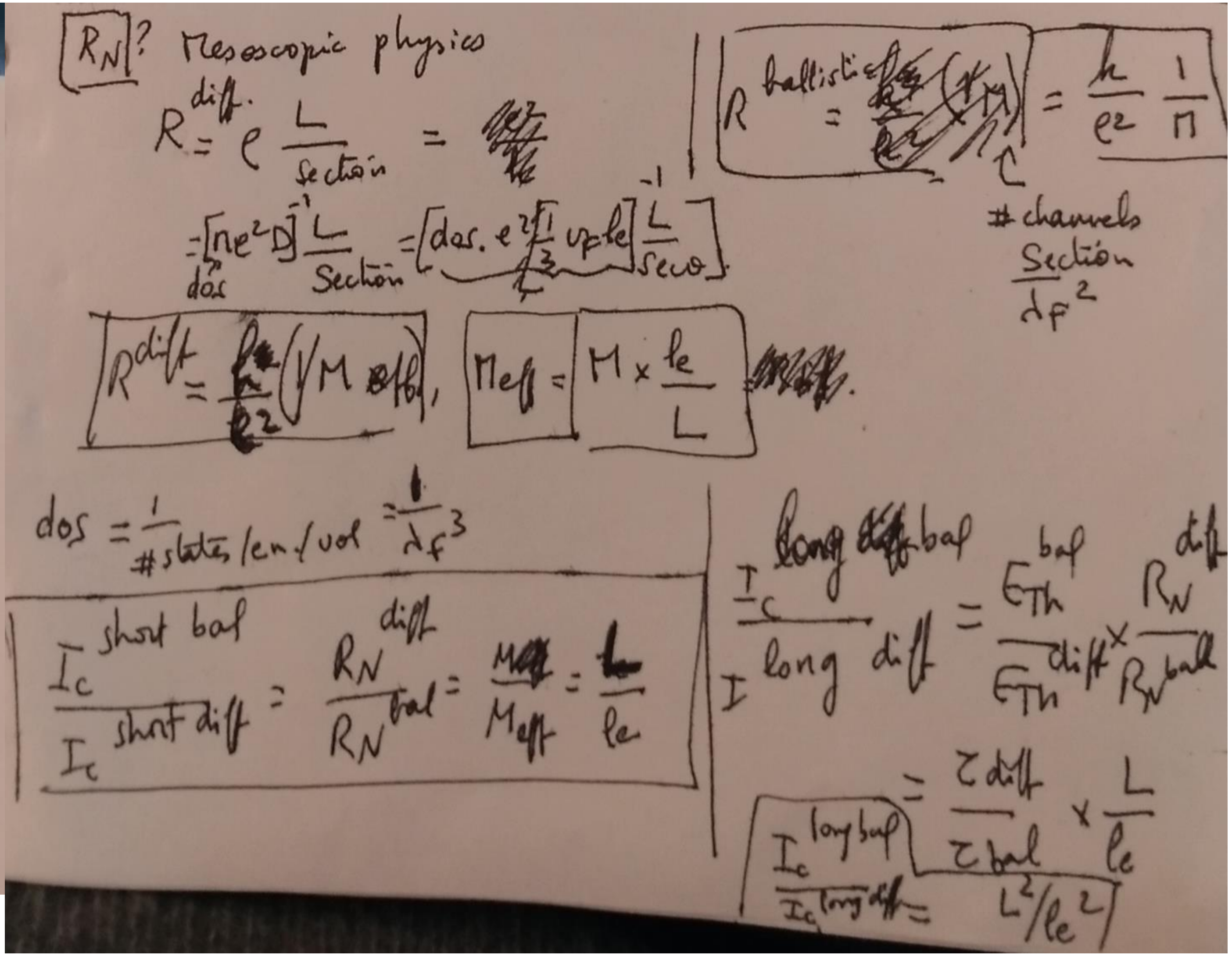
$$x \sim \sqrt{\hbar D/E}$$

Induced minigap, critical current of the order of

$$E_{Th} = \frac{\hbar D}{L^2} = \frac{\hbar}{\tau_D}$$

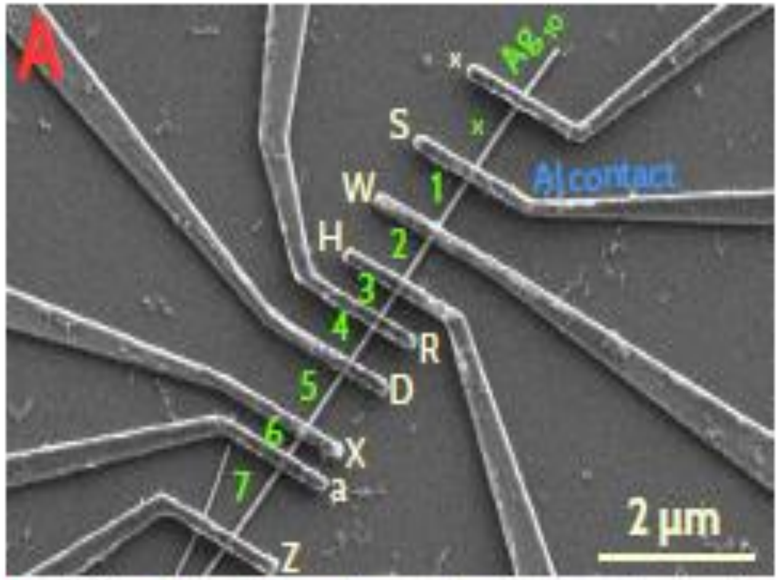
This should help understand why superconductivity helps detect ballistic states, even if many more diffusive states

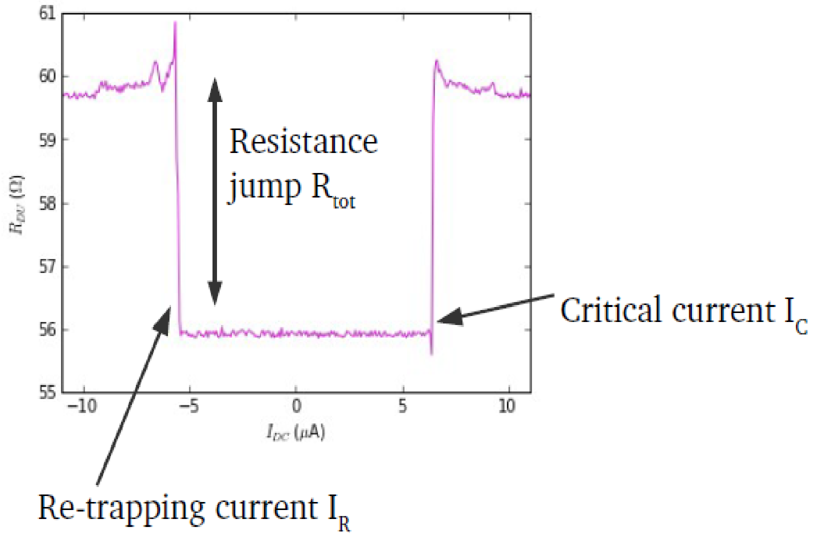


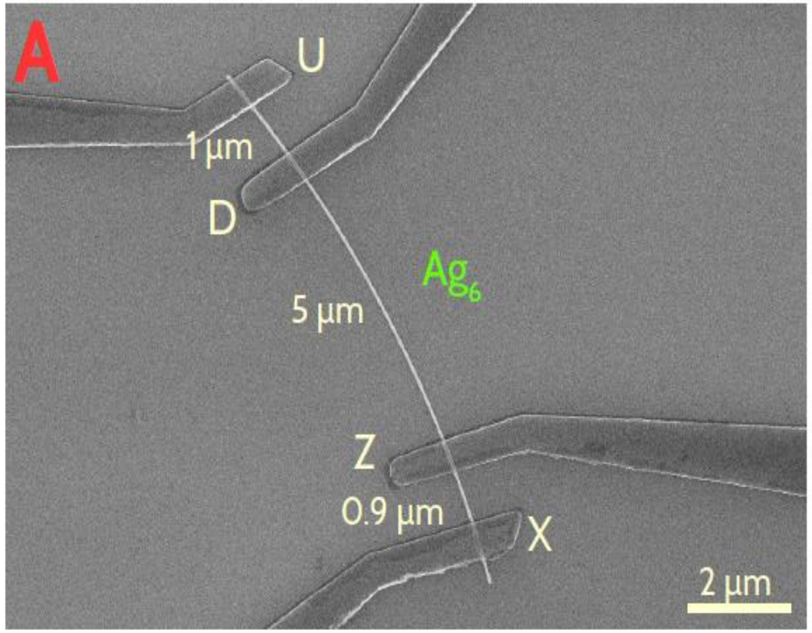


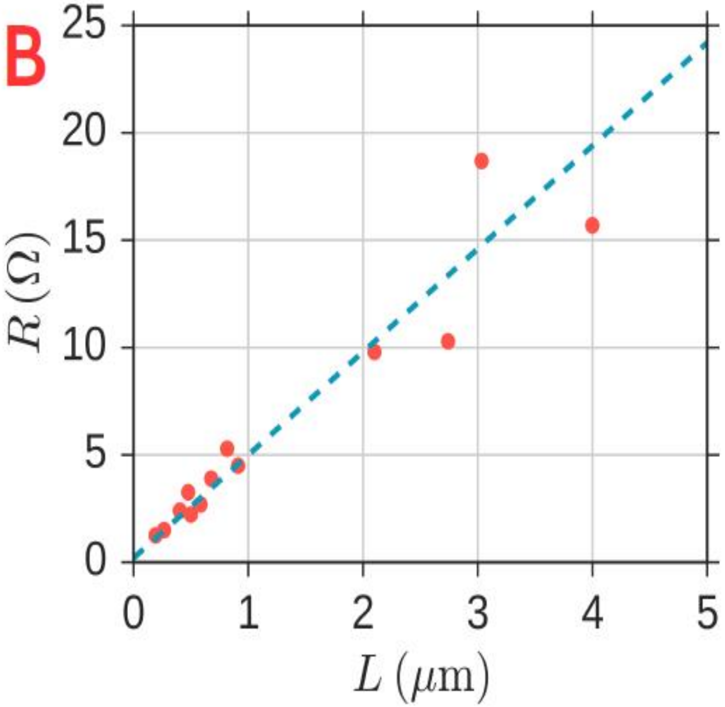
Reference example:
Silver nanowires (50 nm diameter) based Josephson junctions

Murani et al., in preparation







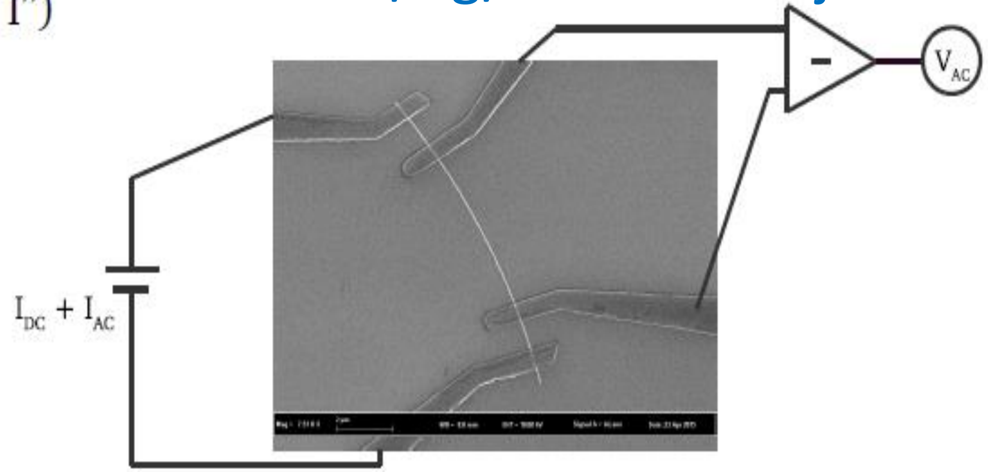


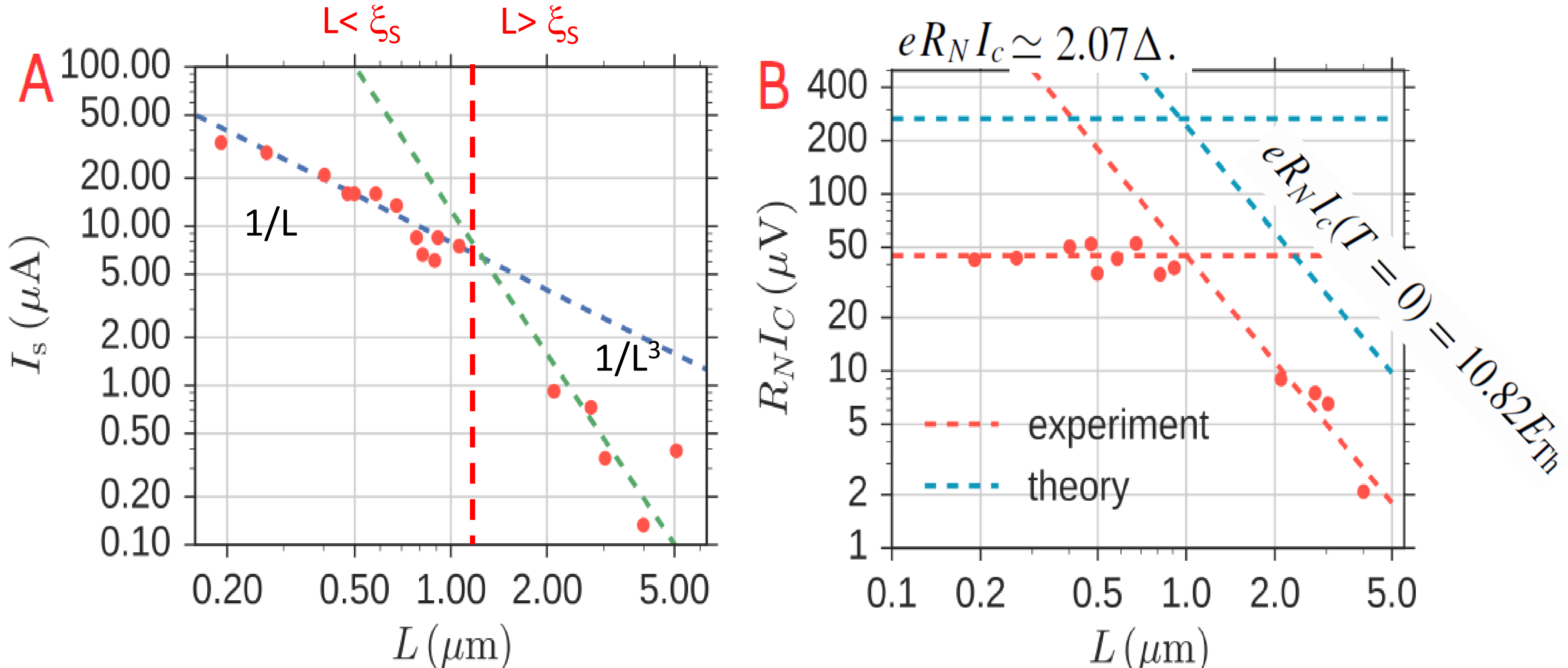
$R = Rc + R_Q Mle/L$

Diffusive normal state conduction

S/Ag/S reference junction, with varying lengths

I")

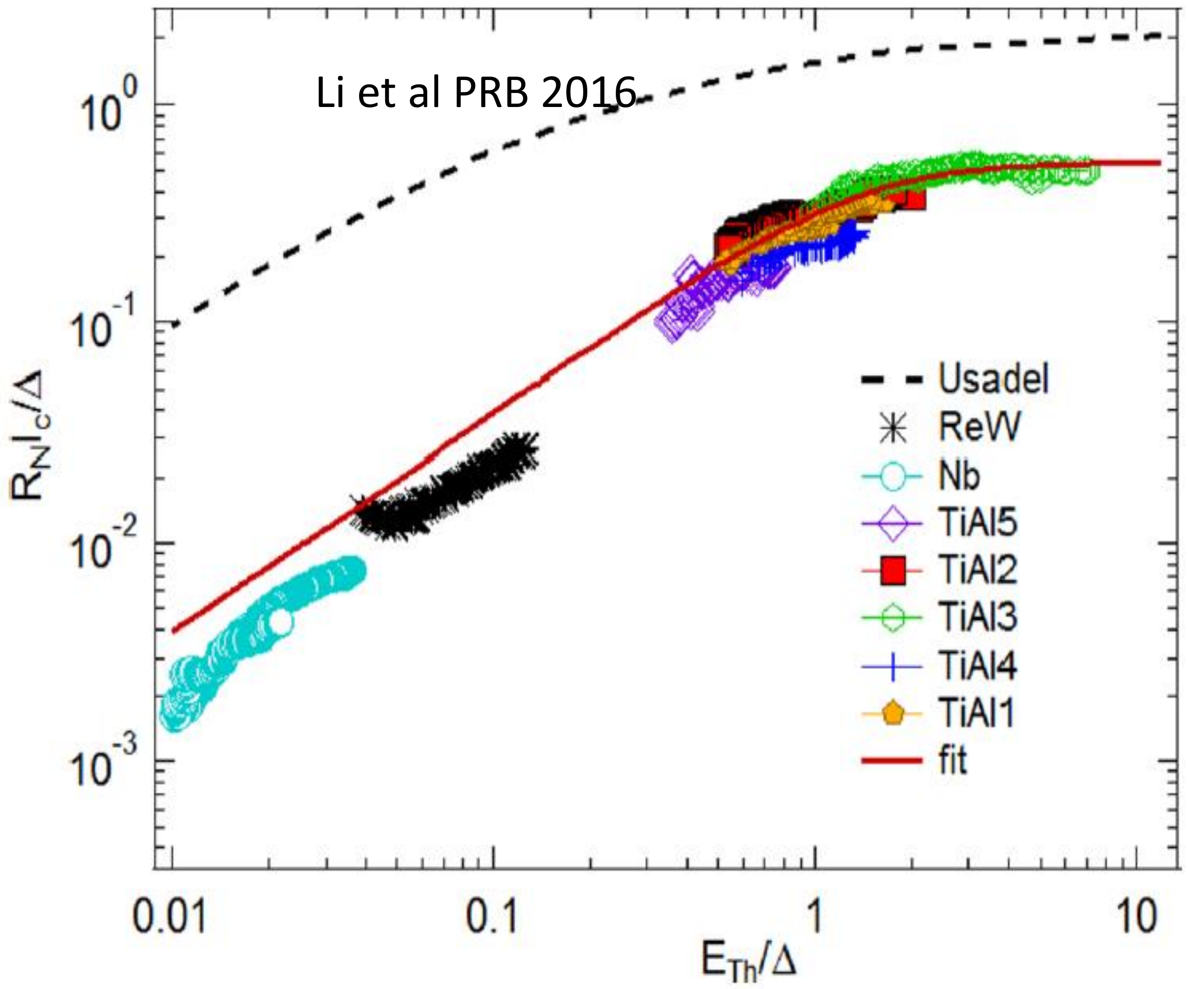


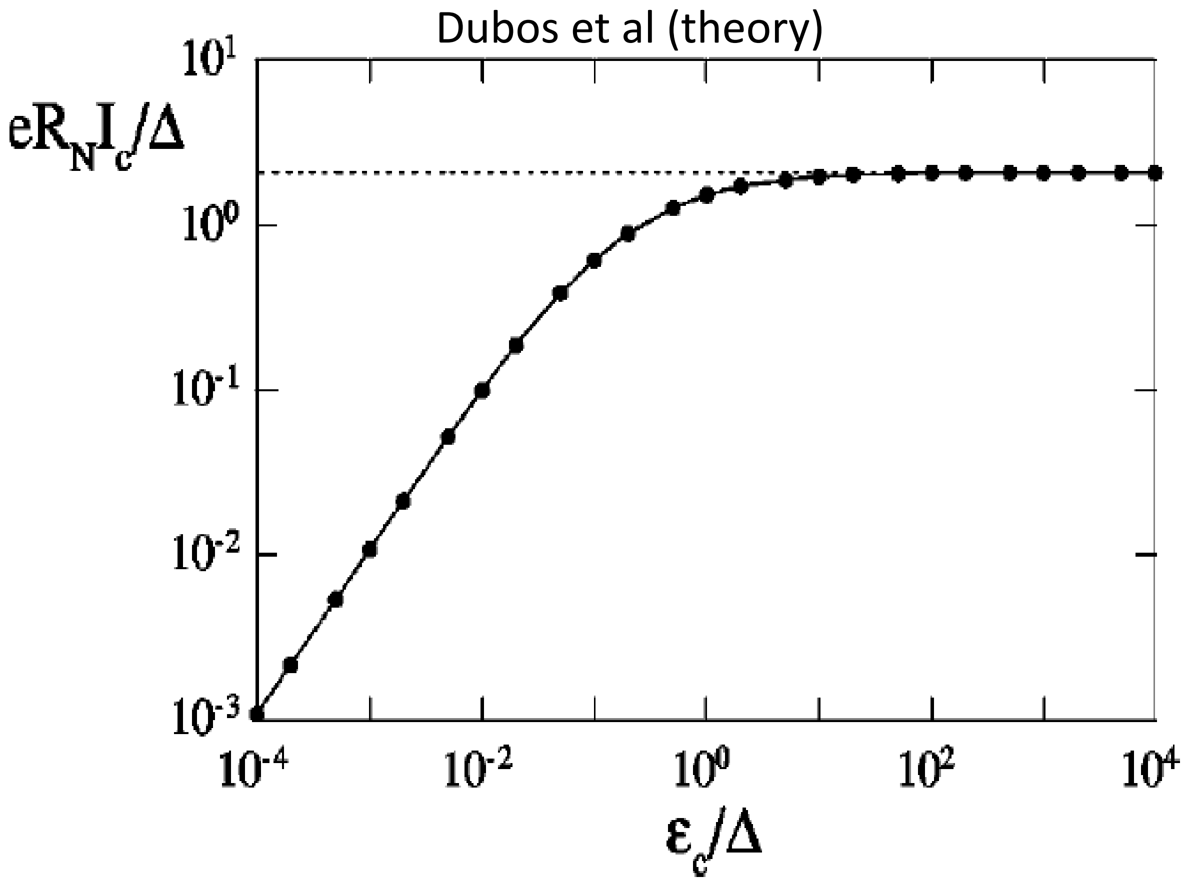


S/Ag/S reference junction: crossover from short to long junction

Crossover from short to long junction also seen in graphene samples

Gate voltage modulates the number of carriers and the diffusion coefficient



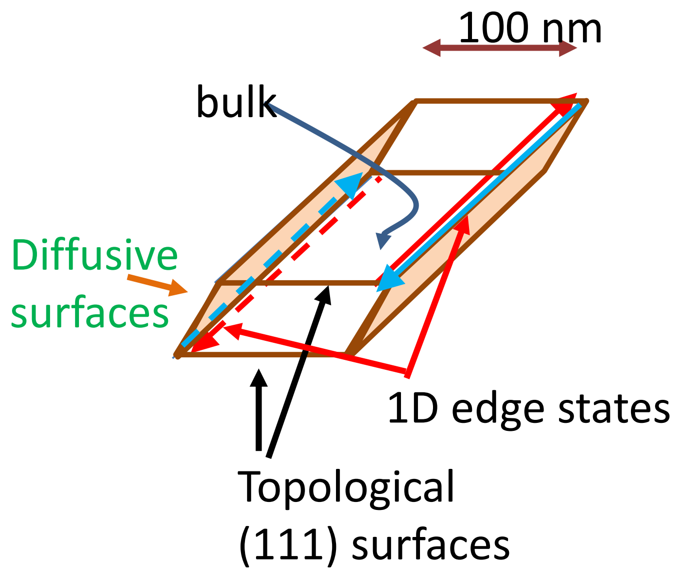


Qualitative agreement but critical currents are lower than expected theoretically

Induced superconductivity enhances contribution of Quantum Spin Hall states

- Critical current carried by diffusive states is much smaller than critical current carried by ballistic states

~ 6 ballistic edge channels, ~ 100 diffusive surface channels, elastic mean free path l_e

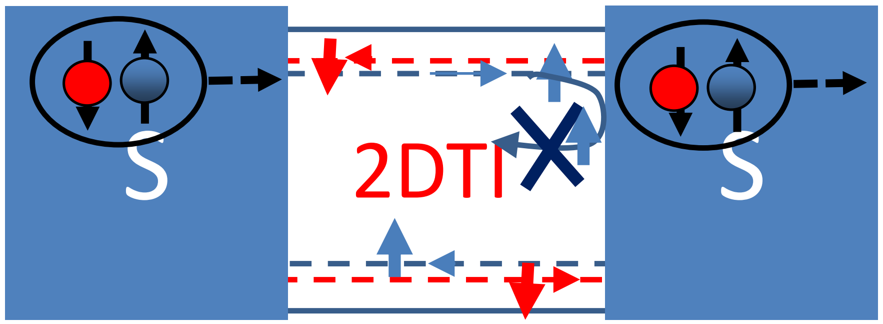


$$I_{c\ \mathrm{1channel,\ ballistic}} \sim \frac{h v_F}{L}\frac{h}{e^2}$$

$$I_{c\ \mathrm{1channel,\ diffusive}} \sim \frac{h v_F}{L}\frac{h}{e^2}\frac{l_e^2}{L^2}$$

100 to 1000 times smaller than ballisitic

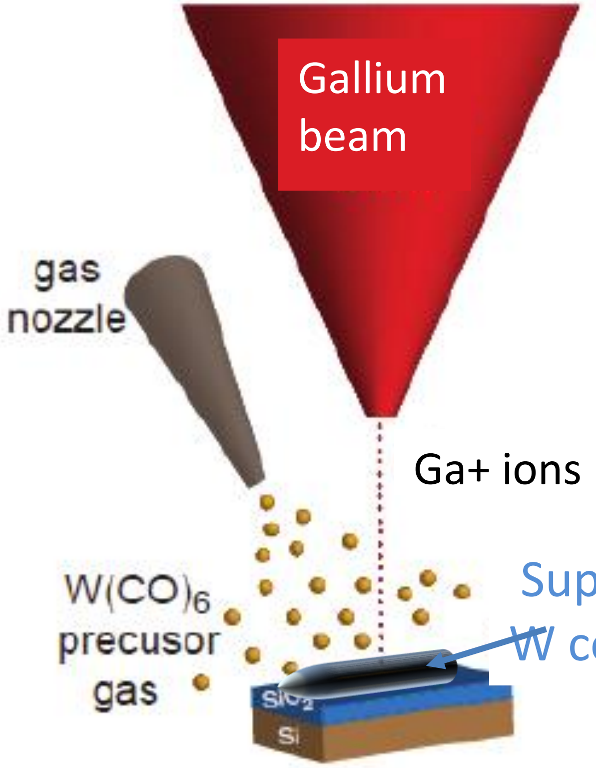
- In addition, Quantum spin Hall edges should have perfect transmission into S (not true of diffusive channels)



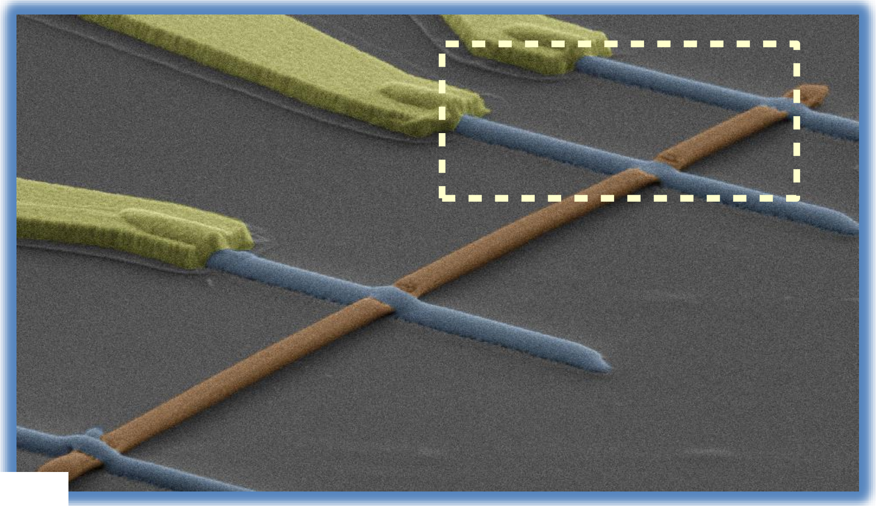
End of first course

Supercurrent through bismuth nanowire below 2 K

Kasumov 2005



Superconducting W compound

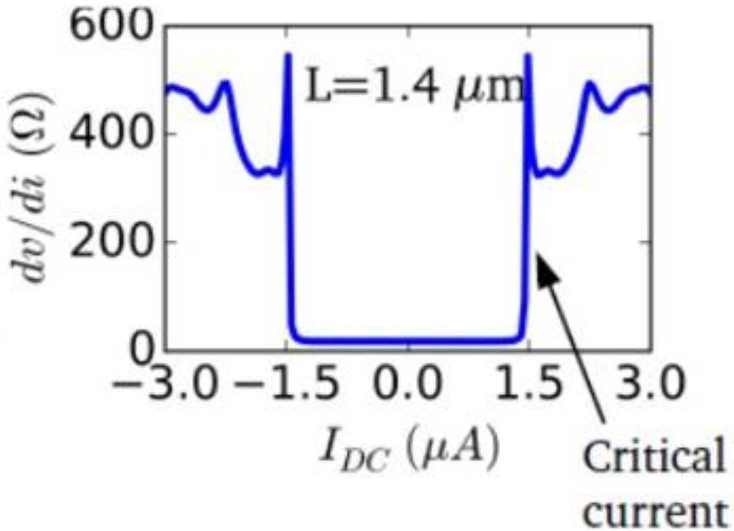


Bismuth nanowire with (111) surfaces

Superconducting W electrodes

T=100 mK

W/Bi/W junction



$I_{c,\ \text{short \& ballistic}} = \Delta/eR_q$ ~100 nA/channel

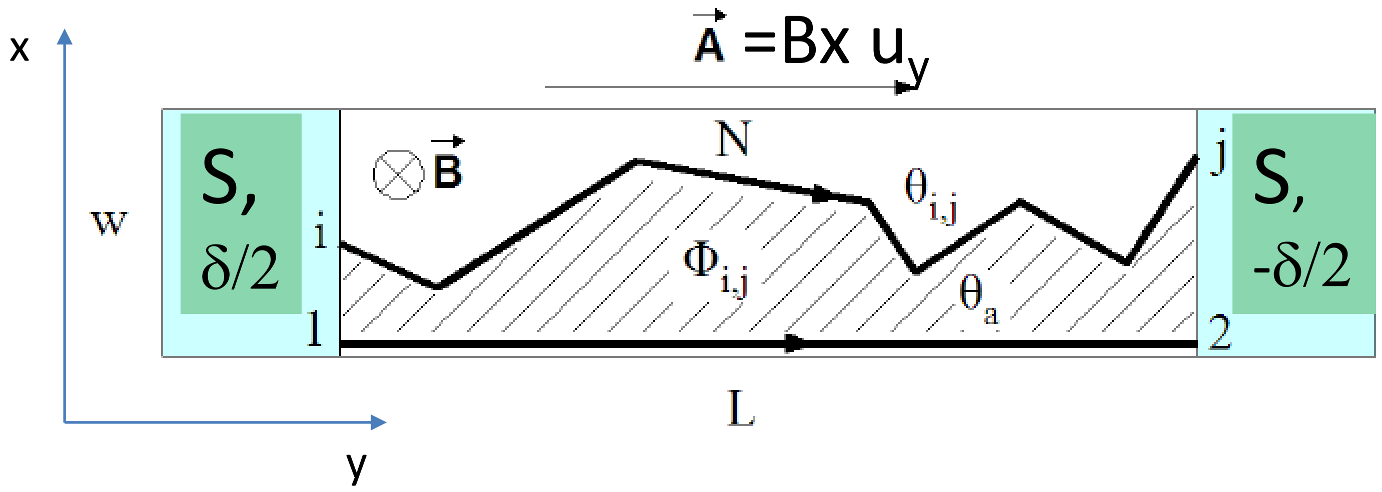
~ 15 well transmitted channels

Where does the supercurrent flow?

Superconducting contacts to exploit macroscopic wavefunction (and its phase): Interference experiments will reveal where supercurrent flows

Gauge invariant Josephson relation:

$$I(\delta) = I_0 \sin\left(\delta - \frac{2e}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{l}\right)$$



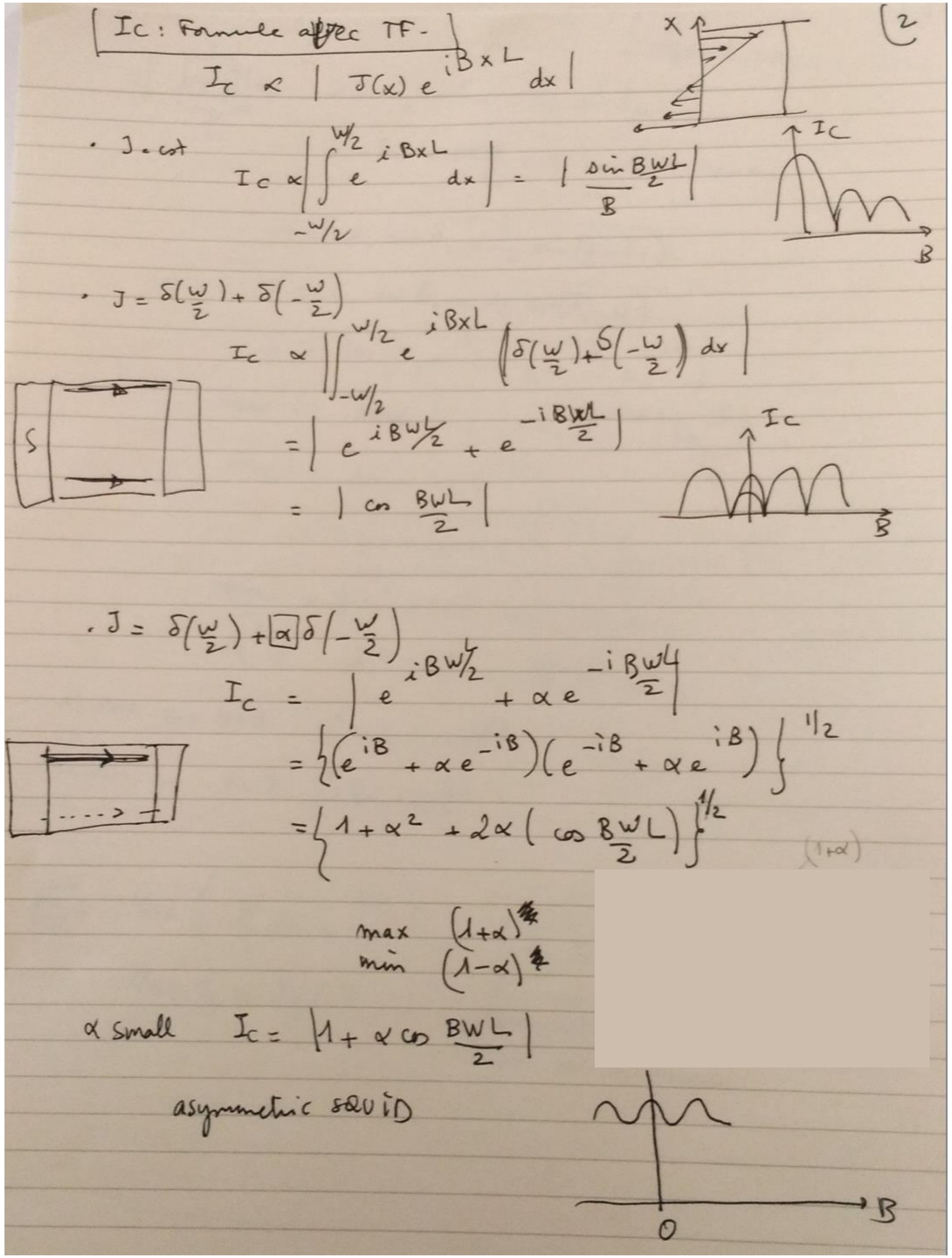
Critical current $I_c(B)$=max of integral over all supercurrent paths: interference terms!

$$I_c(B) = \left| \int_{-W/2}^{W/2} J(x)\cdot e^{2\pi i L B x/\Phi_0}\, dx \right|$$

Critical current $I_c(B)$=|Fourier transform of supercurrent distibution J(x)|

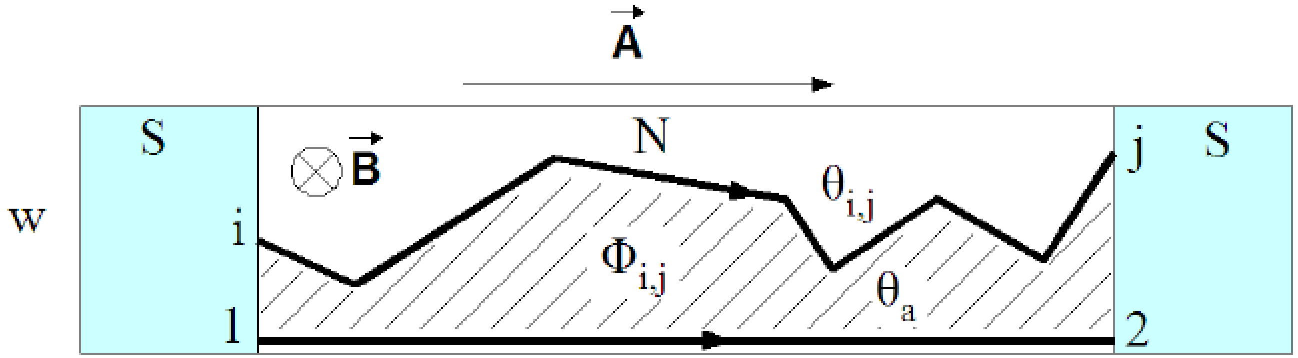
Known interference patterns for :

- Uniform current in wide junctions
- Current at two edges
- Current carried asymmetrically at two edges



- Fraunhofer
- SQUID
- asymmetric SQUID

Narrow diffusive sample with many channels: Flux-dependent phase variation in sample



Cuevas, Montambaux

$$I_c \propto \left| < e^{i\Delta\theta_{i,j}} >_{C_{i,j}} \right| \qquad I_c \propto \left| e^{-<(\Delta\theta_{i,j})^2>_{C_{i,j}}/2} \right|$$

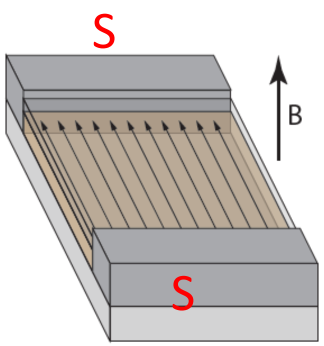
Diffusive trajectories encircle different flux, so pick up different phases

$$\Delta\theta_{i,j} = \frac{2e}{\hbar}\left[\int_i^j A_x dx - \int_1^2 A_x dx\right] = \frac{2e}{\hbar}\oint A_x dx = \frac{2\pi}{\Phi_0} H\, S_{i,j} = 2\pi\frac{\Phi_{i,j}}{\Phi_0} \quad (2.5$$

$$I_c \propto \left| e^{-\,2\pi^2 H^2 \alpha^2/\Phi_0^2} \right|$$

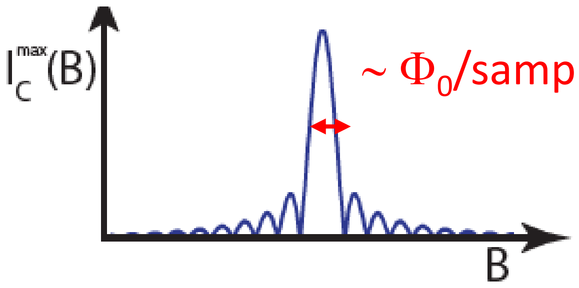
~ Gaussian decay of I_c on scale of Φ_0 because dephasing by field

Critical supercurrent reveals paths taken by pairs (via interference)



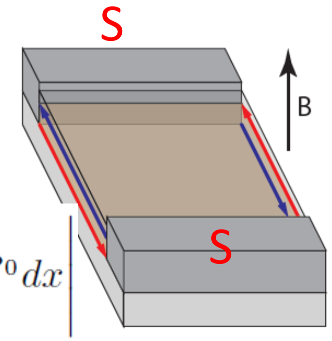
Many paths WIDE
(ballistic or diffusive)

$$I_c(B) = \left| \int_{-W/2}^{W/2} J(x) \cdot e^{2\pi i L B x / \Phi_0} dx \right|$$

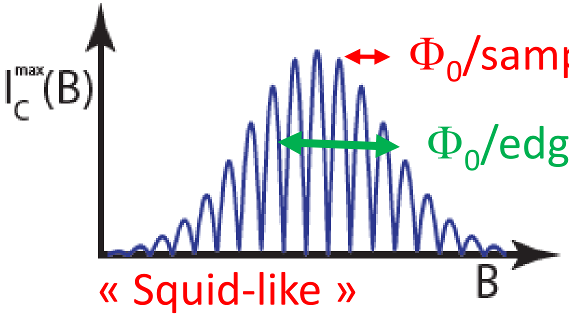


~ Φ_0/sample area

« Fraunhoffer pattern »



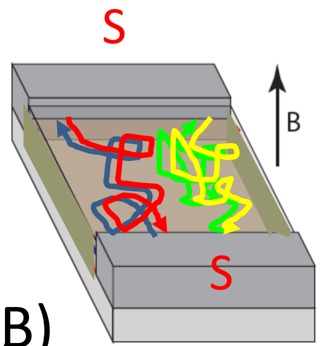
Only 2 paths (edges)
ballistic



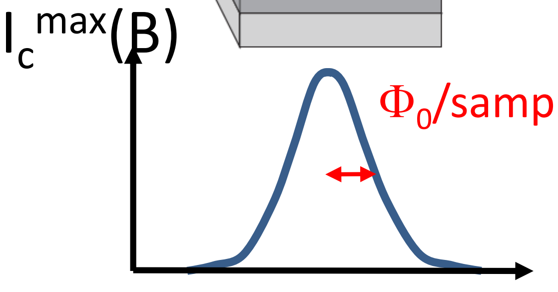
Φ_0/sample area

Φ_0/edge state area

« Squid-like »



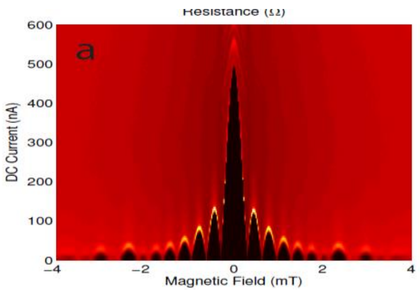
Many paths
NARROW,
diffusive



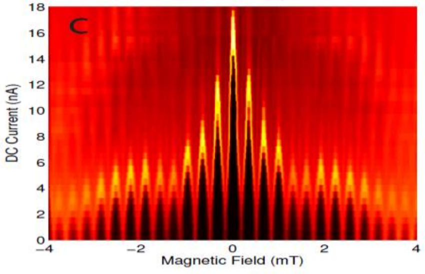
Φ_0/sample area

Many diffusive paths
Gaussian decay

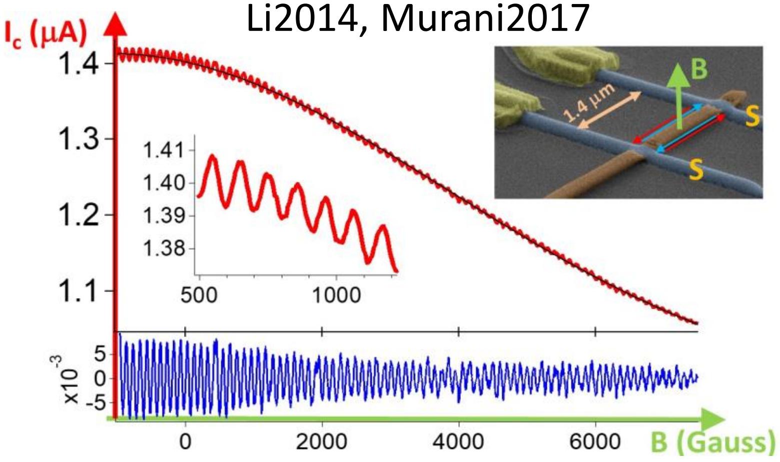
S/Non topo HgTe QW/S,
Hart 2014



S/Topo/S HgTe QW,
Hart 2014

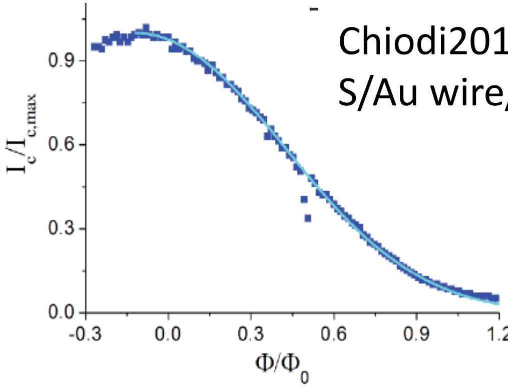


Bismuth nanowire,
Li2014, Murani2017



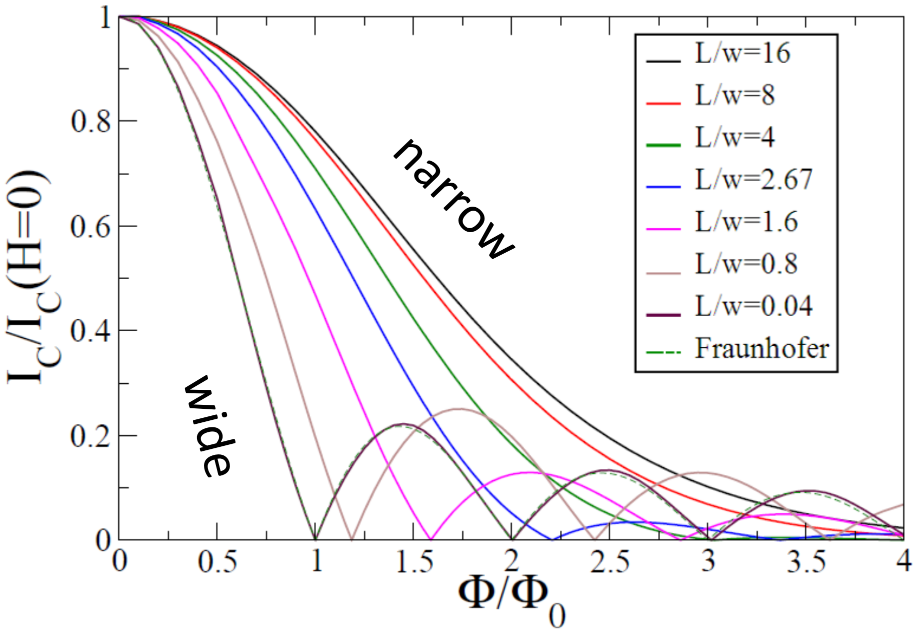
Extremely narrow edge state!!!

Chiodi2012
S/Au wire/S

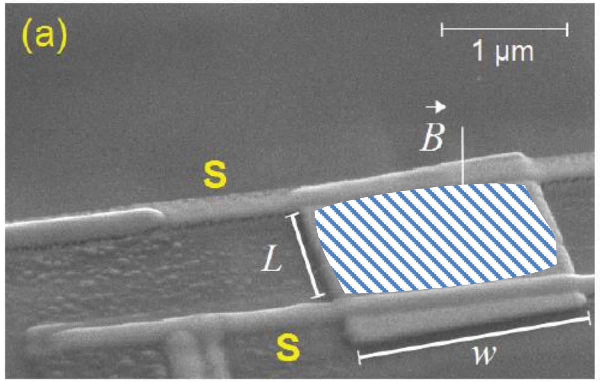


Role of geometry demonstated in (diffusive!) SNS junctions

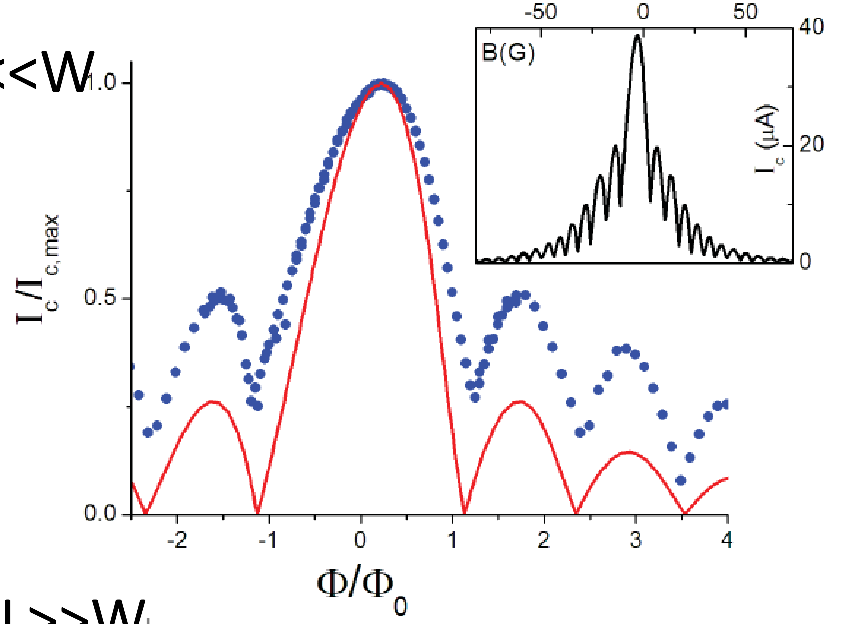
Theory: Bergeret Cuevas 2008

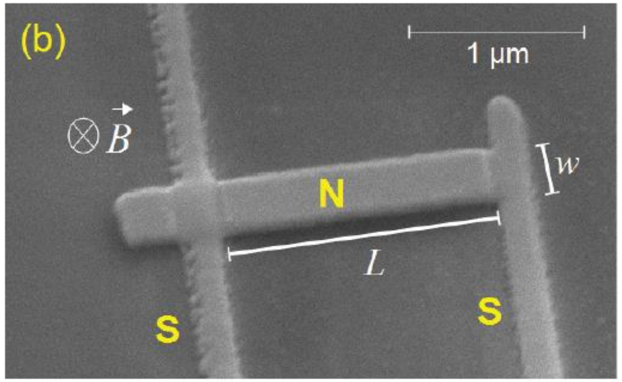


Exp: Chiodi 2012

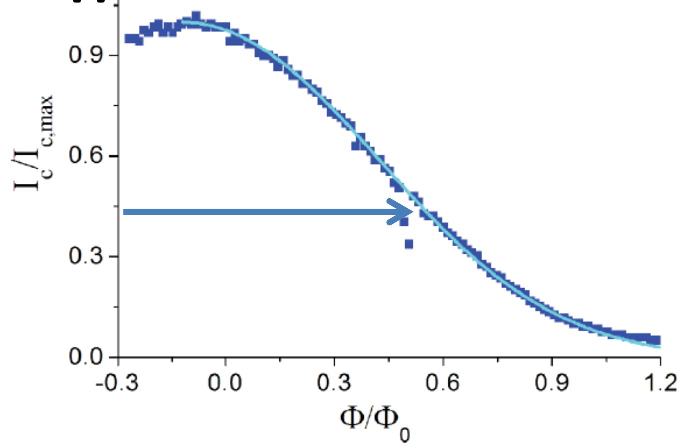


wide: L<<W





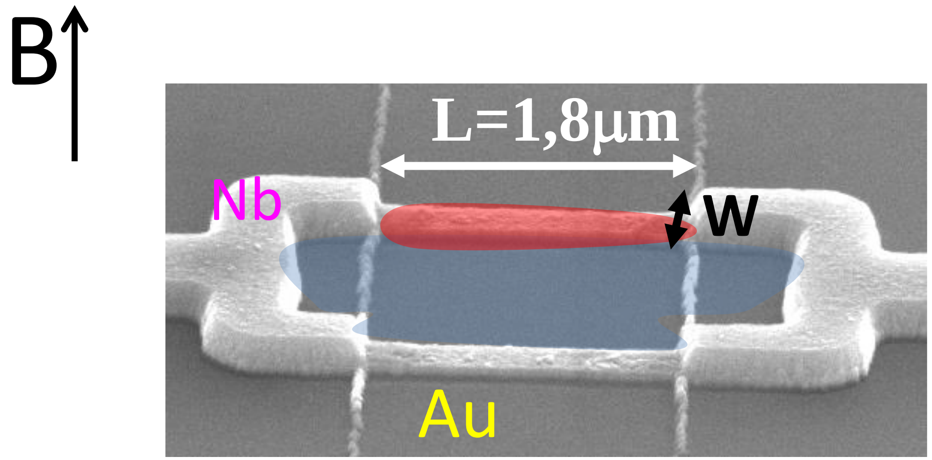
Narrow: L>>W



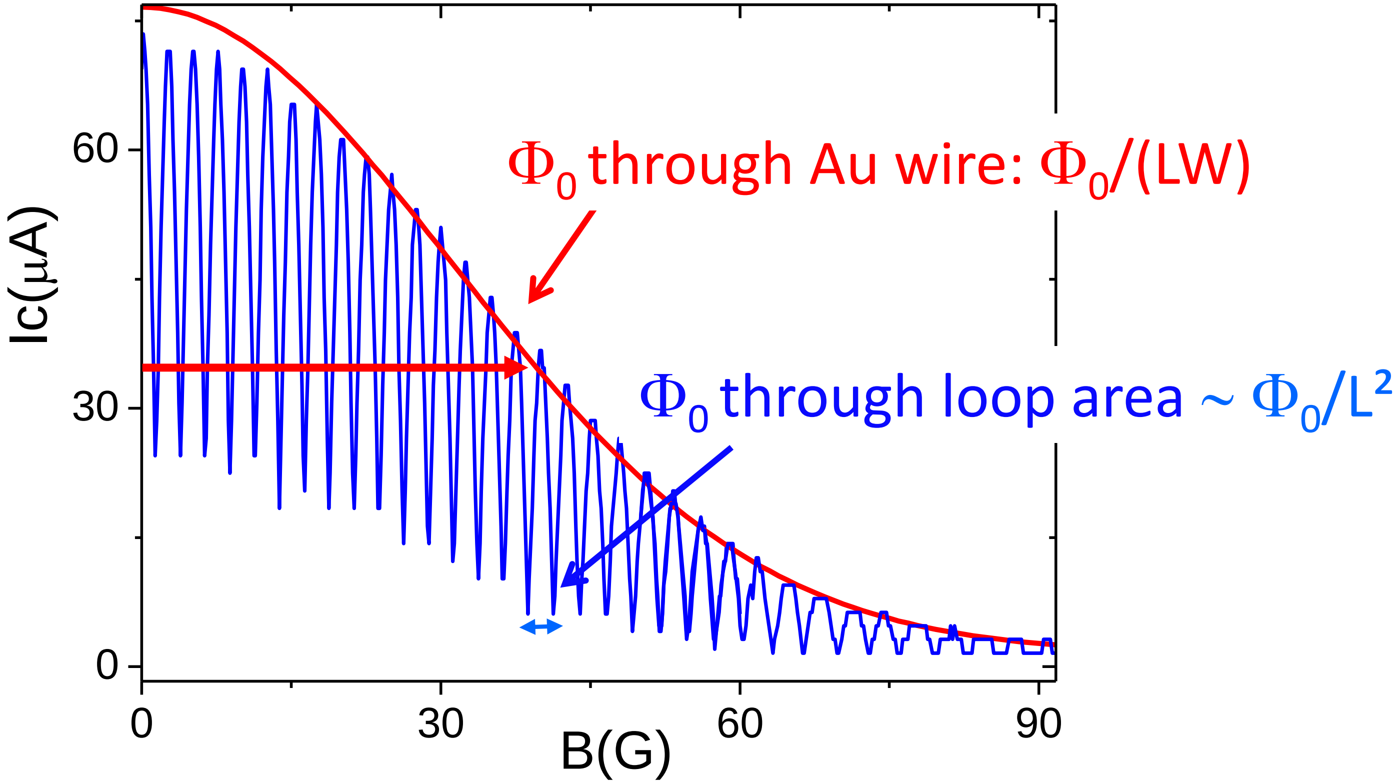
Depends on whether most dephasing by potential vector occurs at S interface or during propagation through N

I_c decays on scale of Φ_0 through sample surface(100 G)

SNS squid junction has it all



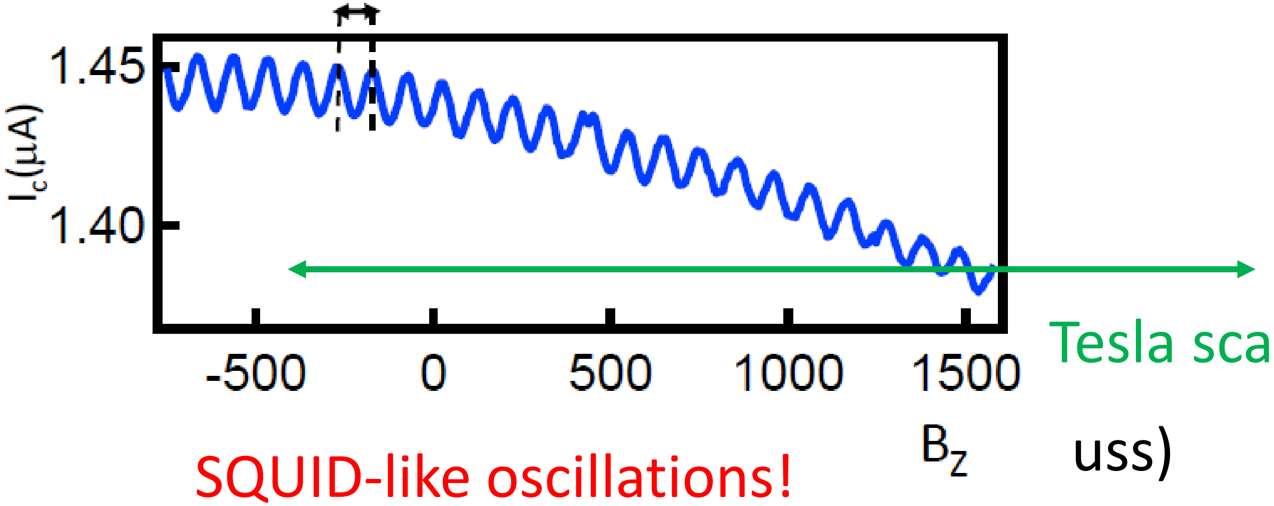
Angers 2008

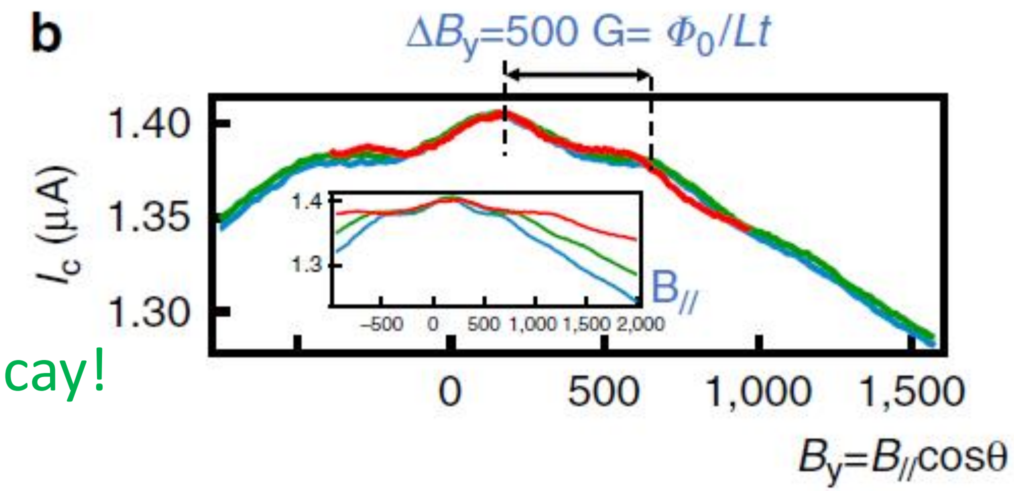


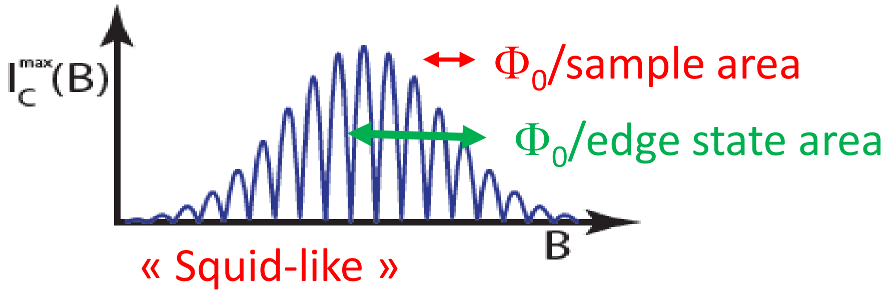
Modulation period a few G : loop area

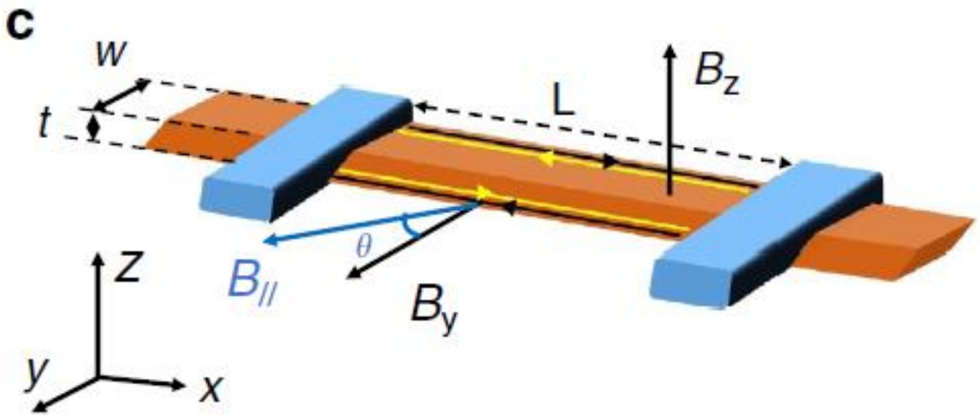
Decay scale ~50 G : current-carrying path area

What about bismuth nanowires?









- Oscillations: supercurrent travels at the two acute wire edges
- High field decay scale : narrow channels (nm!)
- High critical current : well transmitted channels

Beyond interference paths revealed by $I_c(B)$ of SNS junction

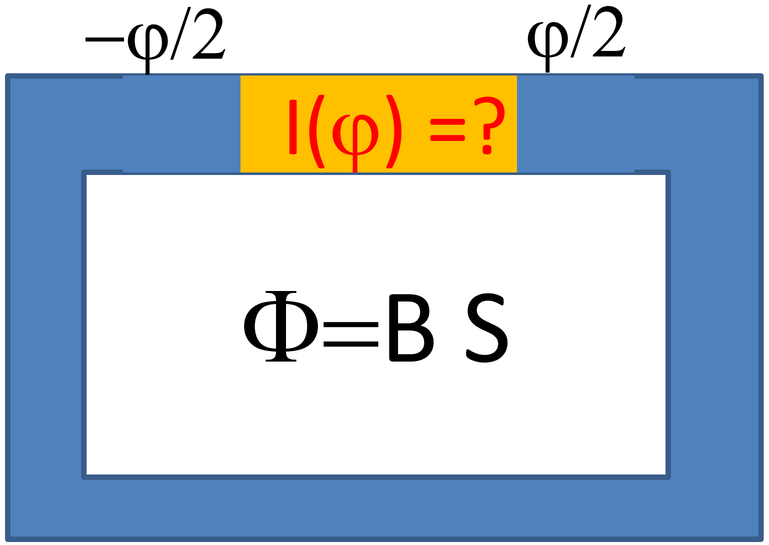
- There is a way to determine the transport regime in the N part (weak link)
- Need to reveal specific Andreev Bound States that form in weak link
- (Short) tutorial on Andreev Bound States and the supercurrent they carry
- The phase-biased configuration is essential

Better than critical current: supercurrent versus phase relation

Usual two contact SNS configuration

I_c= max I(φ), φ not controled

Better: Ring geometry allows «phase biasing»

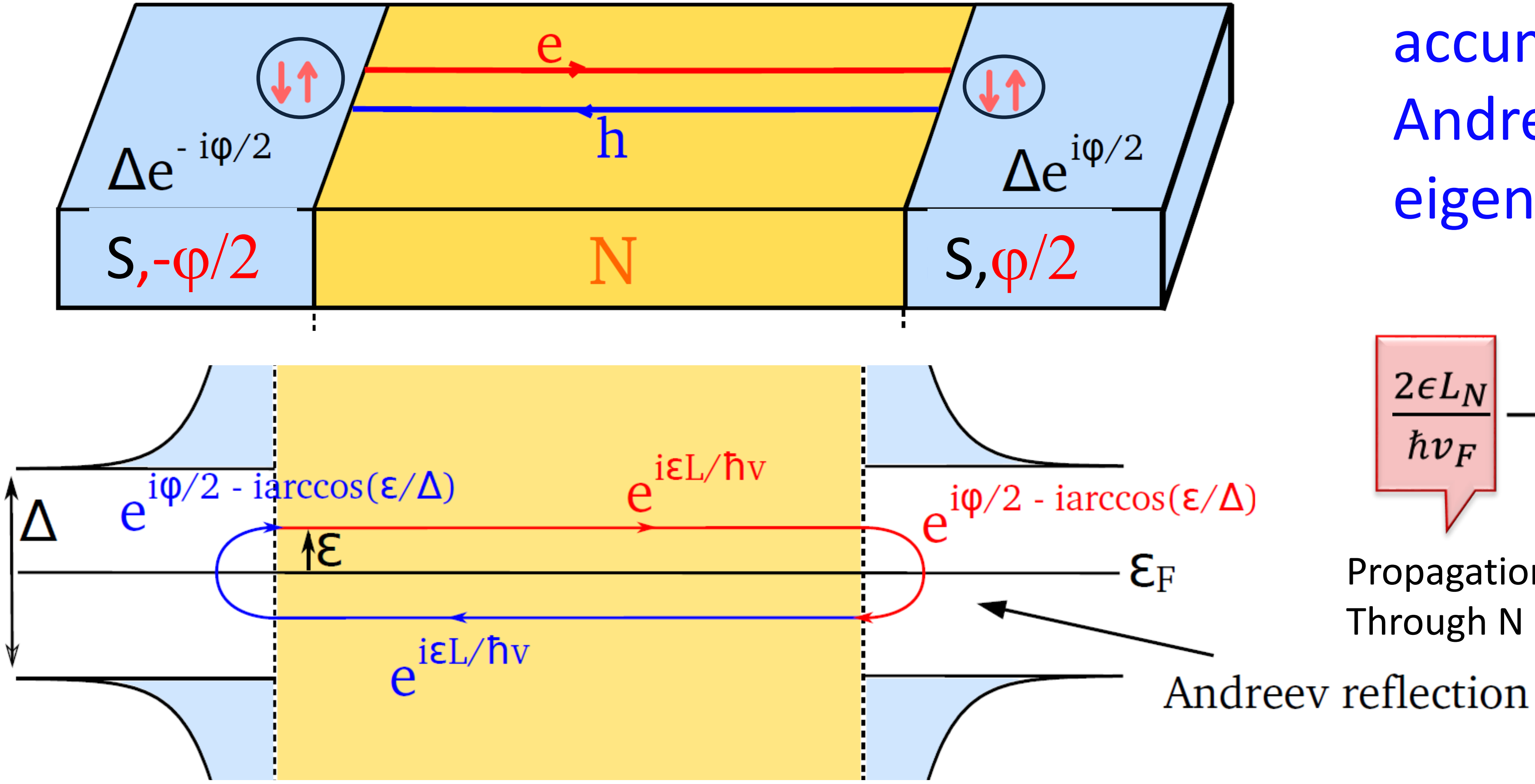


φ controled, proportional to applied magnetic flux

$$\varphi = -2\pi\Phi/\Phi_0$$

I(φ) depends on the transport regime in the N (diffusive, ballistic)

Andreev Bound States in a phase-biased SNS junction



Resonance condition on accumulated phase :
Andreev Bound States with eigenenergies ε_m.

$$\frac{2\epsilon L_N}{\hbar v_F} - 2\,arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

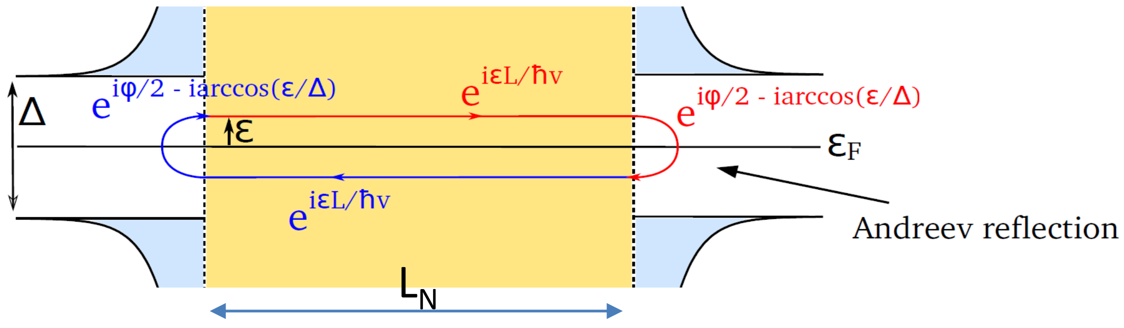
Propagation Through N

Interface reflection

Superconducting phase difference

Andreev bound states carry the supercurrent.
Spectrum, supercurrent, depend on N and phase

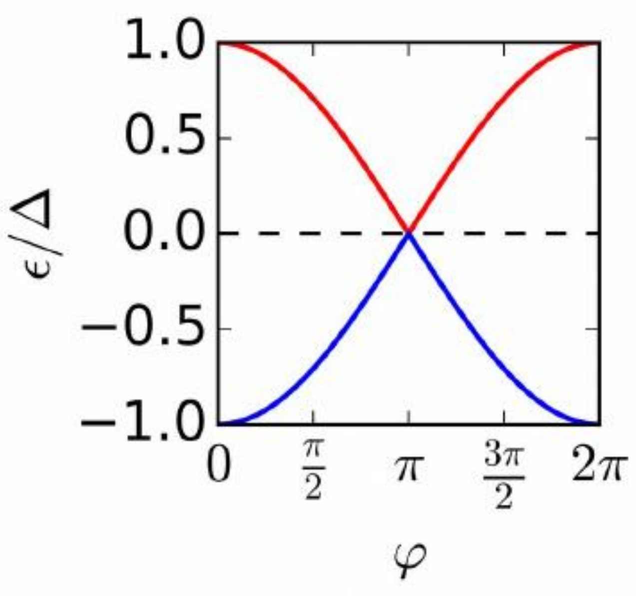
Andreev spectrum and supercurrent in short ballistic junction



$$\frac{2\epsilon L_N}{\hbar v_F} - 2\,arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

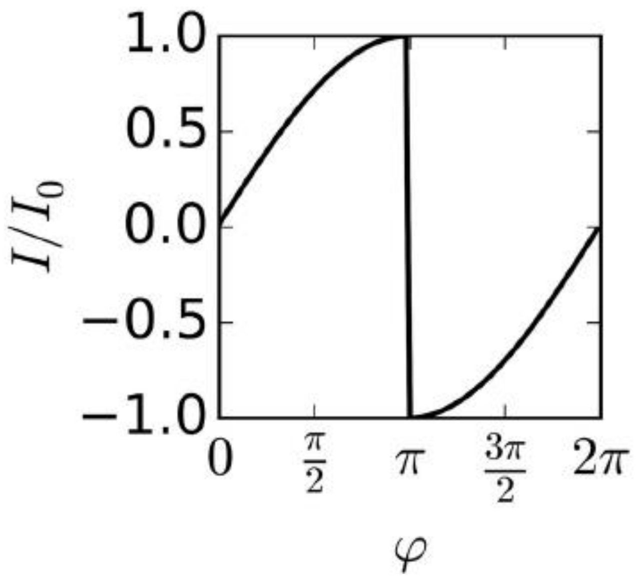
propagation

$\varepsilon_n(\varphi)$~branches of cos(φ/2)



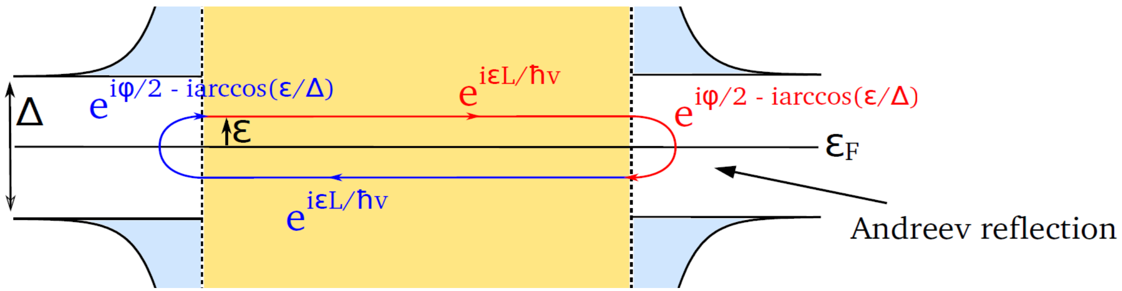
$$I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$

supercurrent



I(φ)~branches of sin(φ/2) with jump at π

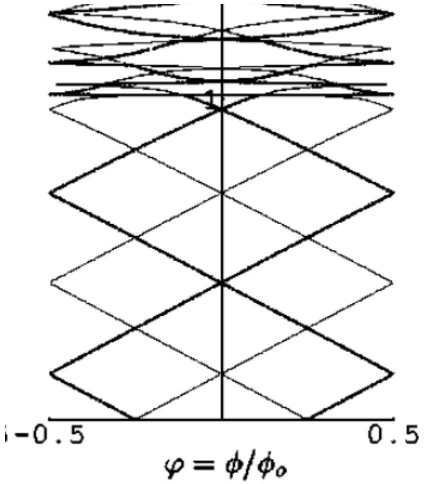
Example: Andreev spectrum and supercurrent in long ballistic SNS junction: L>> $\xi_s = \frac{hV_F}{\Delta}$



$$\frac{2\epsilon L_N}{\hbar v_F} - 2\,\arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

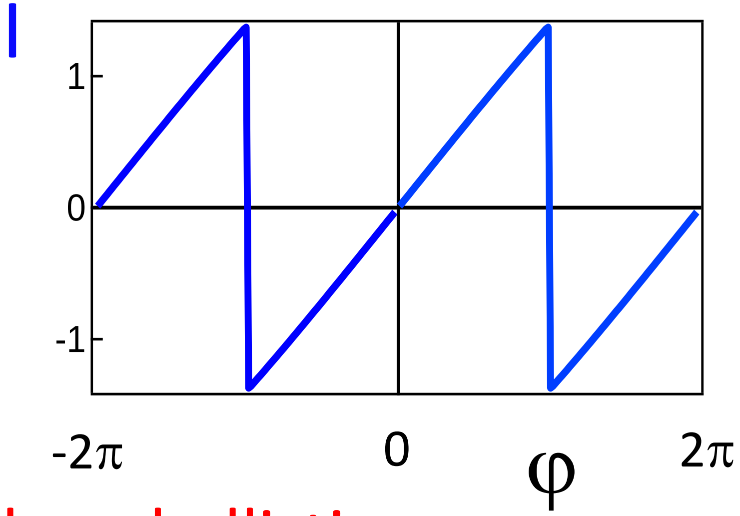
propagation

$\varepsilon_n(\varphi) \sim \varphi$: linear segments



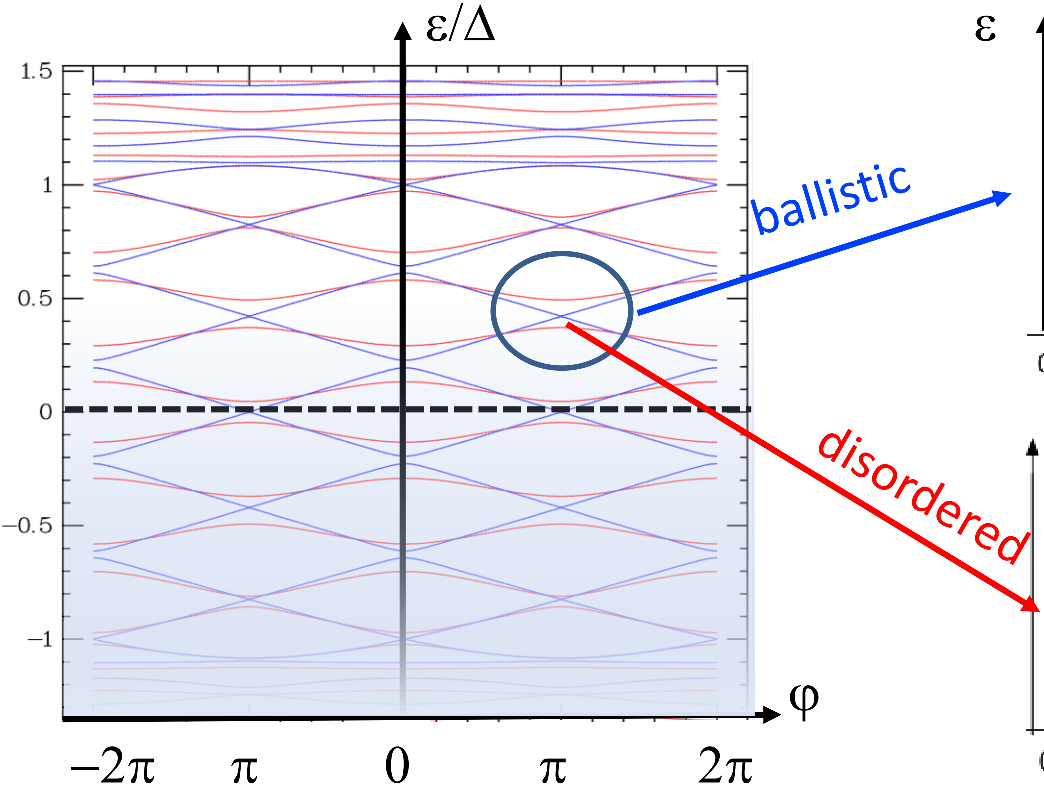
$$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$

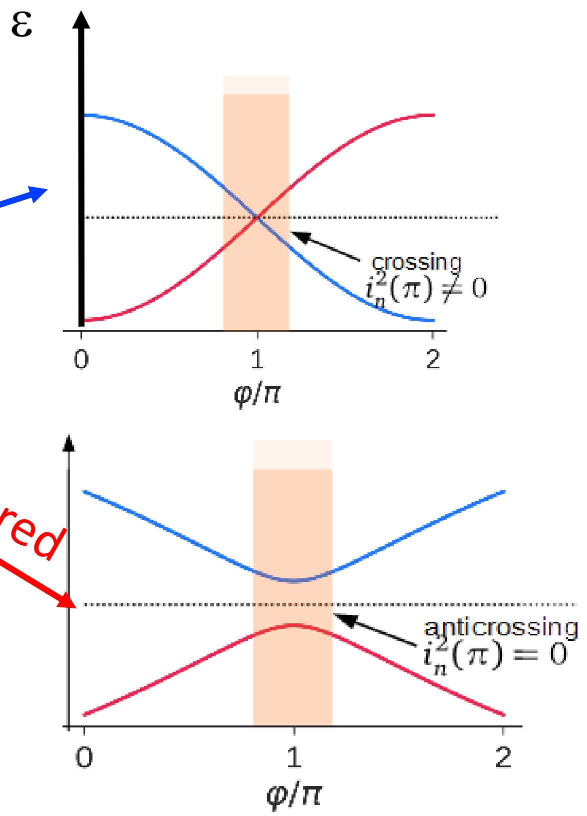
$I(\varphi) \sim$ linear segments with jumps at π



Sawtooth $I(\varphi)$ characteristic of long ballistic

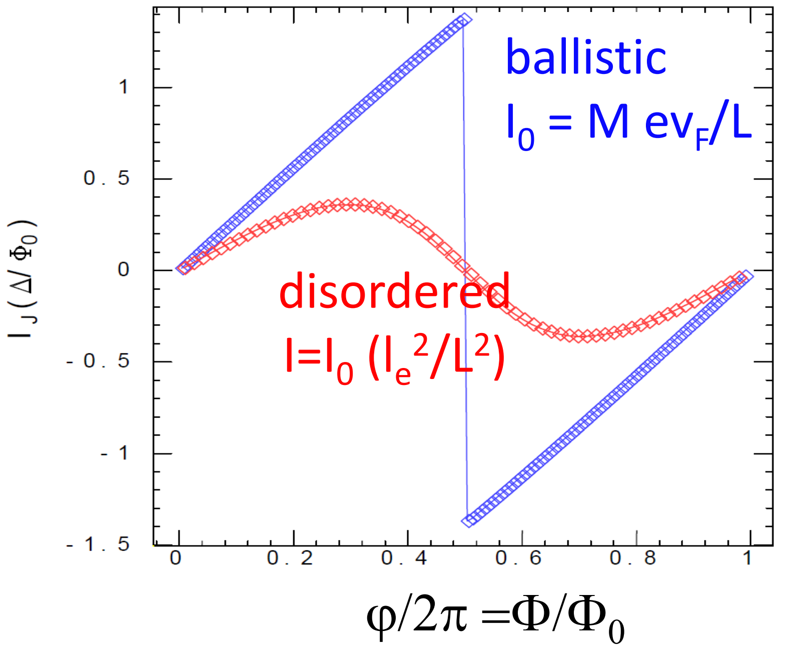
Influence of disorder on Andreev spectrum and supercurrent





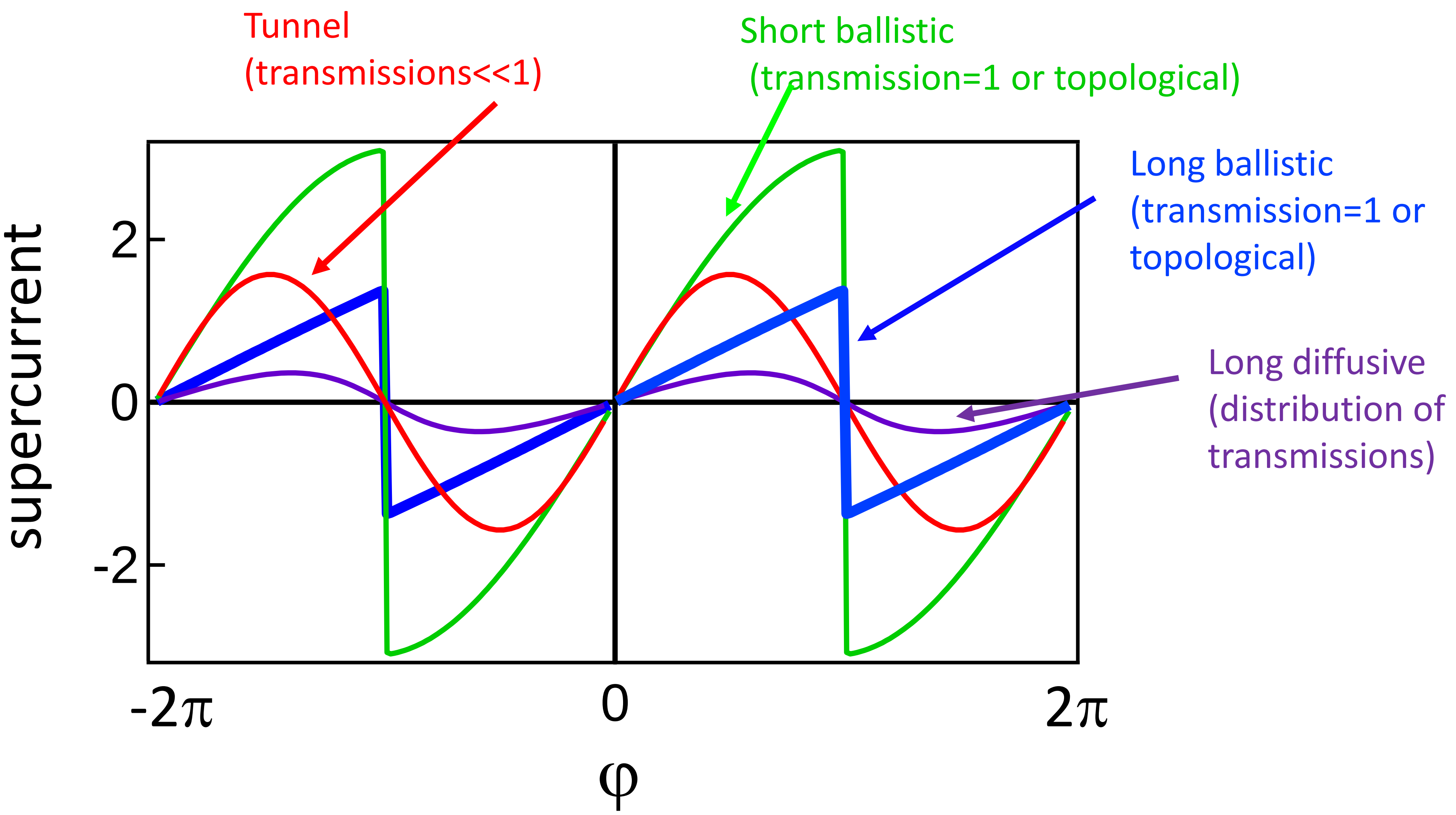
Dc supercurrent versus phase

$$I = \sum_{-\infty}^{\infty} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$



Disorder lifts Andreev level degeneracy at π and rounds I(φ)

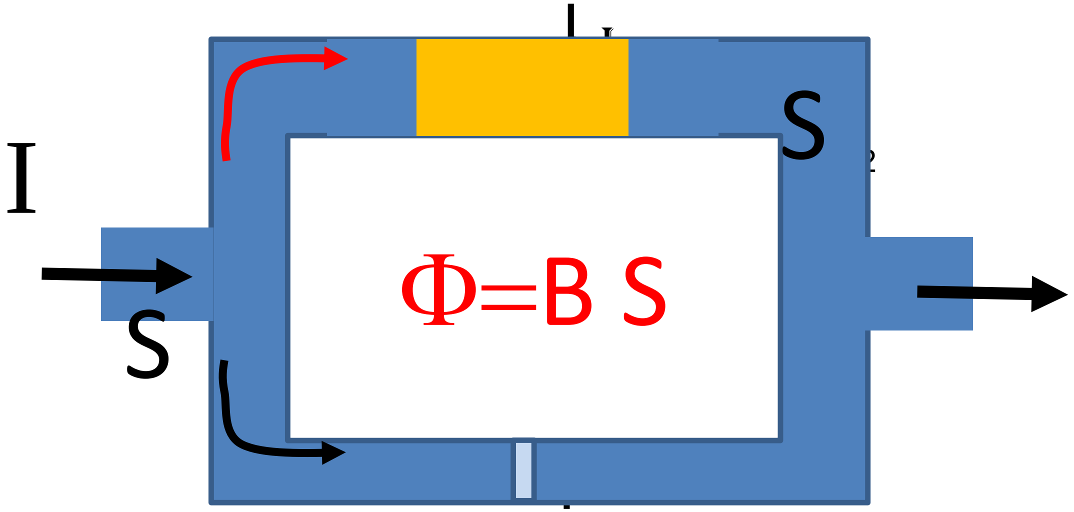
Supercurrent vs phase: expectations



Current-phase measurement with an asymmetric SQUID
Need ring geometry and second junction

$I_2=I_{c2}f(\varphi_2)$, f unknown

Della Rocca et al 2007



$I_1=I_{c1}\sin\varphi_1$ (high I_{c1})

$$I=I_{c1}\sin\varphi_1+I_{c2}f(\varphi_2)$$

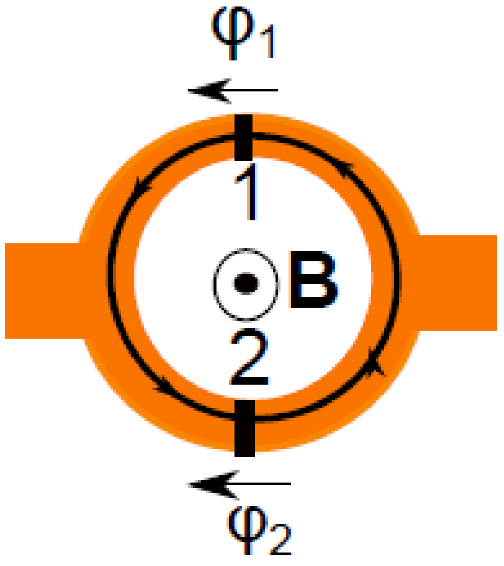
$\varphi_1-\varphi_2=-2\pi\Phi/\Phi_0$

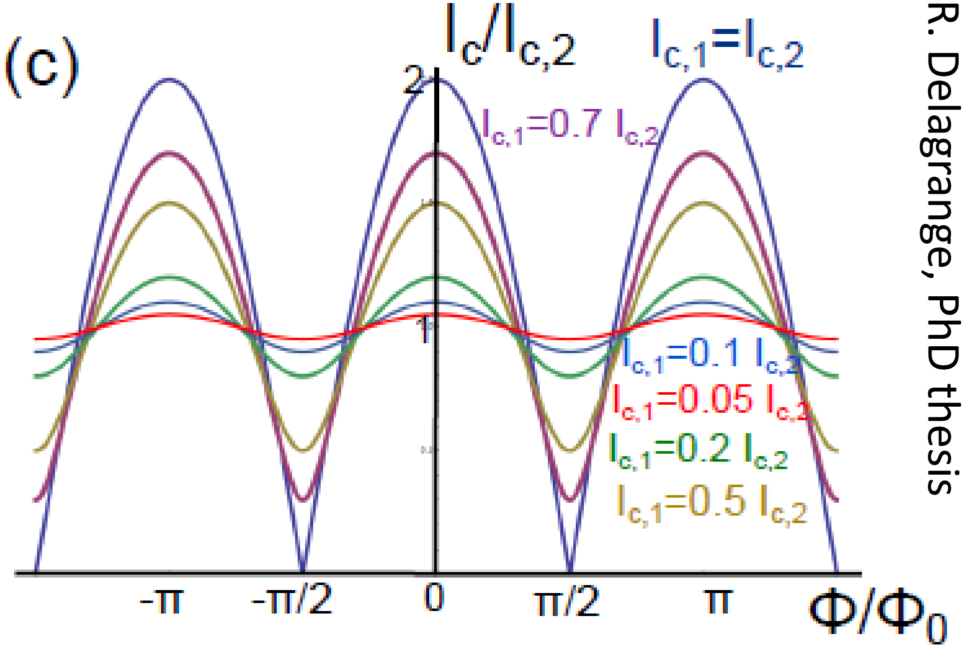
I_c achieved for $\varphi_1\approx\pi/2$

$$I_c \sim I_{c1}+I_{c2}f\left(\frac{\pi}{2}+2\pi\frac{\Phi}{\Phi_0}\right)$$ to first order in I_{c2}/I_{c1}

Critical current I_c of asymmetric SQUID yields current-phase relation $I_{c2}f(\varphi_2)$ of junction with smallest critical current

Critical current of a SQUID: from symmetric to asymmetric



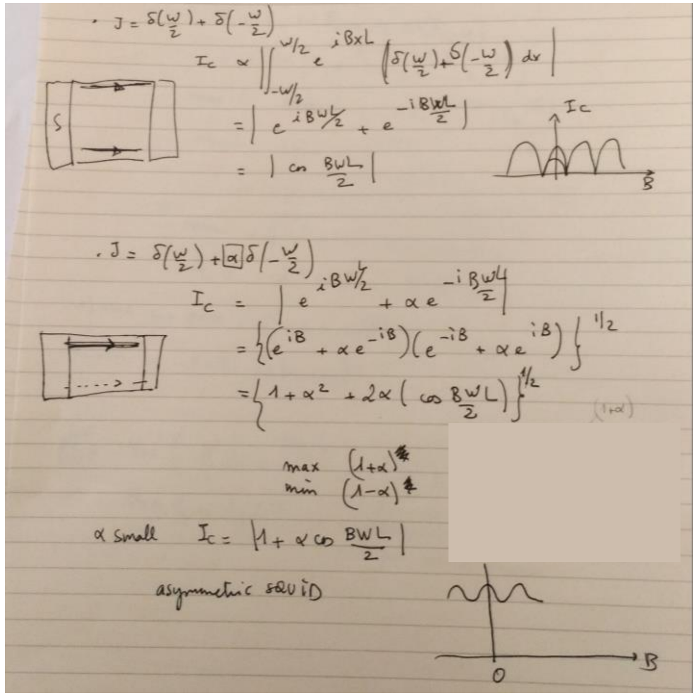


R. Delagrange, PhD thesis

$$I_c^2 = (I_1 - I_2)^2 + 4 I_1 I_2 \cos^2\left(\pi \frac{\Phi}{\Phi_0}\right)$$

Asymmetric: $I_2 << I_1$

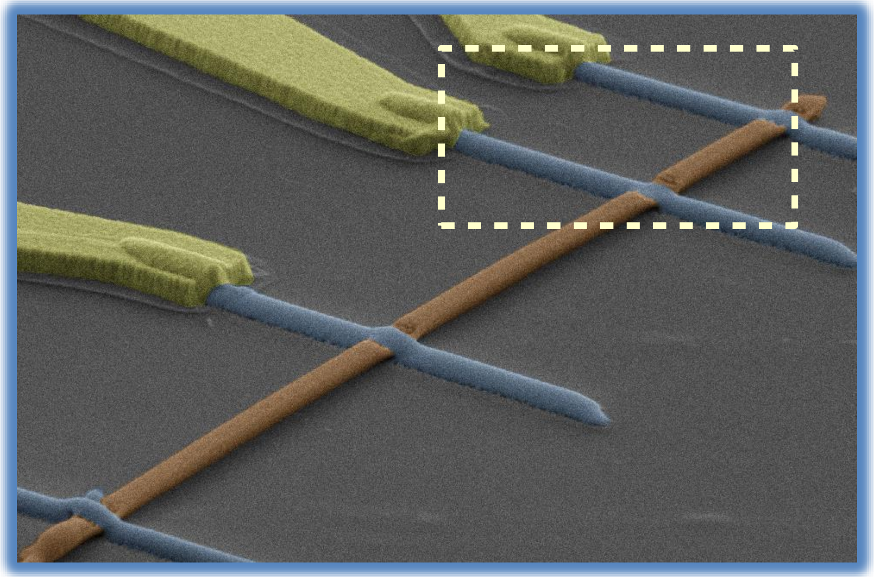
$$I_c = I_2 + I_1 \sin\left(2\pi \,\Phi/\Phi_0 + \pi/2\right)$$

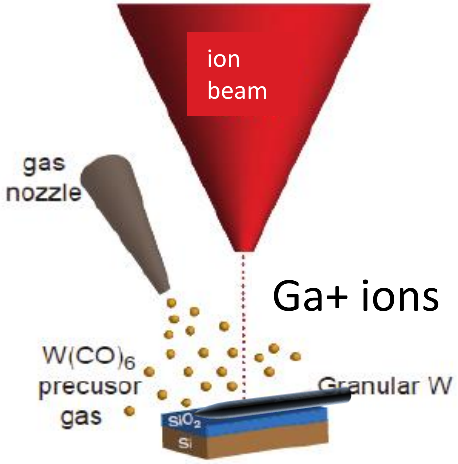


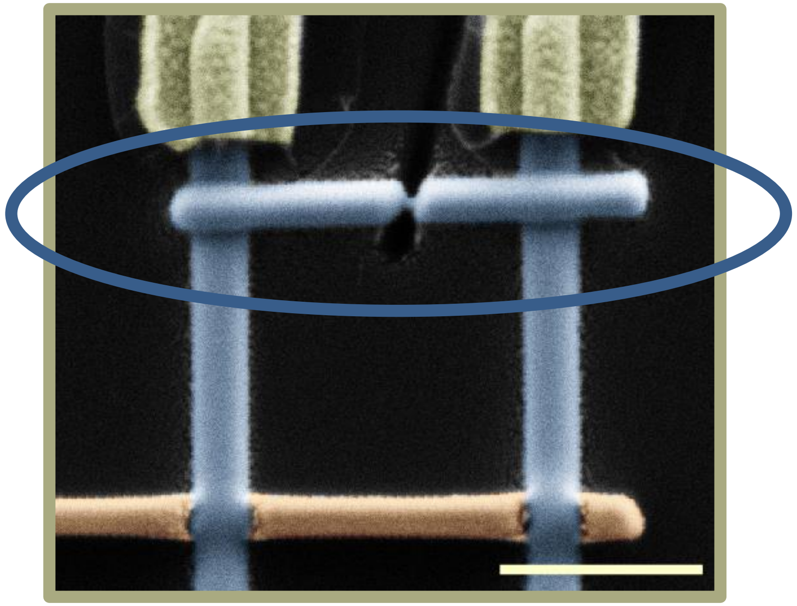
Current-phase relation of junction with smallest critical current (on top of the critical current of largest junction)

Measurement of current-phase relation to test channels that carry the supercurrent (on very same sample)

Add superconducting constriction in parallel



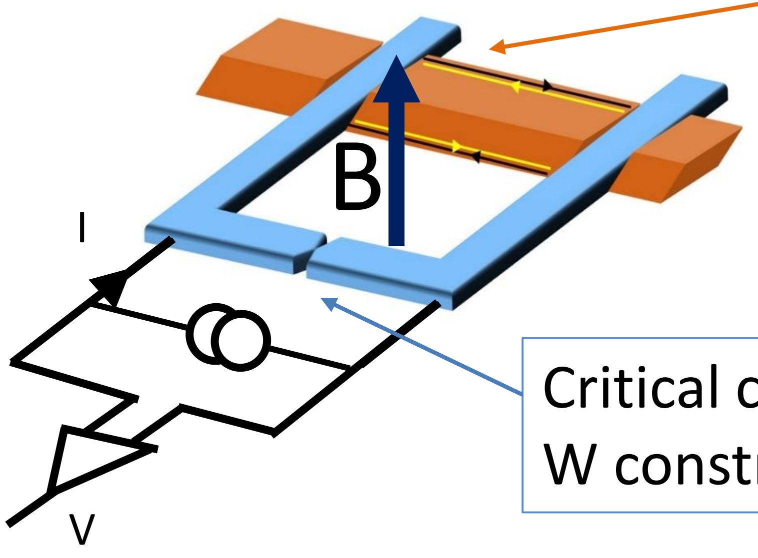


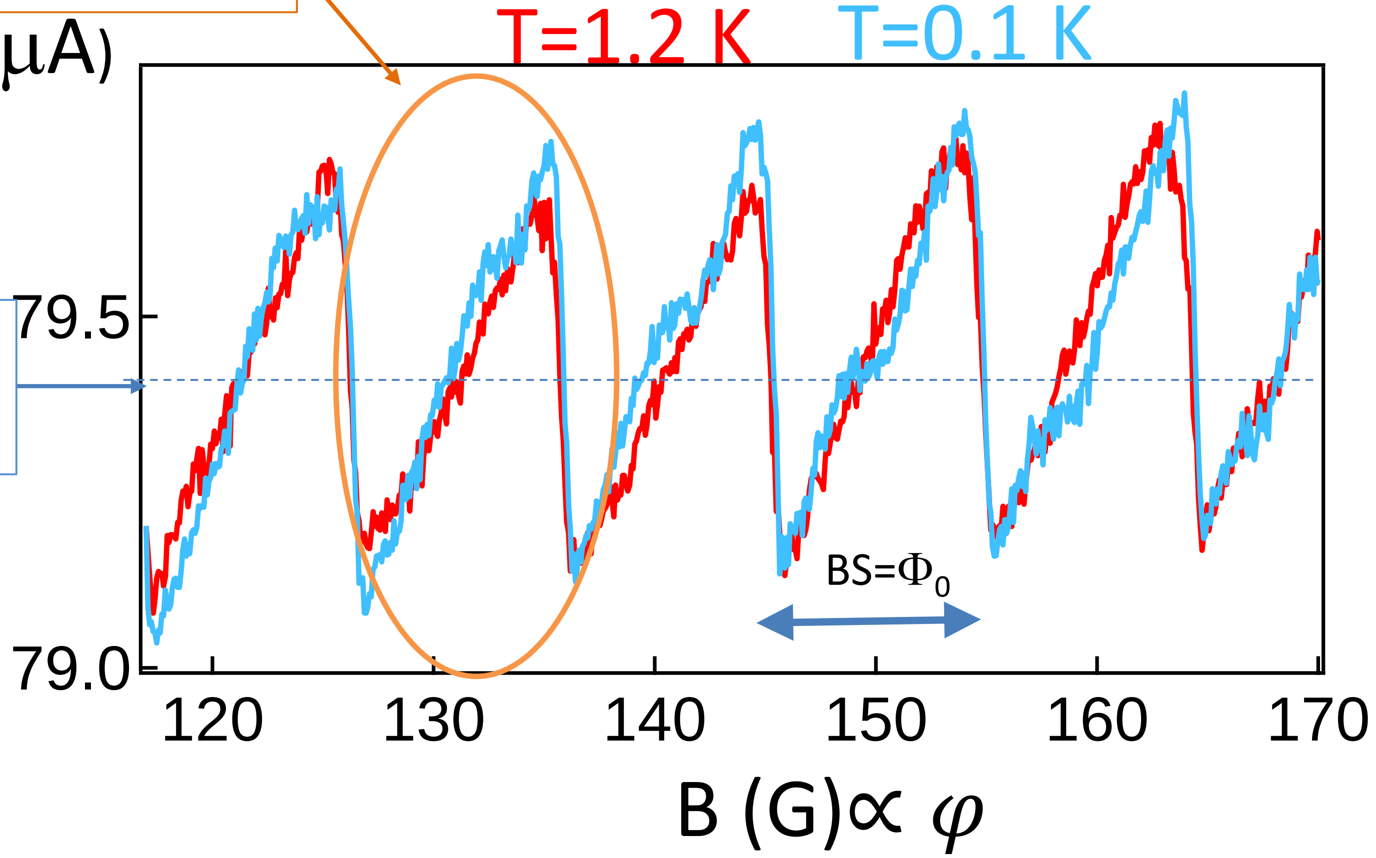


1 μm

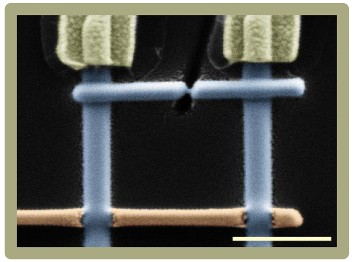
Build an asymmetric SQUID to measure the I(φ) relation

Supercurrent-versus Phase relations of S/Bi/S: switching current as a function of magnetic flux





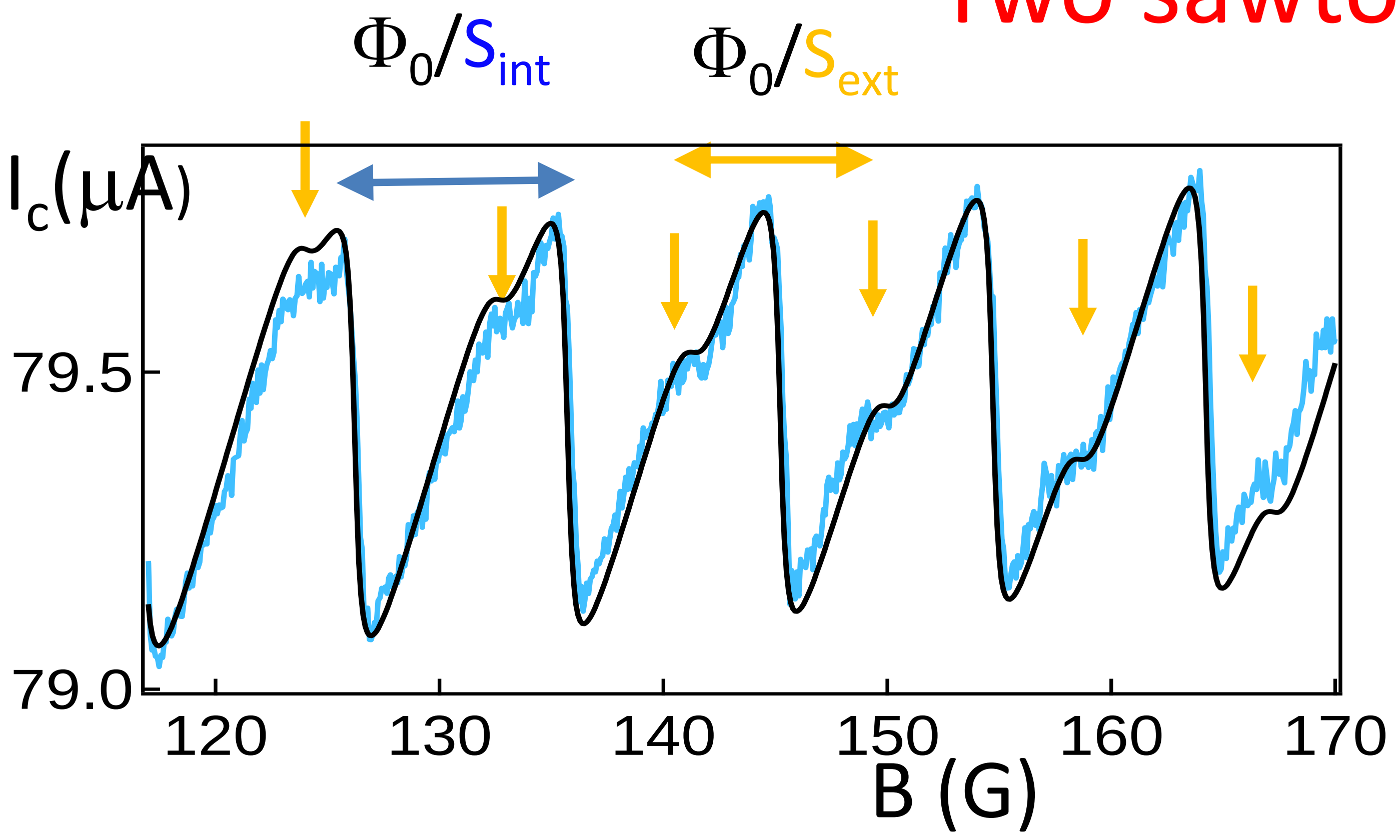
$\varphi = -2\pi BS/\Phi_0$

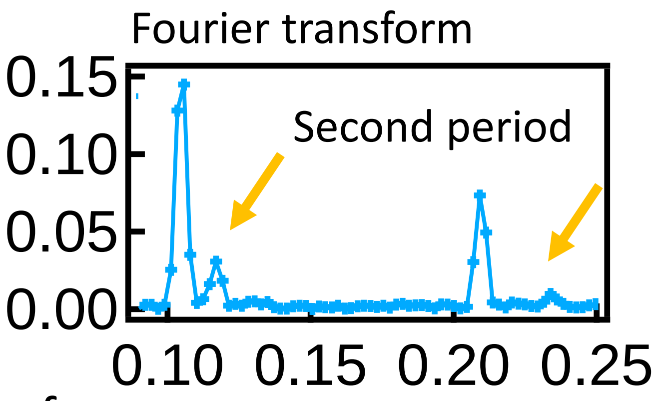


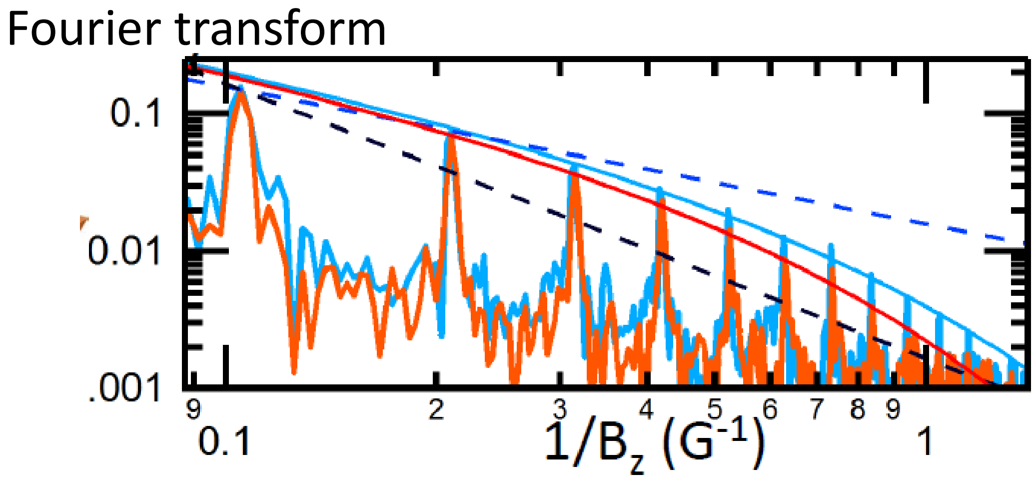
1 μm

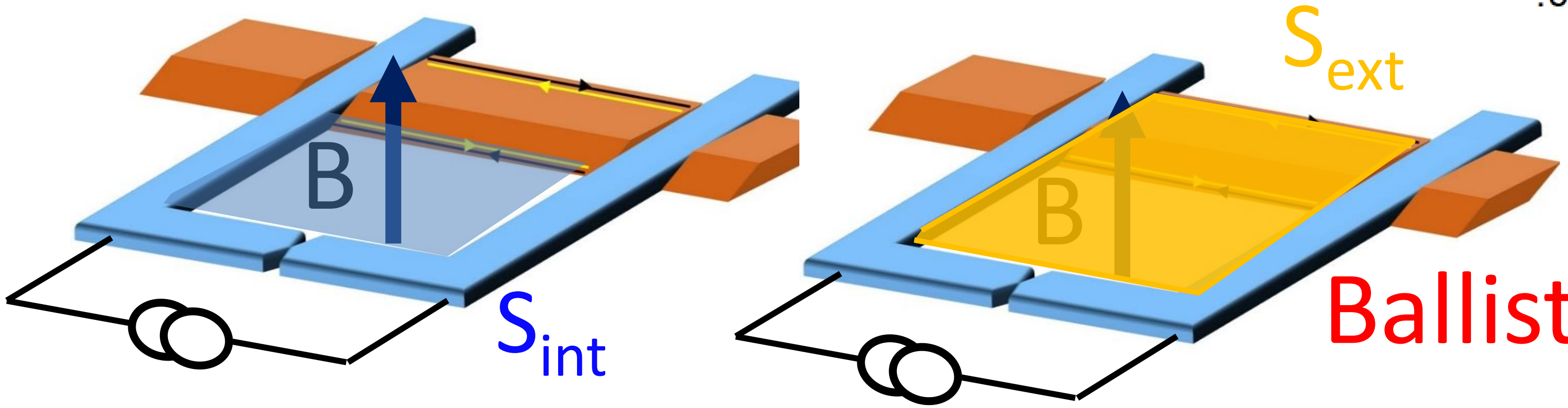
Sawtooth-shaped current phase relation: long ballistic!

Two sawtooths?





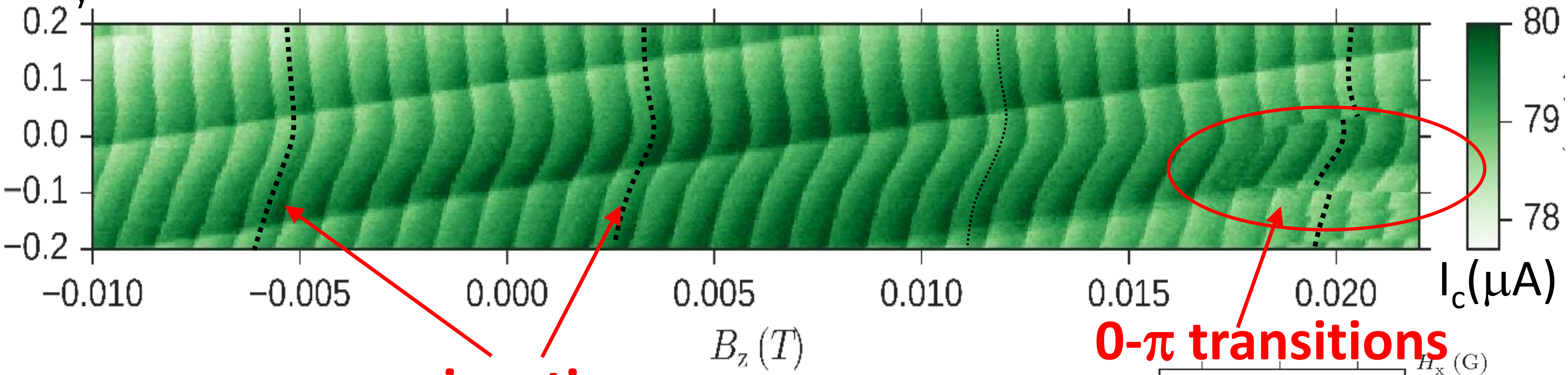




Ballistic states at two edges

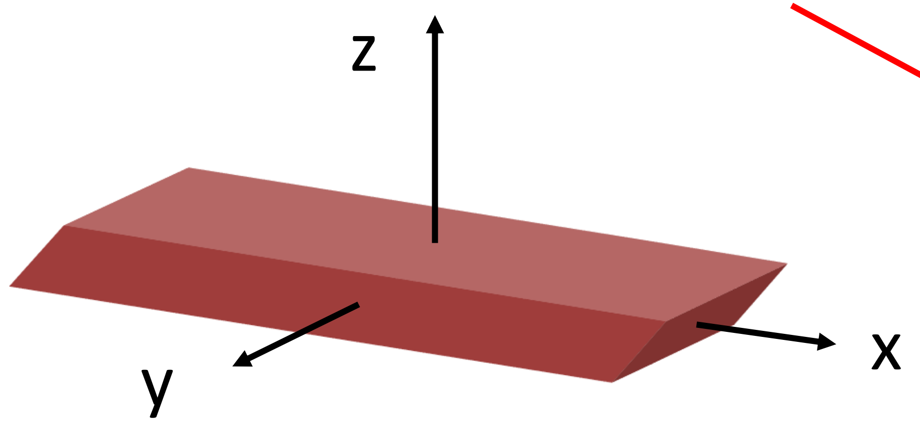
In plane magnetic field affects the phase of the I(φ)

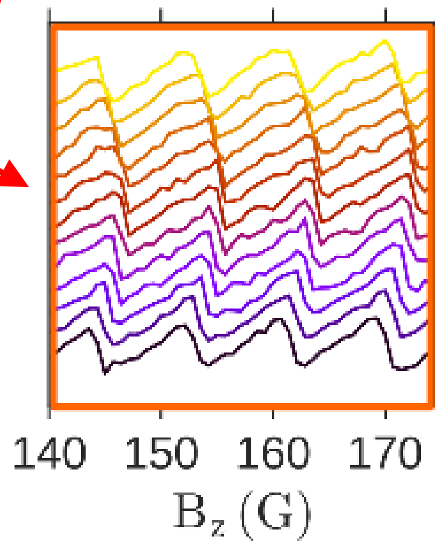
The phase wiggles and jumps (Linear variation of By with Bz subtracted)

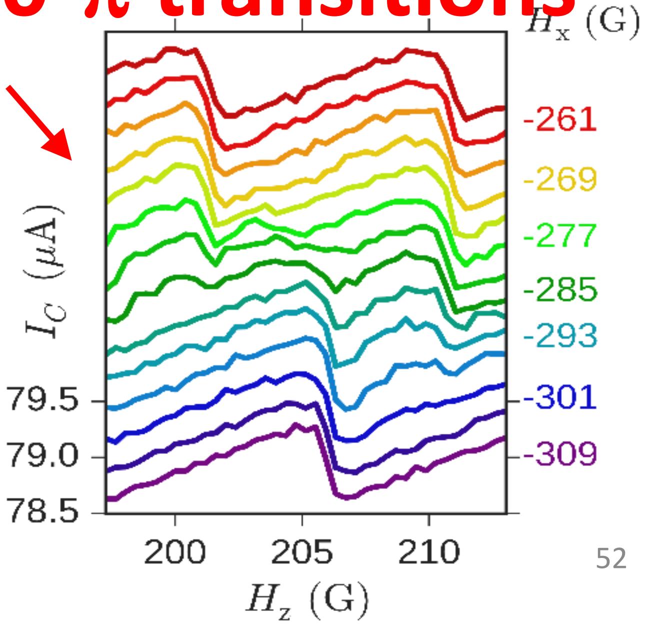


0-π transitions

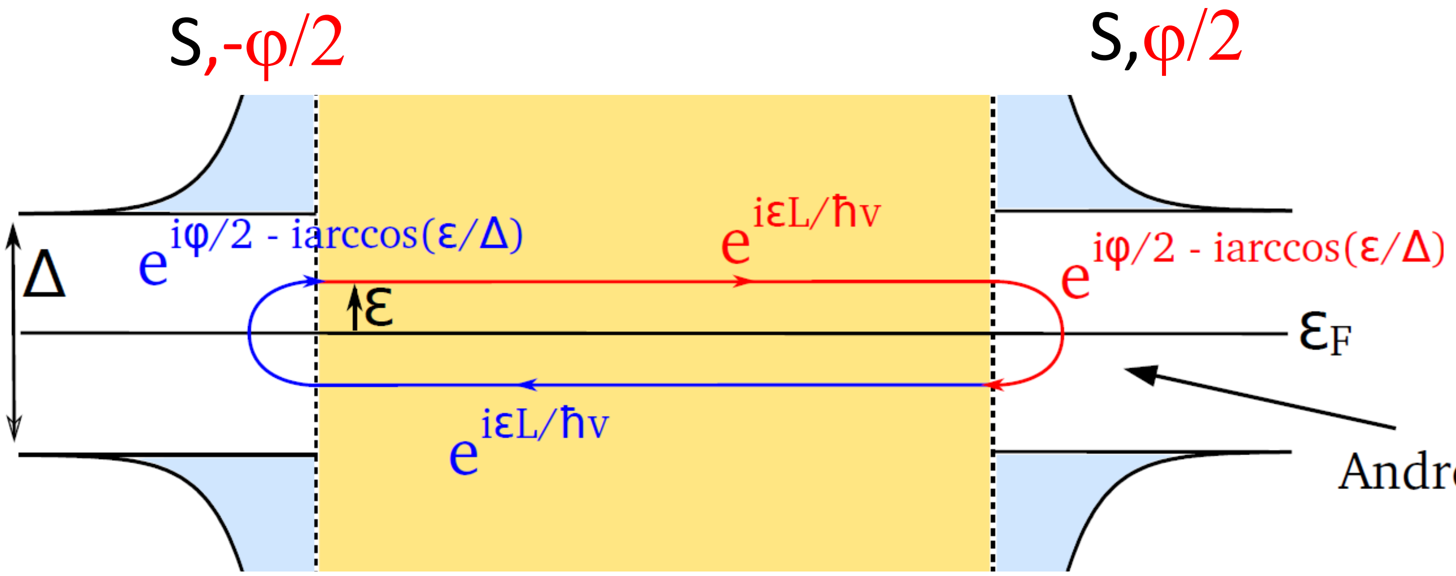
φ_0 junctions







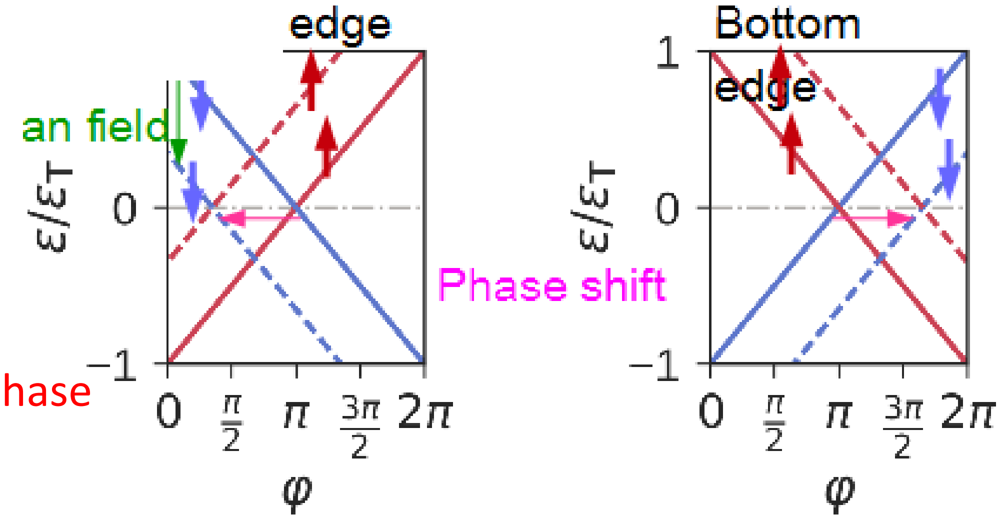
Effect of magnetic field on Andreev states



Resonance condition on accumulated phase : Andreev Bound States

$$\frac{\pm g\mu_B BL}{\hbar v_F} + \frac{2\epsilon L_N}{\hbar v_F} - 2\,arccos\frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

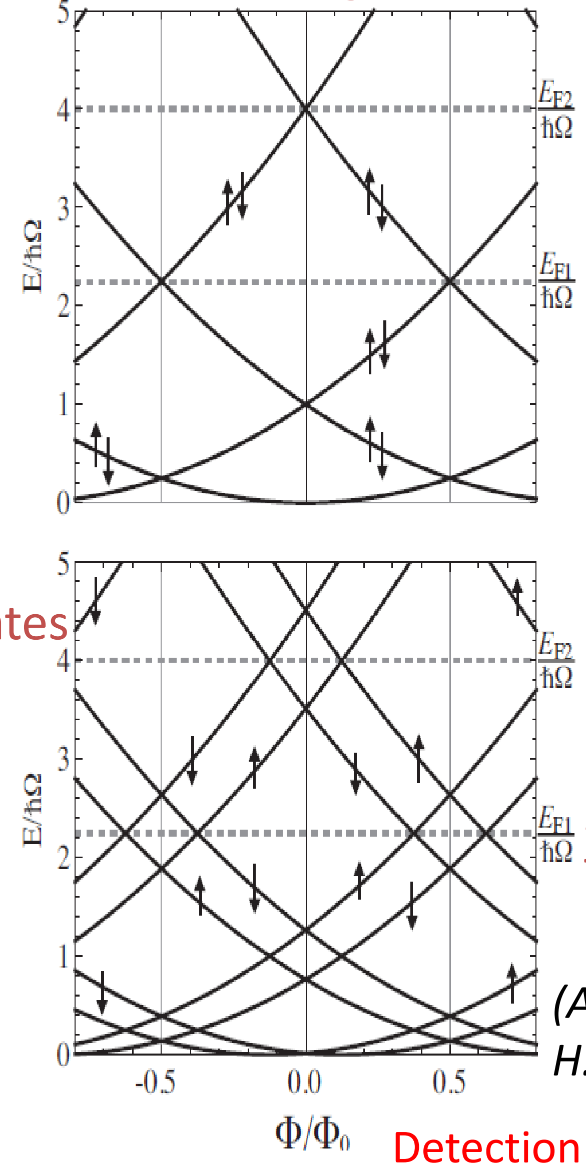
Zeeman effect | propagation | Interface reflection | Superconducting phase difference



Andreev spectrum splits with field,
and shifts if spin-orbit scattering, because spin-dependent v_F

Flux dependent spectrum of a ring in the presence of spin orbit

1D N ring



Phase shifted

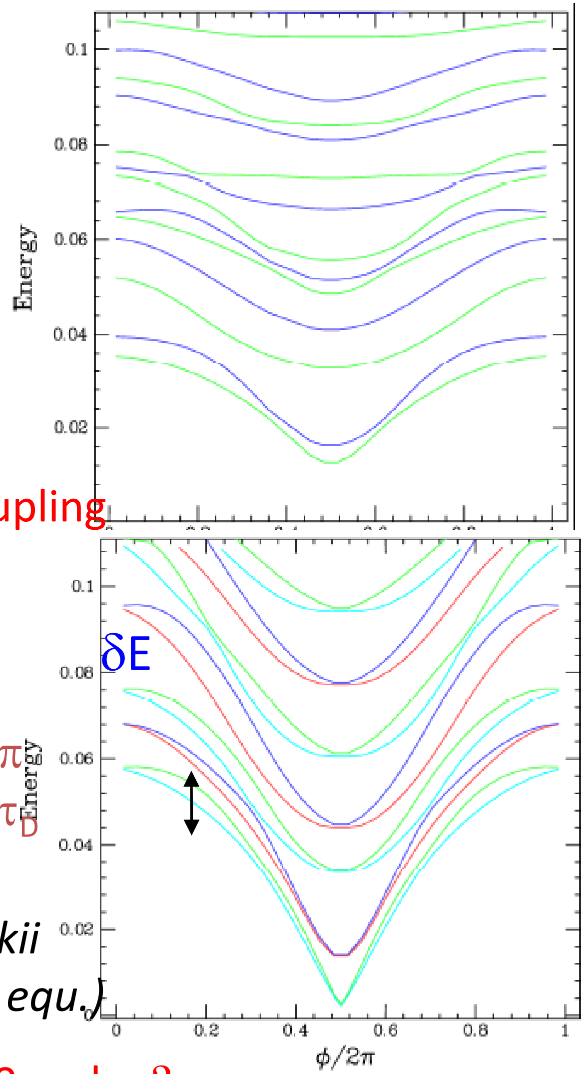
Spin ↑ and ↓ states

$\delta\phi = \pm k_{so}L$

Multichannel

NS ring Andrev states invariant by TRS

No SO coupling



With Rashba SO coupling

$\lambda = E_{SO}/E_F = 0.3$

decreased effective disorder

spin split AS at $\phi \neq 0, \pi$

TRS breaking $\delta E = \lambda\Delta^2\tau_D$

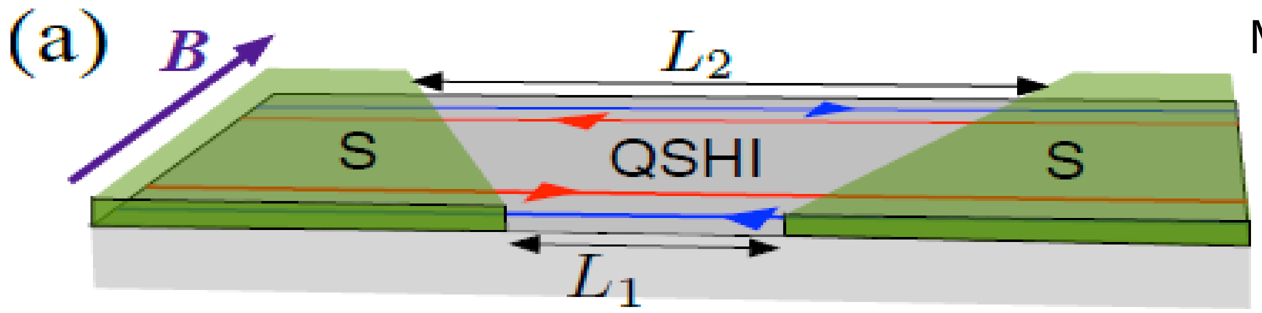
(A.Murani A.Chepelianskii H.B. Similations of BdG equ.)

Detection of level crossings at 0 and π ?

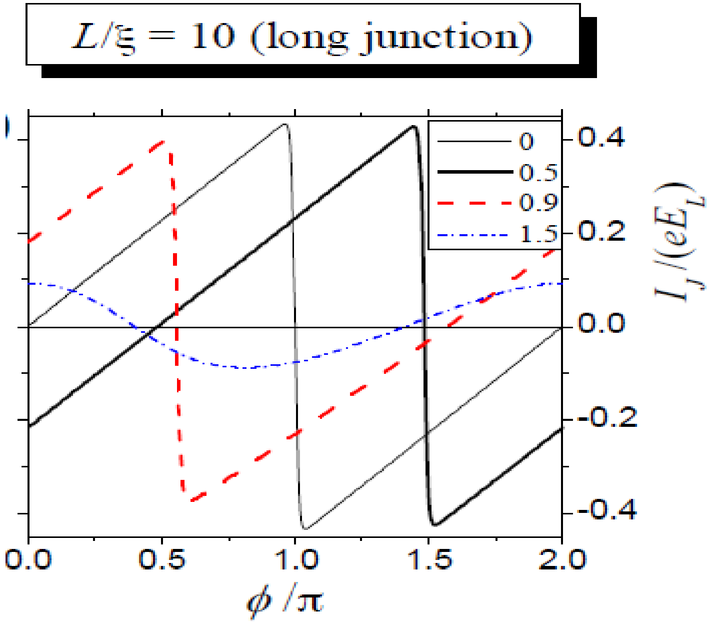
Topological Josephson φ_0 junction expected in field

Dolcini,Houzet, Meyer 2016

...If 2 edges with different lengths



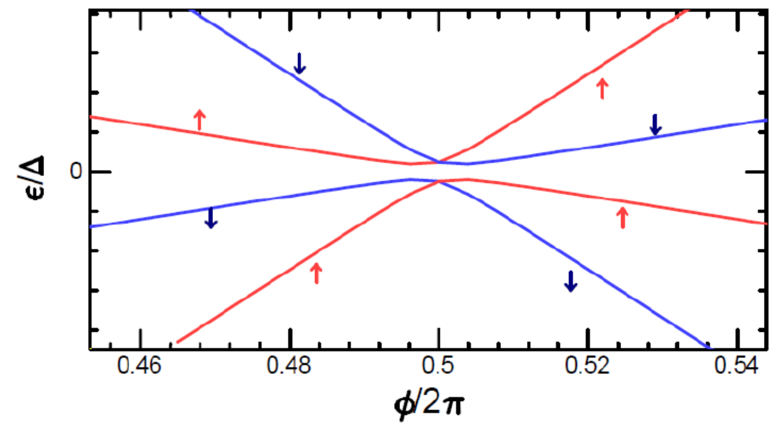
$$2\arccos\left(\frac{E_n+h}{\Delta_0}\right)-\frac{2(E_n+h)}{E_L}=\phi+2\pi n$$

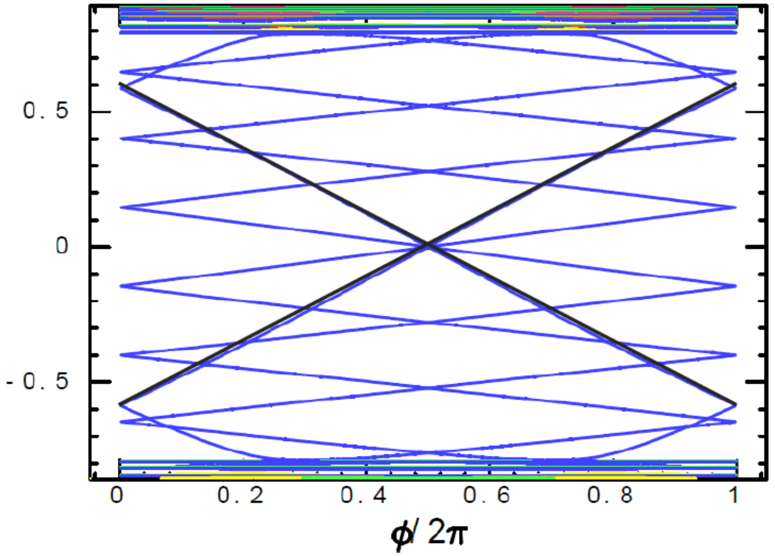


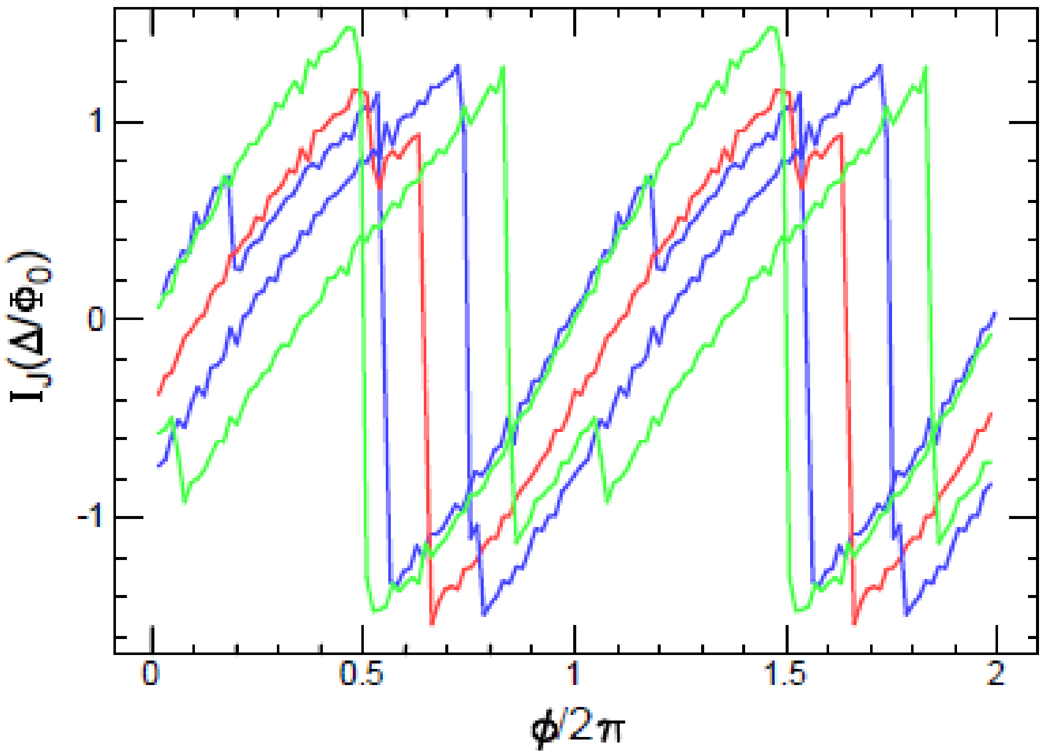
Topological Josephson φ_0 junction expected in field

…If 2 channels with different transmission

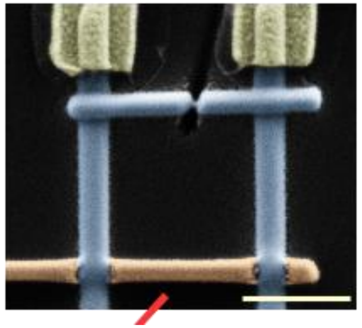
Murani, Chepelianskii, Bouchiat 2017







ac probing of the Andreev spectrum of a Quantum Spin Hall state:



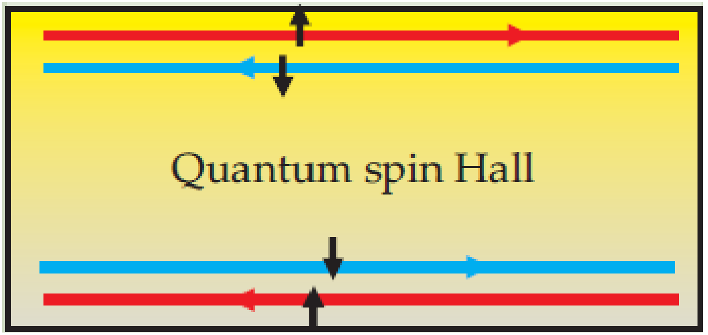
-ac Josephson effect

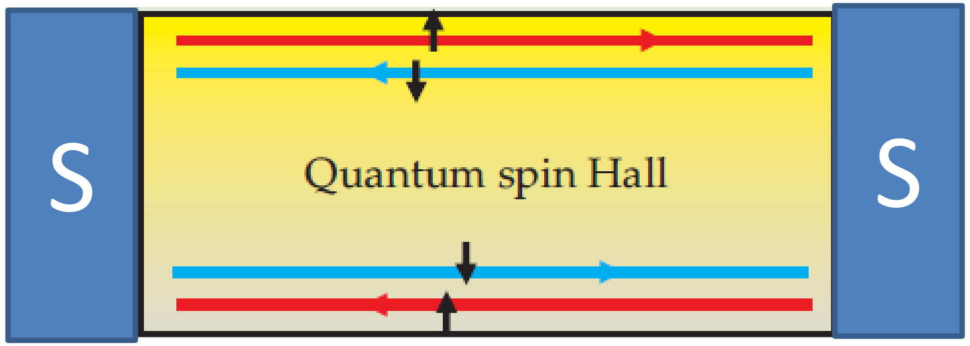
-2pi and 4 pi

-Several ways to probe the dynamic response of a system

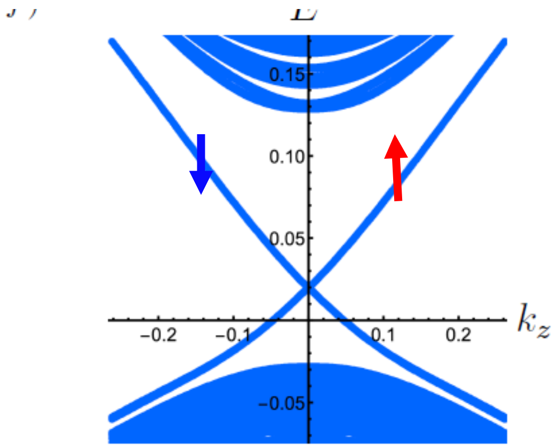
Susceptibility χ=dI/dφ probes topological protection and dynamics

Specificities of the S/Quantum Spin Hall/S junction



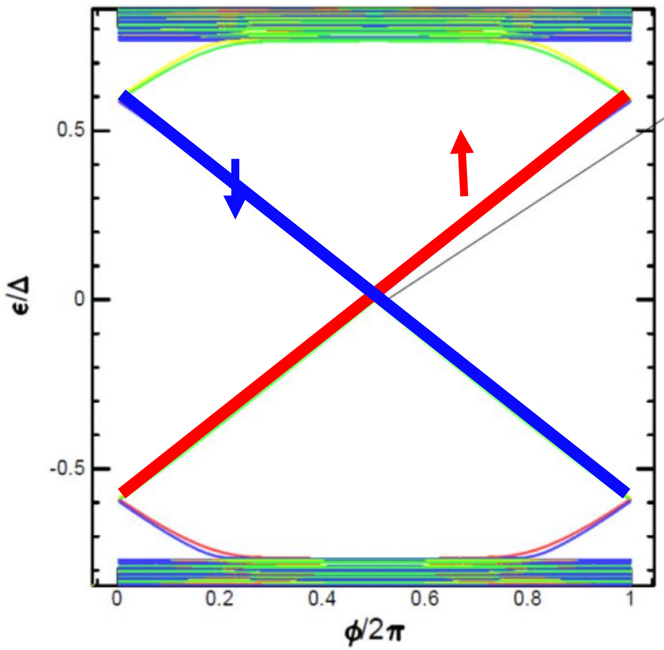


QSH, helical =« half » ballistic 1D



One edge

Andreev spectrum



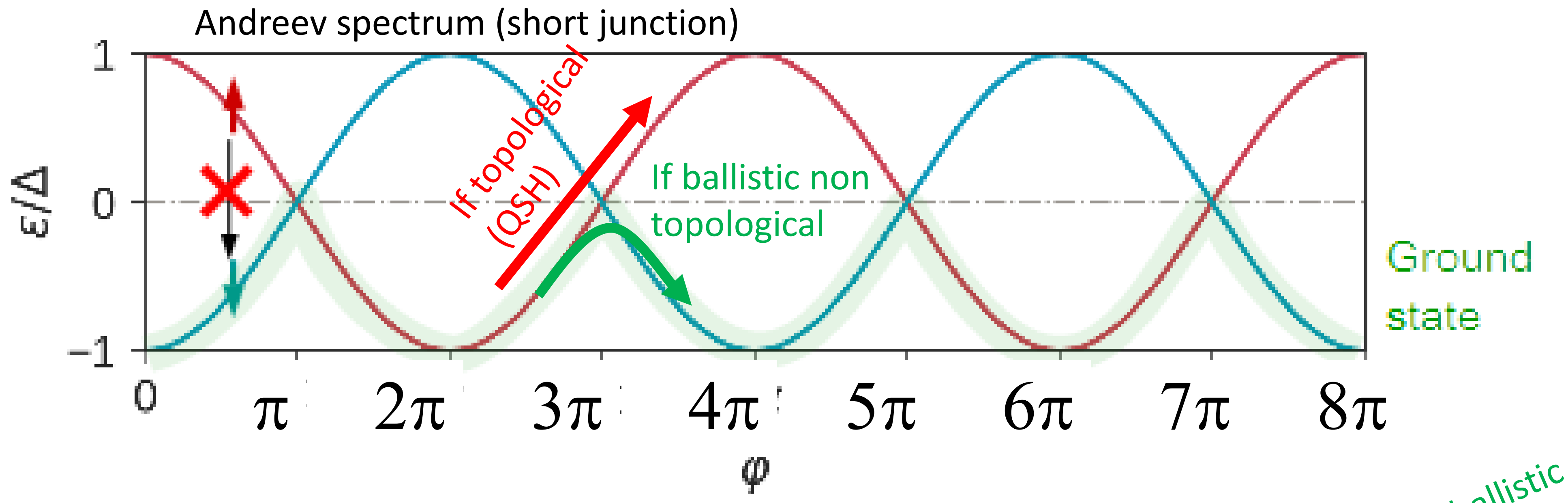
One edge

-Andreev spectrum is « half » of S/1D ballistic/S Andreev spectrum on one edge

-If parity is conserved, no way to backscatter: perfect level crossing at π: $\approx$ no disorder

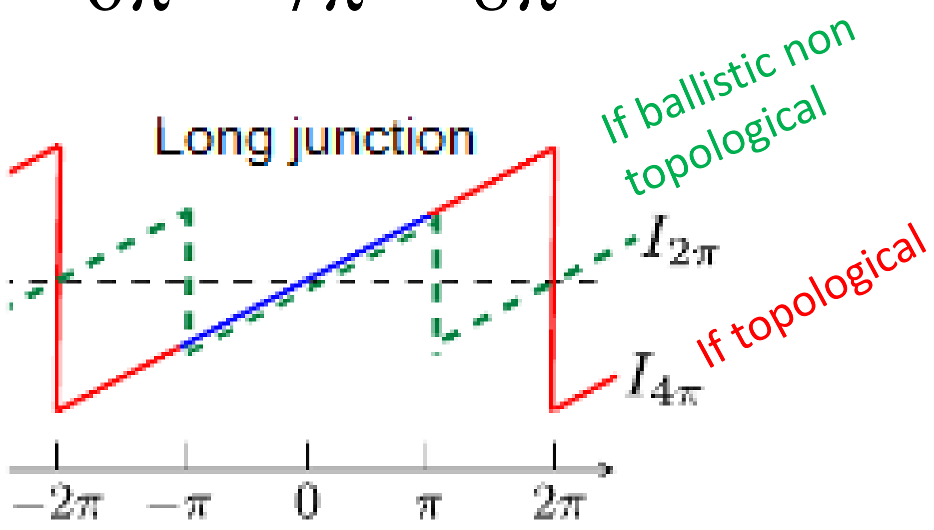
-4π periodicity

Consequence of helicity on current phase relation?



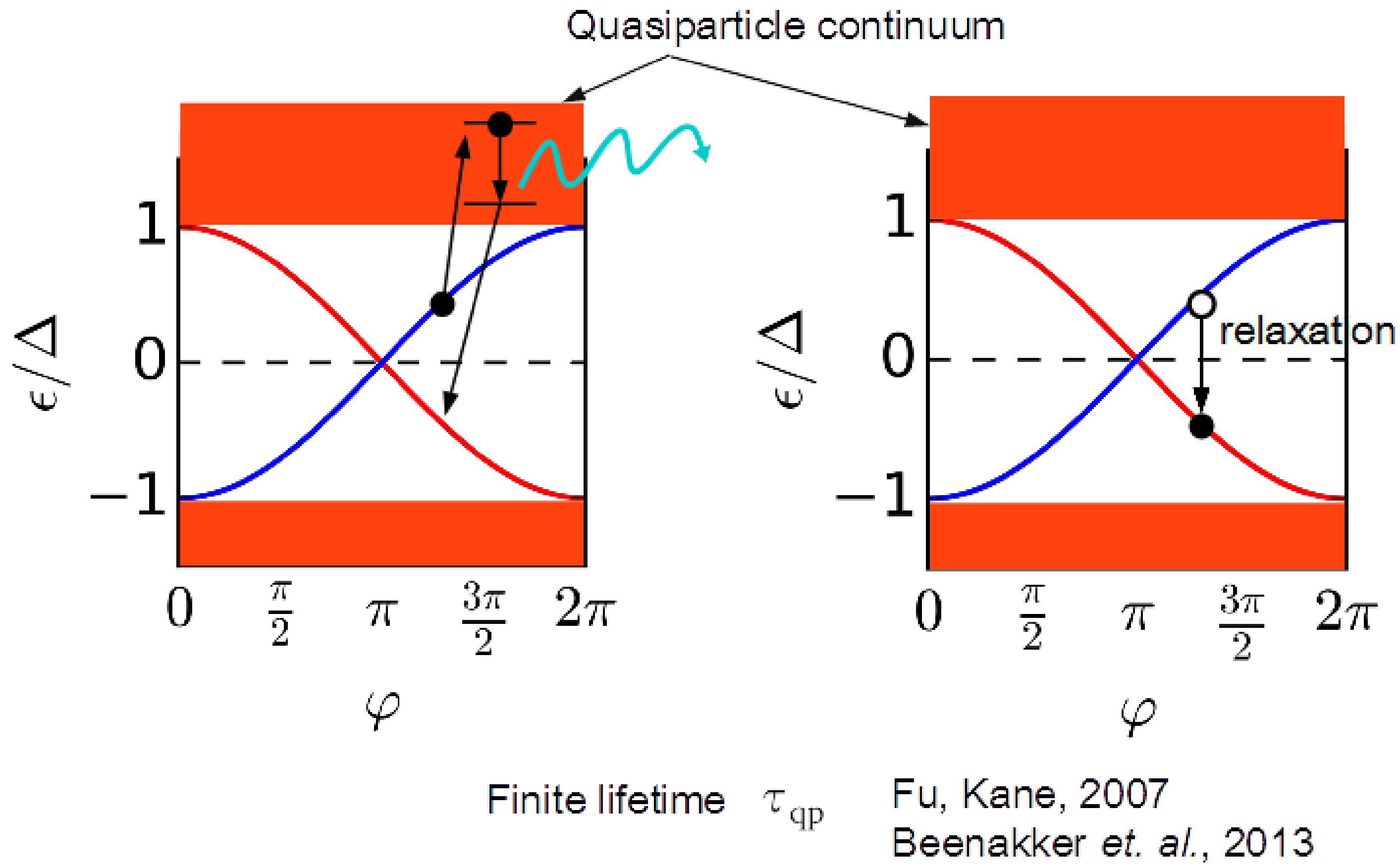
Fu Kane 2009
Beenakker 2013, Trauzettel 2014

$$I = \sum_{-\infty}^{0} \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$



Supercurrent through QSH edge should be 4π periodic, whereas 2π periodicity if ballistic non topological.

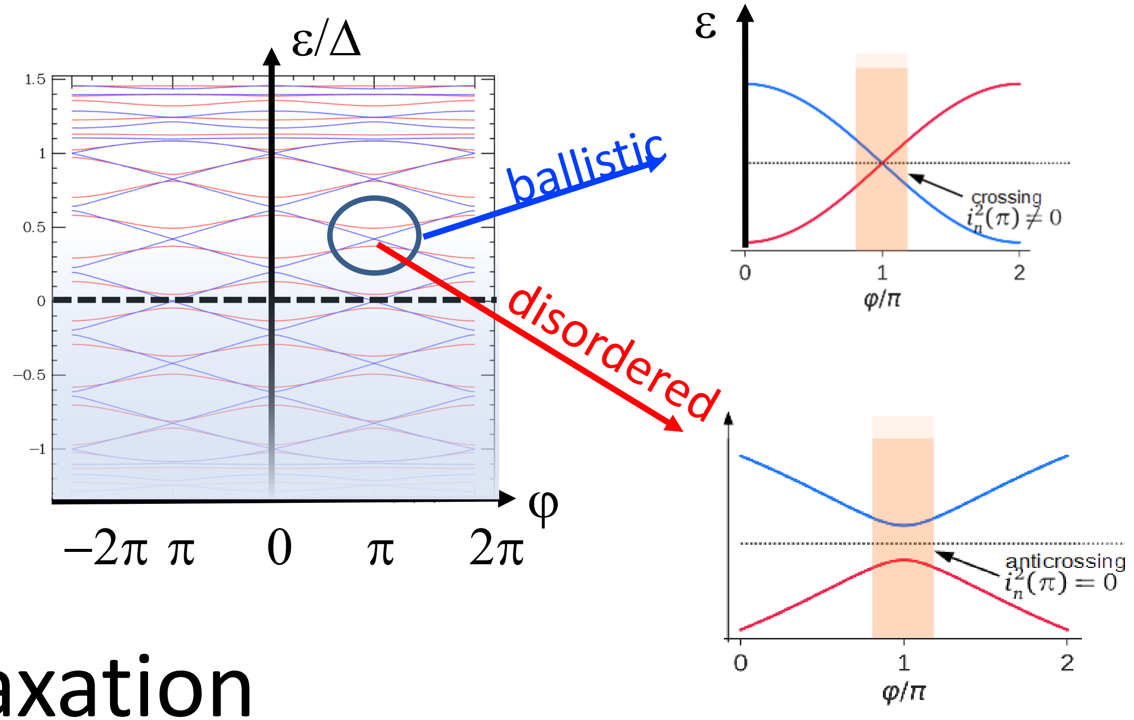
But poisoning can return periodicity to 2π



Need to go beyond dc current phase measurements:
Measure high frequency response (especially near crossings) to beat poisoning/relaxation rate: measure at $\omega >> \gamma_p$!

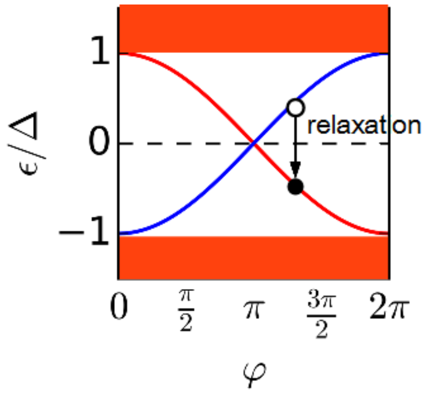
ac experiment can probe:

- Perfect crossing



Splitting due to coupling between two edges

- Rate of parity relaxation



Suggestion to use thermal noise to probe topological crossing

PHYSICAL REVIEW B **79**, 161408(R) (2009)

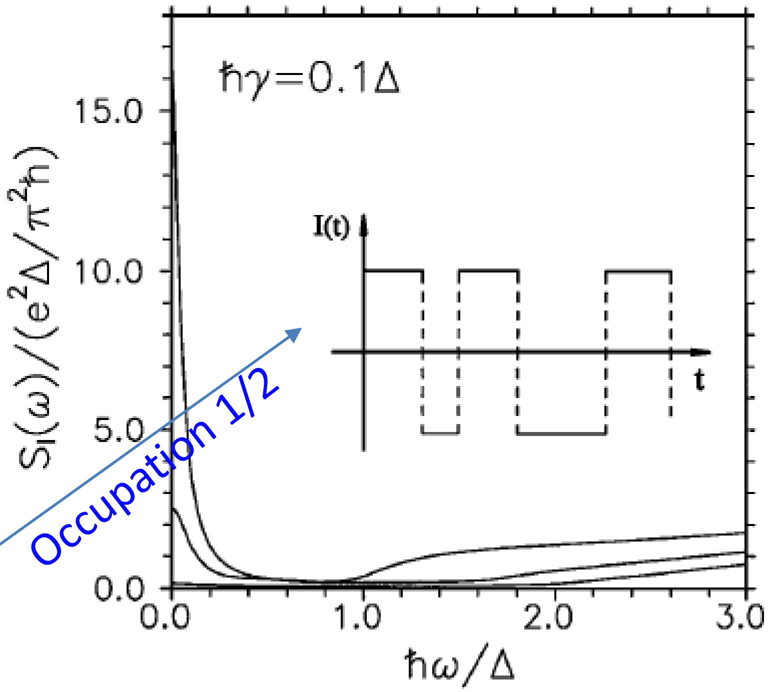
Josephson current and noise at a superconductor/quantum-spin-Hall-insulator/superconductor junction

Liang Fu and C. L. Kane
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 28 April 2008; revised manuscript received 11 February 2009; published 28 April 2009)

We study junctions between superconductors mediated by the edge states of a quantum-spin-Hall insulator. We show that such junctions exhibit a fractional Josephson effect, in which the current phase relation has a 4π rather than a 2π periodicity. This effect is a consequence of the conservation of fermion parity—the number of electron mod 2—in a superconducting junction and is closely related to the Z_2 topological structure of the quantum-spin-Hall insulator. Inelastic processes, which violate the conservation of fermion parity, lead to telegraph noise in the equilibrium supercurrent. We predict that the low-frequency noise due these processes diverges exponentially with temperature T as $T \to 0$. Possible experiments on HgCdTe quantum wells will be discussed.

HUGE telegraphic noise of cupercurrent if perfect crossing and low temperature

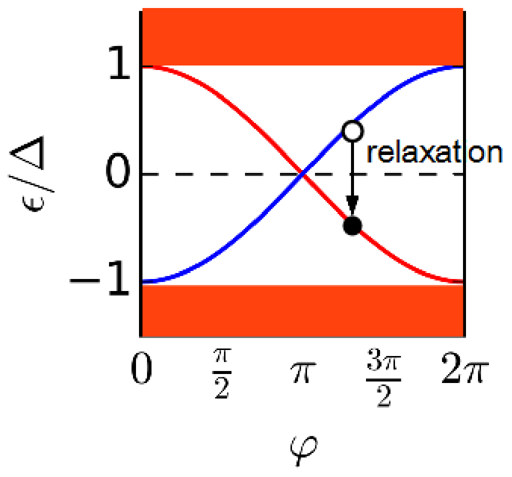
Averin Imam 1996



Noise power S(ω): gives time dependence of current

$$S(\omega) = 2\int_{-\infty}^{\infty} e^{i\omega t}\langle I(t)I(0)\rangle$$

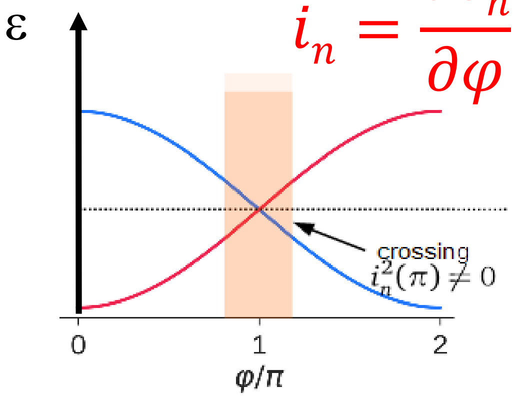
$$S(\omega) = \frac{4I_0^2}{\cosh^2 \epsilon_0(\phi)/2T}\,\frac{\tau}{1+\omega^2\tau^2}$$

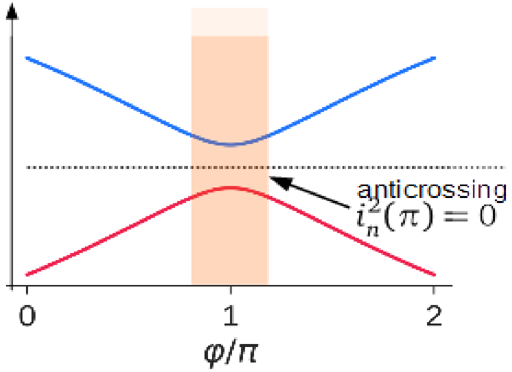


Suggestion to use thermal noise to probe topological crossing

$$S(\omega) = \frac{4I_0^2}{\cosh^2 \epsilon_0(\phi)/2T}\frac{\tau}{1+\omega^2\tau^2}$$

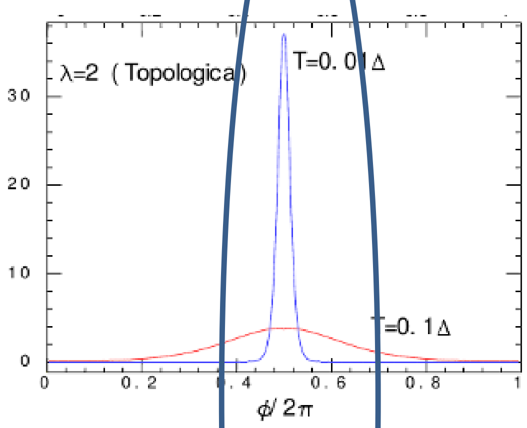
$$i_n = \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$





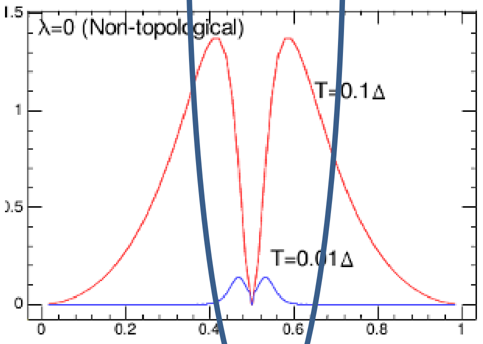
ballistic

$S(\omega)$



disordered

$S(\omega)$



Prediction:

Telegraphic noise of cupercurrent is huge (at π) if perfect crossing and low temperature, and zero if avoided crossing at π.

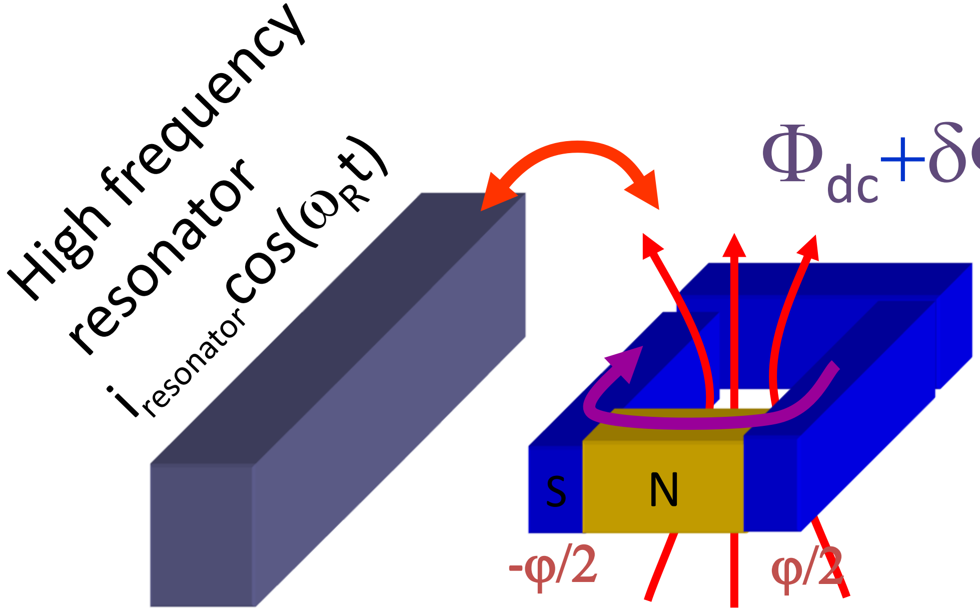
$$S_I(\omega) = \frac{2}{\pi}\frac{k_B T \chi''(\omega)}{\omega}$$

Noise=fluctuations

χ''=dissipation

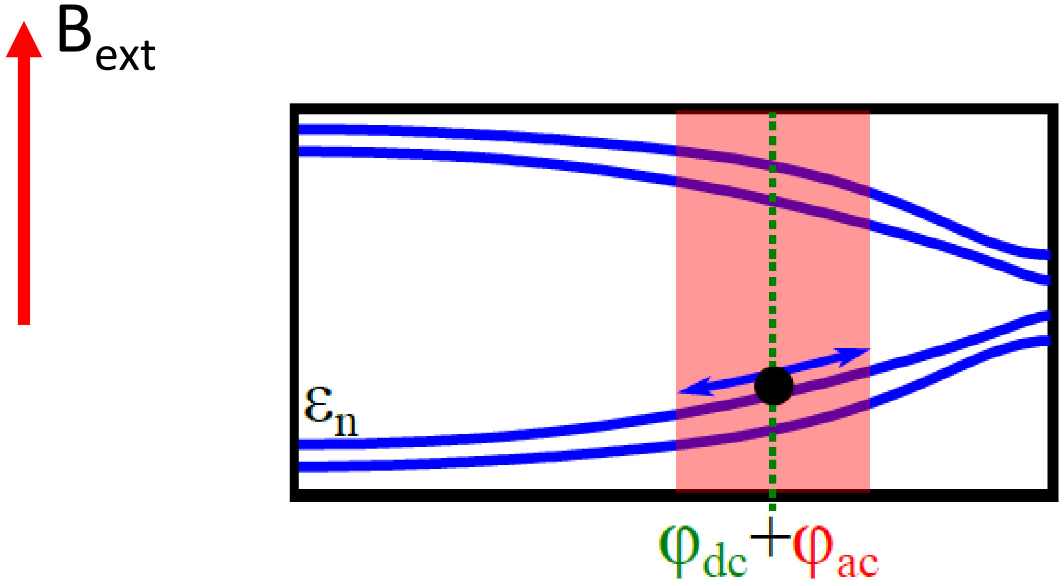
Instead of probing noise, we will probe susceptibility (equivalent via Fluctuation-Dissipation Theorem)

(dc+) ac phase-driven proximity effect



$\Phi_{dc}+\delta\Phi_{ac}\cos\omega t$

$\Phi_0=h/2e$



ac : $\Phi(t)=\Phi_{dc}+\delta\Phi_{ac}\cos\omega t$ $\qquad$ $\varphi(t)=2\pi\,\Phi(t)/\Phi_0$

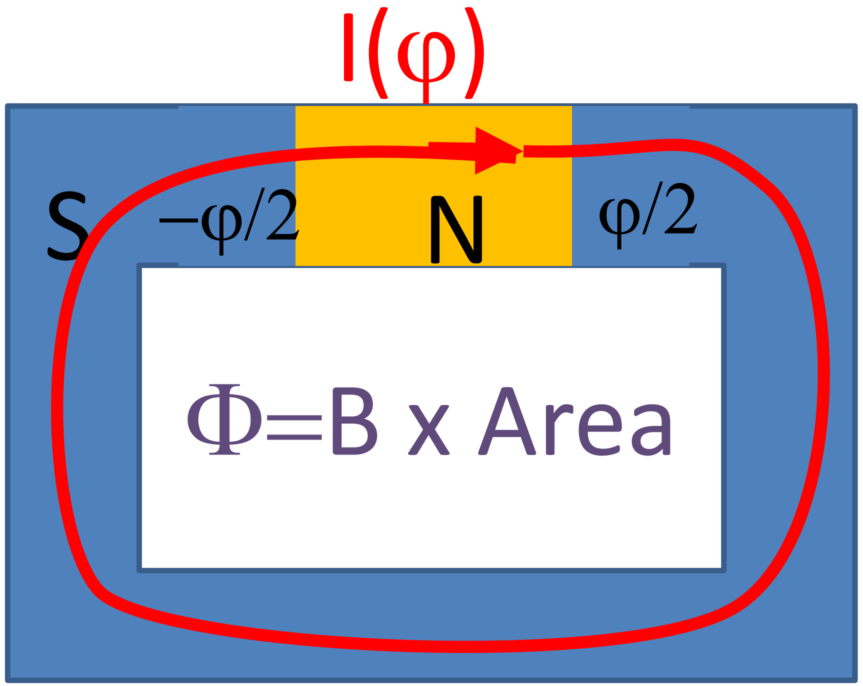
Ring linear response: $I(t,\varphi,\omega)=I_{dc}+\varphi_{ac}(\chi'(\omega)\cos\omega t+\chi''(\omega)\sin\omega t)$

$\chi=\chi'+i\chi''$ $\qquad$ χ' non dissipative $\qquad$ χ'' dissipative

$I(\varphi)$ depends on the transport regime in the N (diffusive, ballistic)

$\chi(\varphi,\omega)$ depends on the spectrum and dynamics

ac phase-biased junction



$\varphi = -2\pi\Phi/\Phi_0$ $\Phi = \Phi_{dc} + \delta\Phi_{ac}\cos\omega t$

Flux-induced current perturbation

$$\hat{I}_s(t) = \hat{I}_s + \delta\hat{I}_s(t)\,,$$

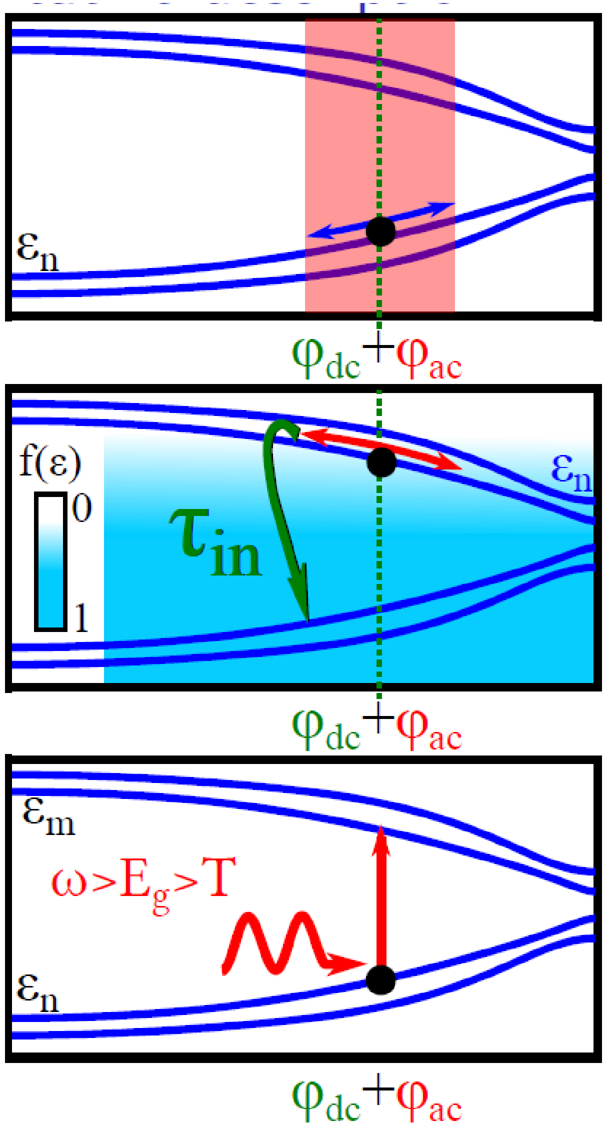
$$\delta\hat{I}_s(t) = -\delta\Phi(t)\frac{\partial^2 H_0}{\partial\Phi^2}\,,$$

$$\langle\hat{I}_s(t)\rangle = \mathrm{Tr}[\hat{I}_s\rho_0] + \mathrm{Tr}[\hat{I}_s\delta\rho(t)] + \mathrm{Tr}[\delta\hat{I}_s(t)\rho_0]$$

$$\chi(\Phi,\omega) = \frac{\delta\langle\hat{I}_s\rangle}{\delta\Phi(\omega)}$$

Susceptibility: Linear response

Contributions to susceptibility



Adiabatic response

Josephson susceptibility: $\chi_J = -\frac{2\pi}{\Phi_0}\frac{\partial I_J}{\partial \varphi}$

Diagonal response: population relaxation

$$\delta f_n = \frac{1}{1 - i\omega\tau_{in}}\frac{\partial f_n}{\partial \varphi}\varphi_{ac}$$

$$\chi_D = -\frac{i\omega}{1/\tau_{in} - i\omega}\sum_n i_n \frac{\partial f_n}{\partial \varphi} = -\frac{i\omega}{1/\tau_{in} - i\omega}\sum_n i_n^2 \frac{\partial f_n}{\partial \epsilon_n}$$

Non diagonal response: Transistions between Andreev levels

$$\chi_{ND} = \sum_{n,m\neq n} J_{n,m}^2 \frac{f_n - f_m}{\epsilon_n - \epsilon_m}\frac{i\hbar\omega}{1/\tau_{nm} + i(\epsilon_n - \epsilon_m - \hbar\omega)}$$

How does a system respond to a high frequency excitation ?

i_n^2

$$\chi(\Phi,\omega) = \underbrace{\frac{\partial I_J}{\partial \Phi}}_{\chi_J} + \underbrace{\sum_n \frac{\omega}{\omega + i\gamma_{nn}} \left(\frac{\partial \epsilon_n}{\partial \Phi}\right)^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n}}_{\chi_D} - \underbrace{\hbar\omega \sum_{n\neq m} \frac{|\langle m|\hat{I}_s|n\rangle|^2}{\epsilon_n - \epsilon_m} \frac{f(\epsilon_n) - f(\epsilon_m)}{\epsilon_n - \epsilon_m - \hbar\omega - i\hbar\gamma_{nm}}}_{\chi_{ND}}$$

Static

Derivative of dc current-phase relation

Delayed response
Population relaxation
Prop to i_n^2

Transitions:Spectroscopy (in some range)

$$I_J(\Phi) = -\sum_n f(\epsilon_n) \frac{\partial \epsilon_n}{\partial \Phi}$$

Most sensitive to avoided/protected crossing

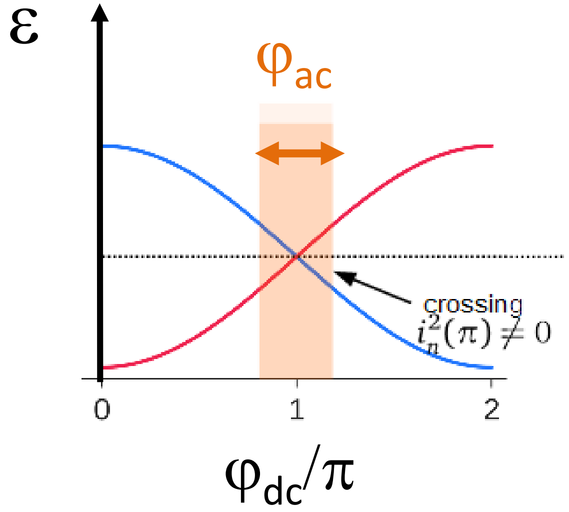
Applied to normal ring (Trivedi Browne PRB 1988), and diffusive SNS ring (Ferrier PRB 2013, Dassonneville 2014)

Terms beyond derivative of dc Josephson relation lead to dissipation!

ac susceptibility (especially diagonal absorption) can distinguish between topo/non topological crossings

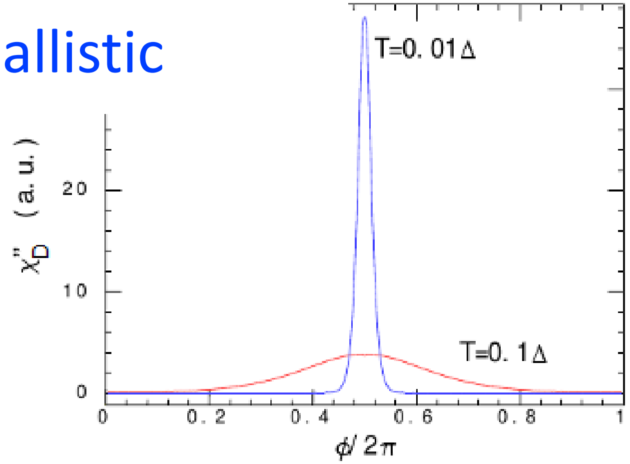
$$\underbrace{\sum_n \frac{\omega}{\omega + i\gamma_{nn}} \left(\frac{\partial \epsilon_n}{\partial \Phi}\right)^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n}}_{\chi_D}$$

$i_n^{\,2}$

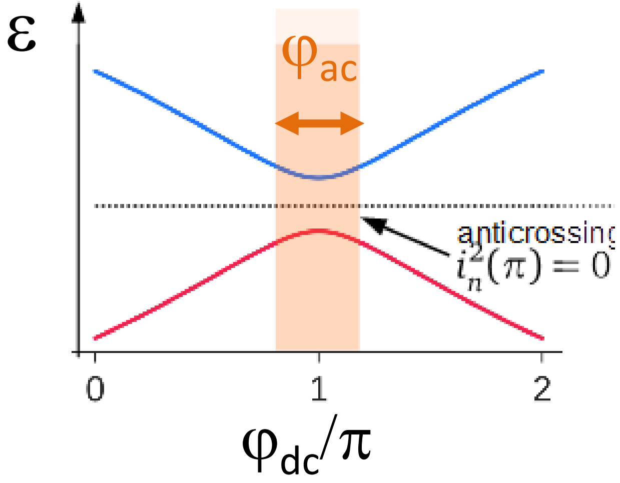


χ_D probes zero energy level crossings!

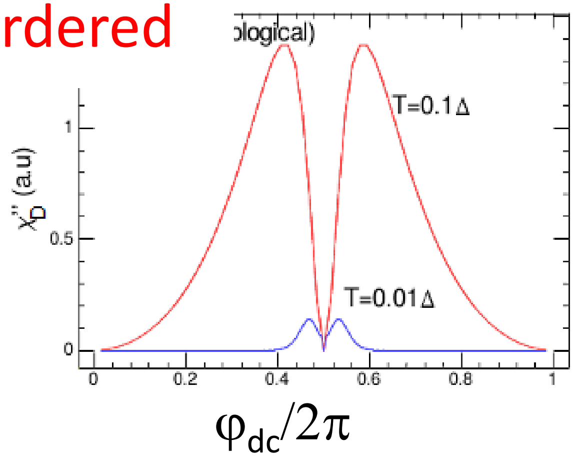
ballistic



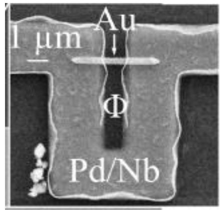
Not seen before…

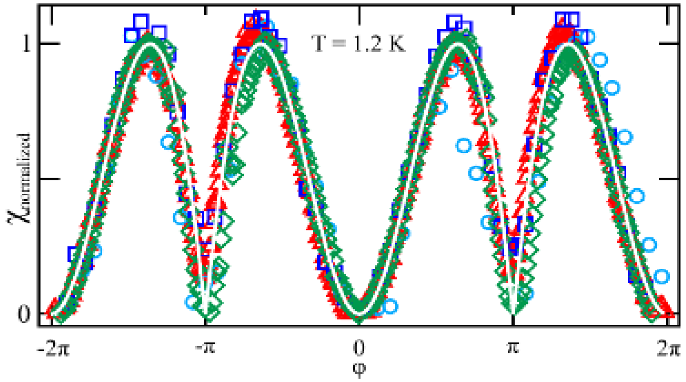


disordered

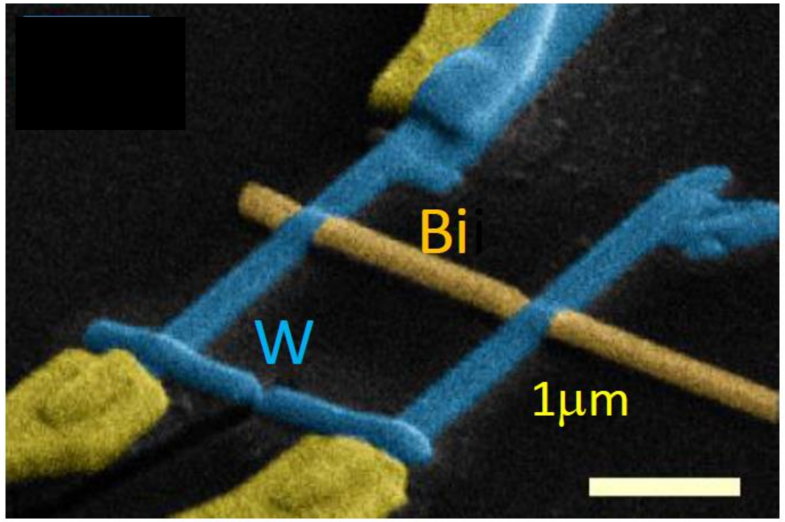


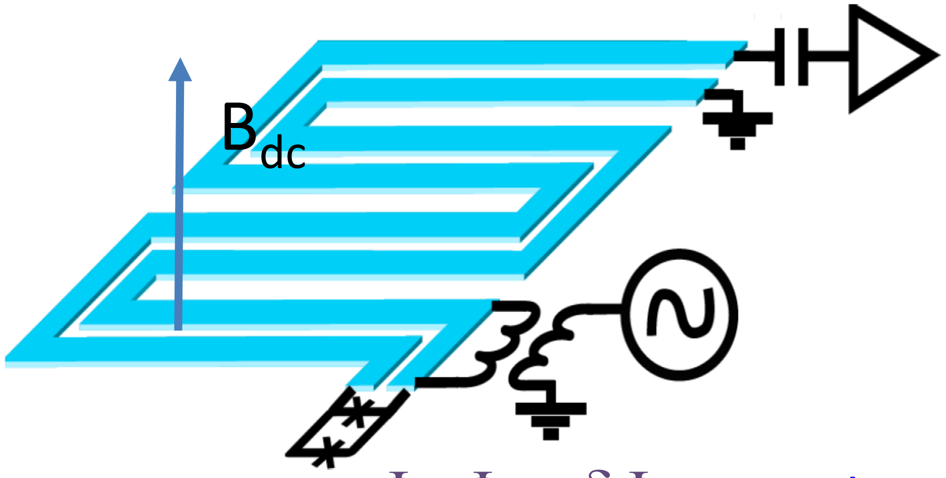
Diffusive SNS ring: Ferrier 2013, Dassonneville 2014, 2018





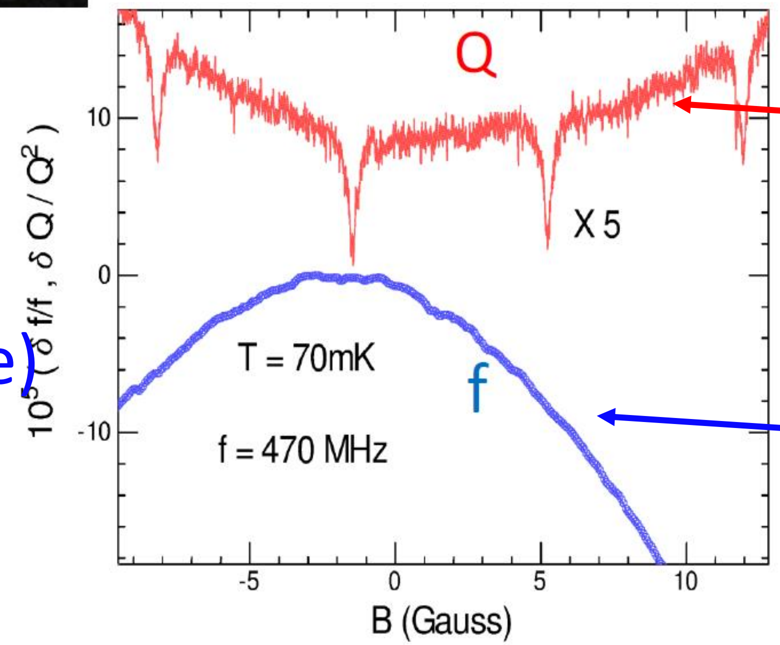
In practice : multimode resonator coupled to S/Bi/S asymmetric SQUID





$\Phi=\Phi_{dc}+\delta\Phi_{ac}\cos\omega t$

Measure Q and f variations with B (at each resonator mode)



Dissipative response

$$\chi''(\varphi) = \frac{L_R}{L_W^2}\delta\left[\frac{1}{Q_n}\right](\Phi)$$

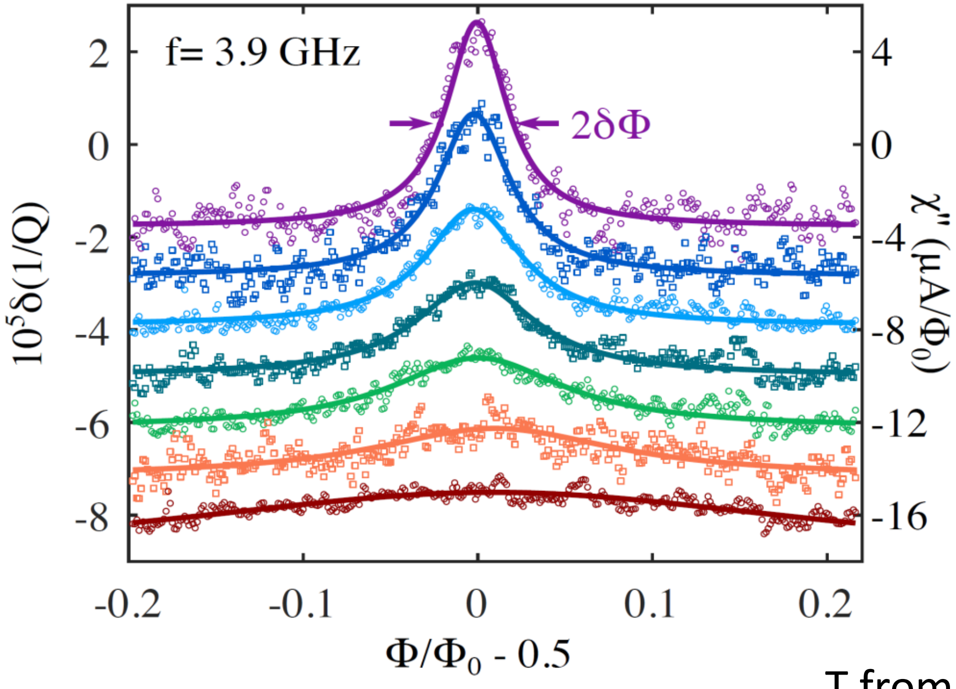
Non-dissipative response

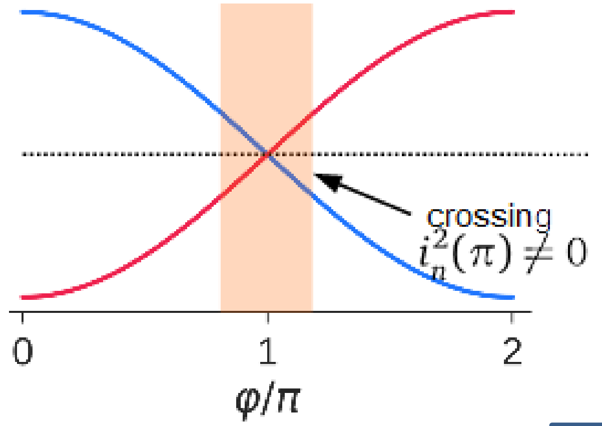
$$\chi'(\varphi) = -\frac{L_R}{L_W^2}\frac{\delta f_n(\Phi)}{2f_n}$$

Absorption peaks at π !

T dependence of absorption peaks at φ=π OK with protected crossing

$\delta\,(1/Q) = L_c^2 \,/\, L_R\, \chi''$





$$\underbrace{\sum_n \frac{\omega}{\omega + i\gamma_{nn}} \left(\frac{\partial \epsilon_n}{\partial \Phi}\right)^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n}}_{\chi_D}$$

i_n^2

$$\chi''_{\mathrm{D}} = i_0^2 \frac{\omega\gamma}{\omega^2+\gamma^2} \frac{1}{2T\cosh^2\left(\frac{\epsilon_T}{2T}(\varphi/\pi - 1)\right)}$$

$i_n = i_0 = E_{Th}/\Phi_0$

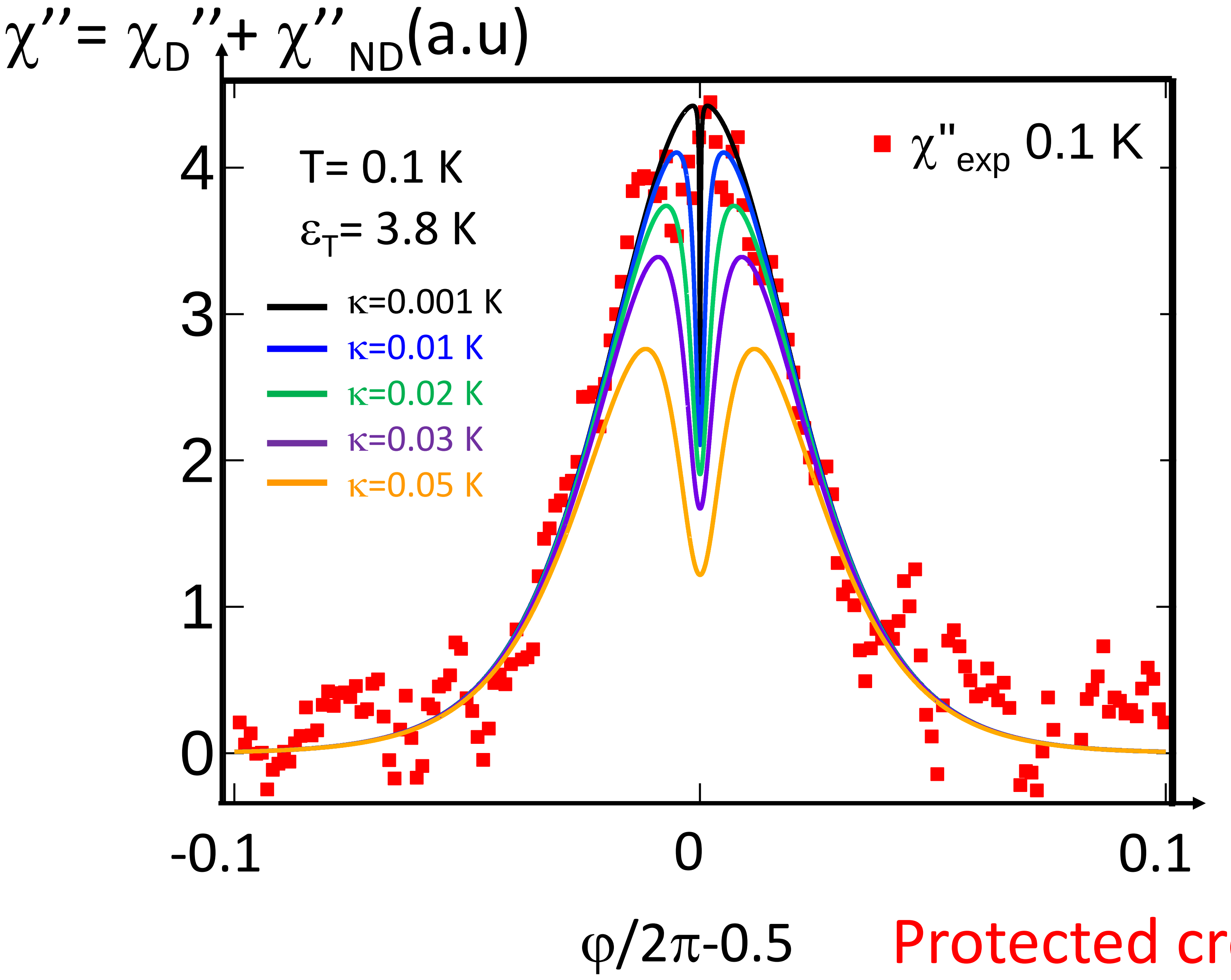
This is the thermal noise of a QSH insulator (Fu Kane)!

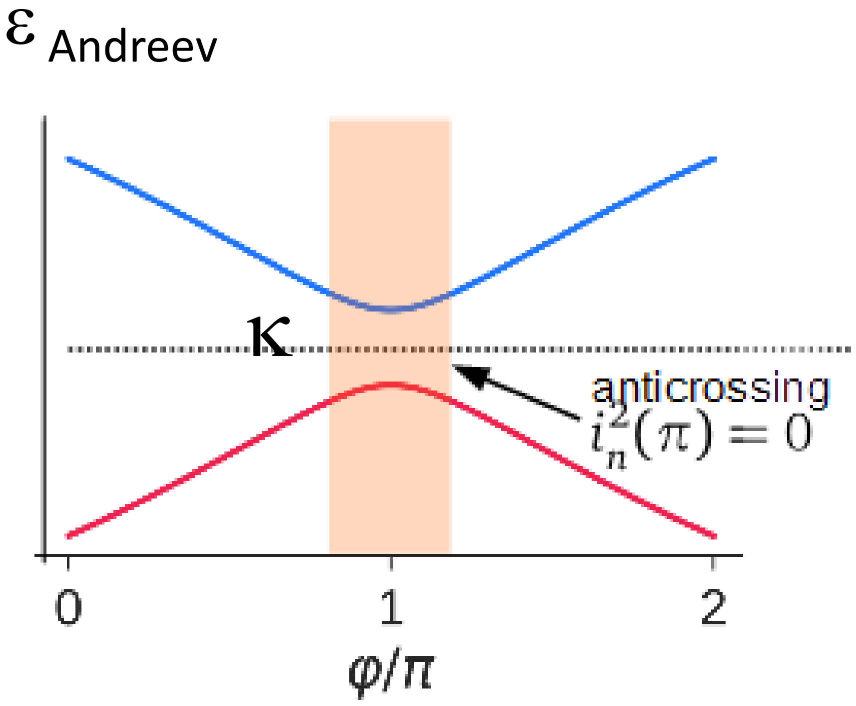
T from 1K to 70 mK

$$S(\omega) = \frac{4 I_0^2}{\cosh^2 \epsilon_0(\phi)/2T} \frac{\tau}{1+\omega^2\tau^2}$$

Noise peaks at level crossing because level occupation fluctuates most (telegraphic noise)

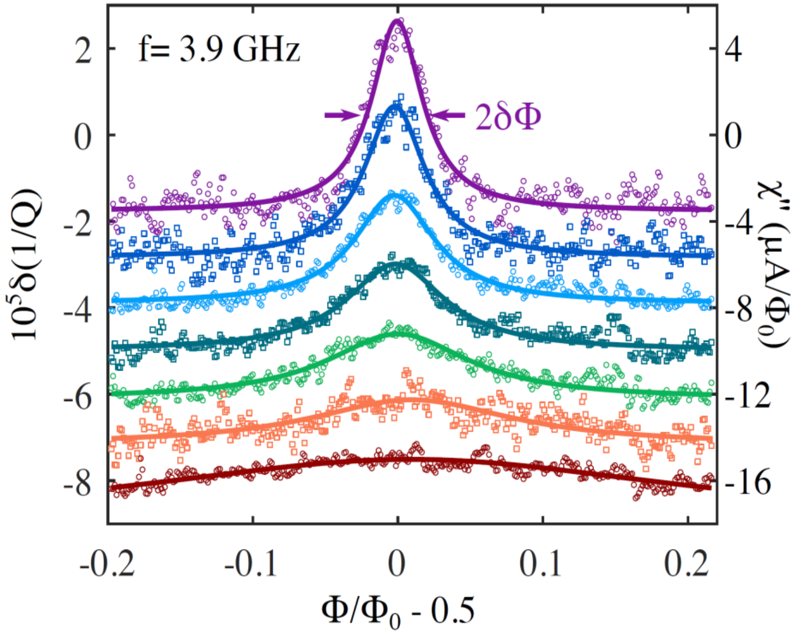
How protected is protected ?

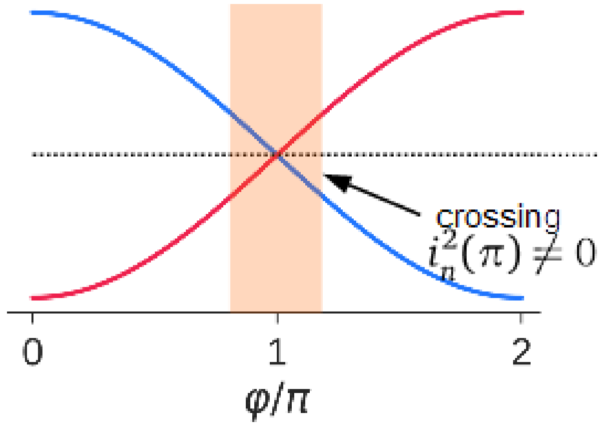




Protected crossing to within 30 mK

Parity is not conserved! Relaxation occurs





$$\underbrace{\sum_n \frac{\omega}{\omega + i\gamma_{nn}} \left(\frac{\partial \epsilon_n}{\partial \Phi}\right)^2 \frac{\partial f(\epsilon_n)}{\partial \epsilon_n}}_{\chi_D}$$

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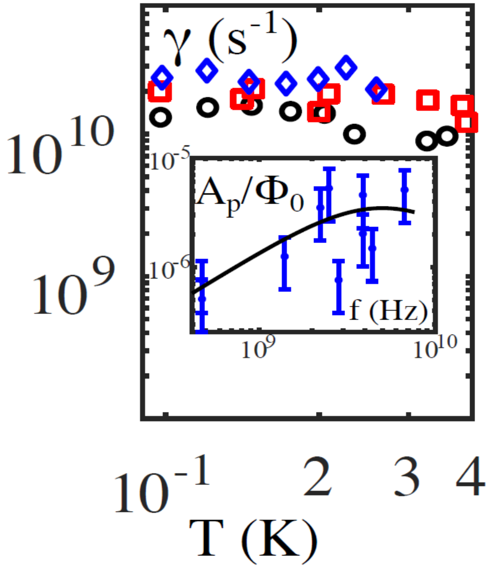
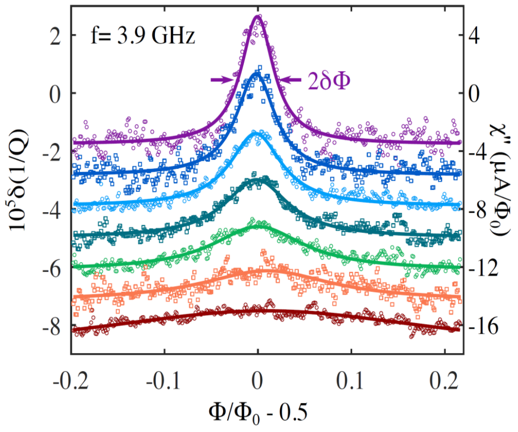
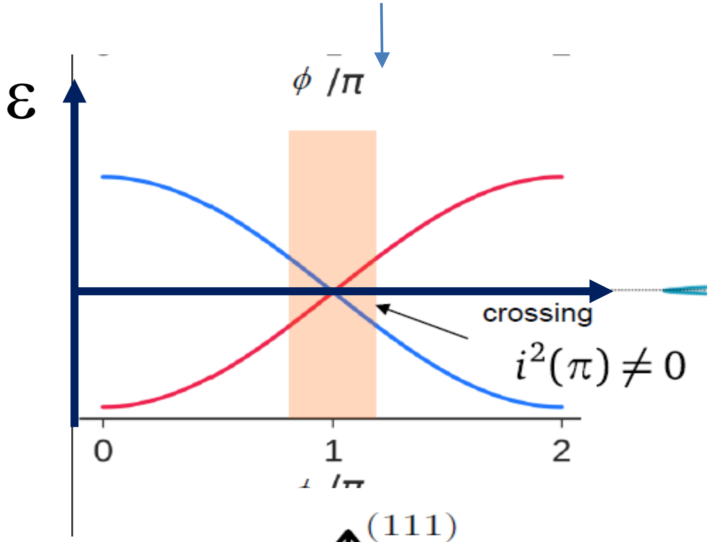


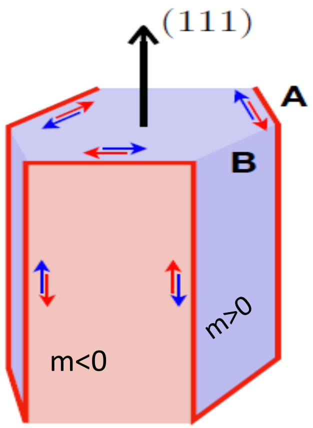
Fast poisoning! ~ 1ns

Due to soft gap, quasiparticles, broadband environment

Enabled us to see a response, but room for improvement…







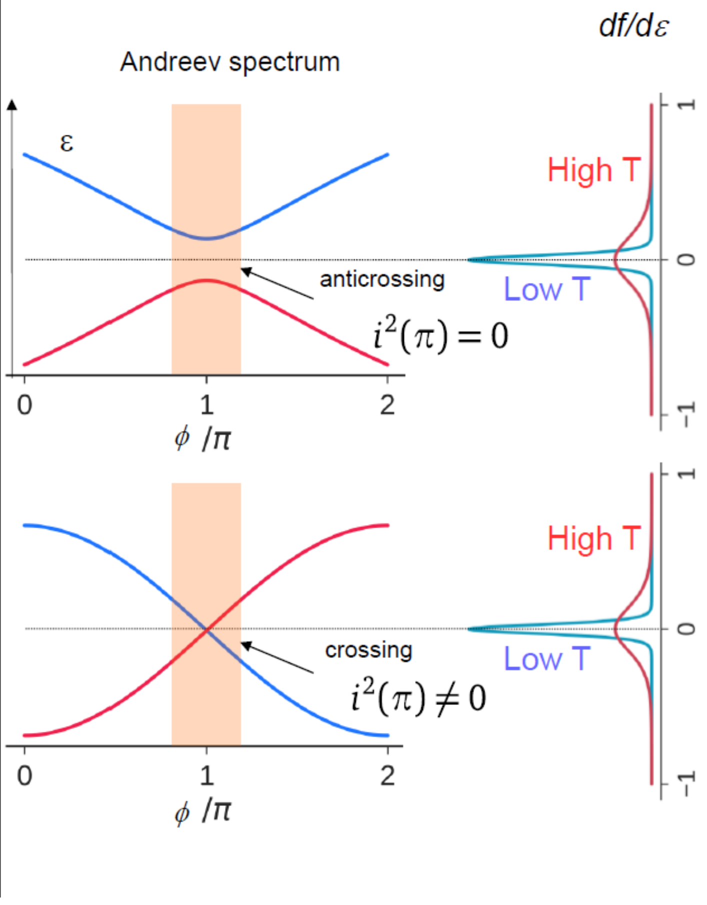
Conclusion

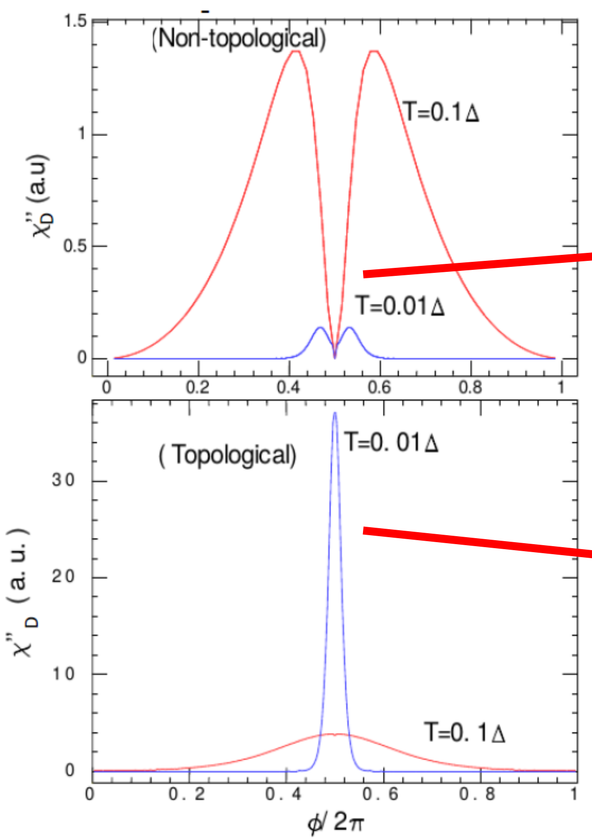
Superconducting proximity effect is a good tool!

Ballistic edge states with protected (topological) crossing (to within 50 mK). Noisy supercurrent at π reveals topology. Bismuth could well be a Second Order Topological Insulator!

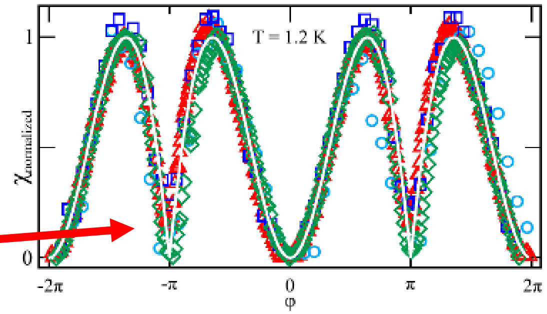
However fast relaxation (parity breaking)
No inductive response… needs further understanding

Comparison of ac susceptibility of S/Bi/S and S/diffusive Au/S

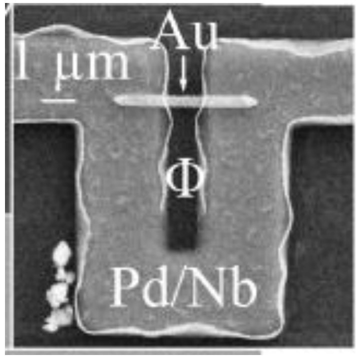




In SNS: zero absorption at π!

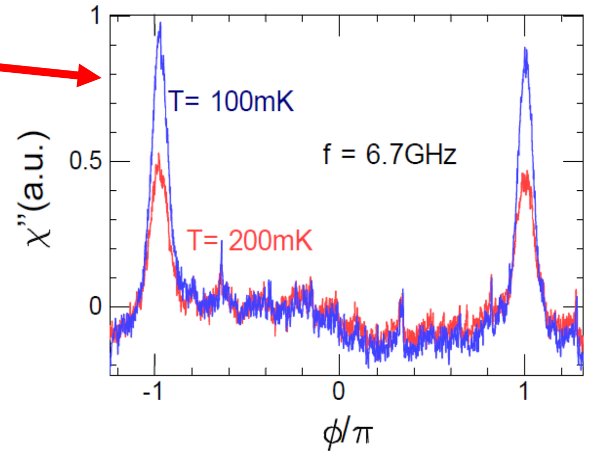


S/diffusive Au/S

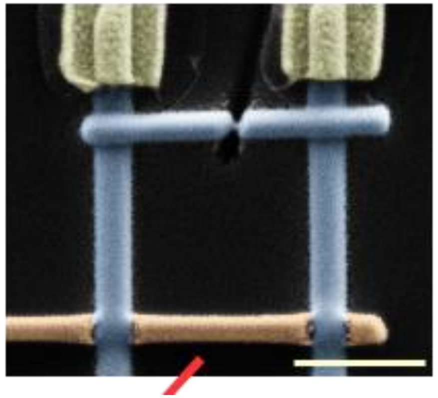


Dassonneville2013

In S/Bi/S: max absorption at π!



S/Bi/S



Murani2018

Open questions/ New experiments/ What next?

Future plans:

- Measure noise at lower frequency (coupling between edges, MHz) to observe restauration of spin degenerate behavior

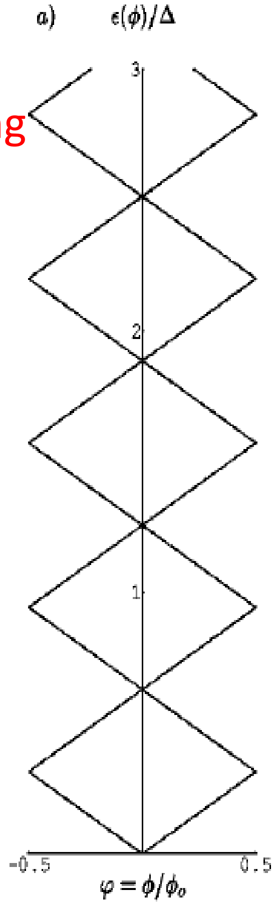
- Investigate difference between SOTI and 2DTI

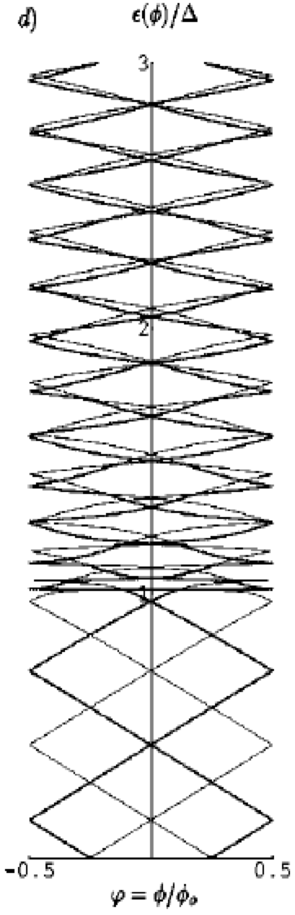
- Other probe of helical edge states? Persistent charge and spin currents of 2DTI/SOTI?

Compare Normal spectrum to Andreev Bound State spectra a)ballistic few channels

Cayssol,Kontos Montambaux 2003.

Normal ring





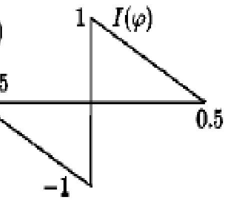
NS ring (long junction)

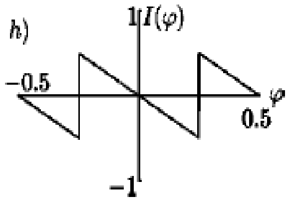
h/e periodicity above gap (like N)

E=Δ

h/2e periodicity in subgap region

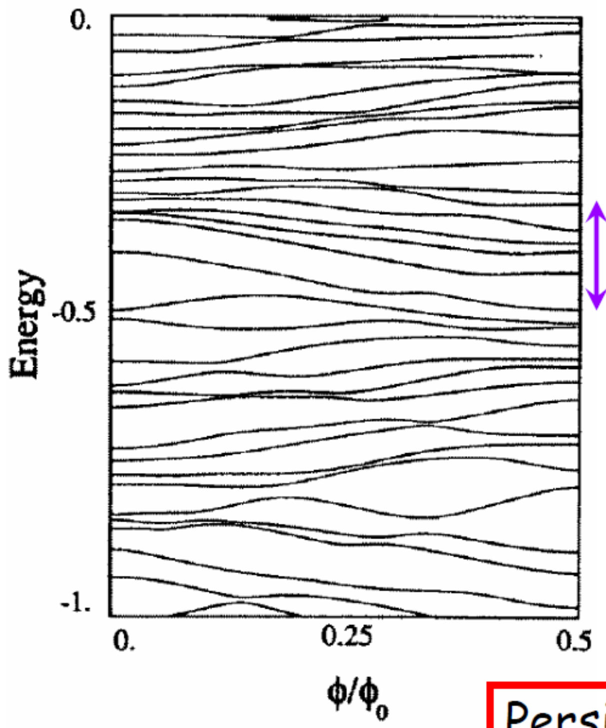
h/e periodicity





h/2e periodicity

SPECTRUM OF A DISORDERED RING



Correlated spectrum

Thouless energy

$E_c = h\,D/L^2 = h/\tau_D$

$\tau_D = L^2/D$ diffusion time around the ring

Spectrum described by random matrix theory on this energy scale.

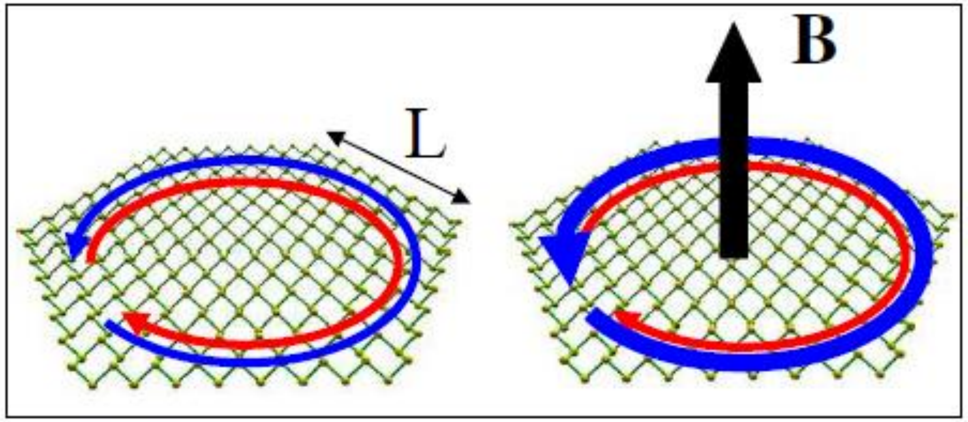
Persistent current: random quantity

$\langle I^2 \rangle^{1/2} = e/\tau_D = I_0\, l_e/L$

- Diffusive states have tiny persistent current ~ evF/L (le/L) (as if only one diffusive channel=)

- Topological 1D edge states have evF/L: 100 nA

Persistent (charge) current best discriminator?



P. Potasz and J. Fernández-Rossier, Nano Lett. (2015).

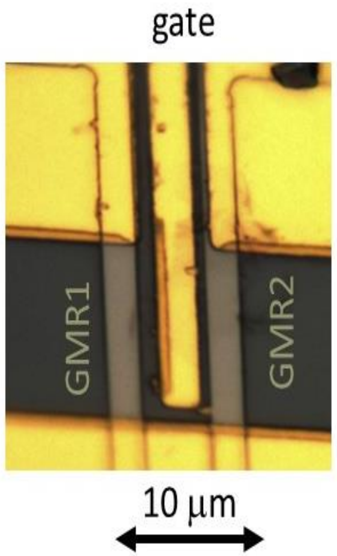
- Take a platelet (no need for a ring), no leads

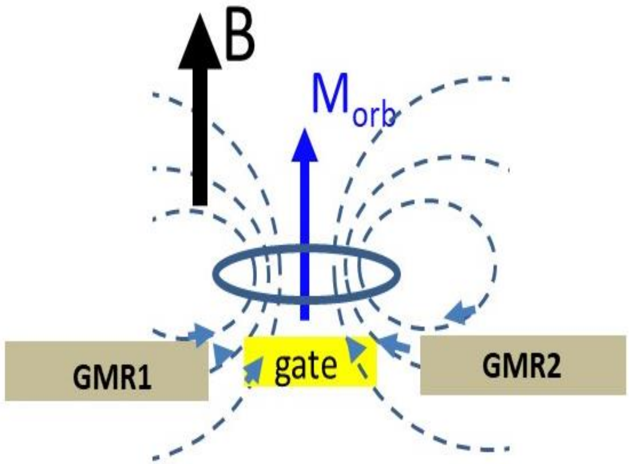
- Diffusive states have tiny persistent current ~ evF/L (le/L) (as if only one diffusive channel=)

- -1D edge states have evF/L: 100 nA

- Only edge states would have a well-defined period

We already have the magnetic probe ready: GMR detector

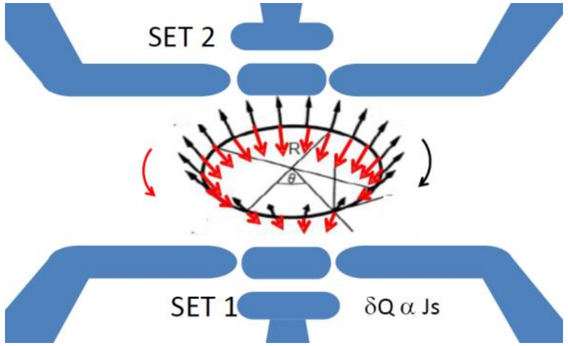




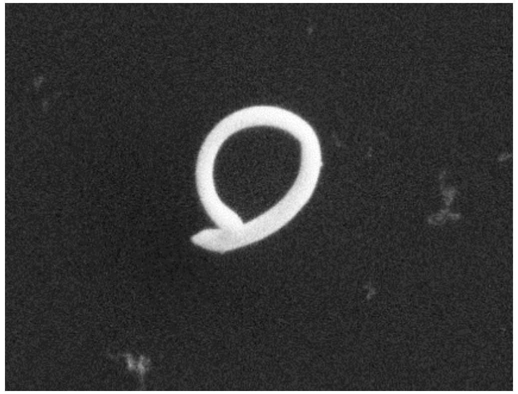
Persistent spin current?

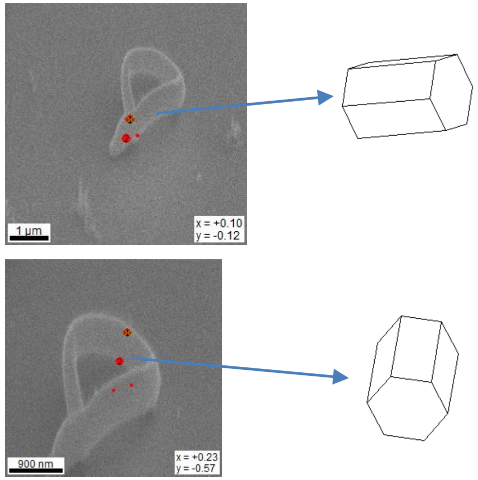
Charge current -> magnetic moment

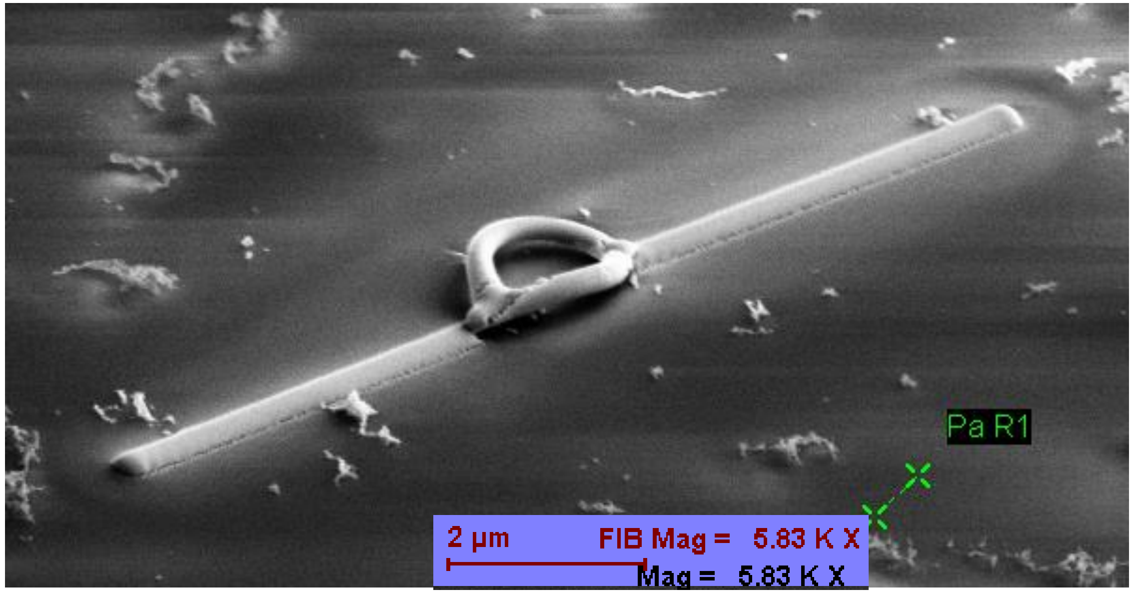
Dual: Spin current -> Electric dipole?

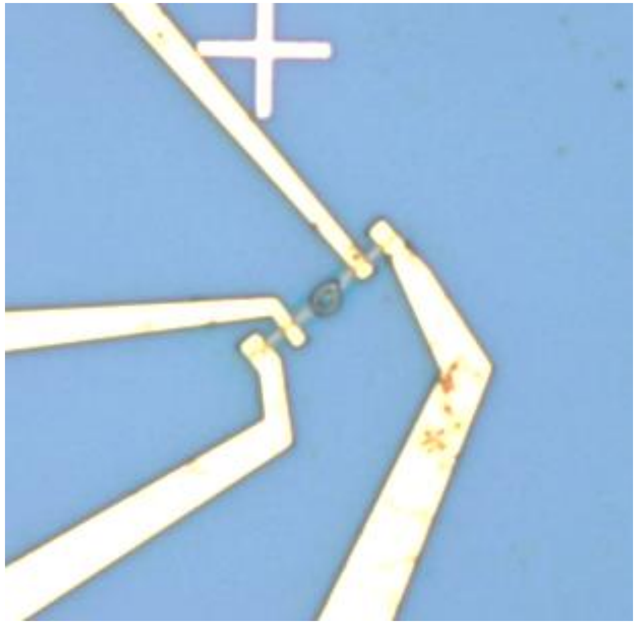


Ongoing experiments on a bismuth ring

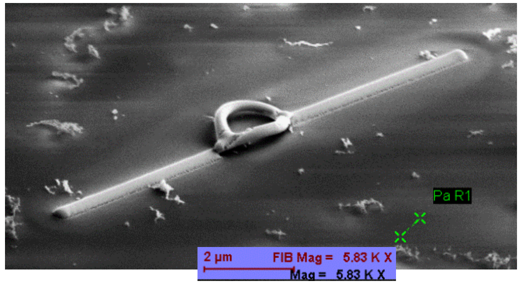


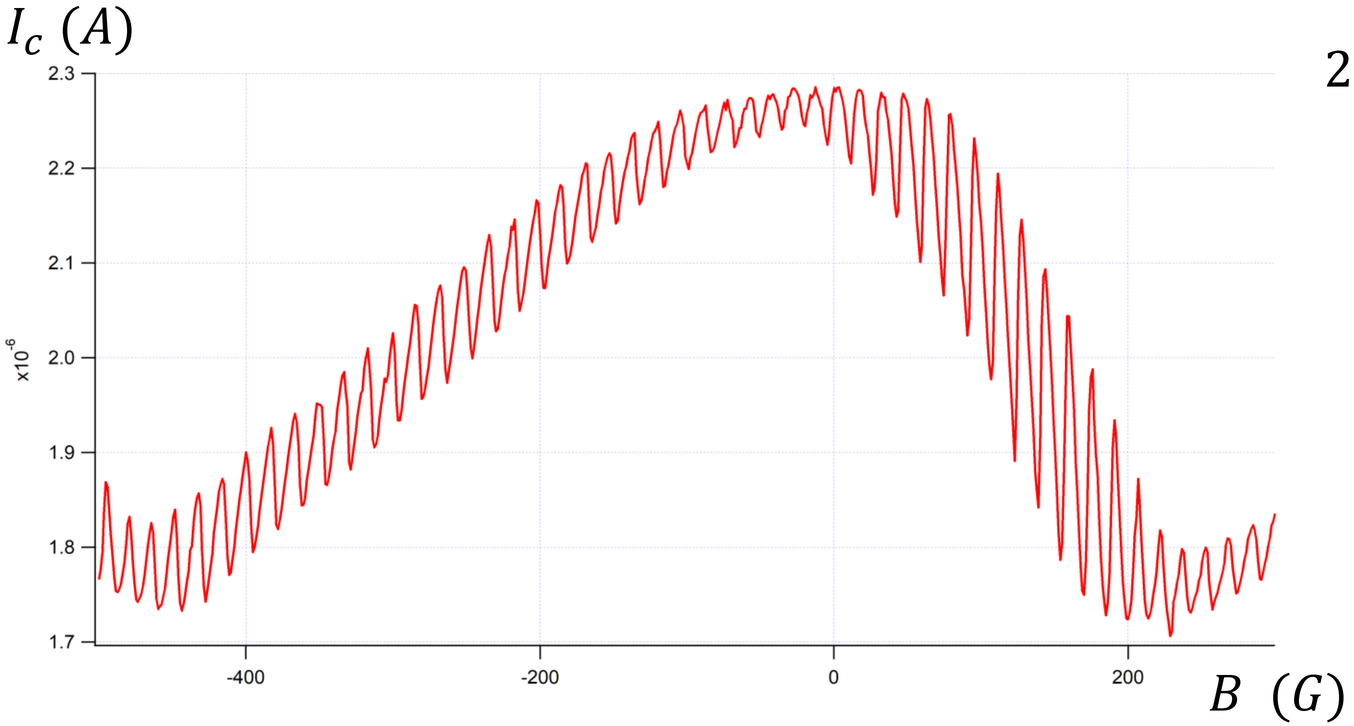


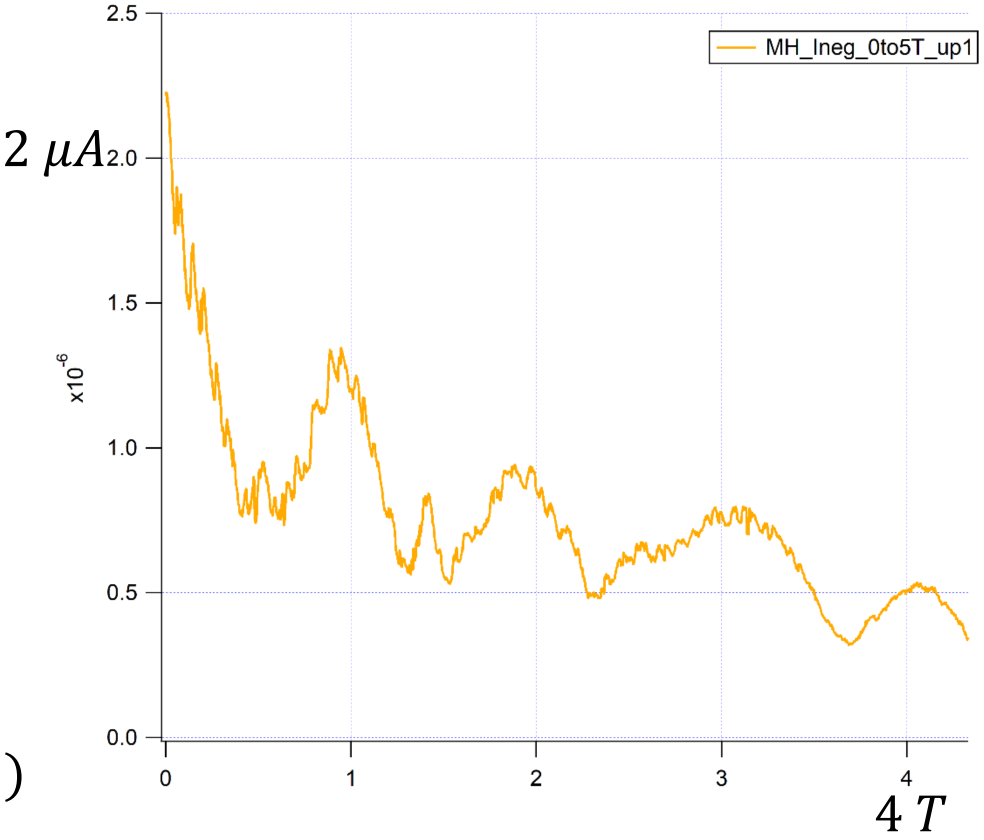




Supercurrent vs phase: ongoing experiments







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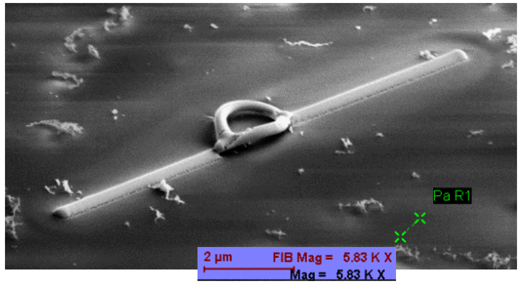
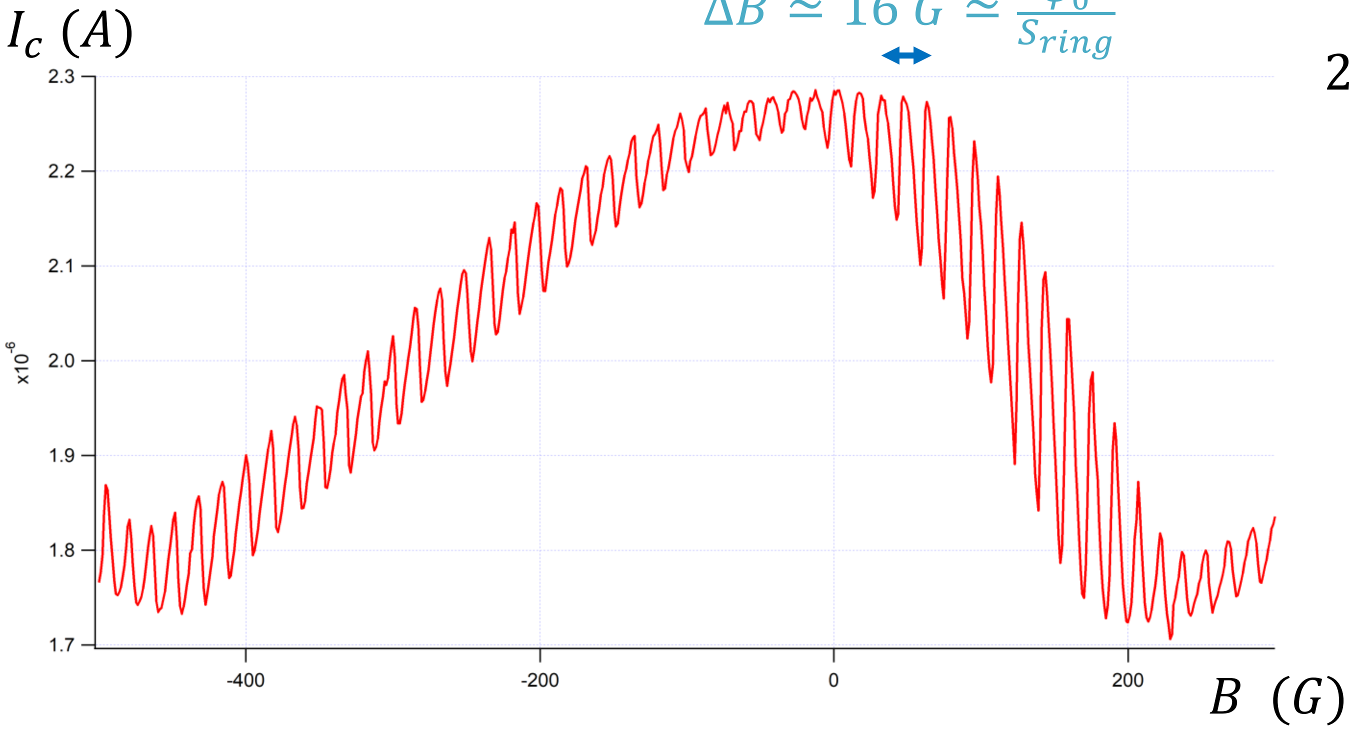
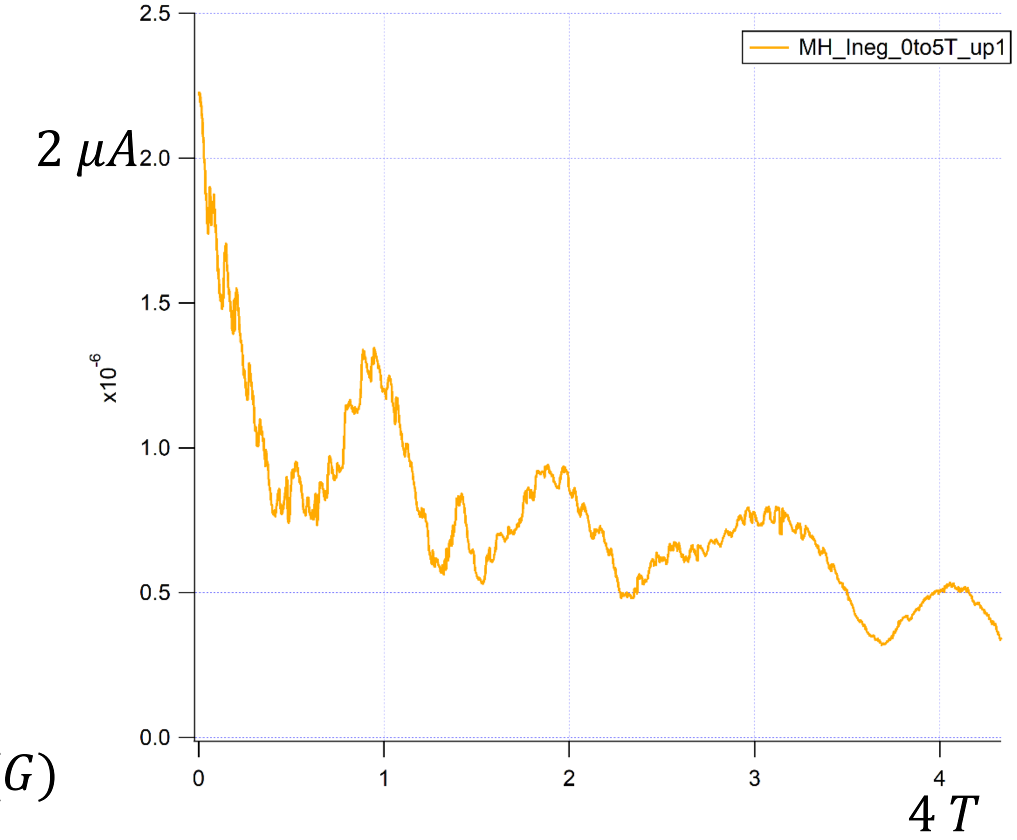


Sawtooth oscillations
=> more like an build-in asymmetric SQUID with phase controlled by the flux inside the ring
=> supercurrent vs phase relation of a long ballistic junction

$$\Delta B \simeq 16\ G \simeq \frac{\phi_0}{S_{ring}}$$





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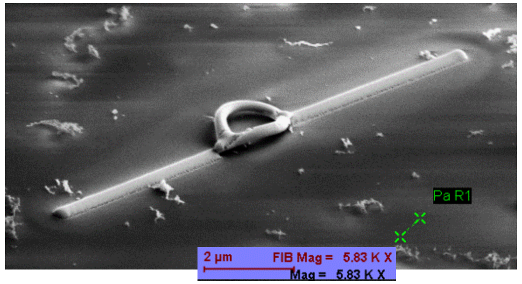

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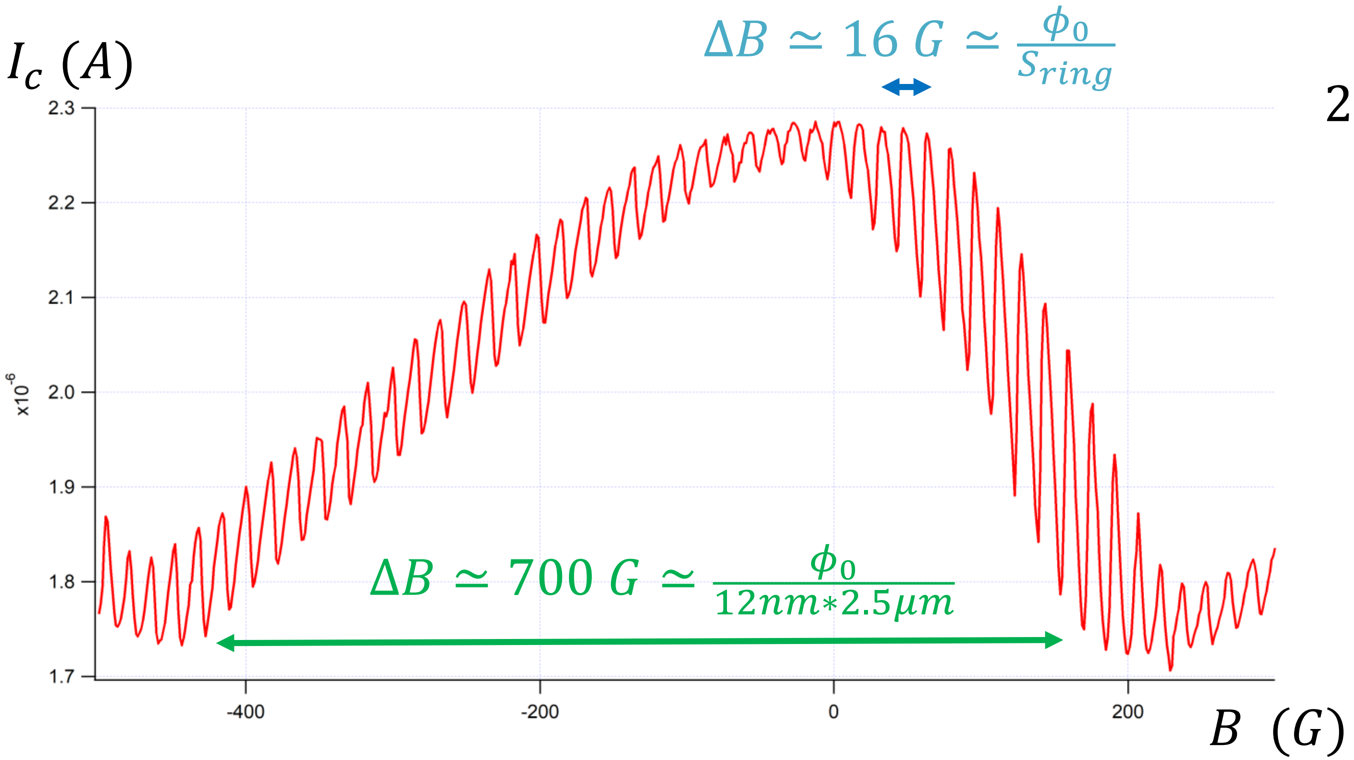
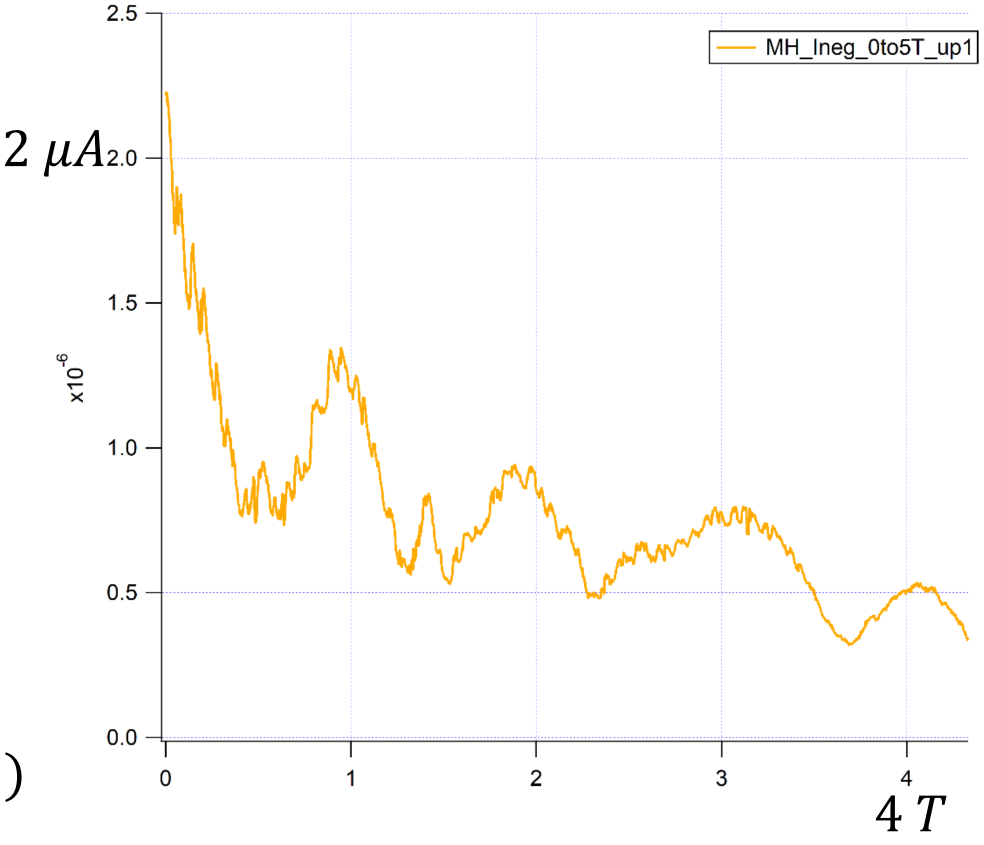


Interferences of $< 1nm$ wide paths,

$\simeq 12nm$ appart, in the same branch



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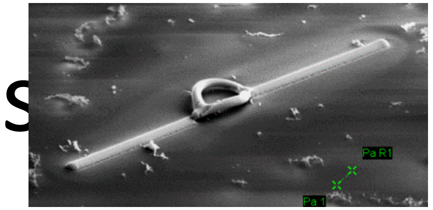
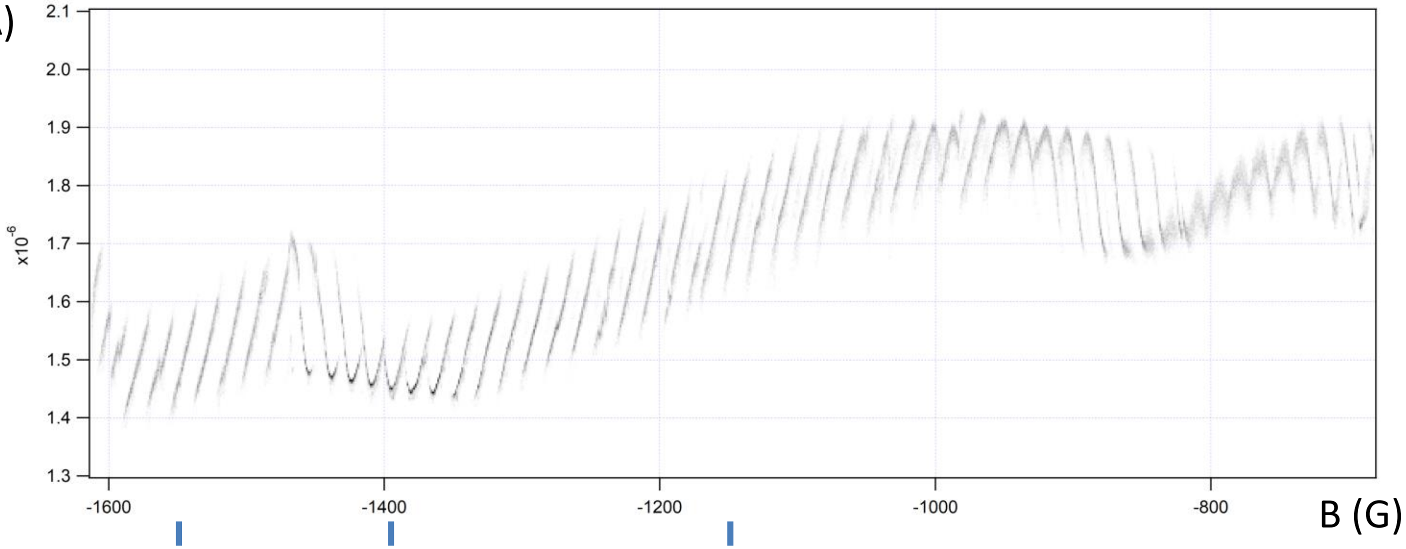
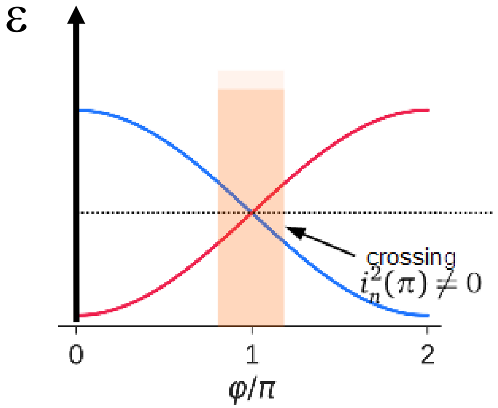
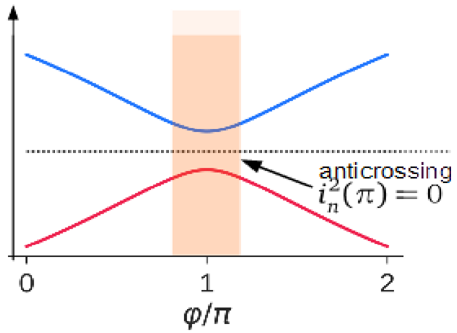


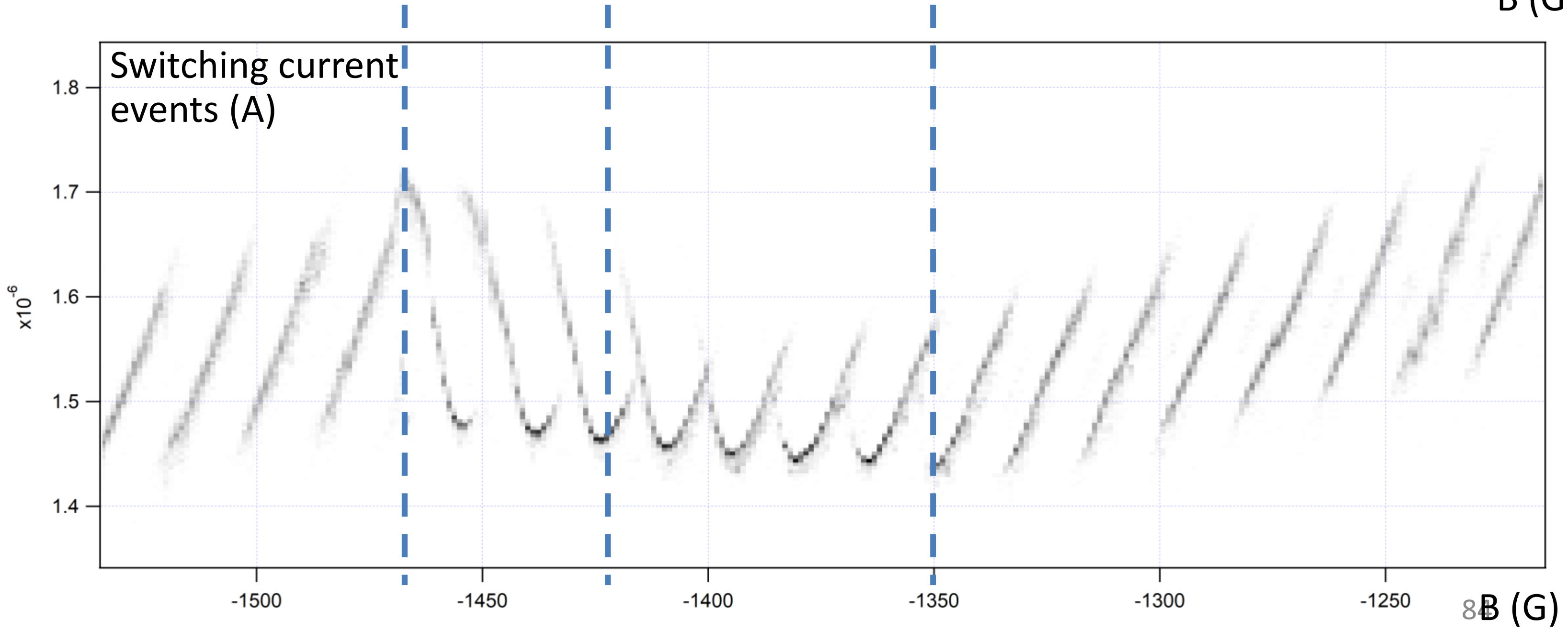
See Peng et al., PRB 2016
related to current fluctuations, see Fu and Kane PRB 2009
related to dissipation, see Murani et al. PRL 2019

Switching current events (A)









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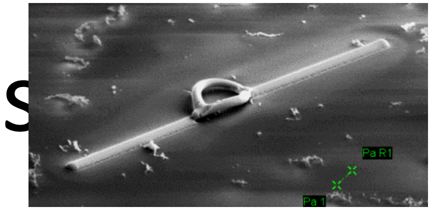

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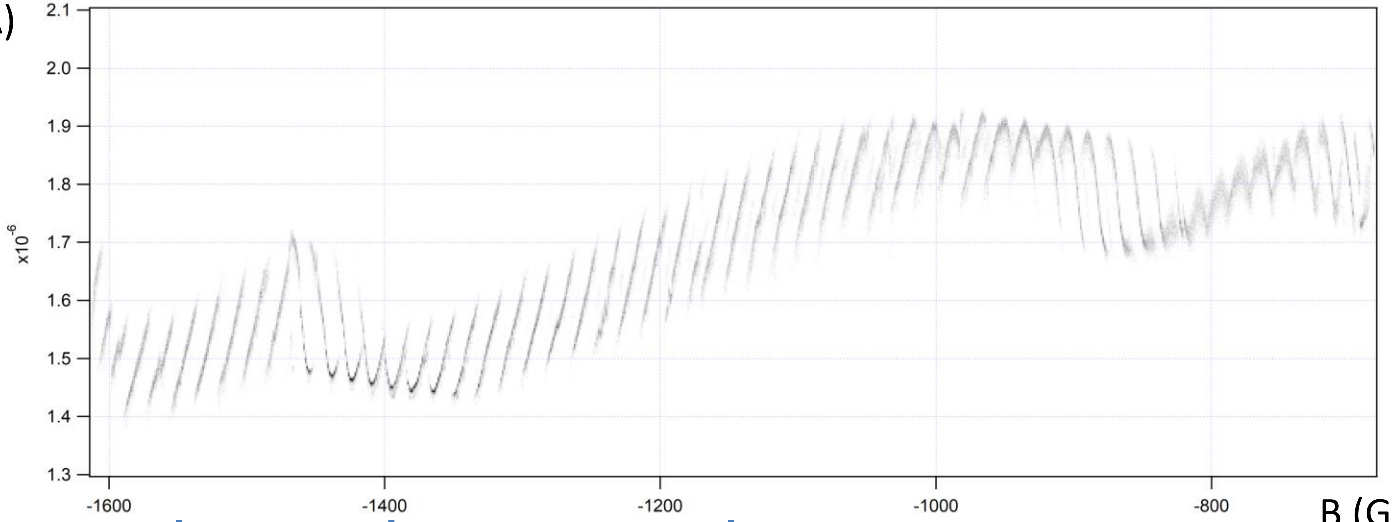
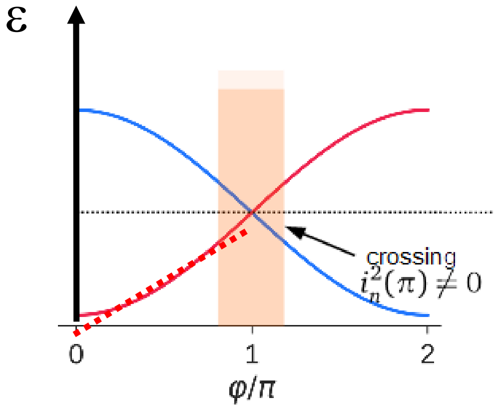
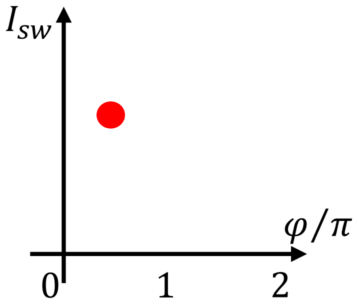
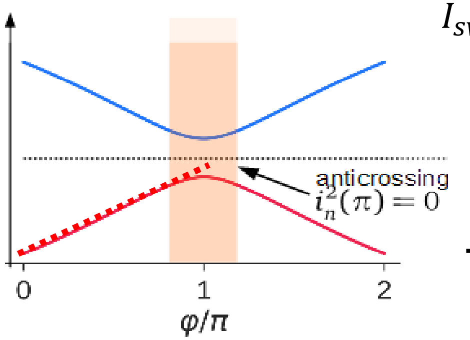
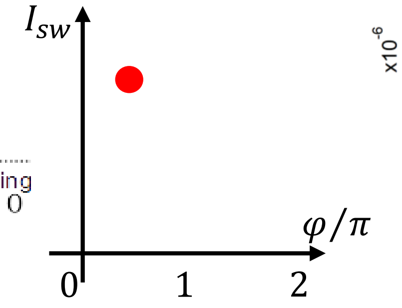


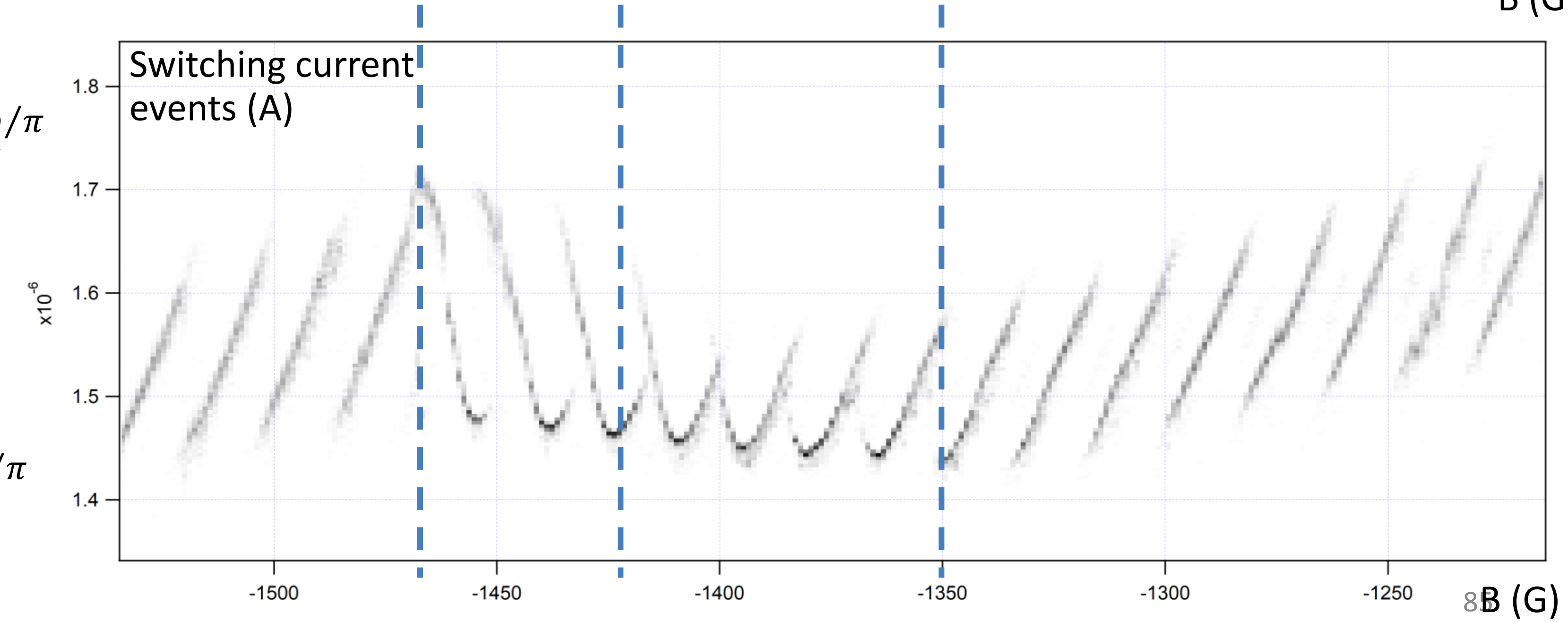
B (G)











B (G)

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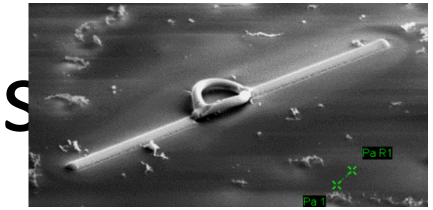

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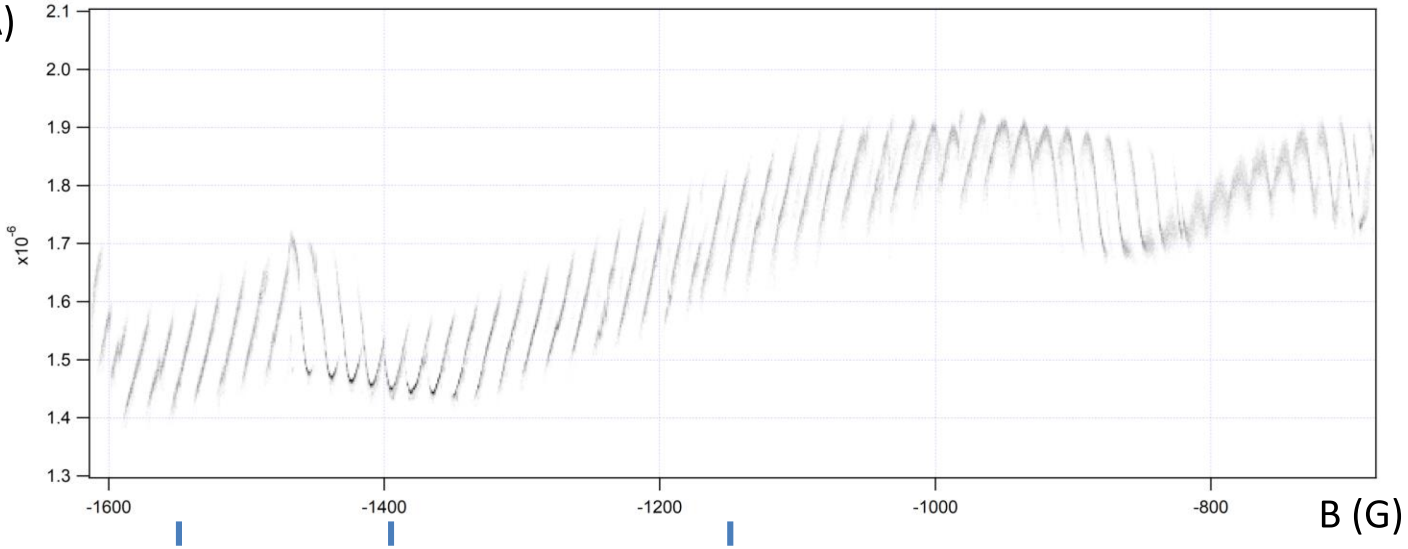
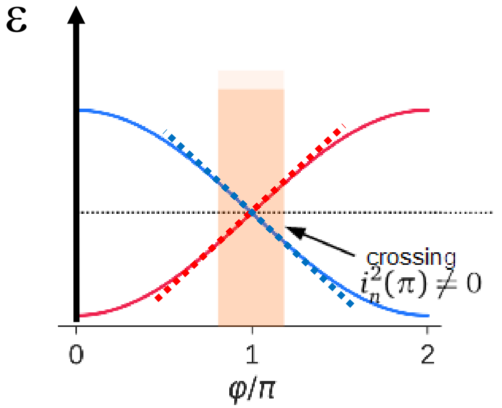
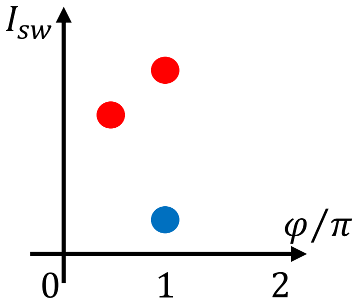
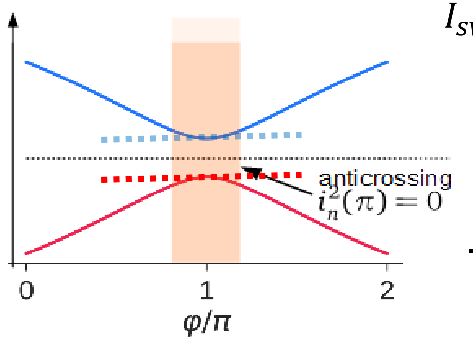
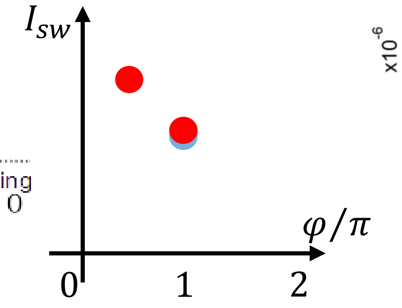
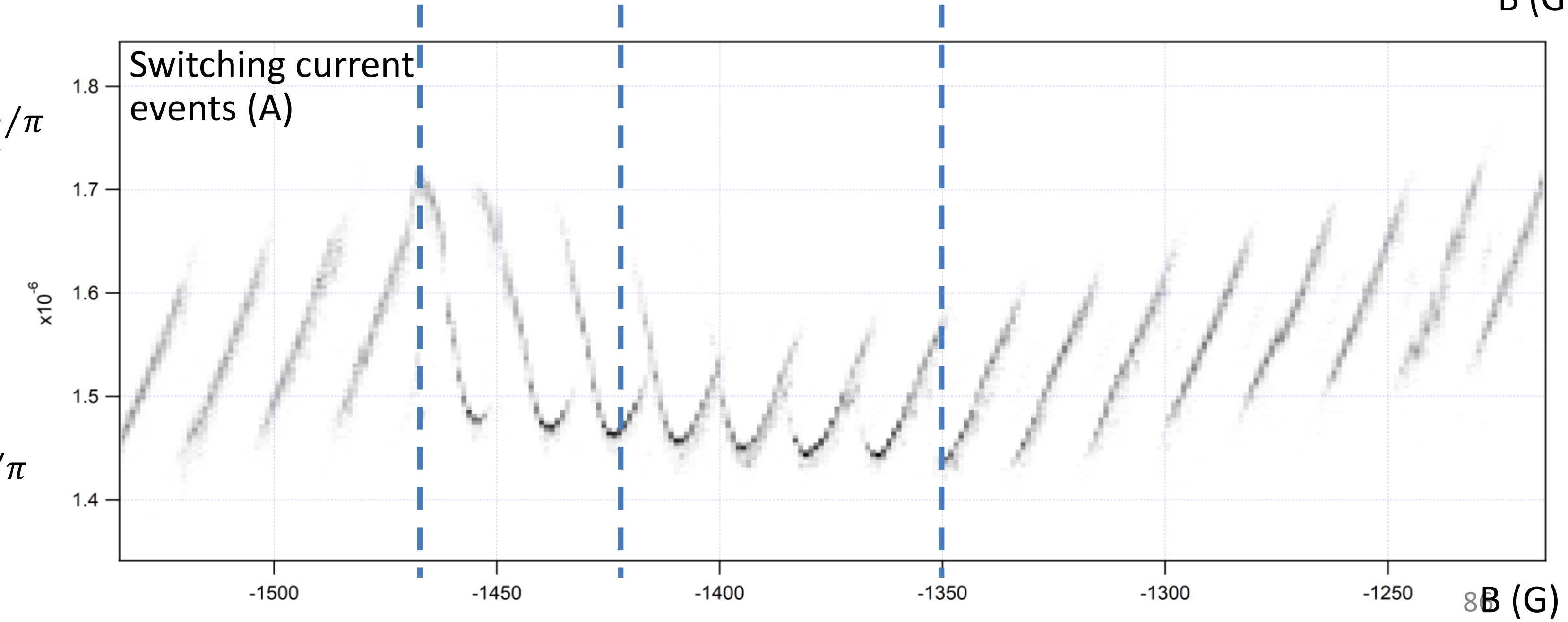




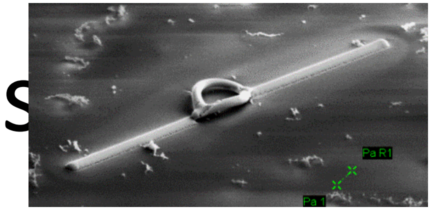




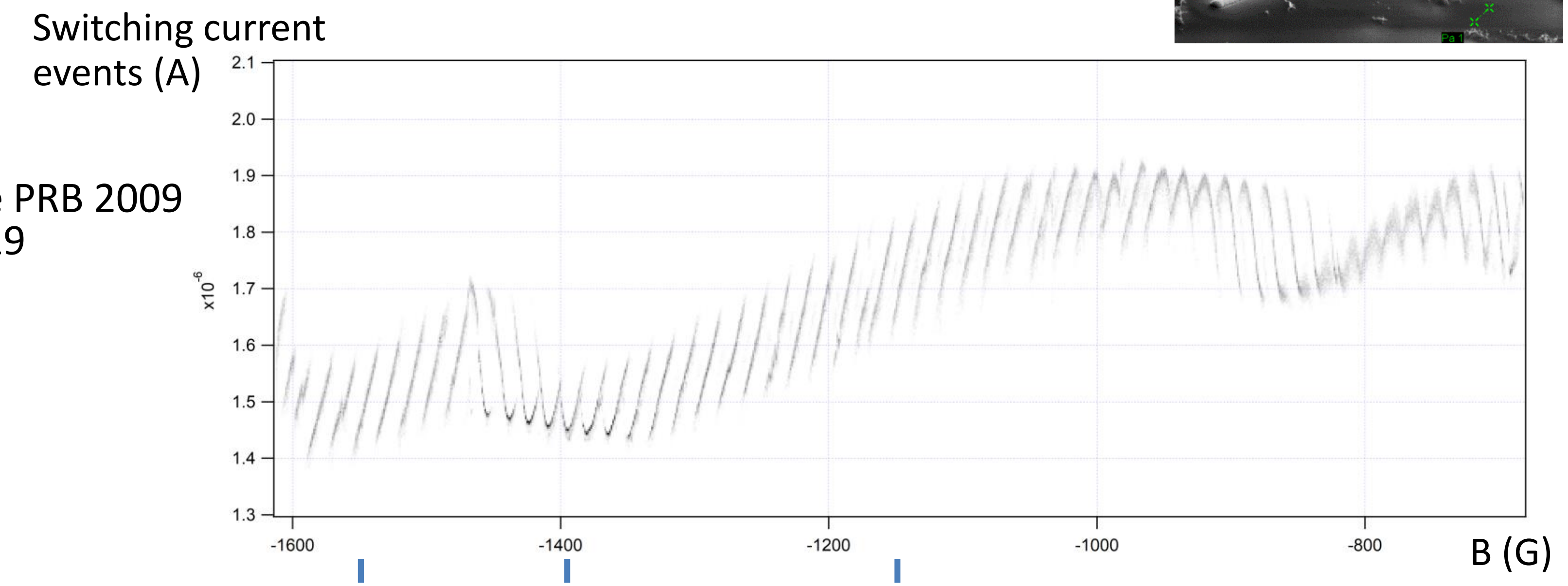


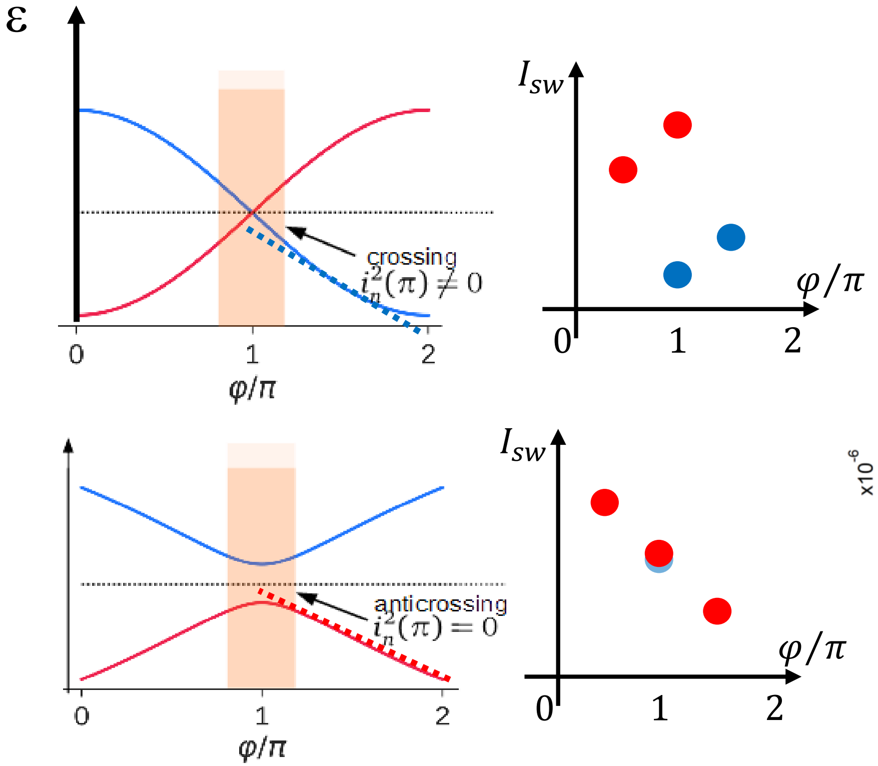


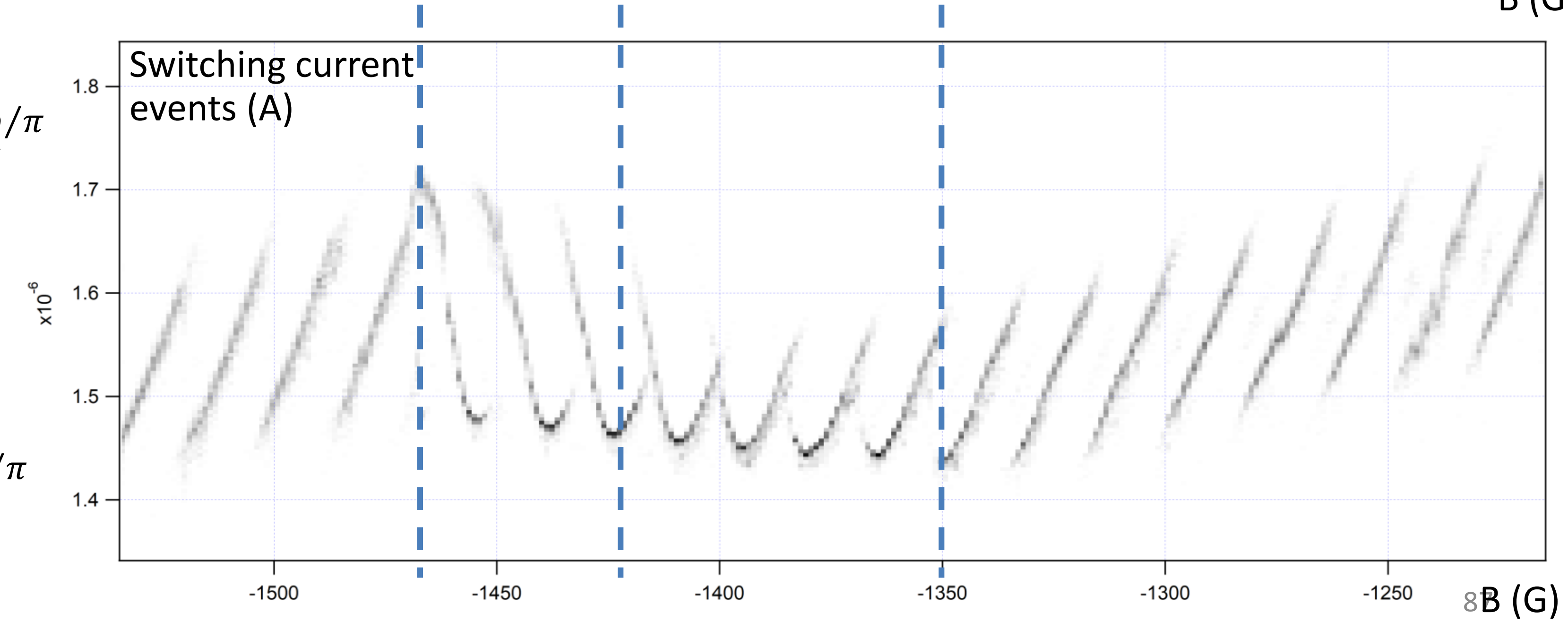
Supercurrent vs phase: ongoing experiments



See Peng et al., PRB 2016
related to current fluctuations, see Fu and Kane PRB 2009
related to dissipation, see Murani et al. PRL 2019



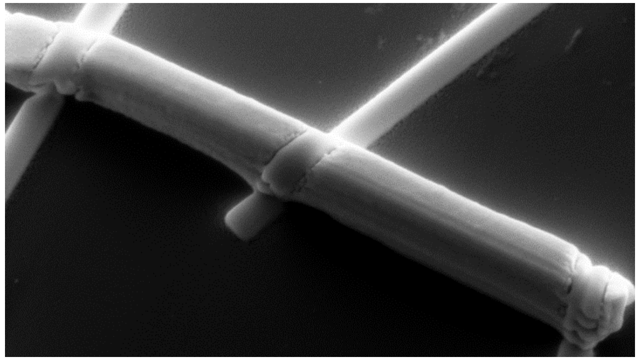


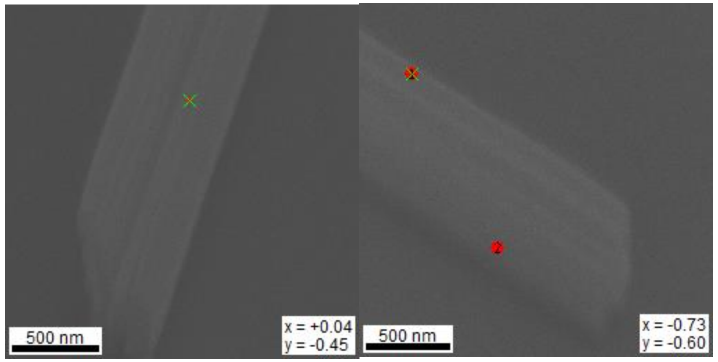


Open questions

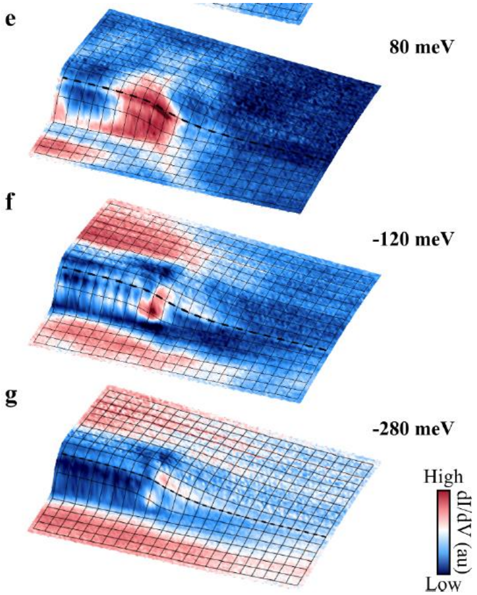
Ishibashi's group, RIKEN

Can a big supercurrent be supported by many hinge states arranged in steps ?

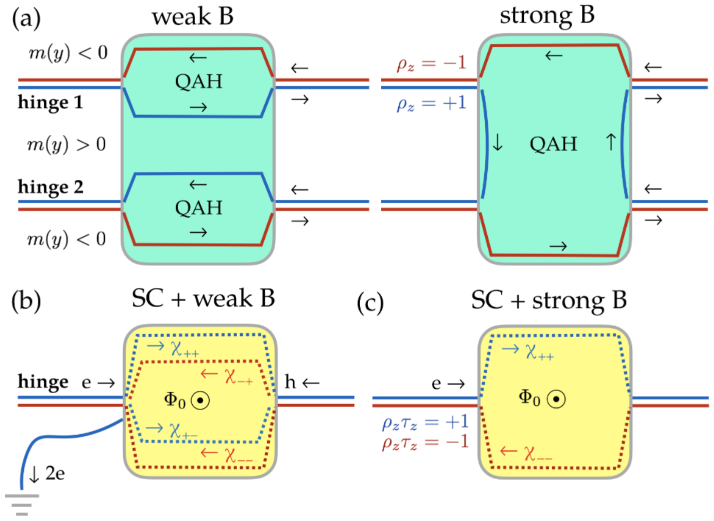




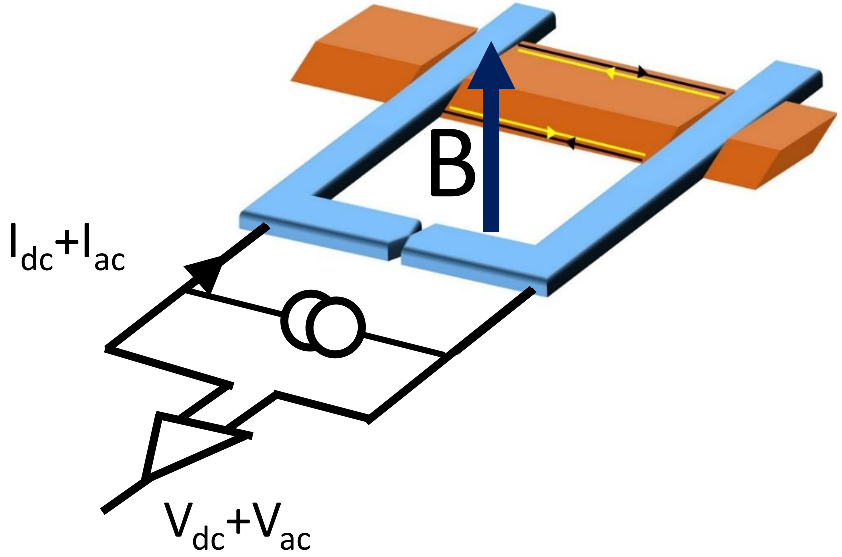
What is the effect of defects on the surface of Bi ?
What about strain ?
Revealing topological nature with screw dislocations ?
(Nayak et al., Cond. Mat. 2019)

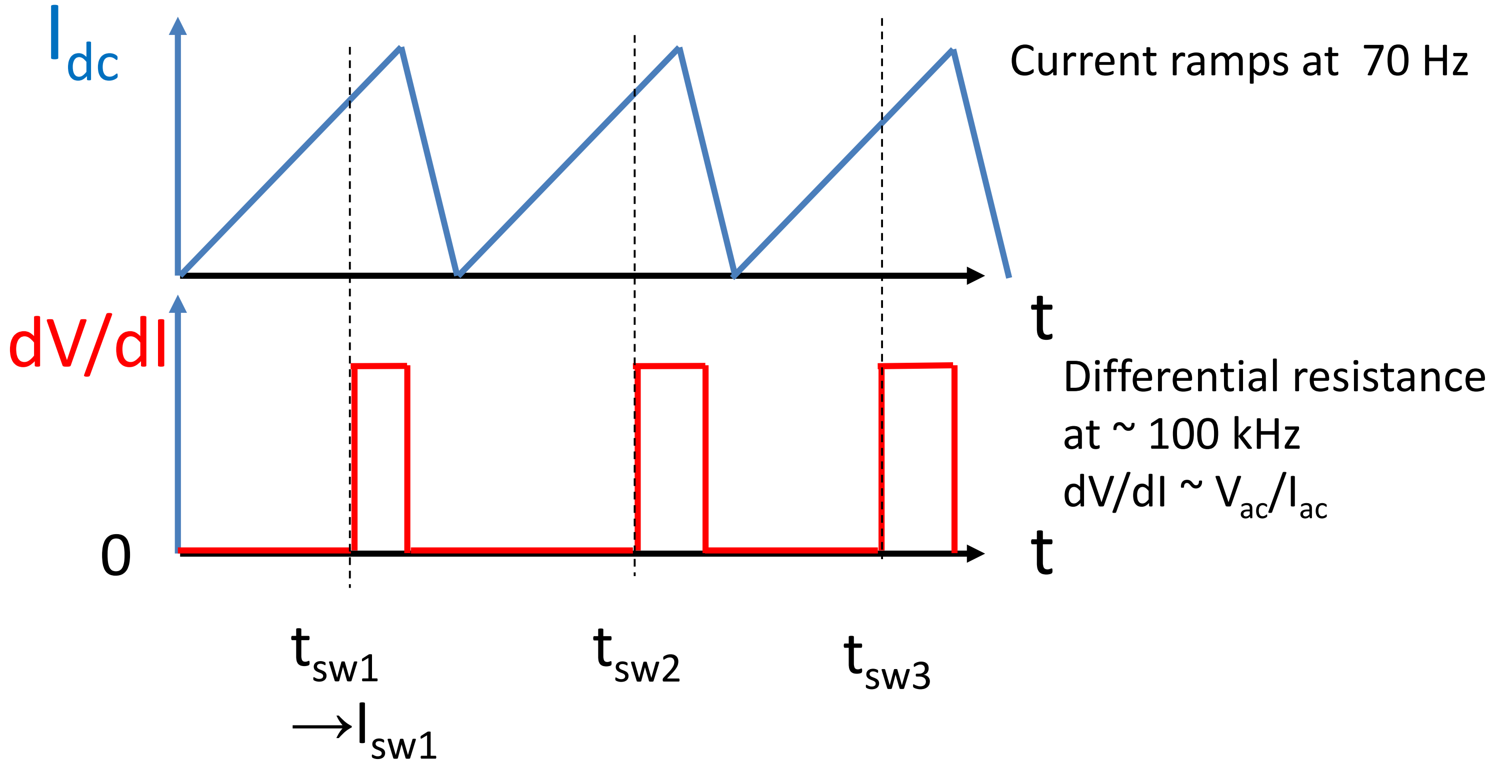


What happens at high magnetic field ?
(Queiroz and Stern, Cond. Mat. 2019)



Switching current measurement





Measure $<I_{sw}>$, averaged over 100 to 400 times, histogram as well