Higher order boundary signatures of crystalline symmetry protected topological phases

Max Geier

Dahlem Center for Complex Quantum Systems
Physics Department
Freie Universität Berlin

  

With: Luka Trifunovic
Max Hoskam
Piet Brouwer

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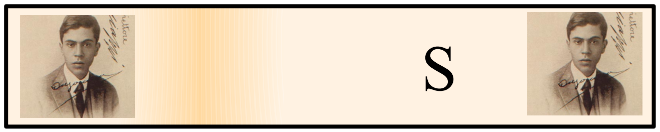
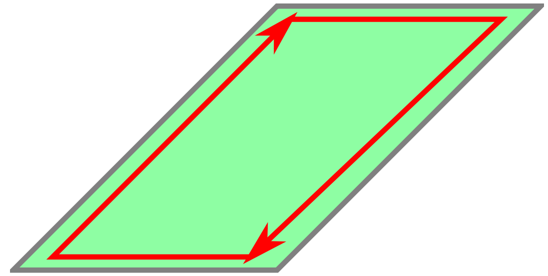
Topological phases of fermions

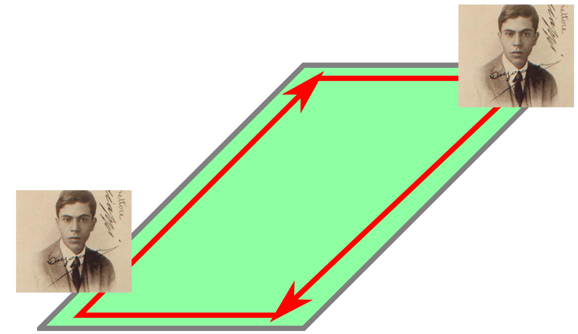
… that can be described by a quadratic (mean field) theory



1d topological superconductor:
Kitaev Wire

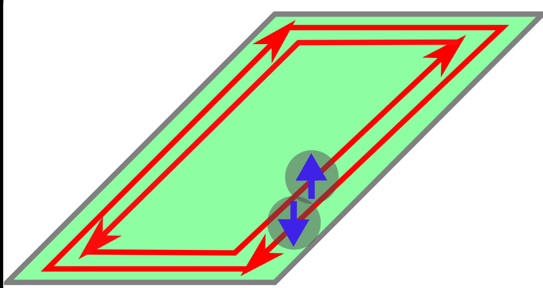


Quantum Hall effect

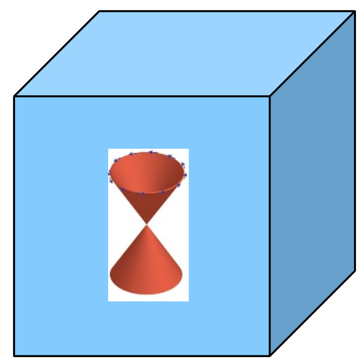


p_x+ip_y wave superconductor

Topological phases protected by a local symmetry: Time reversal



Quantum
spin-Hall effect

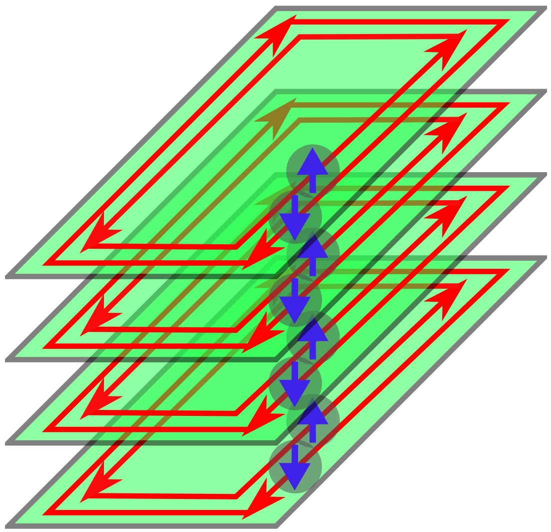


3d topological
insulator (TI)

Bulk boundary correspondence
gapped topological bulk = gapless **anomalous** states on a symmetric boundary

Crystalline symmetry protected topological phases

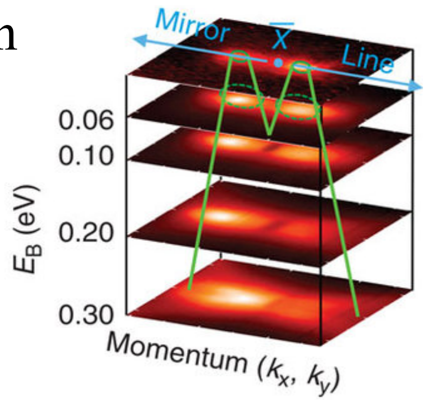
Translation symmetry:
Weak topological insulators



Only boundaries that preserve the protecting translation symmetry are gapless

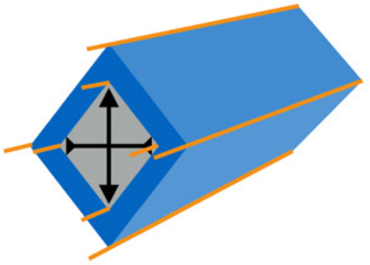
Mirror symmetry: SnTe

Double Dirac cone on reflection-symmetric surface



Hsieh *et al.* (2012)
Xu *et al.* (2012)
Tanaka *et al.* (2012)

Helical hinge modes in a strained lattice

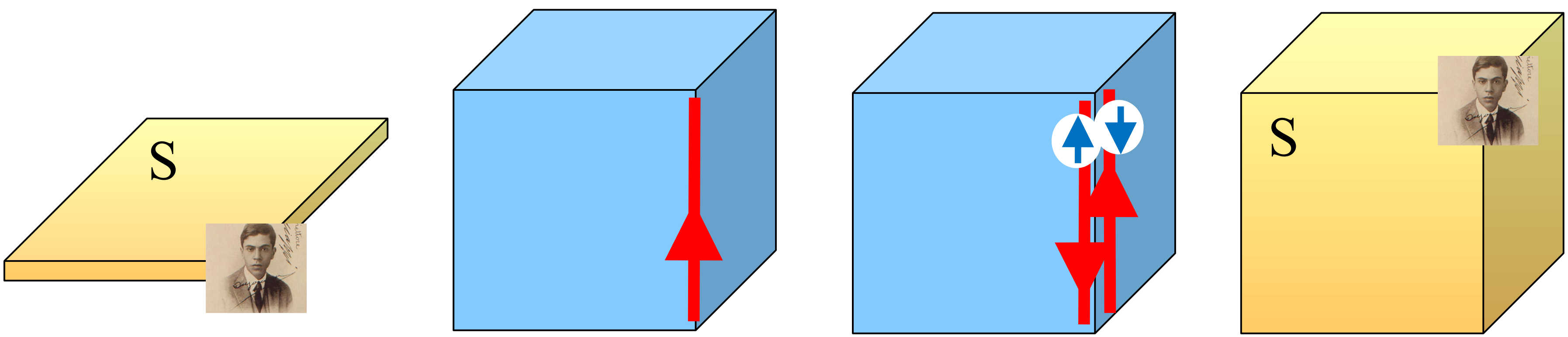


Schindler *et al.* (2018)

Crystalline topological phases with higher order boundary signatures

d-dimensional *n*th order crystalline SPT:

- Anomalous gapless states form a *(d-n)* dimensional manifold on the boundary



n=2 *n=2* *n=2* *n=3*

Signatures of "purely crystalline symmetry protected topological phases"

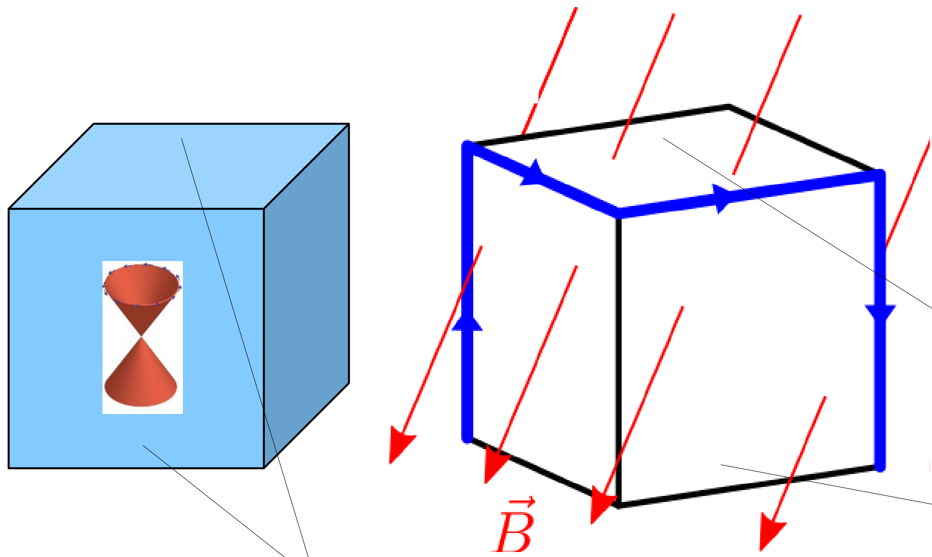
→ Trivial when crystalline symmetry is broken

Not every higher order boundary signature corresponds to a crystalline SPT.

Which higher order boundary signatures correspond to a higher order crystalline SPT?

Extrinsic higher order boundary signatures

Example:
Gapless hinge state of a $3d$ TI in a magnetic field...



Surface Dirac Hamiltonian

$$H_{\mathrm{sf}}(\vec{k}) = k_x \sigma_1 + k_y \sigma_2$$

Crossing protected by TRS: $\mathcal{T} = i\sigma_2 K$

Surface Dirac Hamiltonian

$$H_{\mathrm{sf}}(\vec{k}) = k_x \sigma_1 + k_y \sigma_2 + \vec{B} \cdot \vec{\sigma}$$

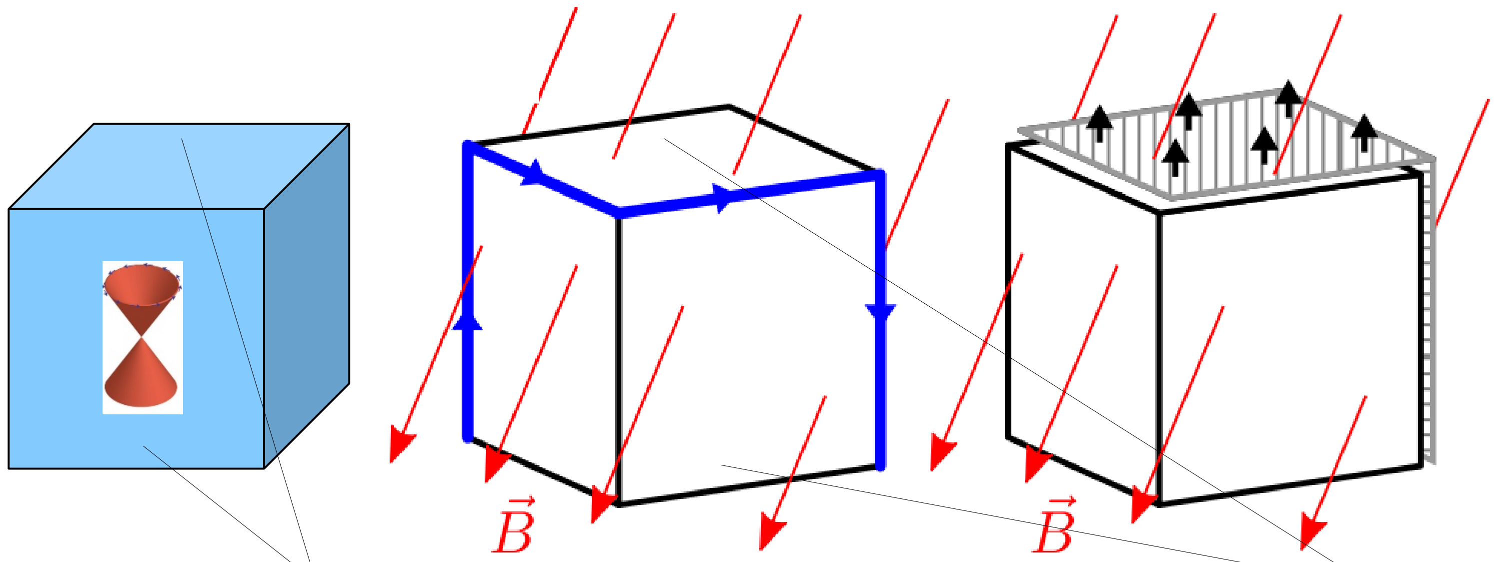
The component $B_z \sigma_z$ normal to the surface gaps the Dirac Hamiltonian!

$$\sigma_{xy} = \pm \frac{1}{2} \frac{e^2}{h}$$

Extrinsic higher order boundary signatures

Example:

Gapless hinge state of a $3d$ TI in a magnetic field... … that can be removed by decorating the TI with a $2d$ ferromagnetic insulator.



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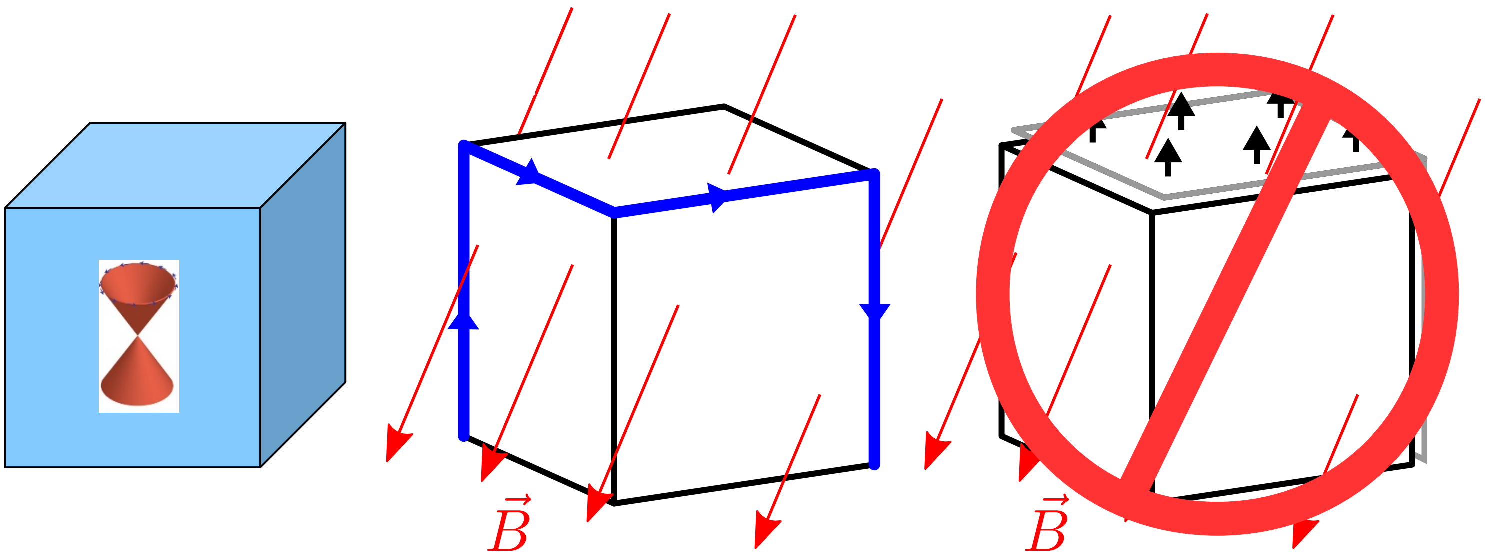
Extrinsic boundary signature: Existence depends on termination.

Intrinsic higher order boundary signatures

Require **inversion symmetry both in the crystal and for the sample**!



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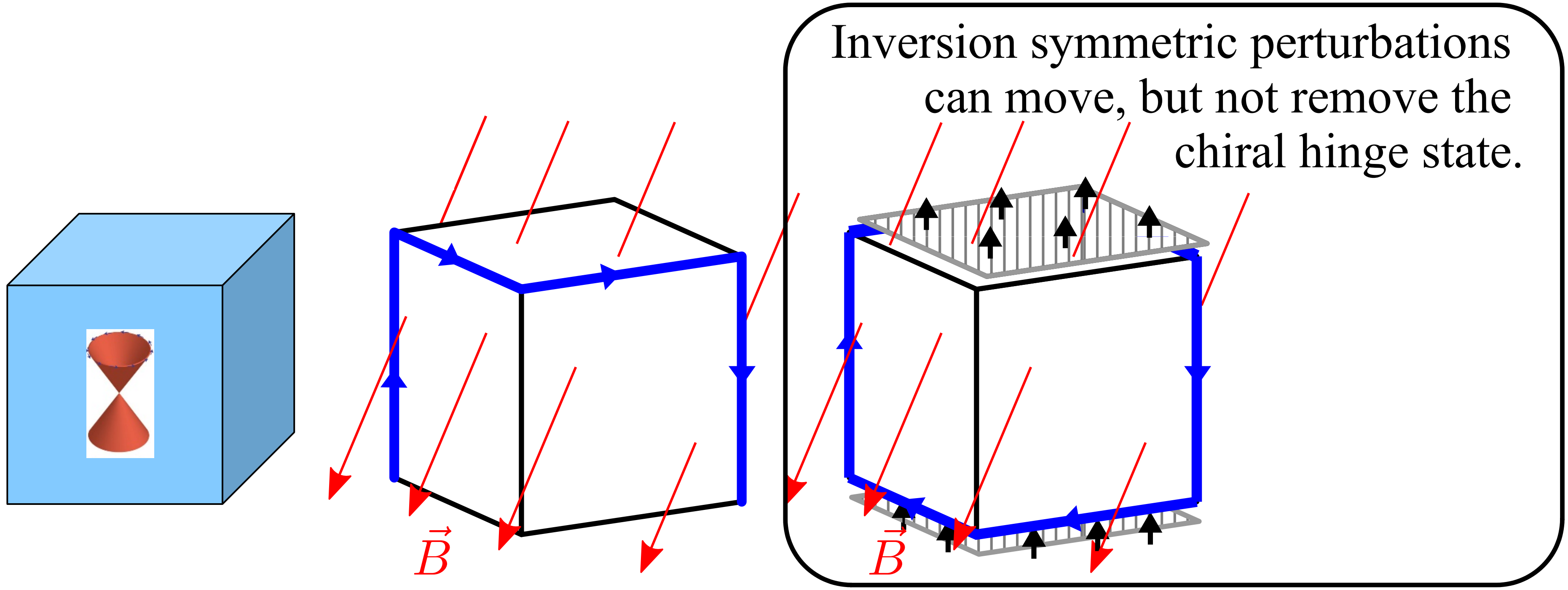
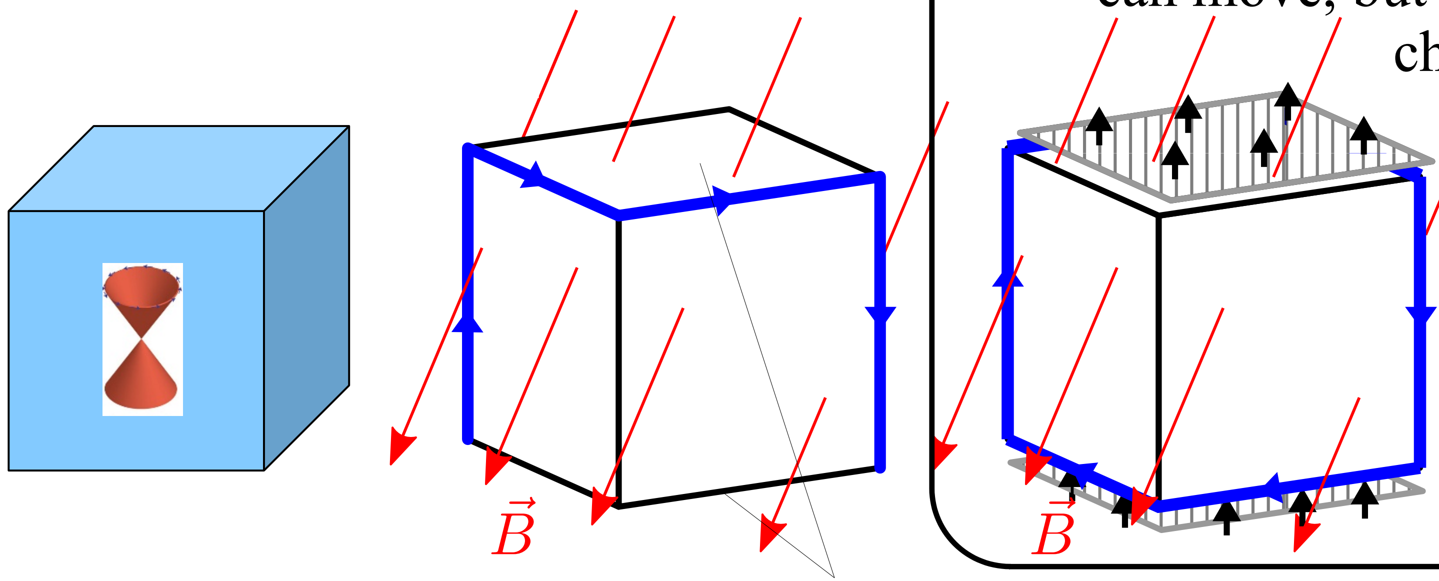


Intrinsic higher order boundary signatures

Require **inversion symmetry both in the crystal and for the sample!**

Inversion symmetric perturbations can move, but not remove the chiral hinge state.



Inversion symmetry requires that inversion symmetry related surfaces have opposite mass!

$$H_{\rm sf}(\vec{k}) = k_x\sigma_1 + k_y\sigma_2 + \vec{B}\cdot\vec{\sigma}$$

Intrinsic boundary signature: Existence independent of termination

Intrinsic higher order boundary signatures

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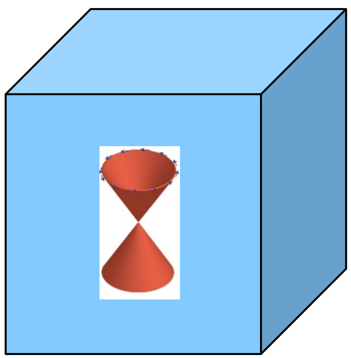

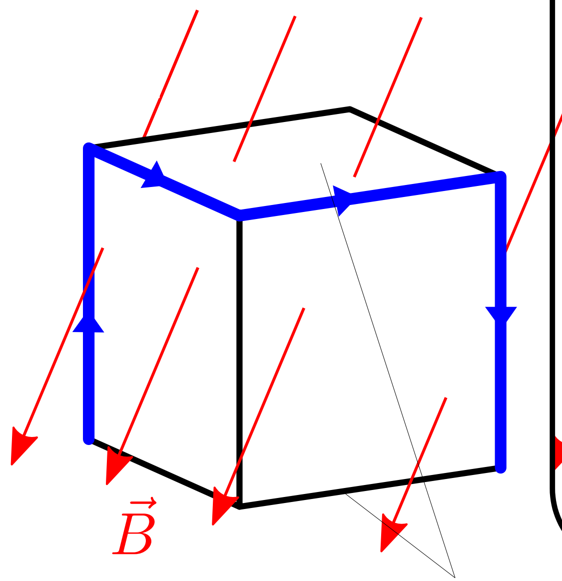

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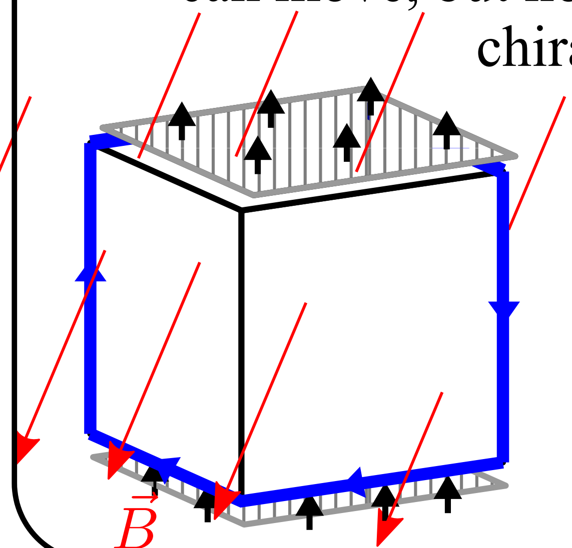

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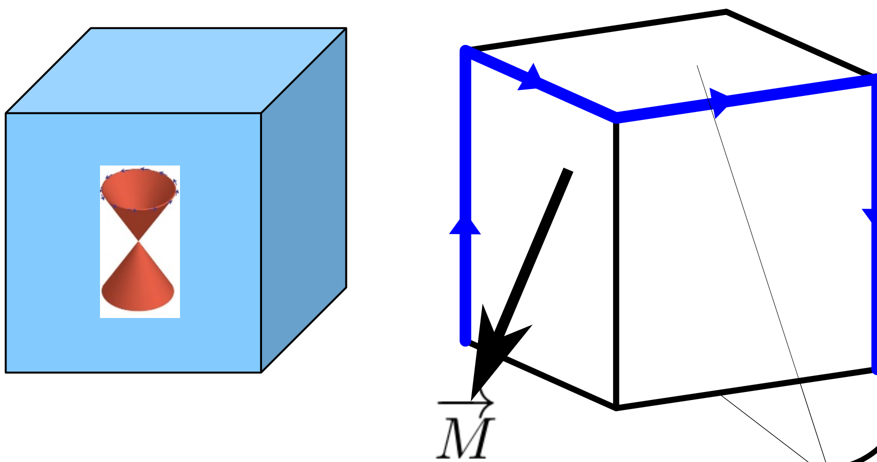
Intrinsic boundary signature: Existence independent of termination

Higher order bulk boundary correspondence:
Intrinsic boundary signatures can not be induced / removed by adding any lower dimensional SPT → Boundary signature of a topological bulk.

Intrinsic higher order boundary signatures

Require **inversion symmetry both in the crystal and for the sample!**

"Magnetic topological insulator"
Cr, V doped $(Bi, Sb)_2Te_3$



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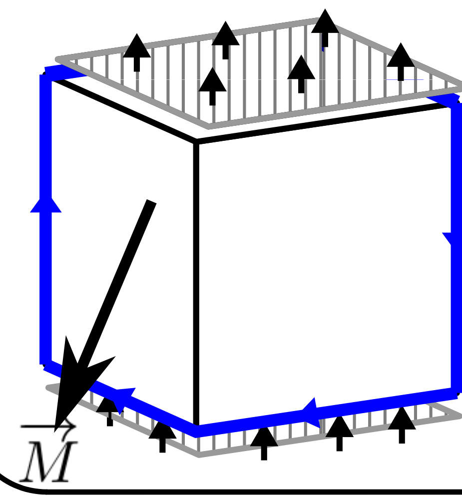


Inversion symmetry requires that inversion symmetry related surfaces have opposite mass!

$$H_{\rm sf}(\vec{k}) = k_x\sigma_1 + k_y\sigma_2 + \vec{M}\cdot\vec{\sigma}$$

Intrinsic boundary signature: Existence independent of termination

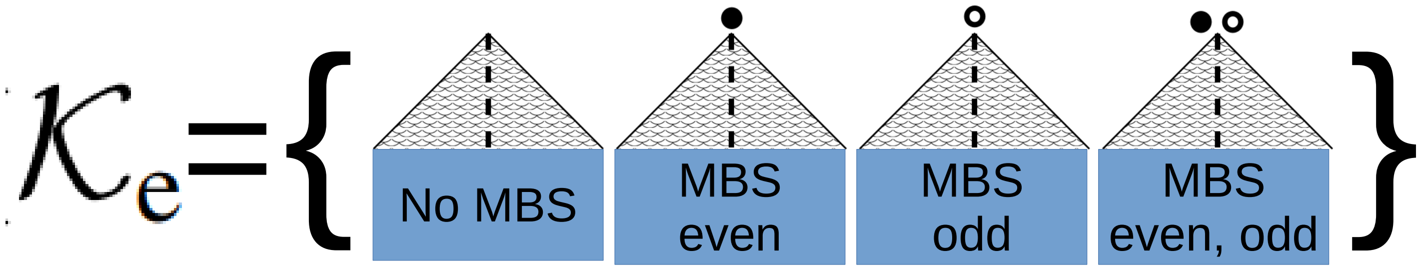
Higher order bulk boundary correspondence:
Intrinsic boundary signatures can not be induced / removed by adding any lower dimensional SPT → Boundary signature of a topological bulk.

Which higher order boundary signatures correspond to a higher order crystalline SPT?

Intrinsic higher order boundary signatures:
Anomalous gapless states on the surface of a crystal
- whose shape respects the space group of the crystal (up to translations)
- that can not be removed by any change of symmetric change of termination

Second order topological superconductor with mirror symmetry: Boundary perspective

Extrinsic classification of a mirror symmetric corner



p-wave / spinless SC:
In Majorana basis:

$$H(k_1, k_2) = -H^*(-k_1, -k_2) = \sigma_1 H(-k_1, k_2)\sigma_1$$

Second order topological superconductor with mirror symmetry: Boundary perspective

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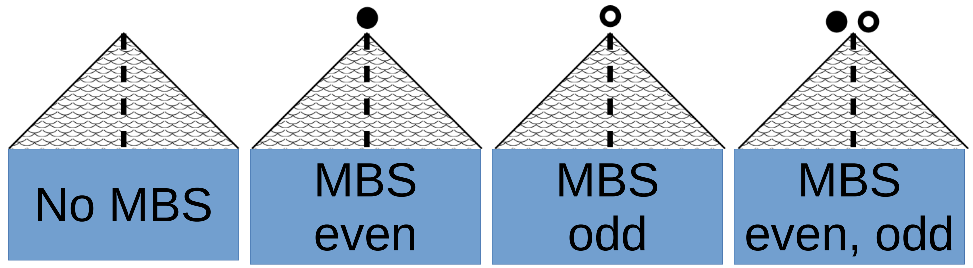
$\mathcal{K}_{\mathrm{e}} = \{$



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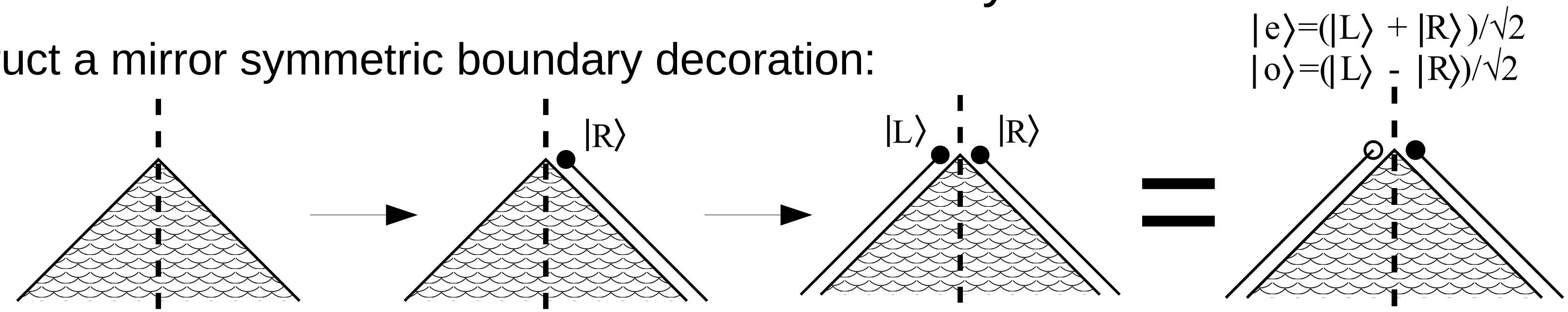
Intrinsic classification of a mirror symmetric corner

Construct a mirror symmetric boundary decoration:



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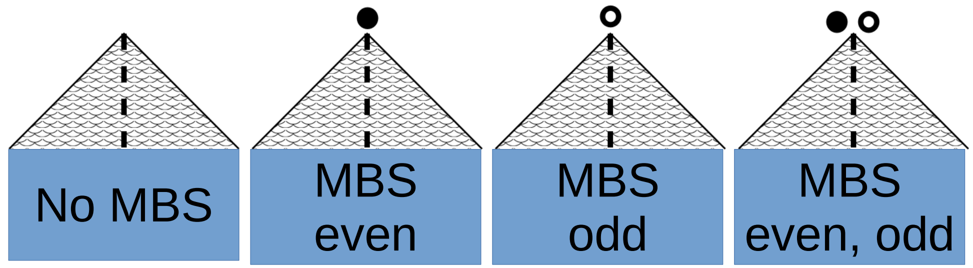

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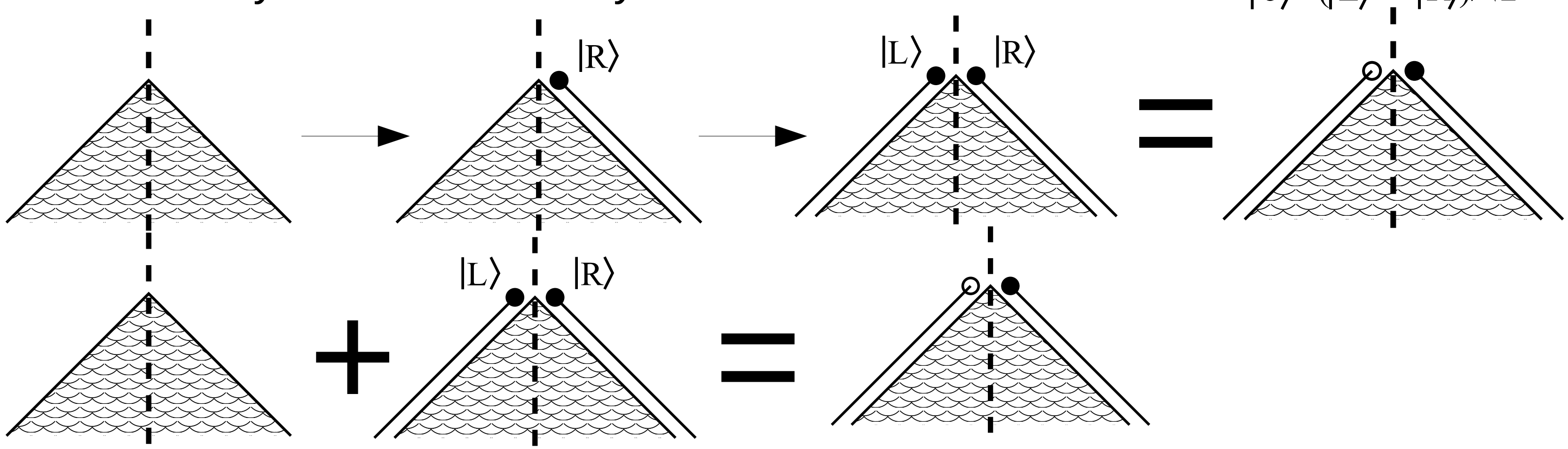
Construct a mirror symmetric boundary decoration:

$|e\rangle = (|L\rangle + |R\rangle)/\sqrt{2}$
$|o\rangle = (|L\rangle - |R\rangle)/\sqrt{2}$



Second order topological superconductor with mirror symmetry: Boundary perspective

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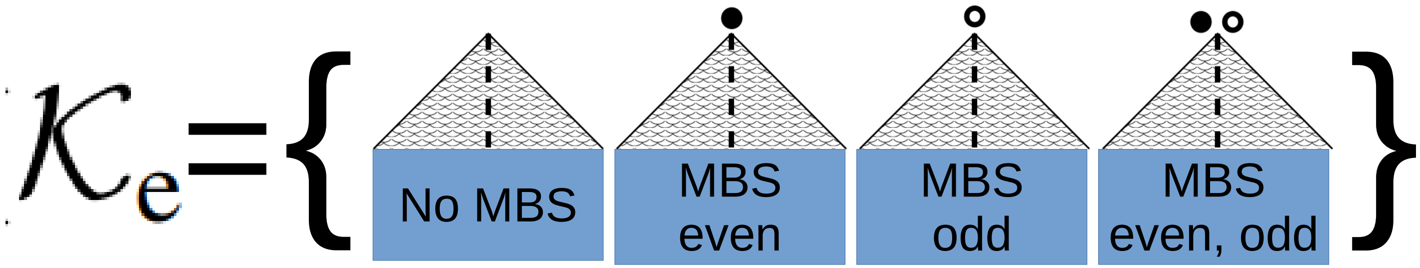

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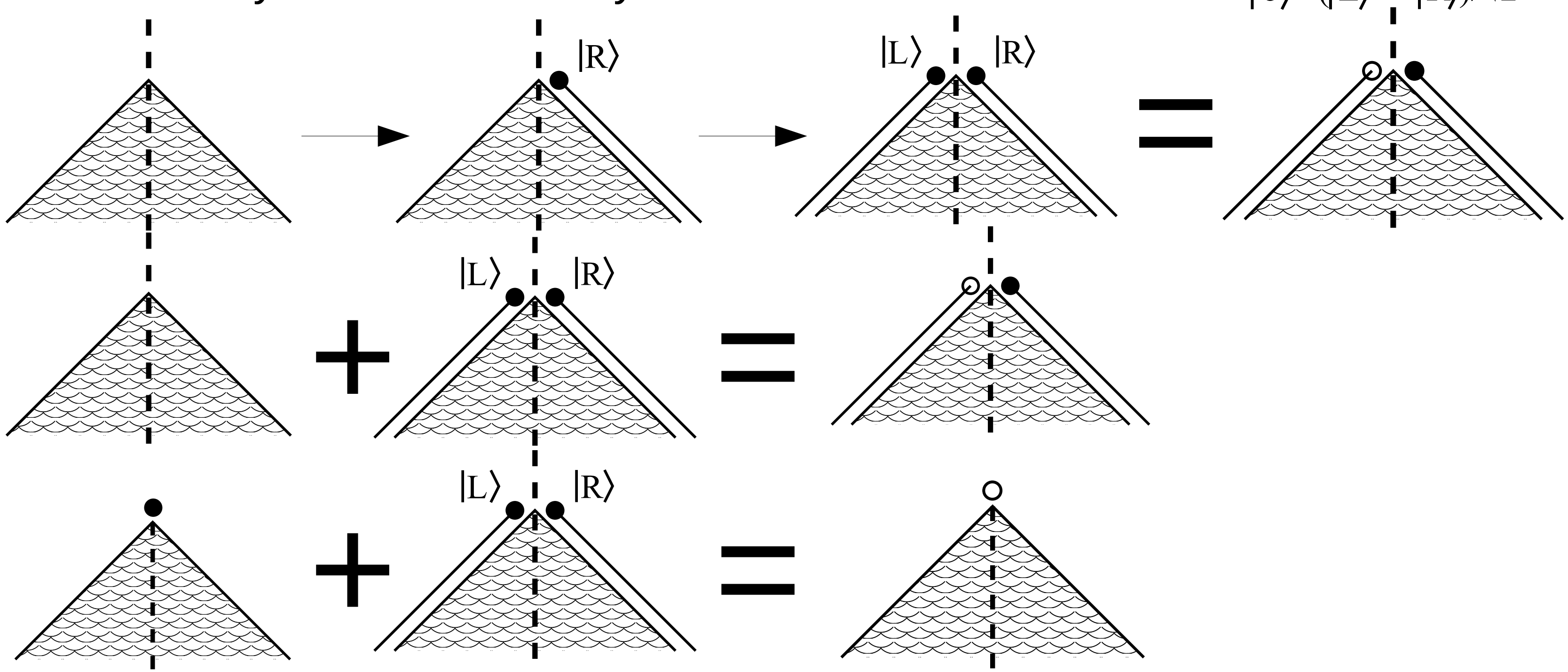

Second order topological superconductor with mirror symmetry: Boundary perspective

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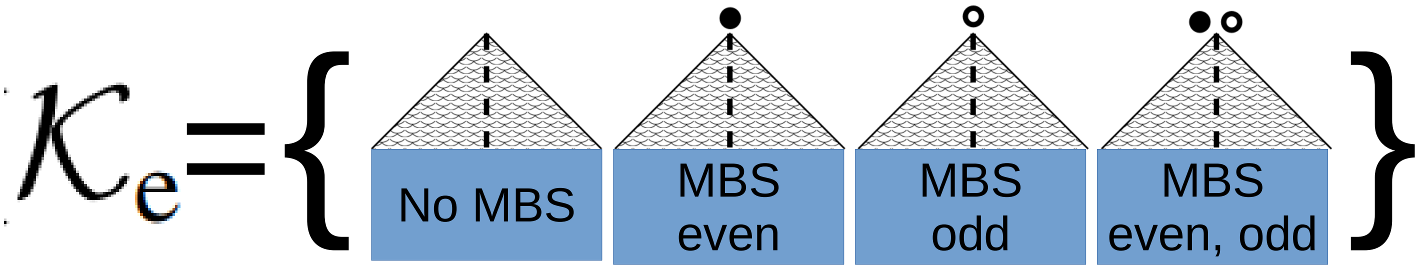

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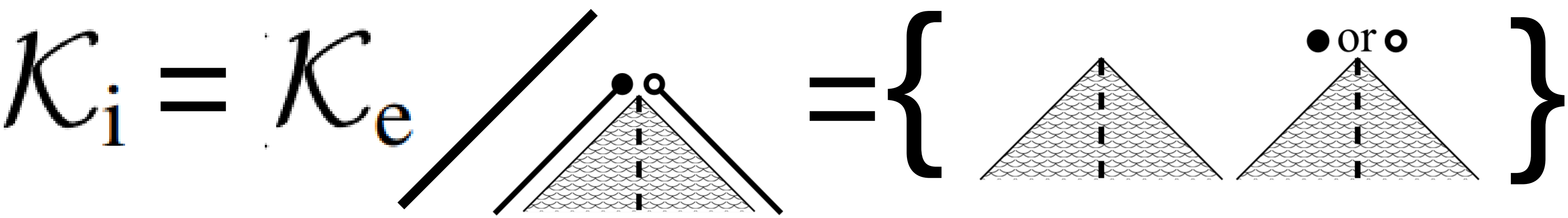

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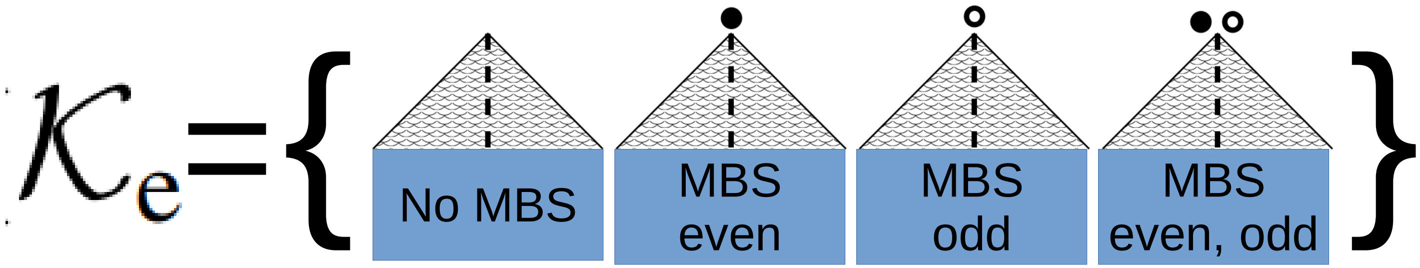

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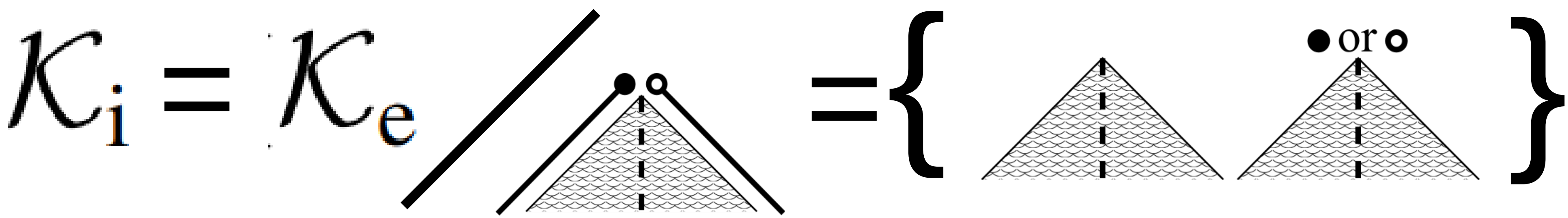
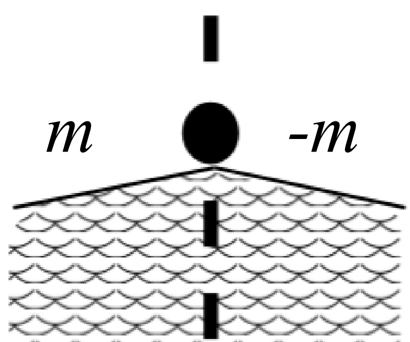


Bulk-boundary-corner correspondence



$$i\partial_x\sigma_3 + m(x)\sigma_2$$
$$m(x) = -m(-x)$$

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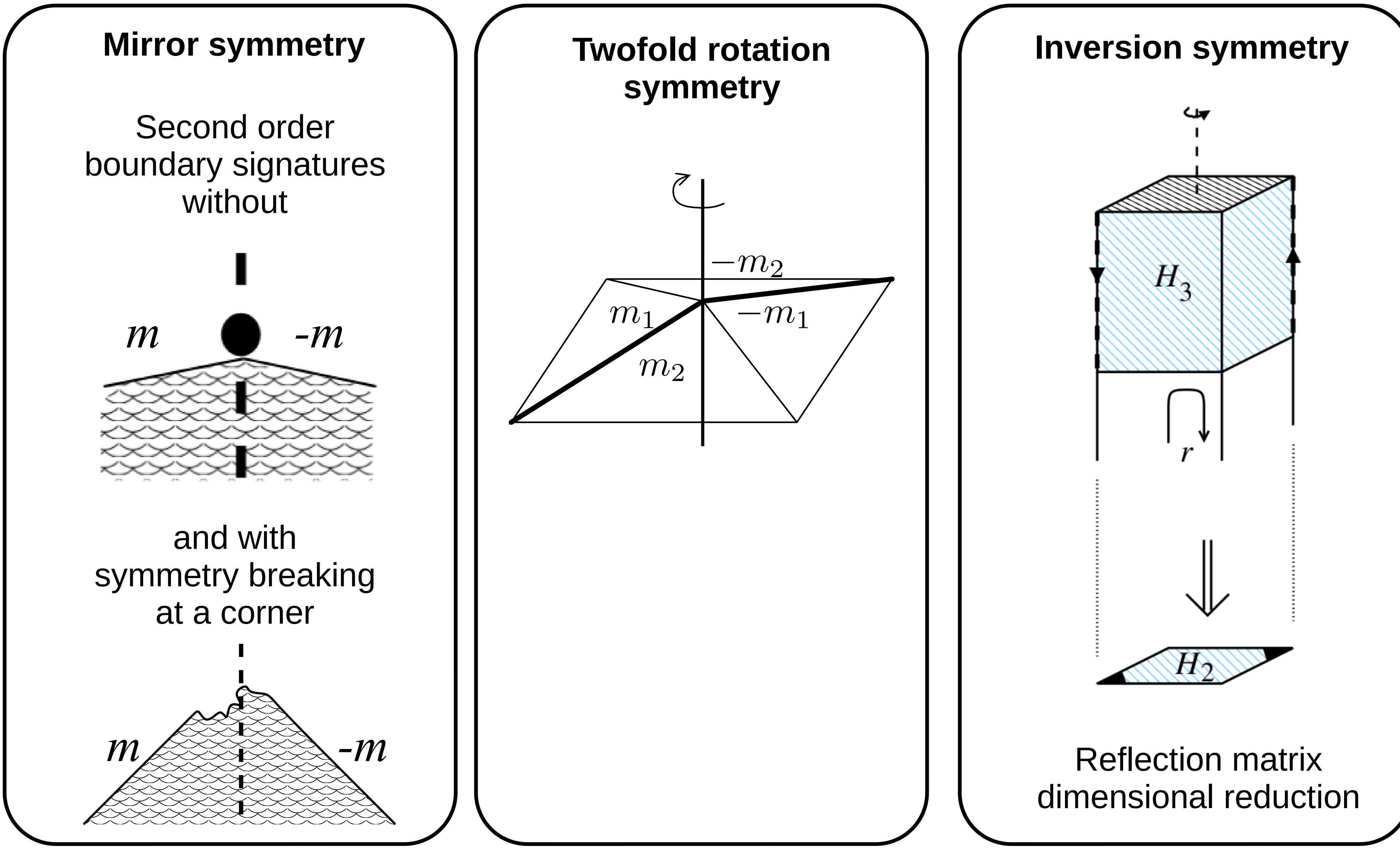
Second-order topological insulators and superconductors with an order-two crystalline symmetry

Max Geier,[1,*] Luka Trifunovic,[1] Max Hoskam,[1,2] and Piet W. Brouwer[1]

[1]*Dahlem Center for Complex Quantum Systems and Physics Department, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany*

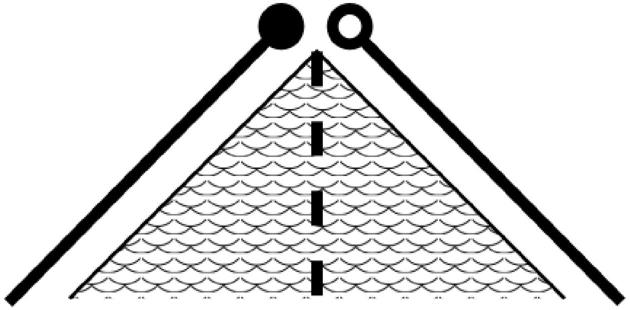
[2]*Department of Applied Physics, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands*



What happens when including interactions?

Within perturbation theory:
Can the gapless boundary modes be gapped by interactions?
$\mathbb{Z}_2$: Majorana, helical mode, … ← NO
$\mathbb{Z}$: Majorana in a spinless TRS superconductor, chiral edge mode, …
Reduce $\mathbb{Z}$ to $\mathbb{Z}_8$!

Beyond perturbation theory:
+ new phases that can not be described with a quadratic theory



Thank you for your attention :)