Realizations of Floquet-Andreev levels in three terminal Josephson junctions with a quantum dot.

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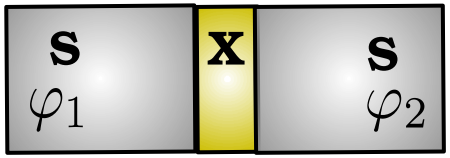
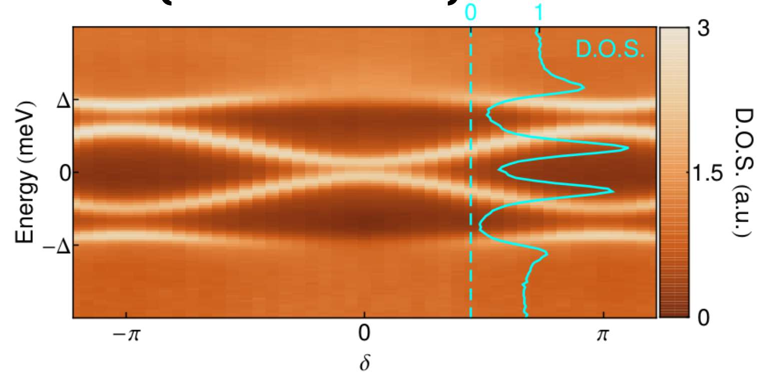
Andreev bound states and topological transconductance

Hybrid S-X-S nanostructure



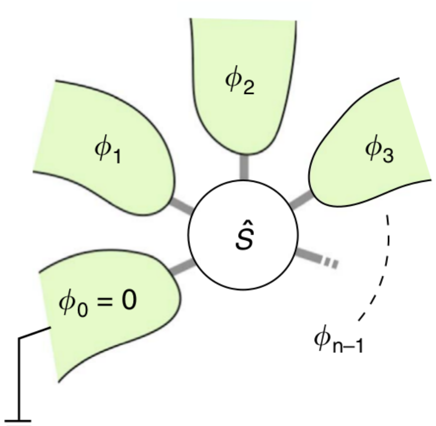
$$\varphi = \varphi_2 - \varphi_1$$

(Pillet 2011)

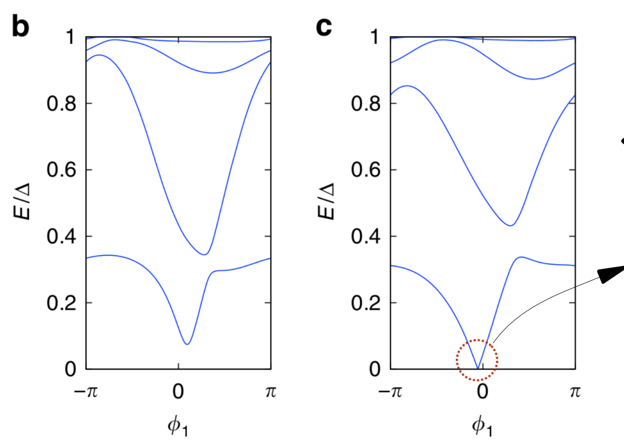


Finite motion of the electron/hole in the nanostructure $\Rightarrow$ **Andreev bound states (ABS)**

N-terminal Josephson junctions $\Rightarrow$ **(N-1) dimensional material**



(Riwar et al, Nat. Comm 2016)



$$\mathcal{F} = \pm \frac{\varphi}{|\varphi|^3}$$

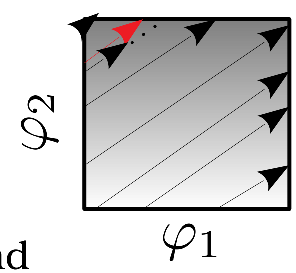
Weyl point (N≥4)

$$\bar{I}_\alpha = G_{\alpha\beta} V_\beta$$

$$G_{\alpha\beta} = -\frac{(2e)^2}{h} C_{\alpha\beta}$$

$$I_\alpha(t) = \frac{2e}{\hbar} \frac{\partial \varepsilon}{\partial \varphi_\alpha} - 2e\mathcal{F}_{\alpha\beta}\dot{\varphi}_\beta$$

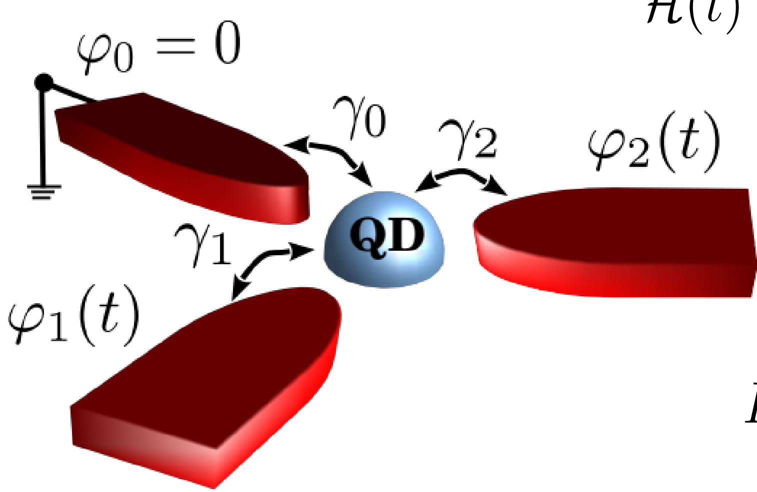
Berry curvature of the ground state, "anomalous" current



$$\dot{\varphi}_\beta = \frac{2eV_\beta}{\hbar}$$

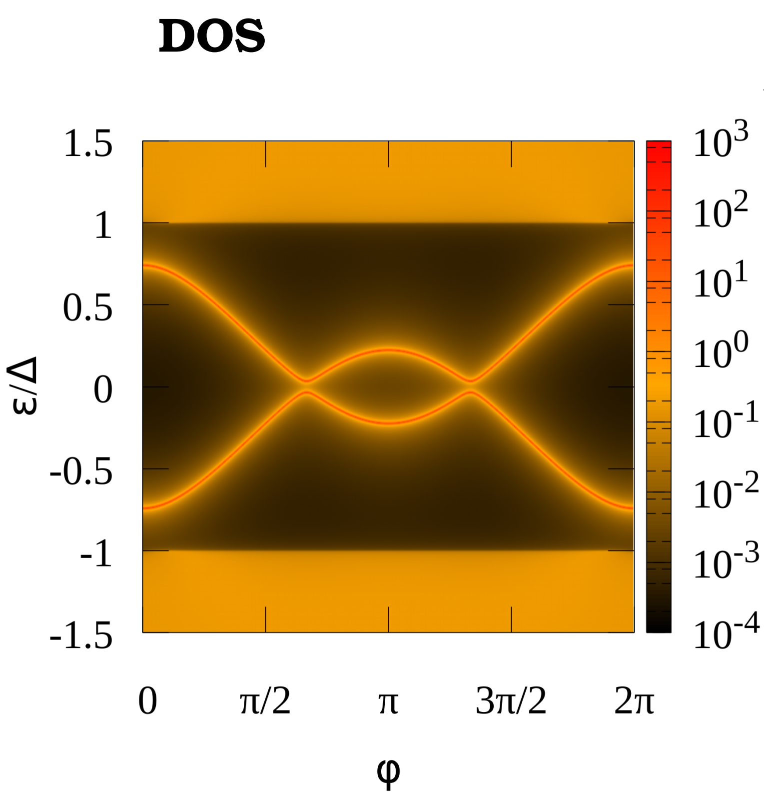
Phase sweeps this 2D "Brillouin zone"

Three terminal Josephson junction + quantum dot

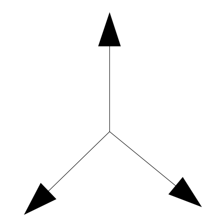


$$\mathcal{H}(t) = \sum_{\boldsymbol{k}\sigma\nu} \xi_{\boldsymbol{k}\nu} c^\dagger_{\boldsymbol{k}\sigma\nu} c_{\boldsymbol{k}\sigma\nu} - \Delta \sum_{\boldsymbol{k}\nu} (c^\dagger_{\boldsymbol{k}\uparrow\nu} c^\dagger_{-\boldsymbol{k}\downarrow\nu} + h.c) + \varepsilon_d \sum_\sigma d^\dagger_\sigma d_\sigma + \sum_{\boldsymbol{k}\sigma\nu} \left(\gamma_\nu e^{-i\varphi_\nu(t)/2} d^\dagger_\sigma c_{\boldsymbol{k}\nu\sigma} + h.c. \right)$$

$$\hat{H}_{\text{eff}}(\boldsymbol{\varphi}) = -\hat{G}^{-1}_{dd}(\omega=0) = \begin{pmatrix} \varepsilon_d & \sum_\nu \Gamma_\nu e^{-i\varphi_\nu} \\ \sum_\nu \Gamma_\nu e^{i\varphi_\nu} & -\varepsilon_d \end{pmatrix}$$



$$\mathcal{F}^{\text{eff}}_{21}(\boldsymbol{\varphi}) = \frac{1}{2\pi} \hat{\boldsymbol{h}}_{\text{eff}} \cdot (\partial_{\varphi_2} \hat{\boldsymbol{h}}_{\text{eff}} \times \partial_{\varphi_1} \hat{\boldsymbol{h}}_{\text{eff}}) \quad \Updownarrow \quad \boldsymbol{k} \cdot \boldsymbol{\delta} = \varphi_{\boldsymbol{\delta}}$$



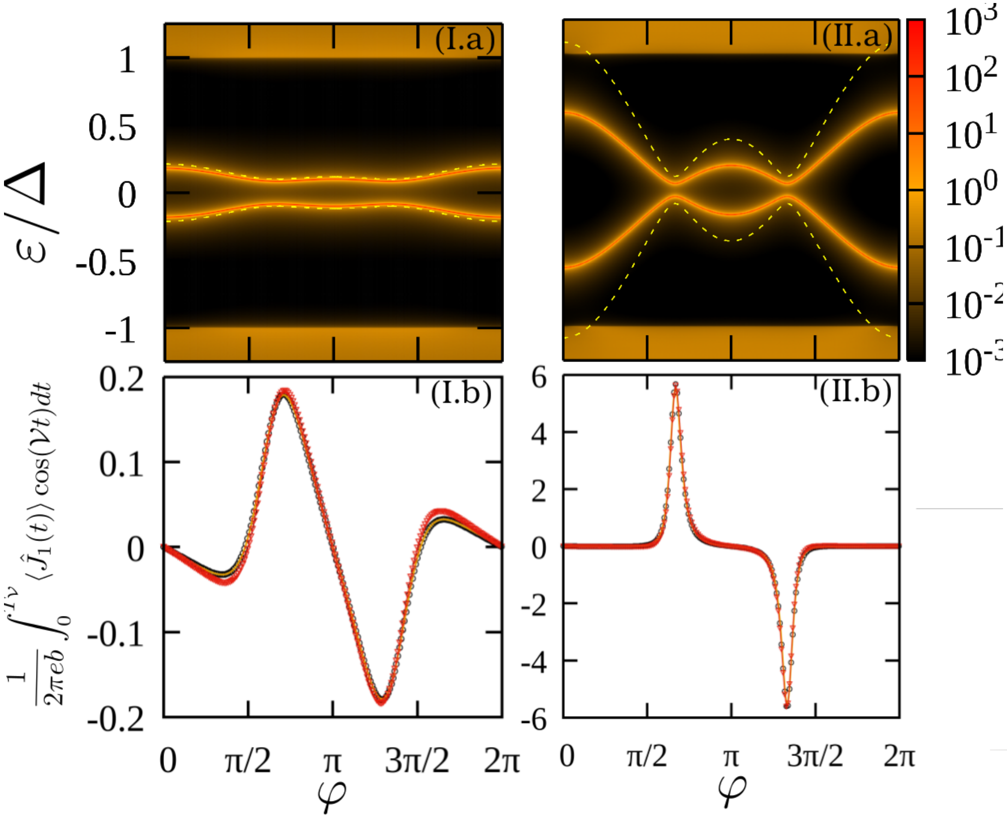
Recall honeycomb lattice...

$$\hat{H}(\boldsymbol{k}) = \begin{pmatrix} \varepsilon_A & \sum_{\boldsymbol{\delta}} t e^{-i\boldsymbol{k}\cdot\boldsymbol{\delta}} \\ \sum_{\boldsymbol{\delta}} t e^{i\boldsymbol{k}\cdot\boldsymbol{\delta}} & \varepsilon_B \end{pmatrix}$$

Non trivial Berry curvature whenever $\varepsilon_d \neq 0$

(Valley Hall effect?)

Protocol to detect Berry curvature

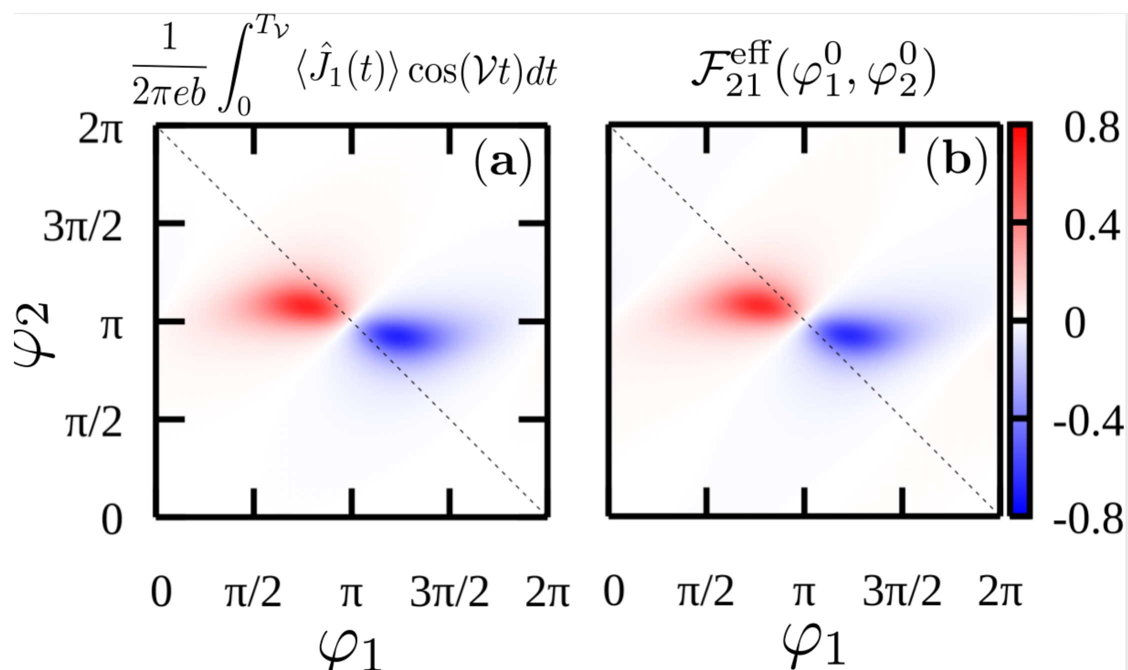


$$\varphi_1(t) = \varphi_1^0$$
$$\varphi_2(t) = \varphi_2^0 + b\sin(\mathcal{V}t)$$

$$\langle\mathcal{J}_1(t)\rangle \simeq \frac{2e}{\hbar}\left(\frac{\partial\varepsilon_g}{\partial\varphi_1} + \frac{\partial^2\varepsilon_g}{\partial\varphi_2\partial\varphi_1} b\sin(\mathcal{V}t)\right)\Bigg|_{\boldsymbol{\varphi}^0} - 2e\mathcal{F}_{12}^g\Big|_{\boldsymbol{\varphi}^0} b\mathcal{V}\cos(\mathcal{V}t)$$

Wigner gradient expansion

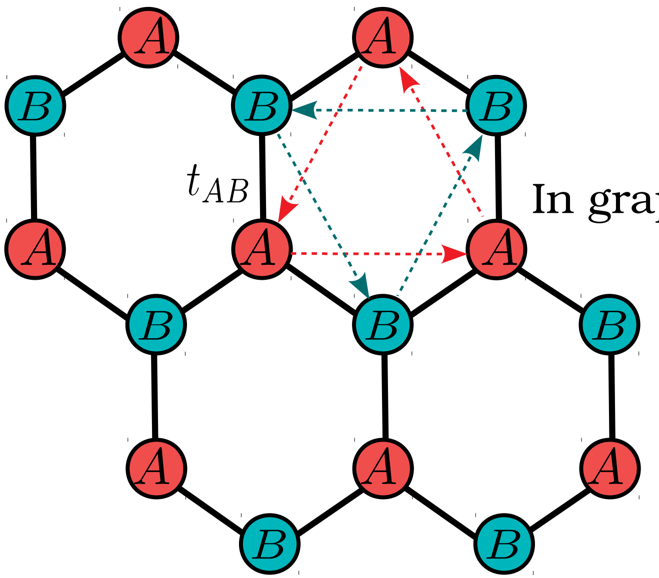
$$\langle\mathcal{J}_\nu(t)\rangle = 2e \lim_{t'-t\to\epsilon^+}\left\{-\frac{i}{\hbar}\int\frac{d\omega}{2\pi}\mathrm{Tr}\left[\frac{\partial H}{\partial\varphi_\nu}\tilde{\mathcal{G}}_0^c\right] - \sum_\rho\int\frac{d\omega}{4\pi}\mathrm{Tr}\left[\epsilon^{\nu\rho}\tilde{\mathcal{G}}_0^{c\,-1}\frac{\partial\tilde{\mathcal{G}}_0^c}{\partial\varphi_\nu}\cdot\tilde{\mathcal{G}}_0^{c\,-1}\frac{\partial\tilde{\mathcal{G}}_0^c}{\partial\varphi_\rho}\cdot\tilde{\mathcal{G}}_0^{c\,-1}\frac{\partial\tilde{\mathcal{G}}_0^c}{\partial\omega}\right]\dot{\varphi}_\rho\right\}$$
$$= \frac{2e}{\hbar}\frac{\partial\varepsilon_g(t)}{\partial\varphi_\nu} - 2e\sum_\rho\mathcal{F}_{\nu\rho}^g(t)\dot{\varphi}_\rho(t),$$



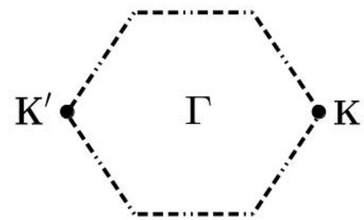
Are anomalous Hall signals possible in graphene?

Time reversal $\mathcal{T} \to \mathcal{F}(\boldsymbol{k}) = -\mathcal{F}(-\boldsymbol{k})$

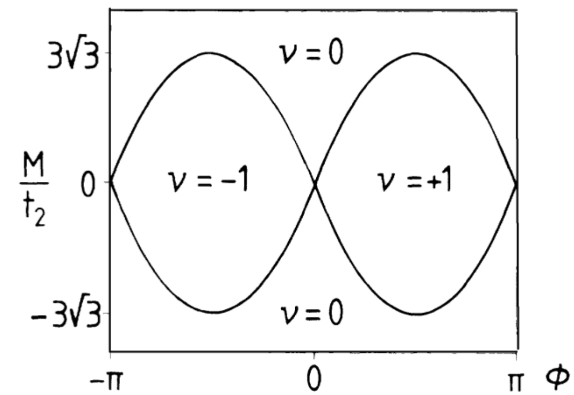
Spatial inversion $\mathcal{I} \to \mathcal{F}(\boldsymbol{k}) = \mathcal{F}(-\boldsymbol{k})$



In graphene the curvature is strictly zero! $\mathcal{T} + \mathcal{I} \Rightarrow \mathcal{F} = 0$



$$\mathbf{K}_\xi = \left(\xi \frac{4\pi}{3\sqrt{3}a}, 0\right)$$



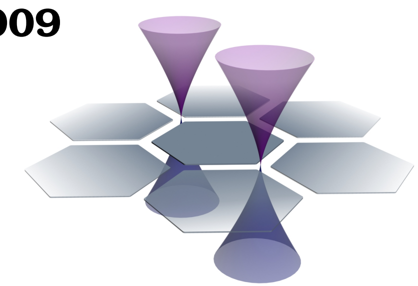
Haldane (PRL 1988) $\Longrightarrow$ **Break TRS** with complex hoppings up to NNN

Kane and Mele (2005) $\Longrightarrow$ SOC preserves TRS leads to QSH

$\hbar\Omega$

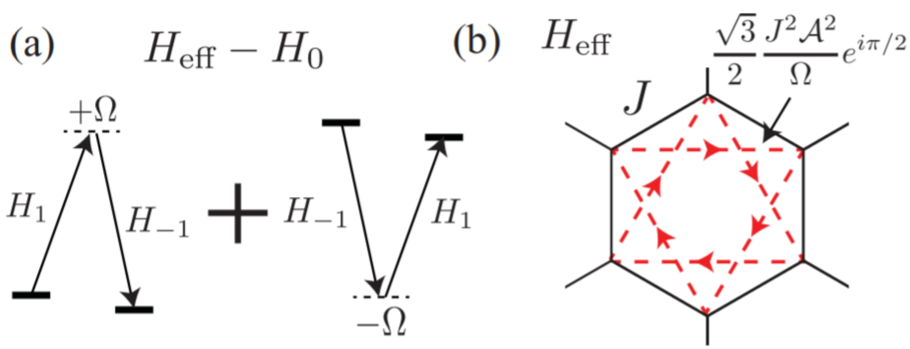
Can be engineered by driving!

Oka & Aoki, PRB 2009

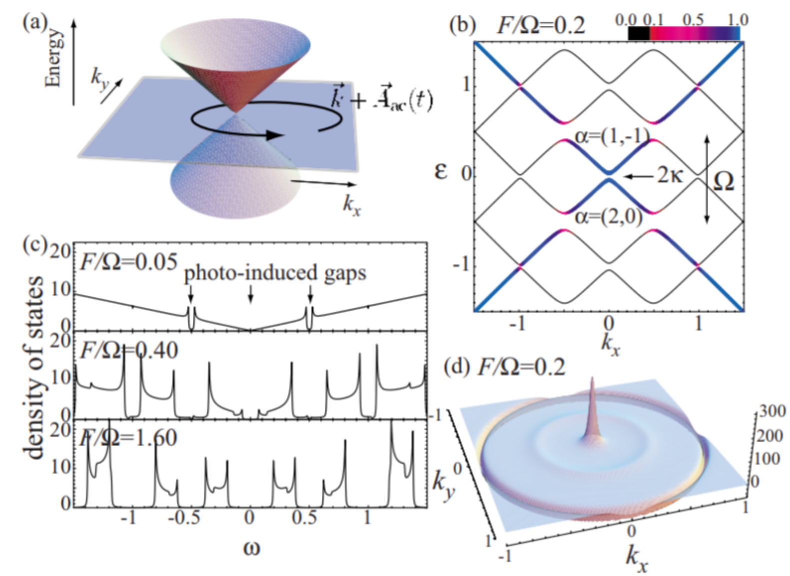


$$\hat{H}[\boldsymbol{k}(t)] = \begin{pmatrix} \varepsilon_A & \sum_{\boldsymbol{\delta}} t e^{-i[\boldsymbol{k} - e/c\boldsymbol{A}(t)]\cdot\boldsymbol{\delta}} \\ \sum_{\boldsymbol{\delta}} t e^{i[\boldsymbol{k} - e/c\boldsymbol{A}(t)]\cdot\boldsymbol{\delta}} & \varepsilon_B \end{pmatrix}$$

Dynamically realized Haldane model as a Floquet Hamiltonian



Kitagawa et al, PRB 2011



Oka & Aoki, PRB 2009

LETTER

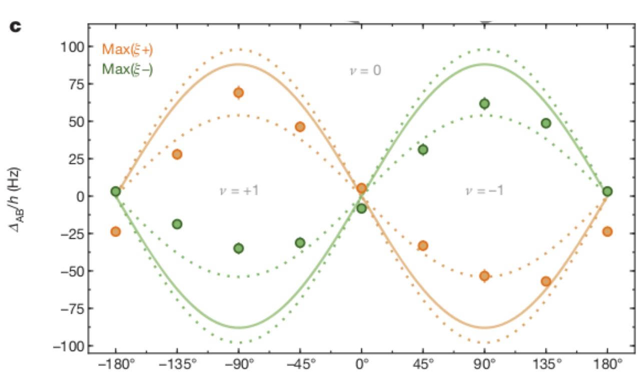
doi:10.1038/nature13915

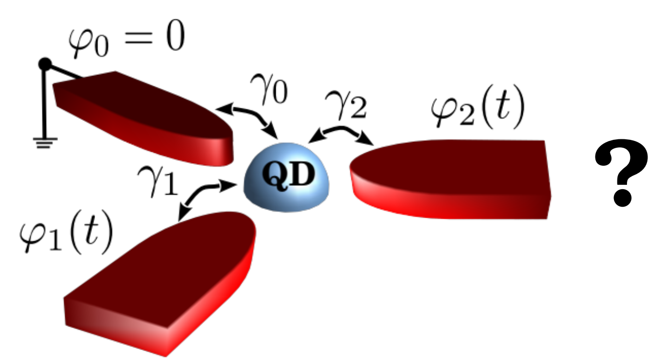
Experimental realization of the topological Haldane model with ultracold fermions

Gregor Jotzu[1], Michael Messer[1], Rémi Desbuquois[1], Martin Lebrat[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

Jotzu et al, Nature 2014

$$\varphi_1(t) = \varphi_1^0 + A_0\cos(\Omega t + \chi_1)$$
$$\varphi_2(t) = \varphi_2^0 + A_0\cos(\Omega t + \chi_2)$$



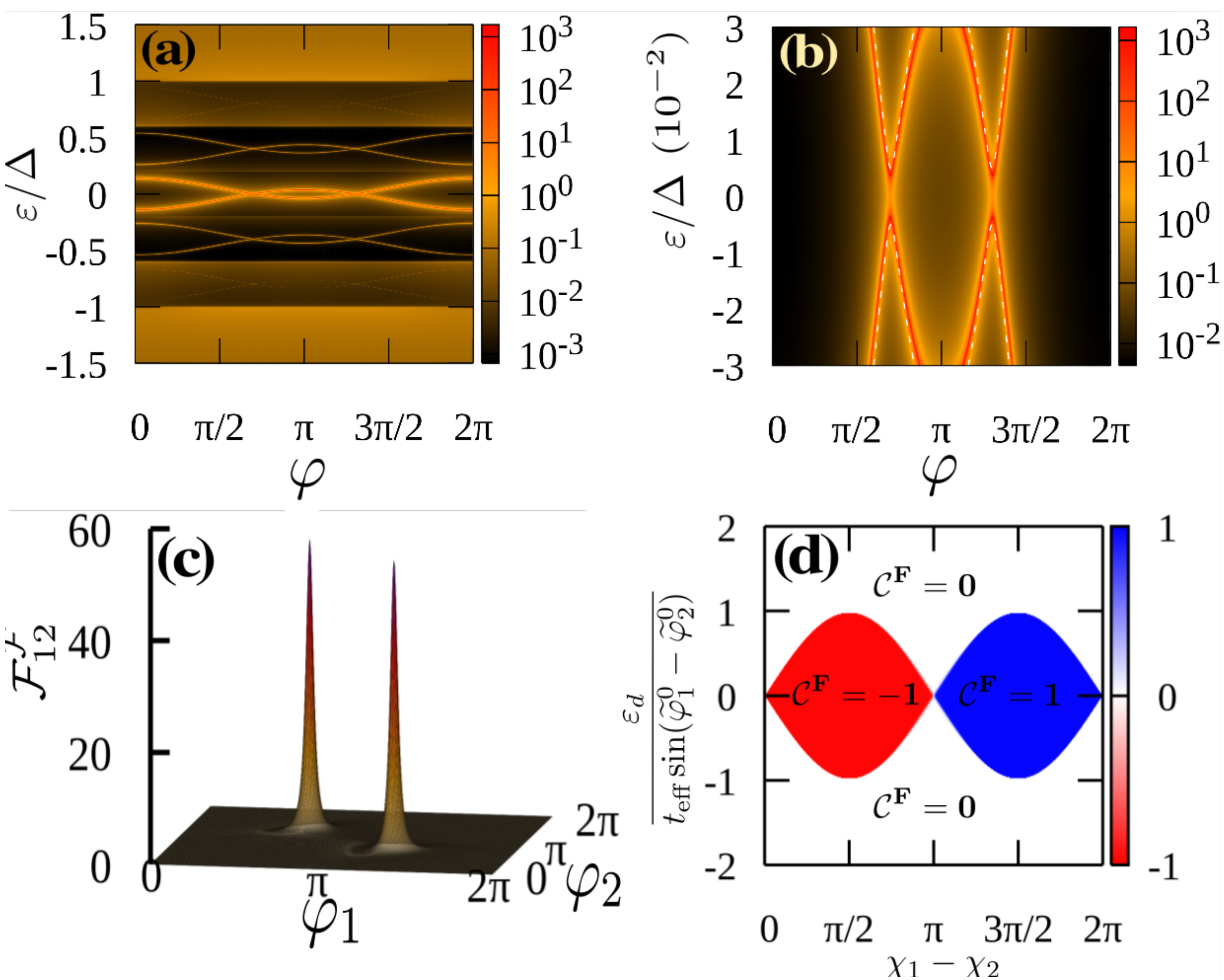


Floquet-Andreev bands: Realization of a Haldane model in a solid state device

$$\hat{H}^{\mathcal{F}}_{\text{eff}}(\boldsymbol{\varphi}) = \begin{pmatrix} \tilde{\varepsilon}_d(\boldsymbol{\varphi}^{\mathbf{0}}) & \sum_\nu \tilde{\Gamma}_\nu e^{-i\varphi_\nu} \\ \sum_\nu \tilde{\Gamma}_\nu e^{i\varphi_\nu} & \tilde{\varepsilon}_d(\boldsymbol{\varphi}^{\mathbf{0}}) \end{pmatrix}$$

$$\begin{aligned} \tilde{\varepsilon}_d(\boldsymbol{\varphi}^{\mathbf{0}}) &= \varepsilon_d - t_{\text{eff}} \sin(\varphi_1^0 - \varphi_2^0) \sin(\chi_1 - \chi_2) \\ \tilde{\Gamma}_{1,2} &= \Gamma_{1,2} J_0(A_0), \end{aligned}$$

$$t_{\text{eff}} = 4\Gamma_1\Gamma_2 J_1^2(A_0)/\hbar\Omega$$



Same topological charge when TRS is broken.

Haldane-like topological phase diagram of the driven junction

Concluding remarks and take home message

- Local measurements of the **Berry curvature** (not so easy in crystalline solids!)

- Winding number in terms of the Green's function as theoretical tool to obtain topological invariants. **Correctly accounts the effect of the continuum states.**

- Inducing topology in three terminal junctions by driving: **Floquet-Andreev states as a realization of a topological Haldane model**.

Berry curvature tomography and realization of topological Haldane model in driven three-terminal Josephson junctions. **ArXiv:1804.04755**