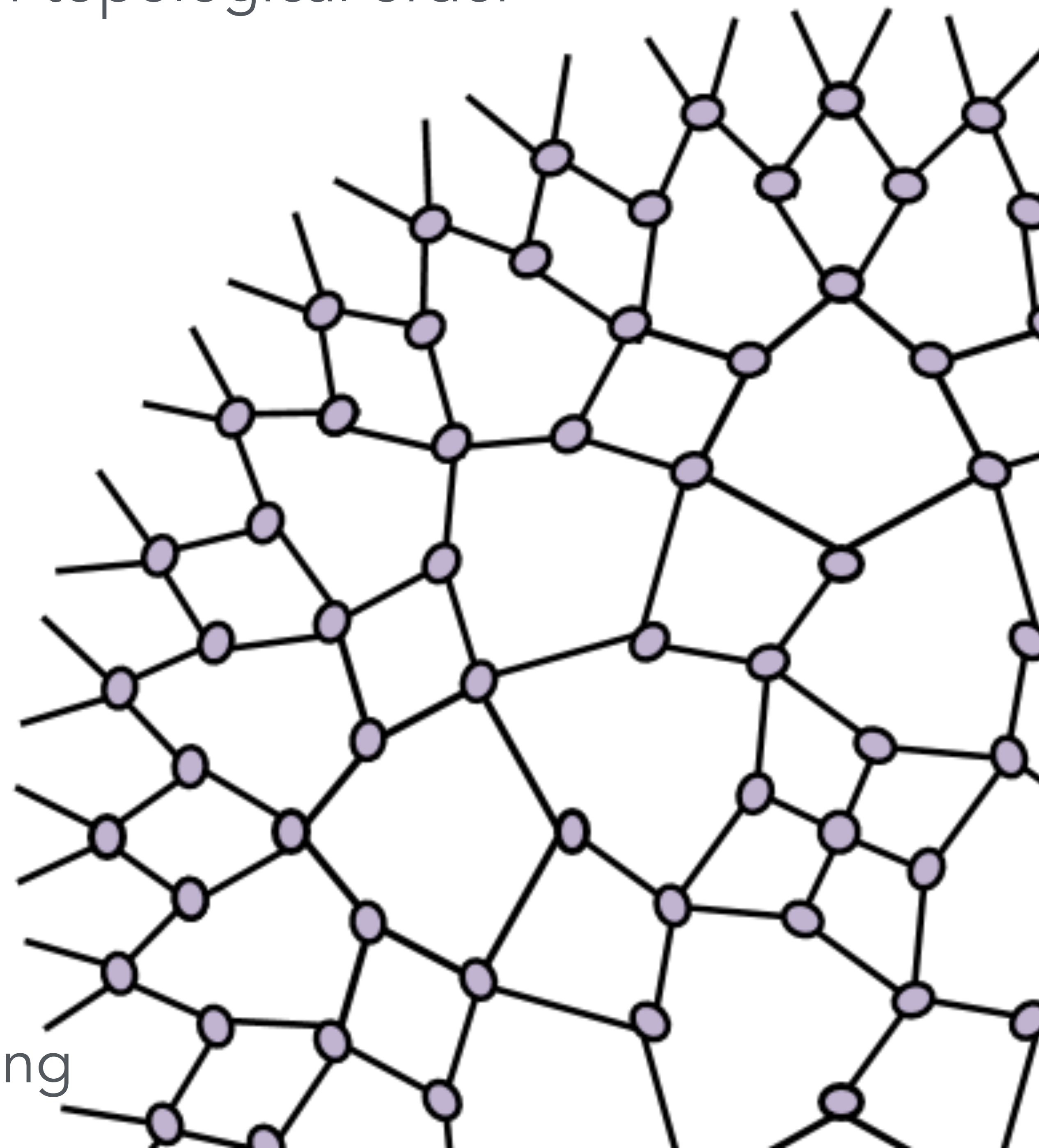
Tensor network states

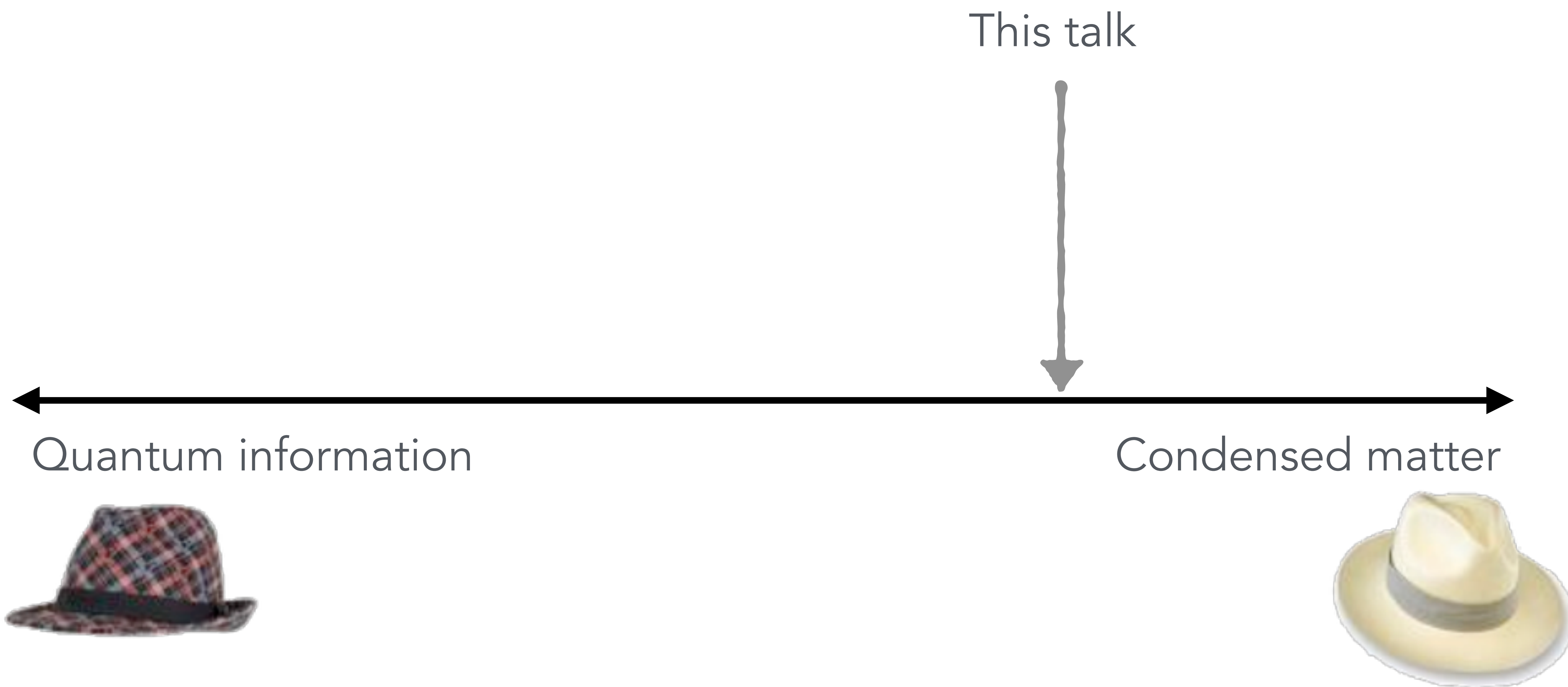
An entanglement based approach to numerical simulations of strongly correlated matter and analytical studies of topological order



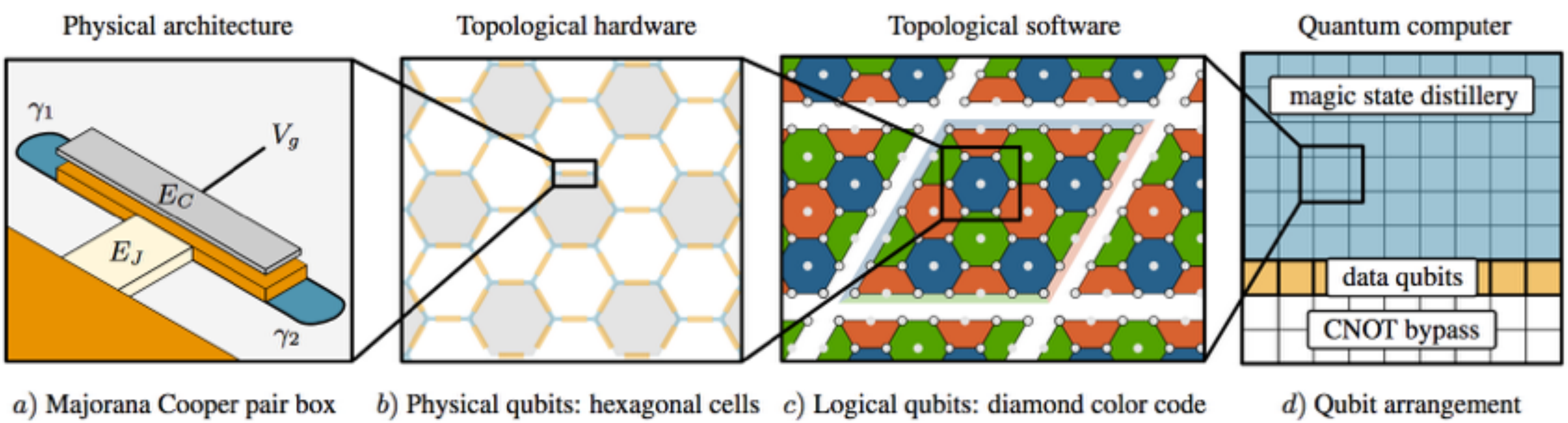
Jens Eisert, Freie Universität Berlin

The Capri Spring School 2017

Solid-state quantum information processing

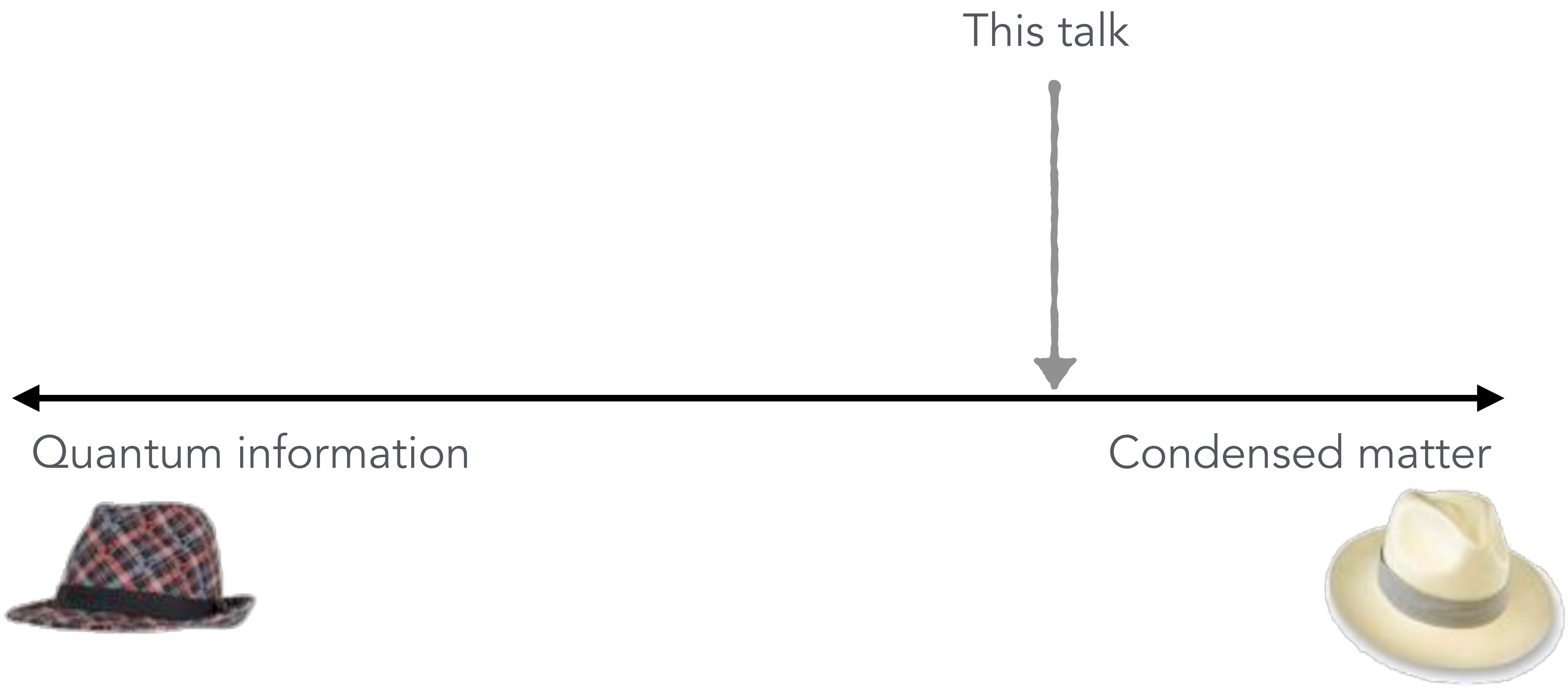

This talk
Quantum information
Condensed matter

- Daniel Litinski's talk (today 18:30)



a) Majorana Cooper pair box *b*) Physical qubits: hexagonal cells *c*) Logical qubits: diamond color code *d*) Qubit arrangement

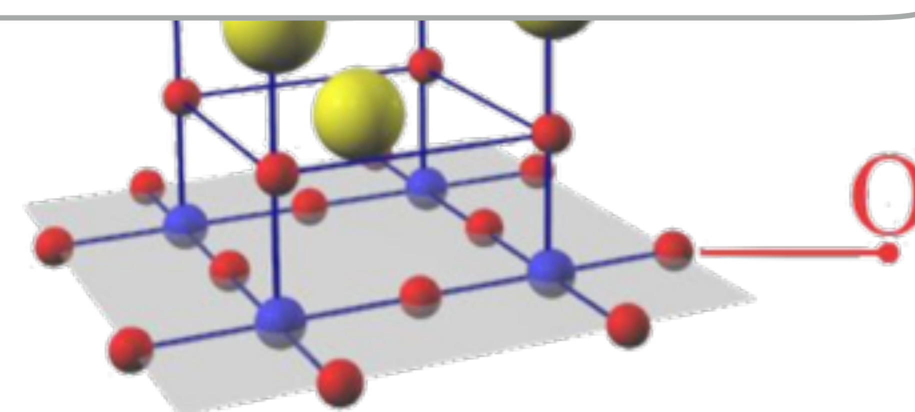
Litinski, Kesselring, Eisert, von Oppen, arXiv:1704.01589


This talk
Quantum information
Condensed matter

- Natural ground states of quantum many-body systems are very little entangled in a precise sense. This allows for computational methods based on tensor networks as well as new ways for their mathematical study."

- **This talk:** Find out what that means

- **Lecture 1:**
  - Area laws for entanglement entropies
  - Matrix-product states and operators
  - Applications: Grounds states, open systems, many-body localisation



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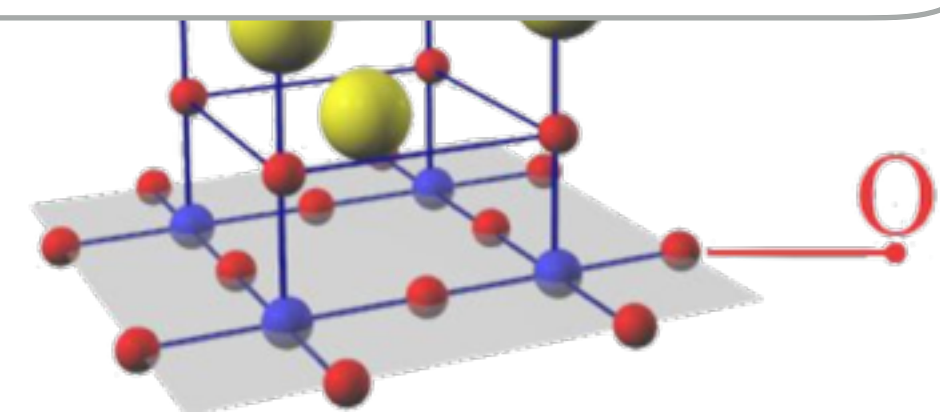
- **This talk:** Find out what that means

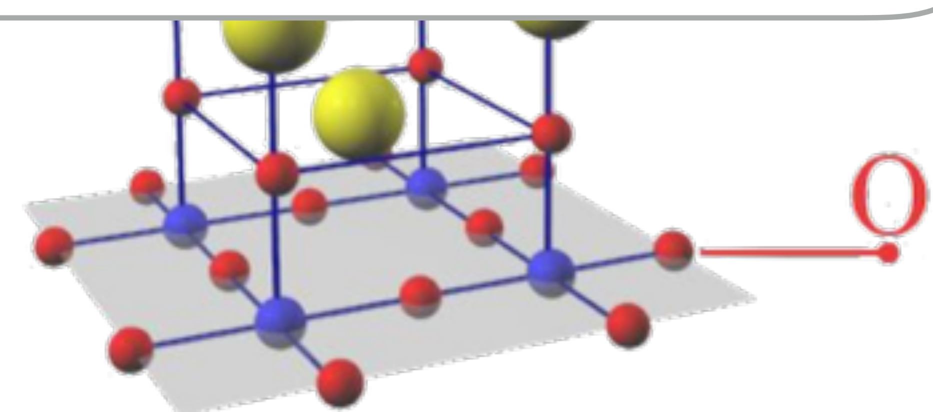
- **Lecture 2:**
  - Symmetries
  - Classification of phases
  - Projected entangled pair states, application: t-J model



- Natural ground states of quantum many-body systems are very little entangled in a precise sense. This allows for computational methods based on tensor networks as well as new ways for their mathematical study."

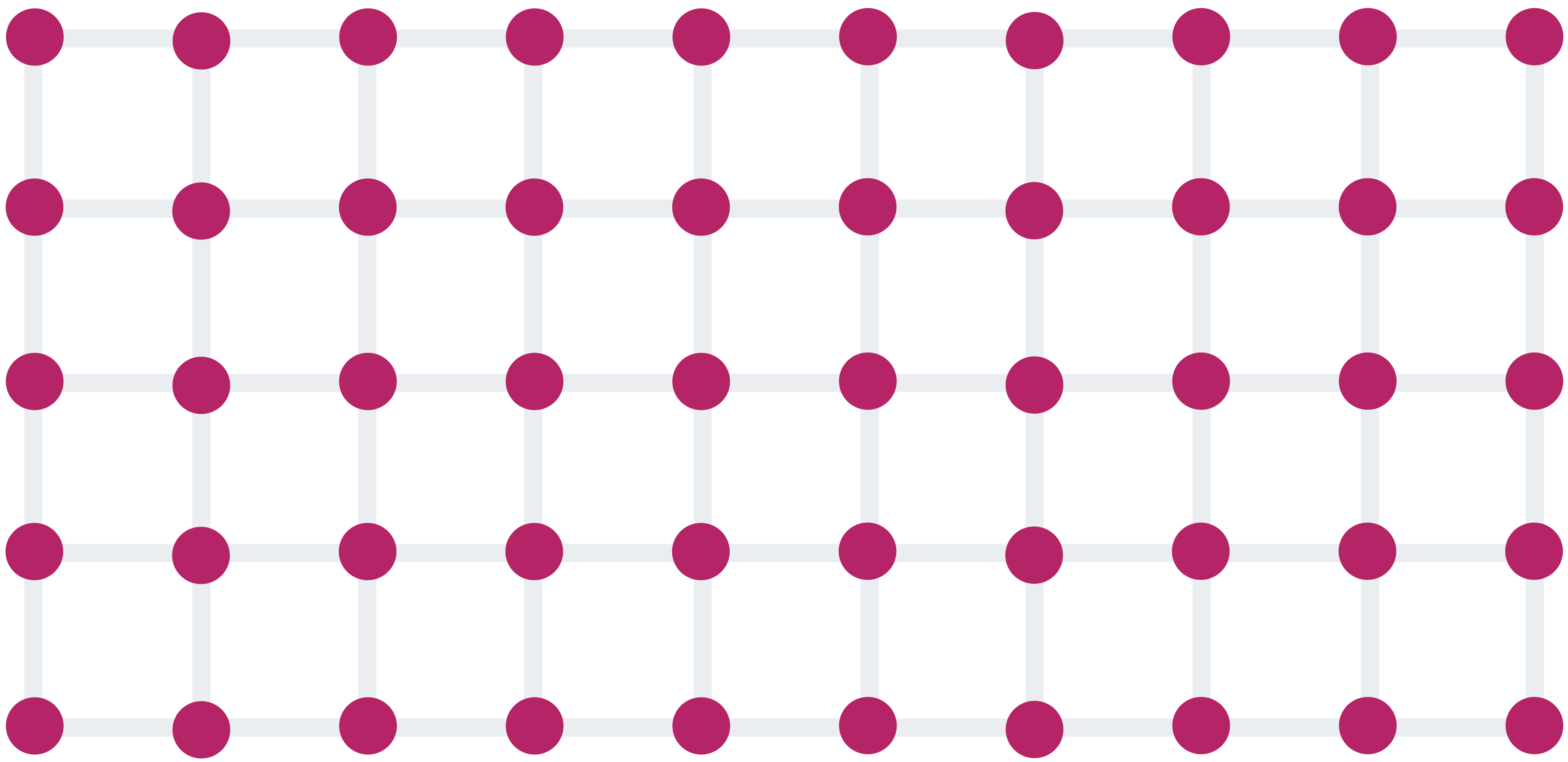
- **This talk:** Find out what that means

- **Lecture 3:**
  - Notions of topological order
  - Toric codes and topological quantum memories
  - Fermionic models and topological order



Area laws for the entanglement entropy

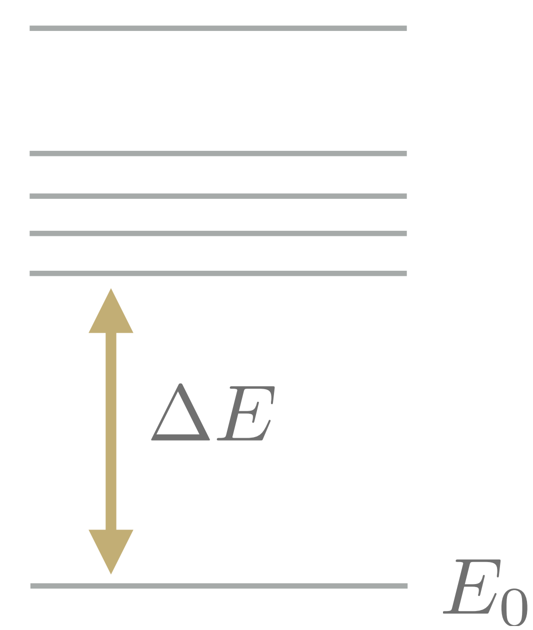
Clustering of correlations



$$H = \sum_j h_j$$

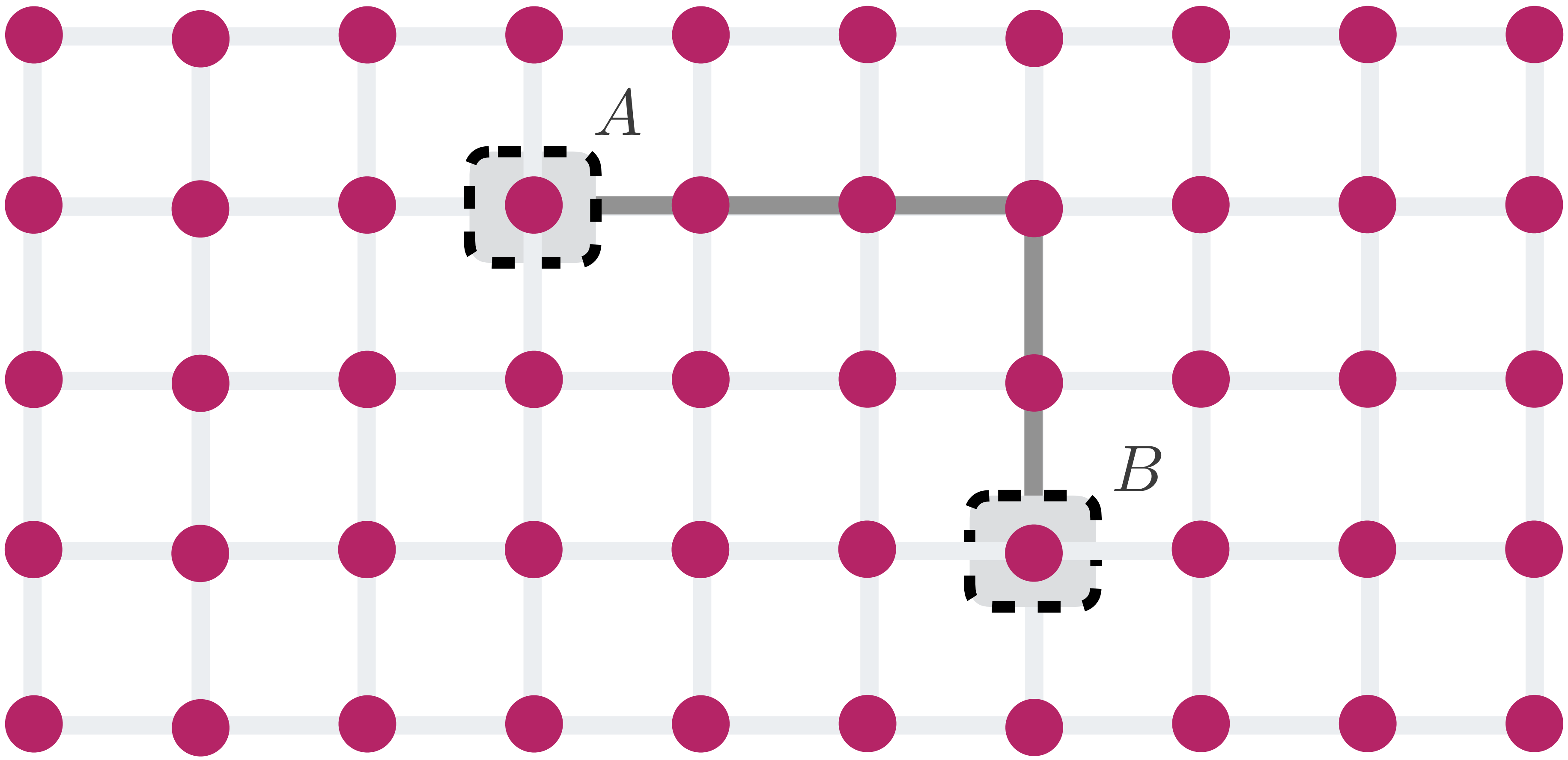
- Ground states of local gapped models

$$\Delta E = \inf_{|\psi\rangle \in \mathcal{H} \setminus \mathcal{G}} \langle \psi | H | \psi \rangle - E_0$$



Hastings, Koma, Commun Math Phys 3, 443 (1992)

Clustering of correlations



$$H = \sum_j h_j$$

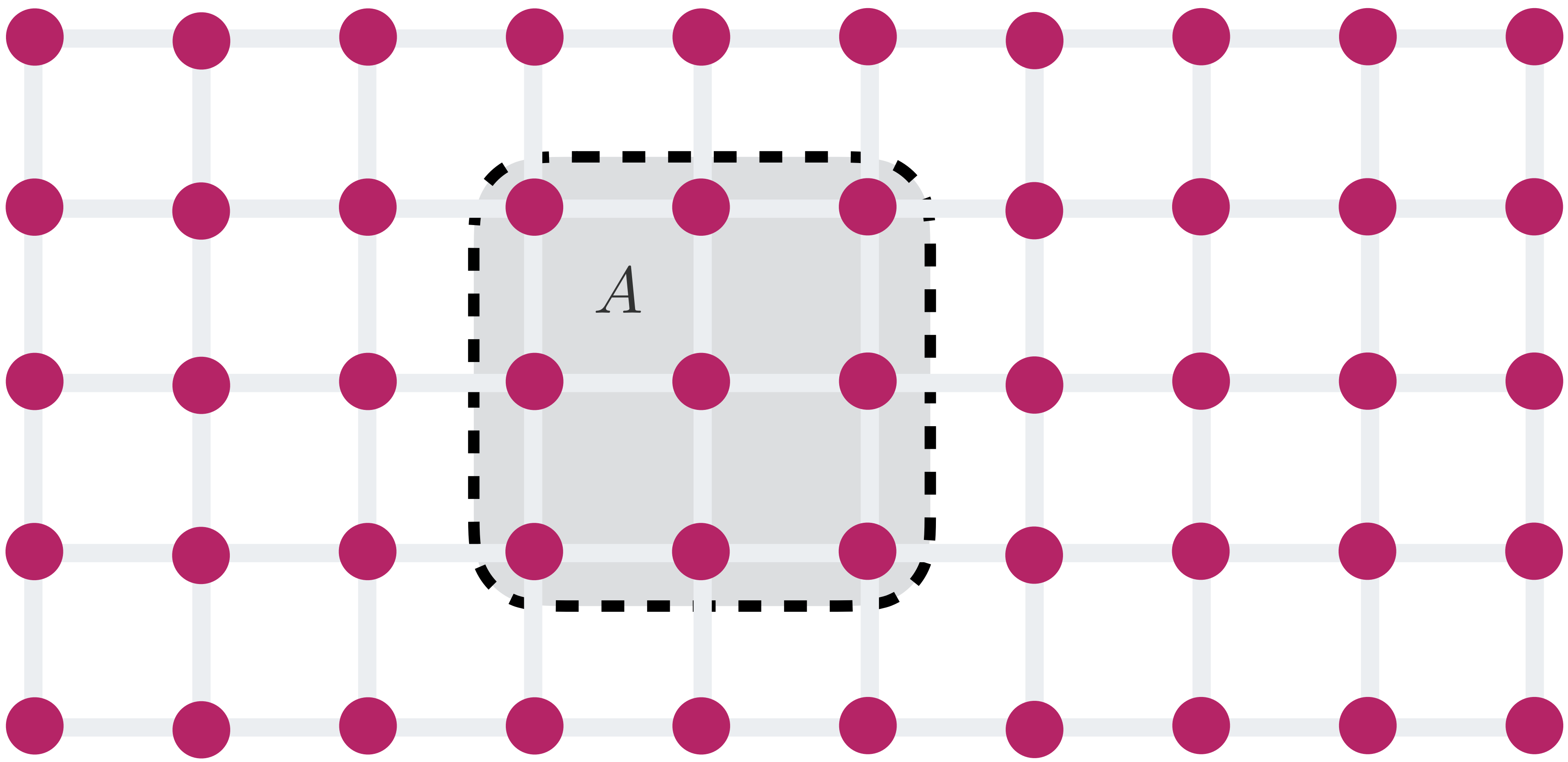
- Ground states of local gapped models

$$|\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle| \leq C e^{-\mathrm{dist}(A,B)\Delta E/(2v)} \|O_A\| \, \|O_B\|$$

- Exponentially clustering correlations

Hastings, Koma, Commun Math Phys 3, 443 (1992)

Area laws for entanglement entropies

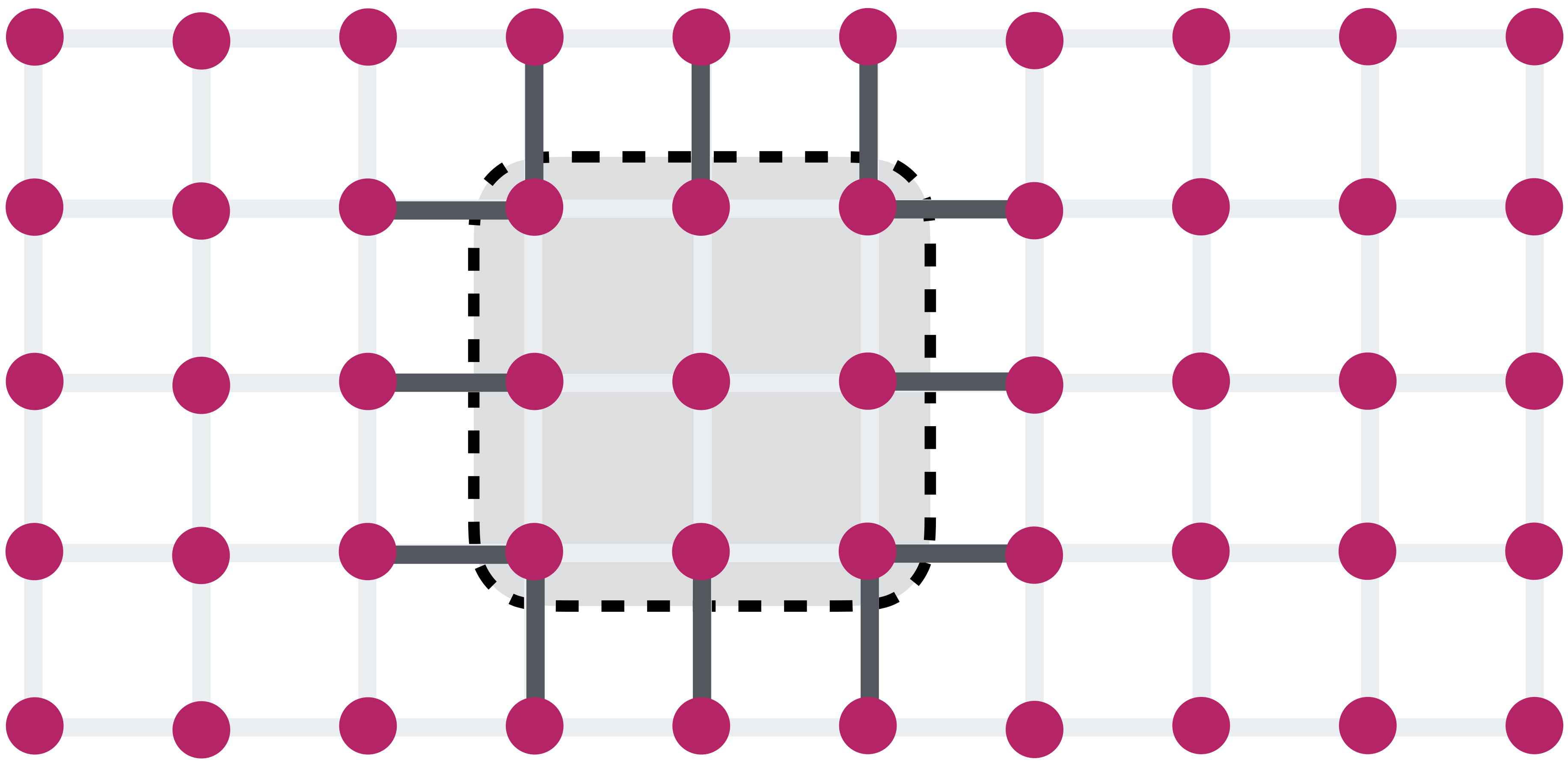


- **Area law** for the entanglement entropy $S(\rho_A)$:

$$S(\rho_A) = O(|\partial A|)$$

Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)
Hastings, JSTAT, P08024 (2007)

Area laws for entanglement entropies



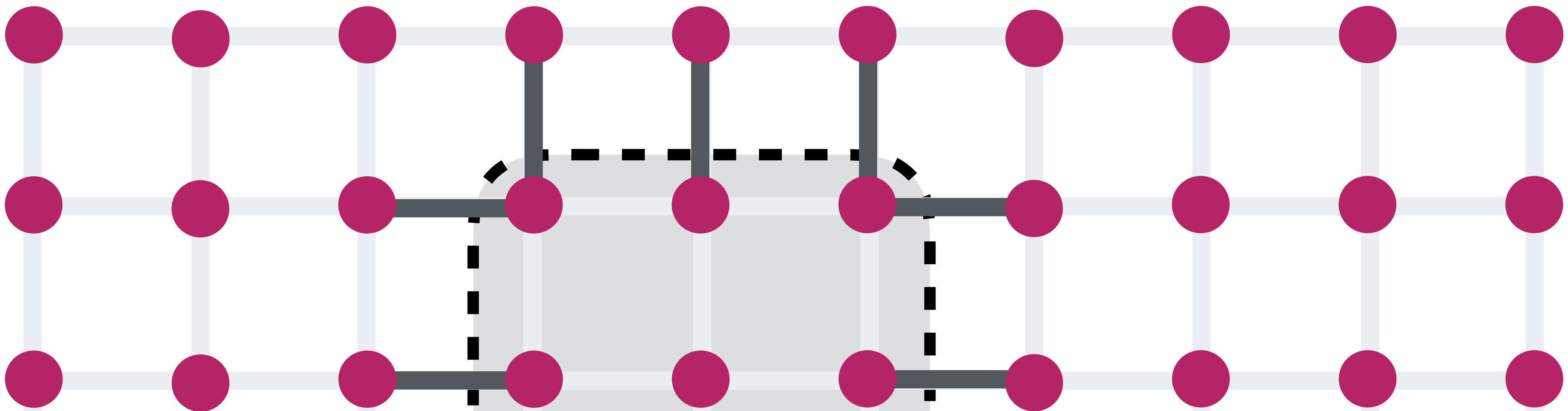
- **Area law** for the entanglement entropy $S(\rho_A)$:

$$S(\rho_A) = O(|\partial A|)$$

- Scale like boundary area, not volume: Much less entangled than possible!

Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)
Hastings, JSTAT, P08024 (2007)

Area laws for entanglement entropies



- **Theorem:** Area laws hold true for

  1. arbitrary gapped models in 1D
  2. free bosonic and fermionic gapped Hamiltonians in any D
  3. Stabiliser Hamiltonians (as in quantum codes)

- **Are**

$$S(\rho_A) = O(|\partial A|)$$

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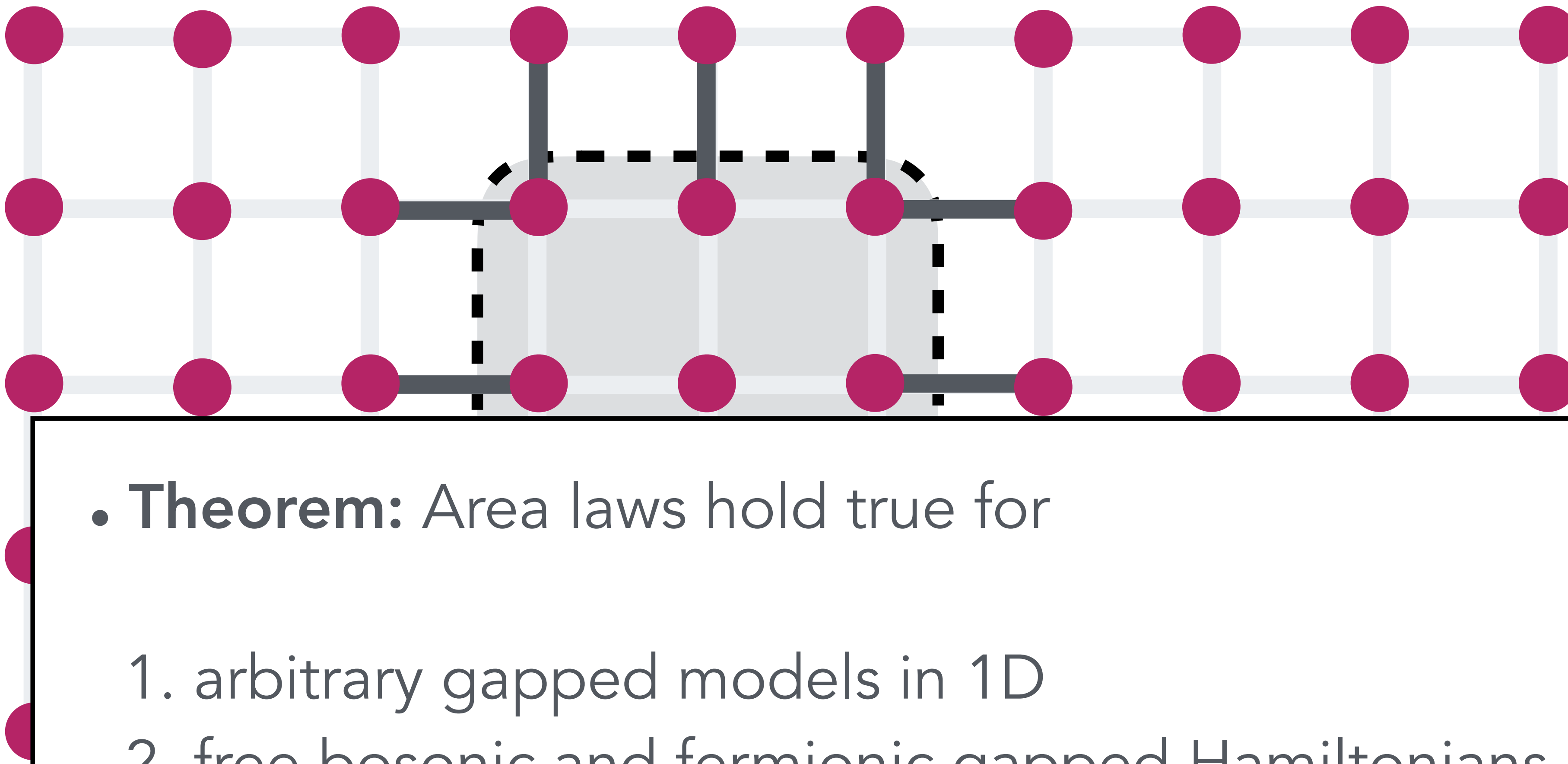
Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)
Hastings, JSTAT, P08024 (2007)

Area laws for entanglement entropies



- **Theorem:** Area laws hold true for

  1. arbitrary gapped models in 1D
  2. free bosonic and fermionic gapped Hamiltonians in any D
  3. Stabiliser Hamiltonians (as in quantum codes)

- Entanglement entropies tool for detecting topological entropy γ

$$S(\rho_A) = \alpha|\partial A| - \gamma + O(|\partial A|^{-\beta})$$

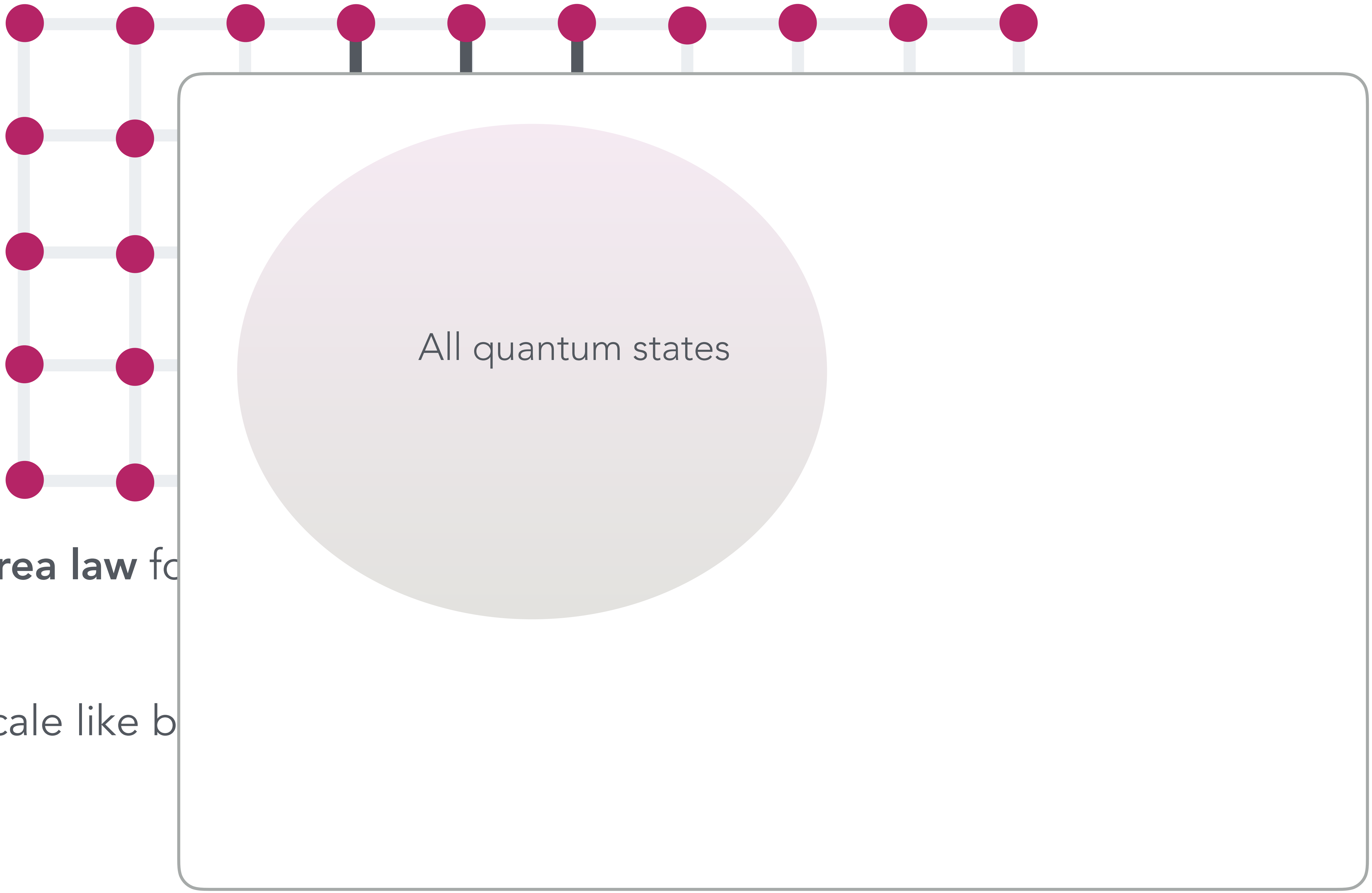
Kitaev and J. Preskill, Phys Rev Lett 96, 110404 (2006)
Levin and Wen, Phys Rev Lett 96, 110405 (2006)
Bauer, Cincio, Keller, Dolfi, Vidal, Trebst, Ludwig, Nature Comm 5, 5137 (2014)

- **Are**

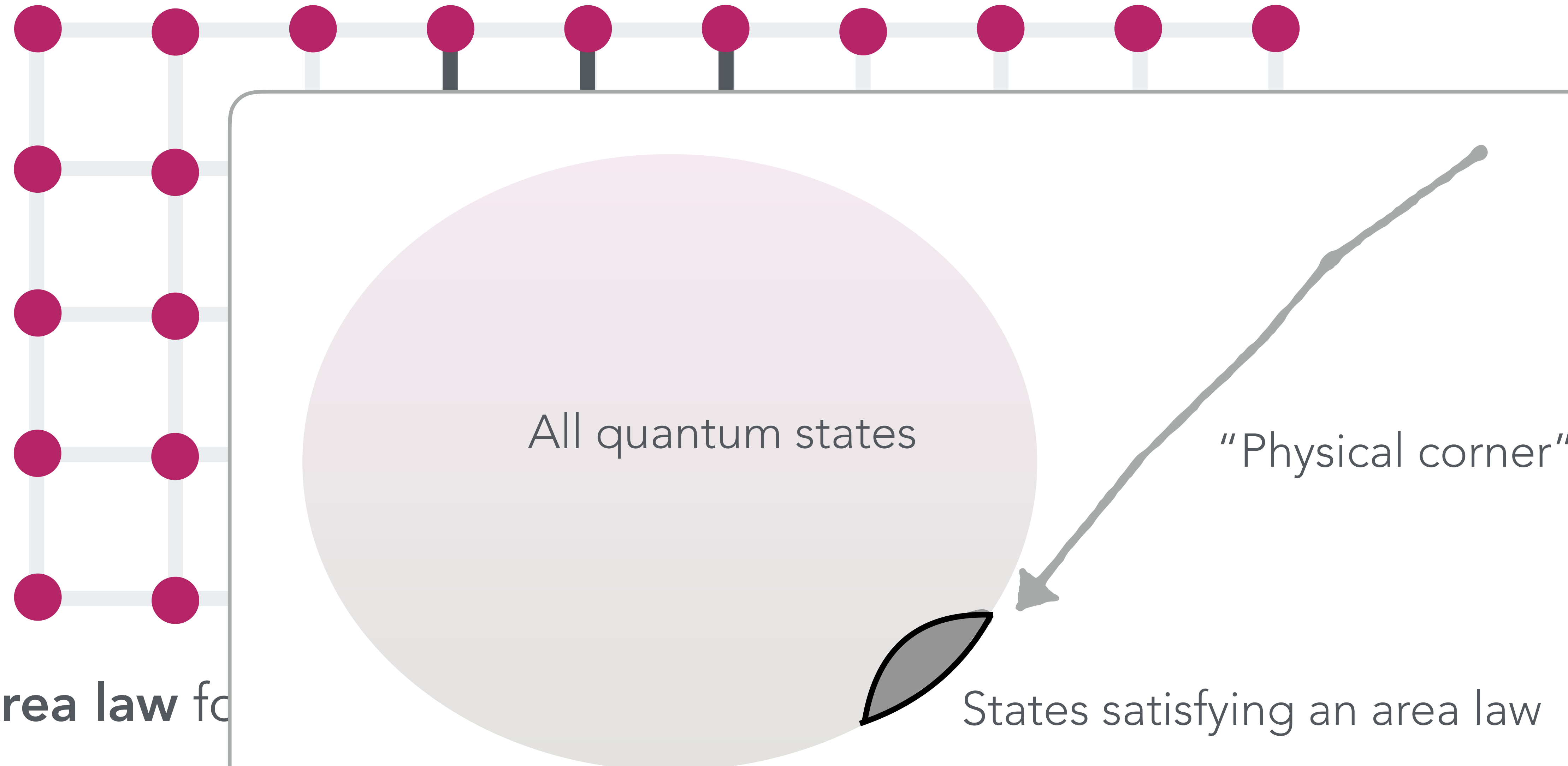
- Sca ... le!

Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)
Hastings, JSTAT, P08024 (2007)

Area laws for entanglement entropies



- **Area law** fo
- Scale like b



- **Area law** fo
- Scale like b

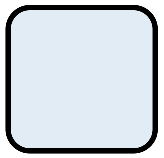
- **Entanglement** captures "essential degrees of freedom, hugely removes redundancy"
- Can this be used to largely "**parametrise**" **states**?

Matrix-product states

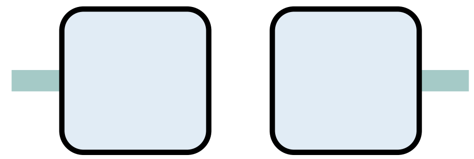
Tensor networks

- Graphical notation of tensors of varying degree

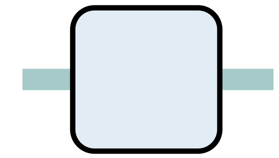
- This is a scalar



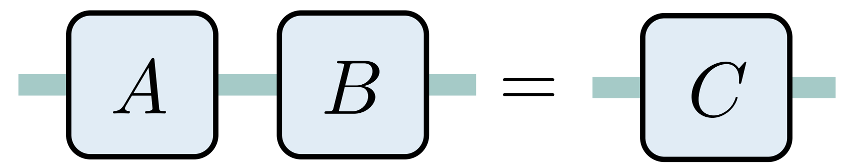
- Vectors and dual vectors



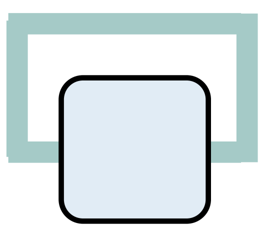
- This is a matrix



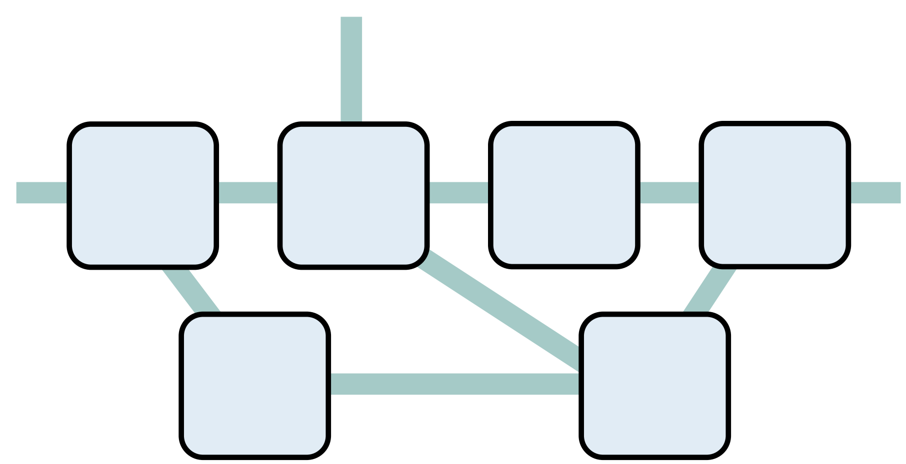
- Contraction of edge: Summation
  - E.g., matrix product $C_{\alpha,\beta} = \sum_{\gamma=1}^{N} A_{\alpha,\gamma} B_{\gamma,\beta}$

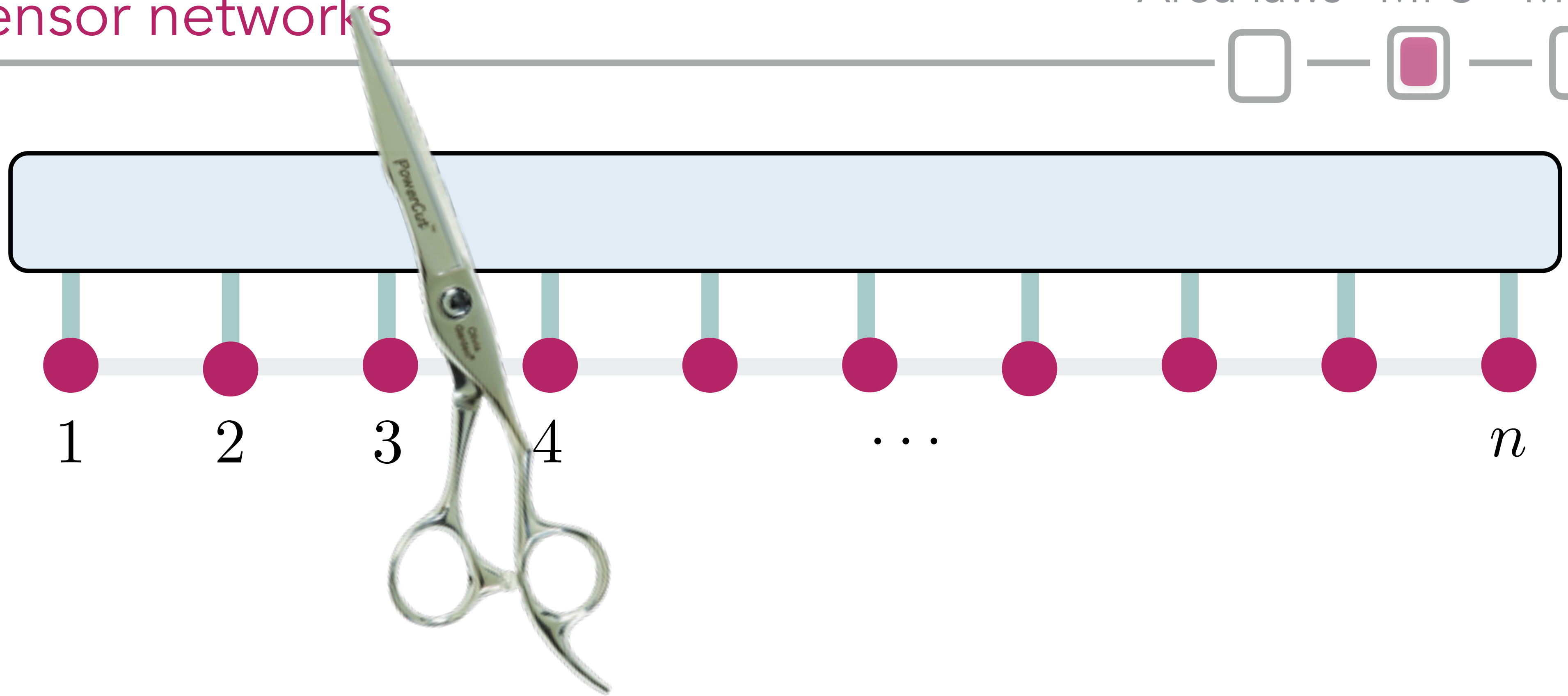


  - Trace

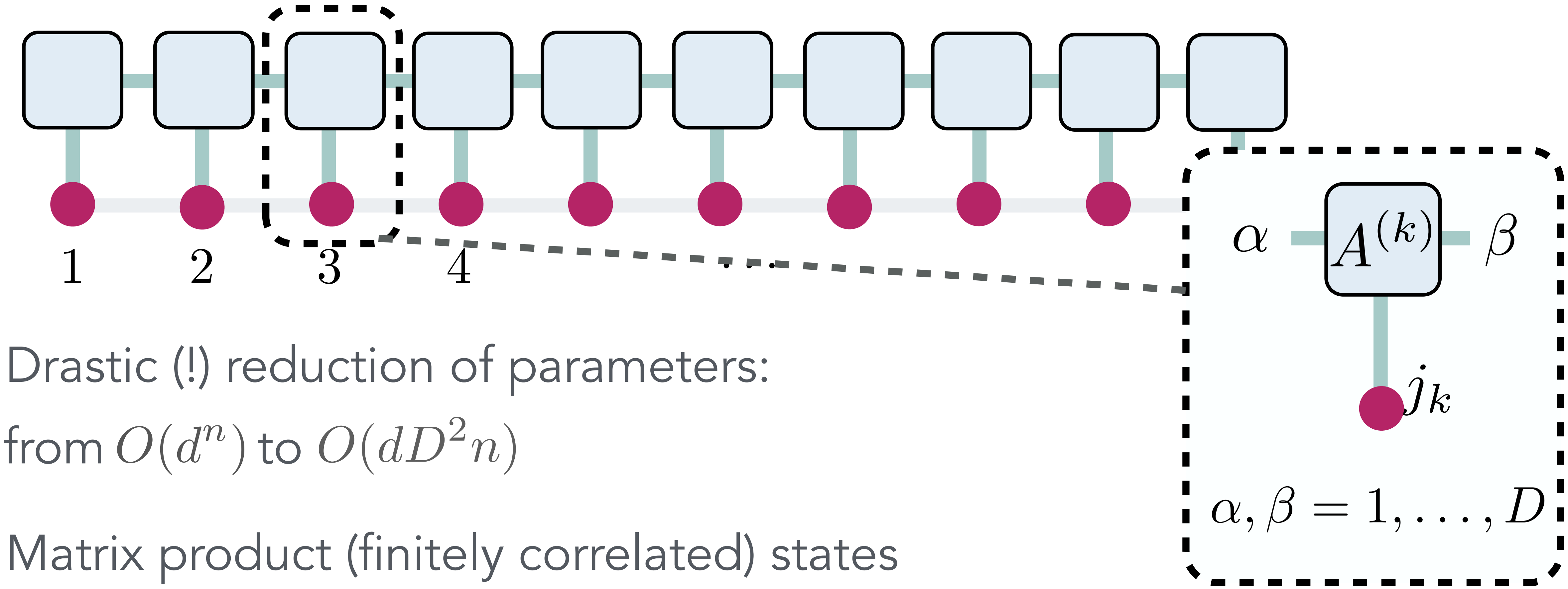


- Tensor network with open edges




1
2
3
4
$\cdots$
n

Matrix-product states (MPS)

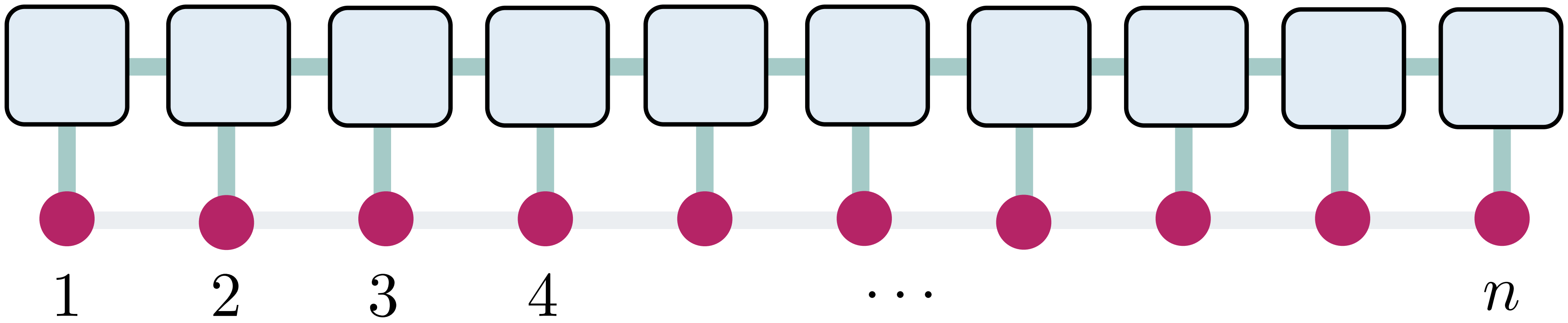


- Drastic (!) reduction of parameters: from $O(d^n)$ to $O(dD^2n)$
- Matrix product (finitely correlated) states

- Why would this be any good?

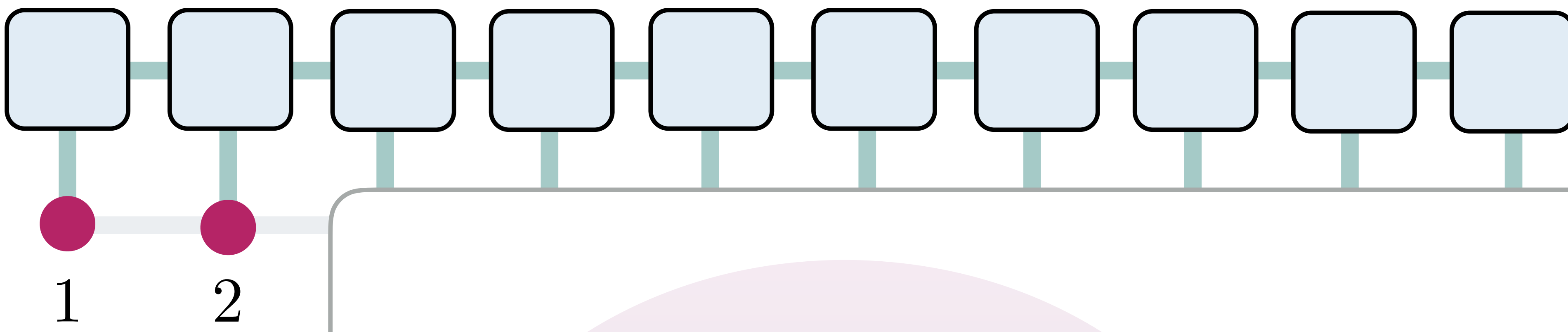
White, Phys Rev Lett 69, 2863 (1992)
Fannes, Nachtergaele, Werner, Commun Math Phys 3, 443 (1992)

Matrix-product states (MPS)

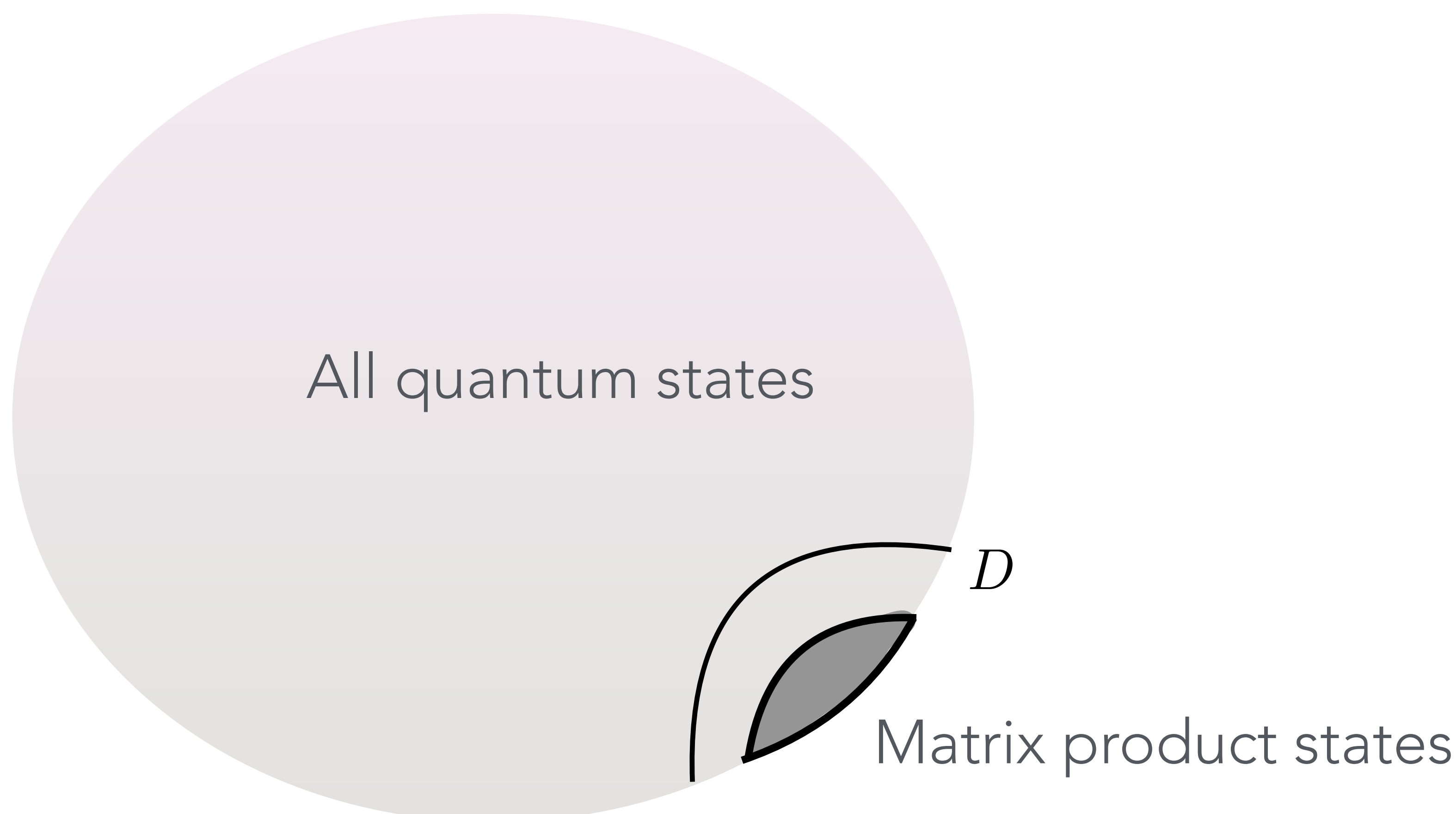


- **Theorem:** • All MPS satisfy area laws $S(\rho_A) = O(\log(D))$

Matrix-product states (MPS)



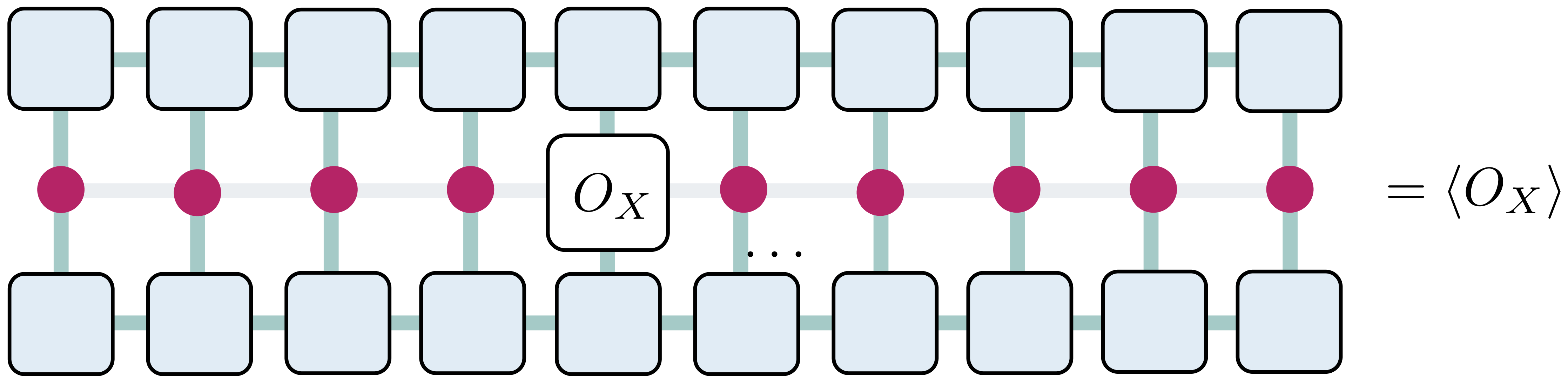
- **Theor**



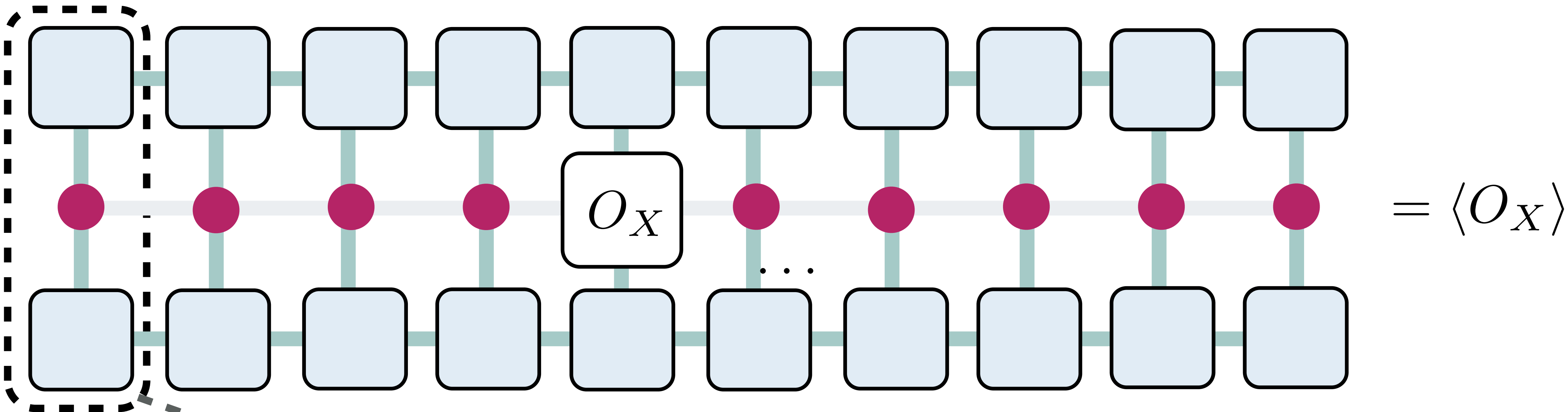
- MPS “largely parametrize the physical corner”

Getting it to work: Ground states

Contraction of MPS



- How can local expectation values be computed?
- Naively not!



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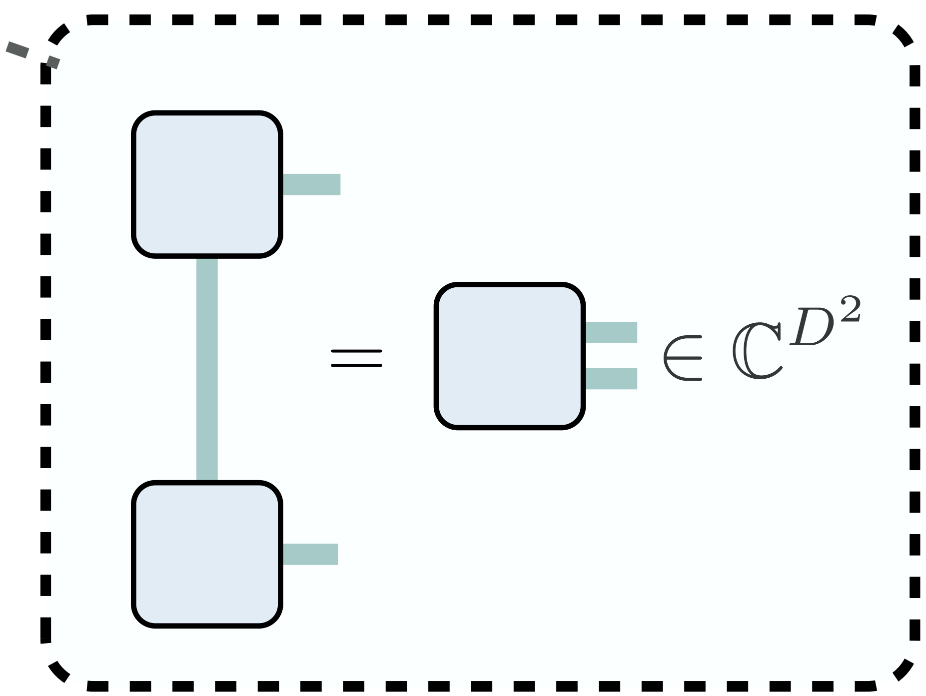
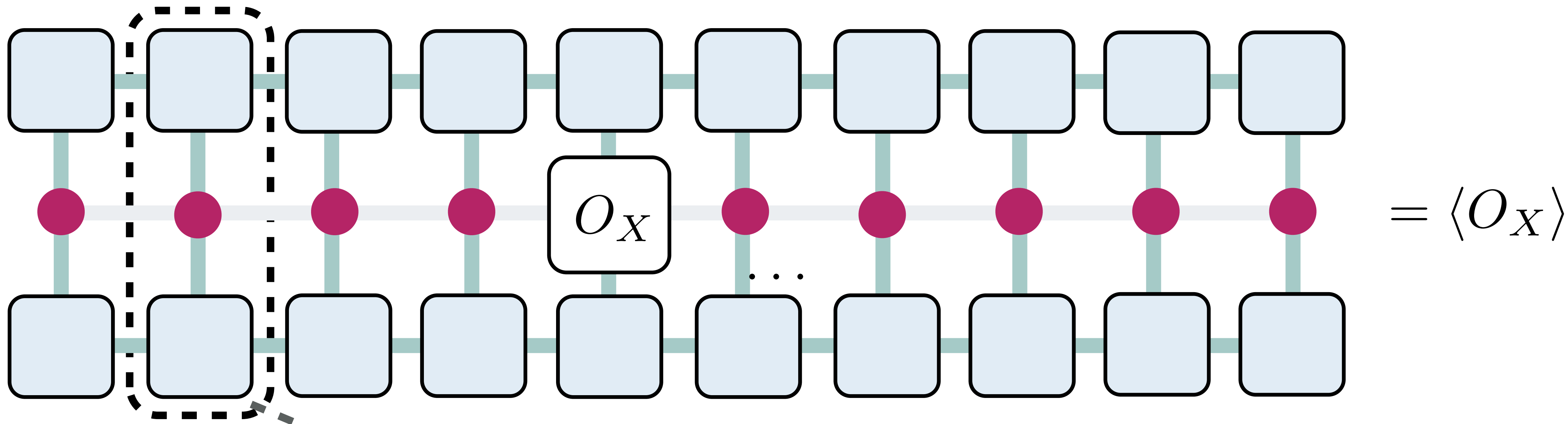


Contraction of MPS



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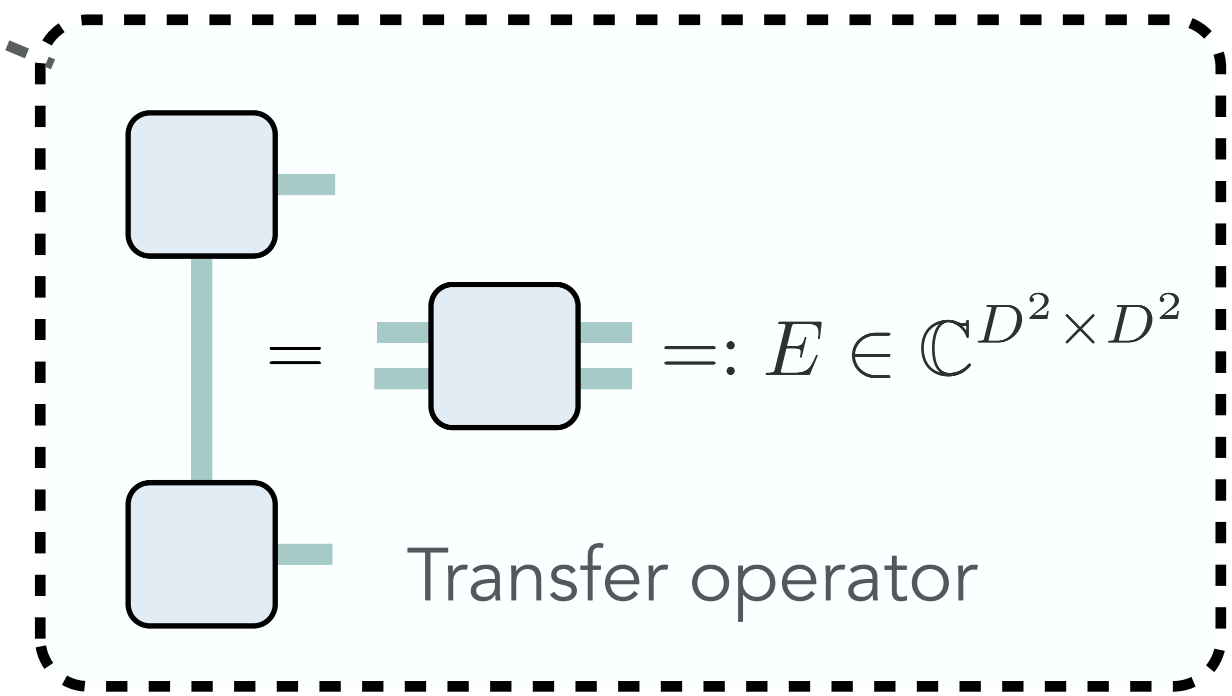
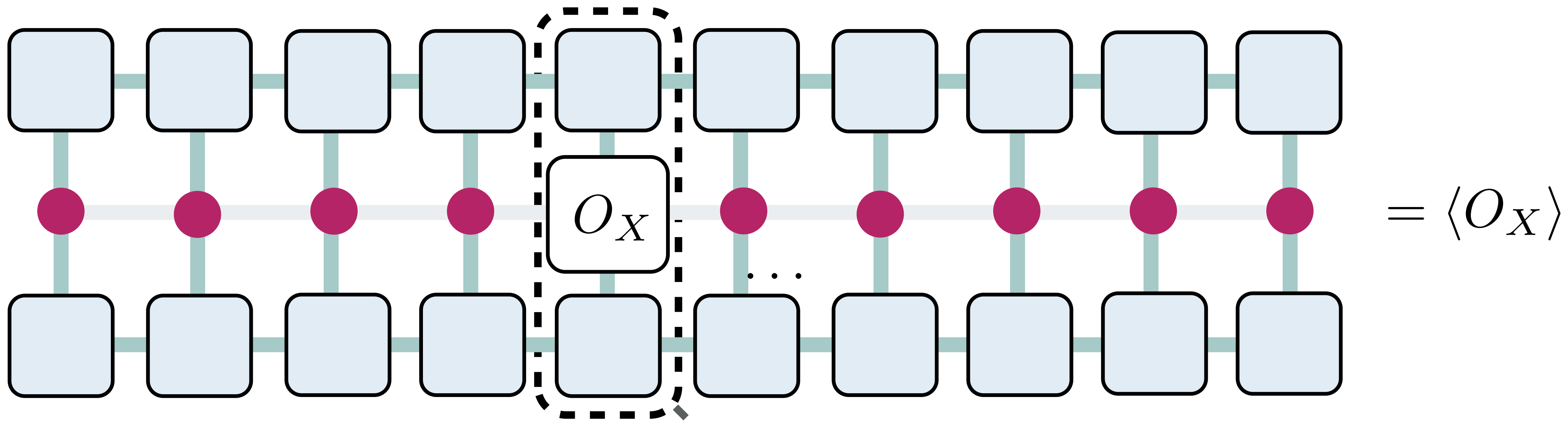


Contraction of MPS



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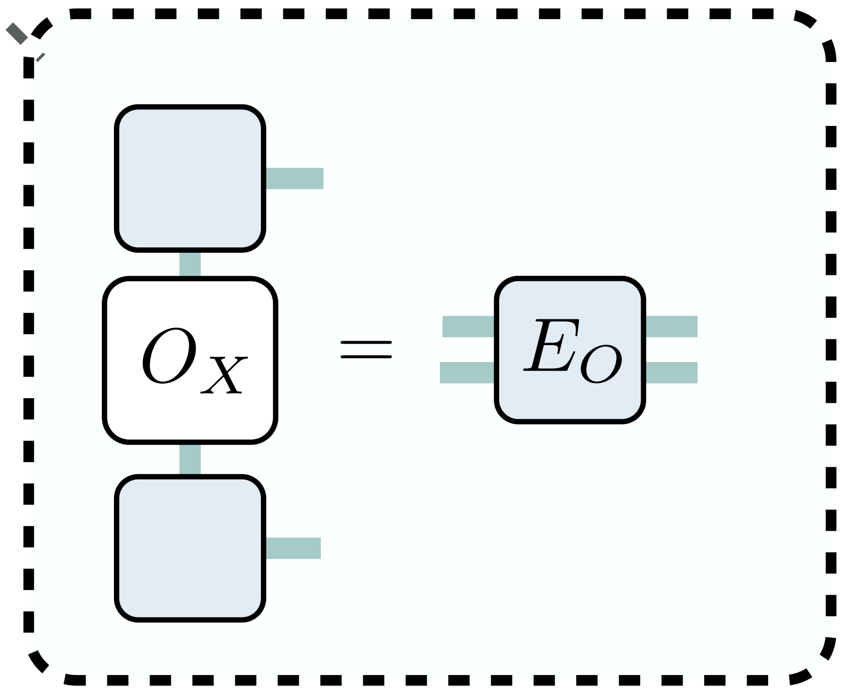
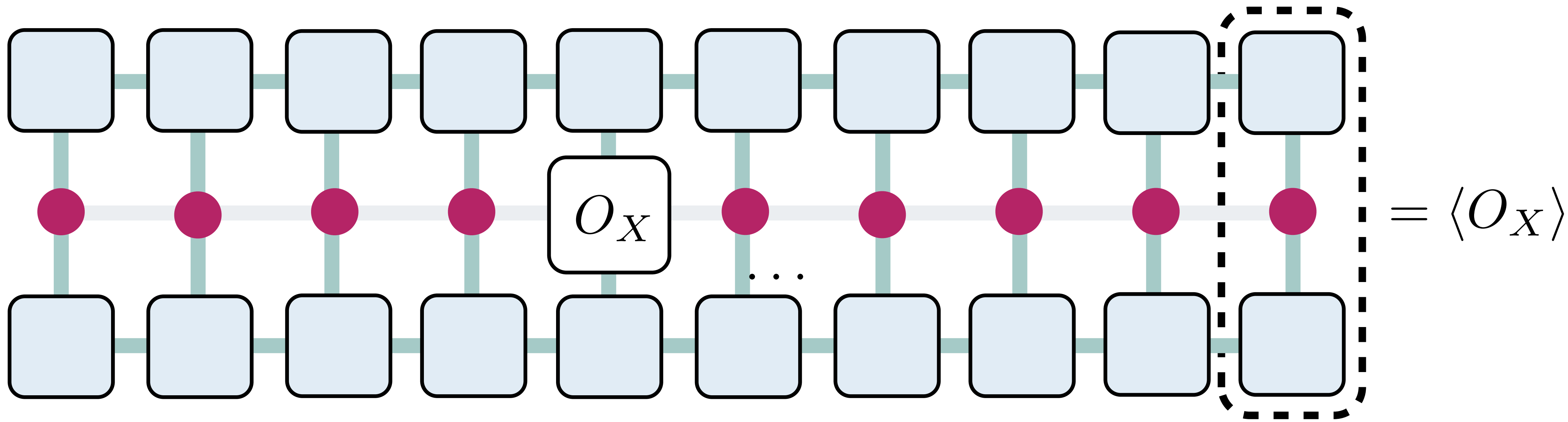
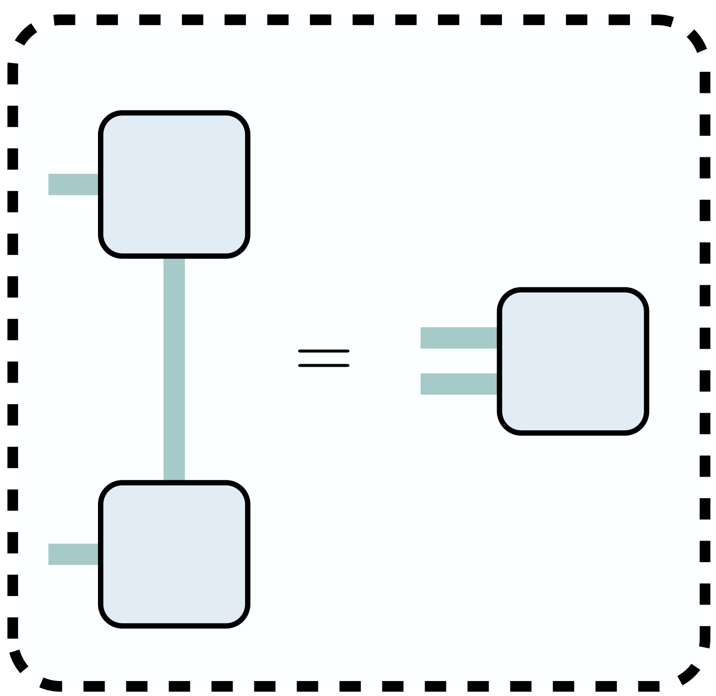


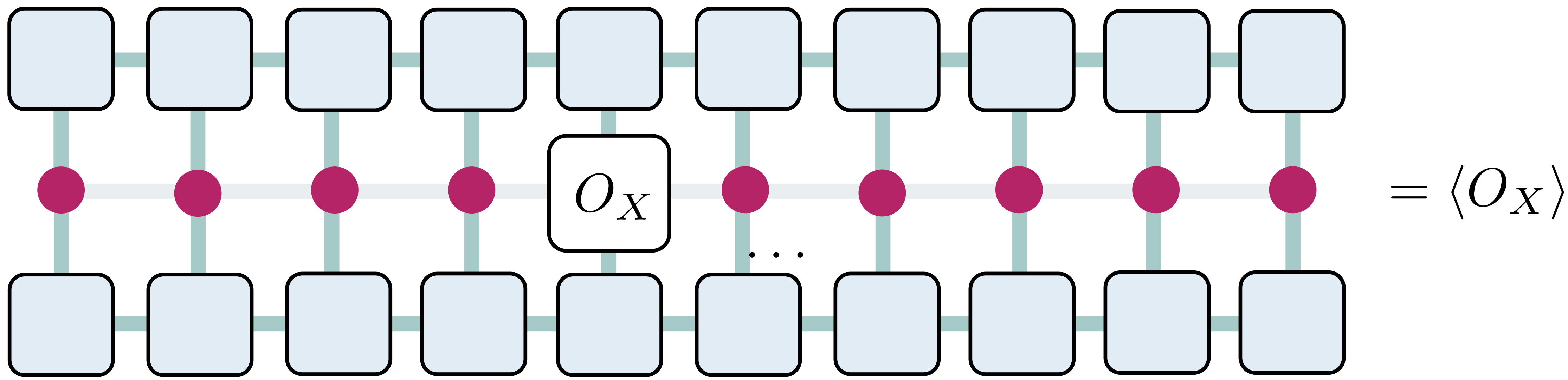
Contraction of MPS



- How can local expectation values be computed?

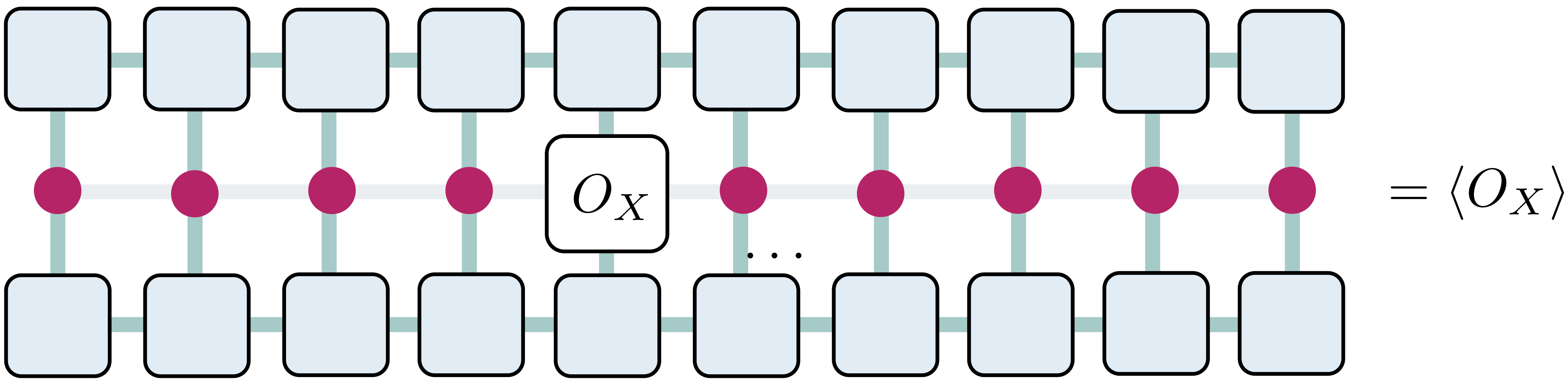


Contraction of MPS



- How can local expectation values be computed?
- Gives rise to effort of $O(D^4)$ - perfectly efficient!

Contraction of MPS

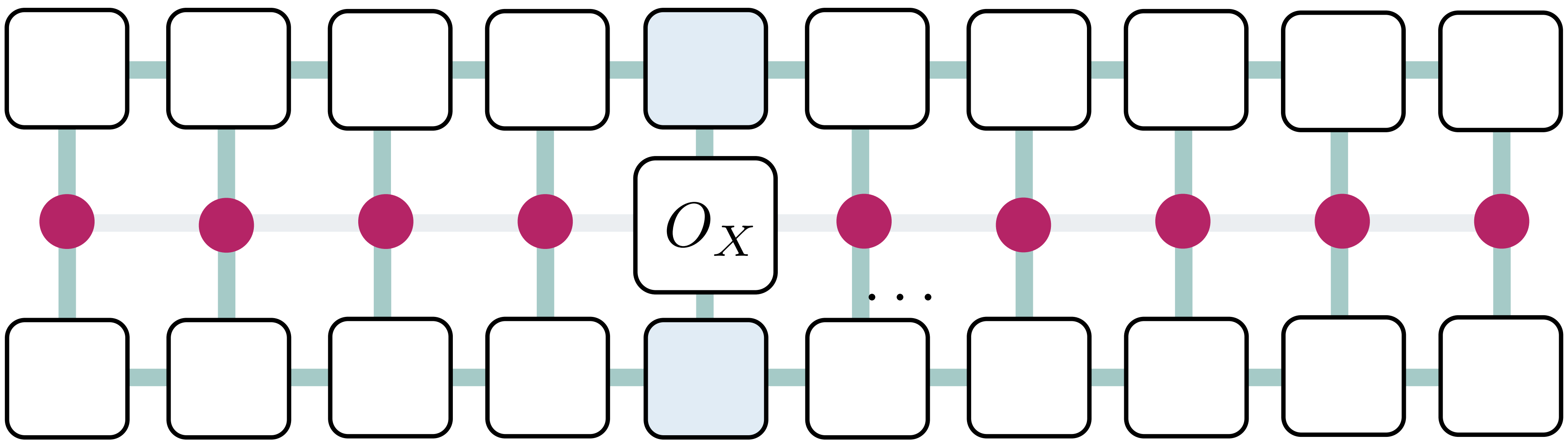


- Basis of DMRG (density matrix renormalisation group method)

White, Phys Rev Lett 69, 2863 (1992)

Landau, Vazirani, Vidick, Nature Physics 11, 566 (2015)

Contraction of MPS

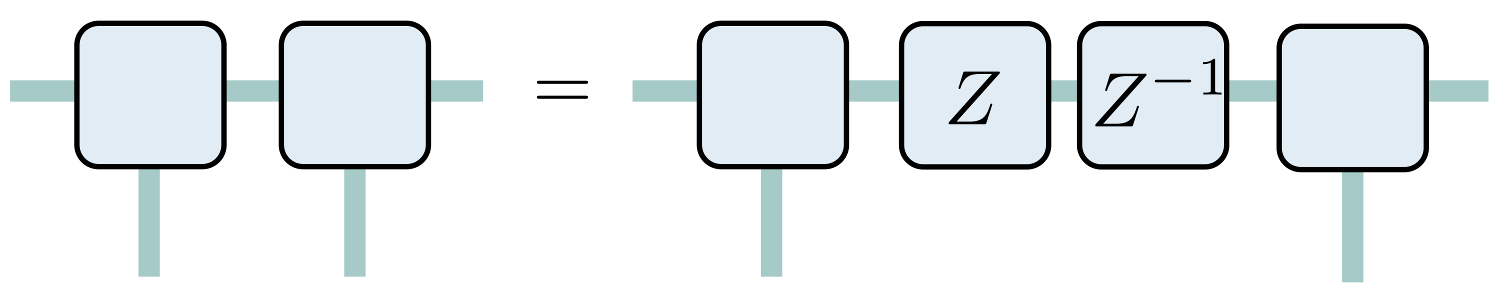


$= \langle O_X \rangle$

- Basis of DMRG (density matrix renormalisation group method)

White, Phys Rev Lett 69, 2863 (1992)

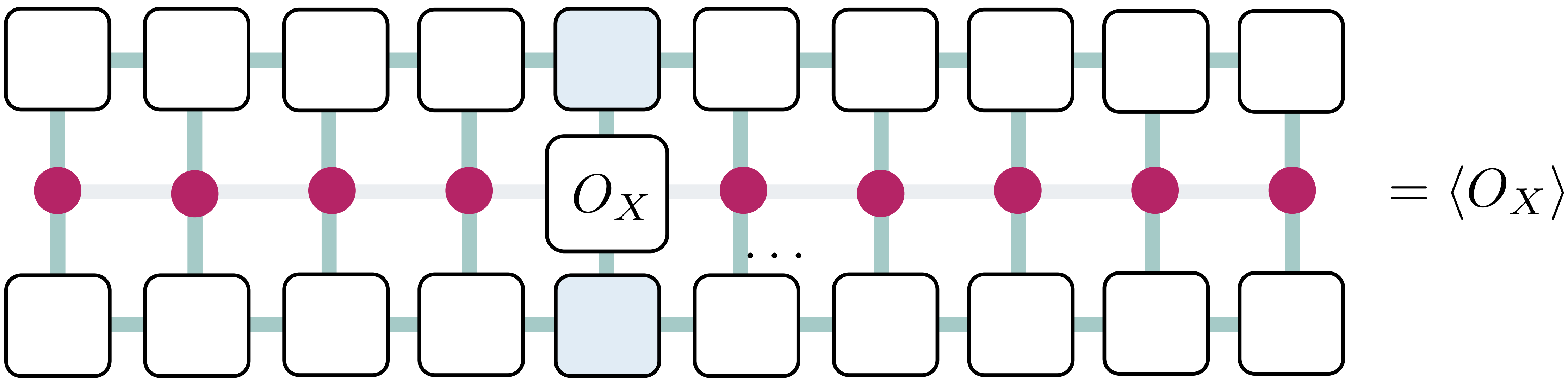
- Lots of variants: B.B (eigenvalue problem)
- Can be reduced to $O(D^3)$, exploiting gauge freedom



Schollwoeck, Ann Phys 326, 96 (2011)

Landau, Vazirani, Vidick, Nature Physics 11, 566 (2015)

Contraction of MPS



- Basis of DMRG (density matrix renormalisation group method)

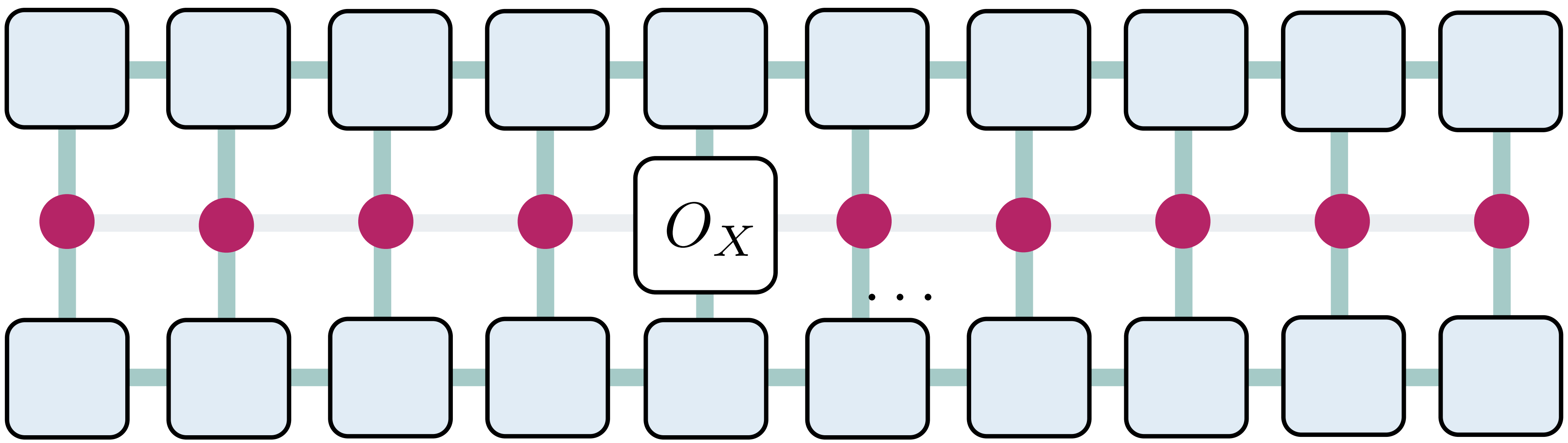
White, Phys Rev Lett 69, 2863 (1992)

- Lots of variants: B.B (eigenvalue problem), B..B (SVD), etc

- Rigorous efficient approximation for gapped models

Landau, Vazirani, Vidick, Nature Physics 11, 566 (2015)

Contraction of MPS

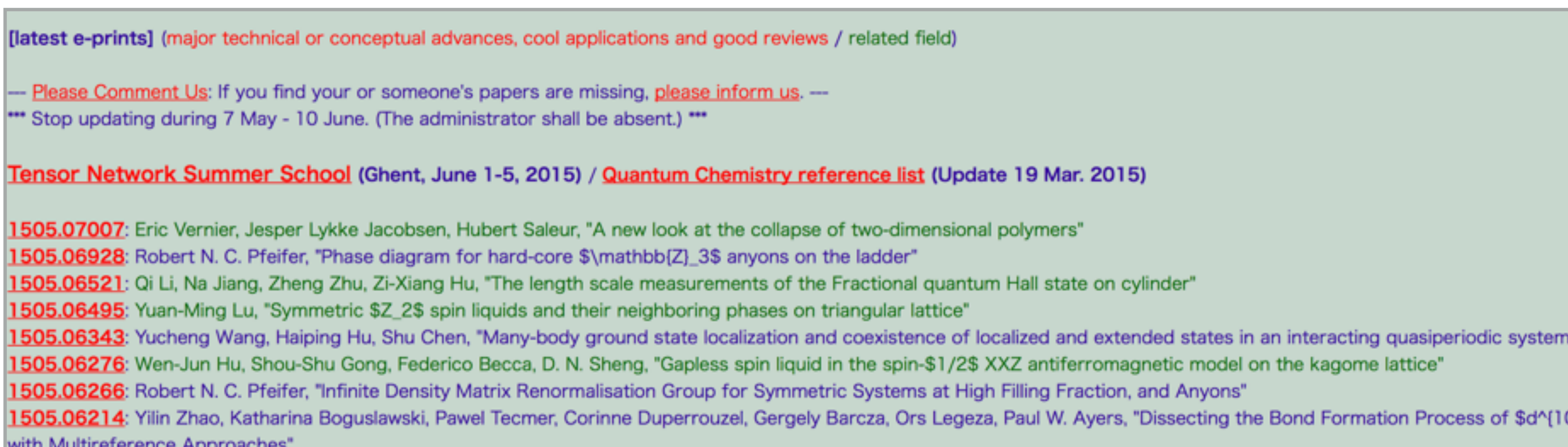


$$= \langle O_X \rangle$$

- Basis of DMRG (density matrix renormalisation group method)

White, Phys Rev Lett 69, 2863 (1992)

- Extremely well-developed in 1D



[latest e-prints] (major technical or conceptual advances, cool applications and good reviews / related field)

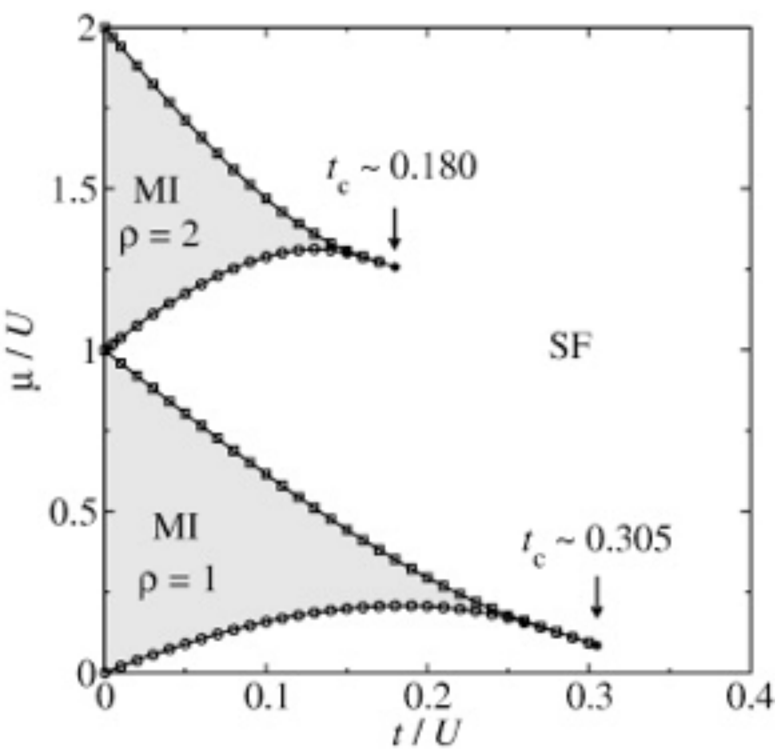
--- Please Comment Us: If you find your or someone's papers are missing, please inform us. ---
*** Stop updating during 7 May - 10 June. (The administrator shall be absent.) ***

Tensor Network Summer School (Ghent, June 1-5, 2015) / Quantum Chemistry reference list (Update 19 Mar. 2015)

1505.07007: Eric Vernier, Jesper Lykke Jacobsen, Hubert Saleur, "A new look at the collapse of two-dimensional polymers"
1505.06928: Robert N. C. Pfeifer, "Phase diagram for hard-core $\mathbb{Z}_3$ anyons on the ladder"
1505.06521: Qi Li, Na Jiang, Zheng Zhu, Zi-Xiang Hu, "The length scale measurements of the Fractional quantum Hall state on cylinder"
1505.06495: Yuan-Ming Lu, "Symmetric Z_2 spin liquids and their neighboring phases on triangular lattice"
1505.06343: Yucheng Wang, Haiping Hu, Shu Chen, "Many-body ground state localization and coexistence of localized and extended states in an interacting quasiperiodic system"
1505.06276: Wen-Jun Hu, Shou-Shu Gong, Federico Becca, D. N. Sheng, "Gapless spin liquid in the spin-$1/2$ XXZ antiferromagnetic model on the kagome lattice"
1505.06266: Robert N. C. Pfeifer, "Infinite Density Matrix Renormalisation Group for Symmetric Systems at High Filling Fraction, and Anyons"
1505.06214: Yilin Zhao, Katharina Boguslawski, Pawel Tecmer, Corinne Duperrouzel, Gergely Barcza, Ors Legeza, Paul W. Ayers, "Dissecting the Bond Formation Process of d^{10} with Multireference Approaches"

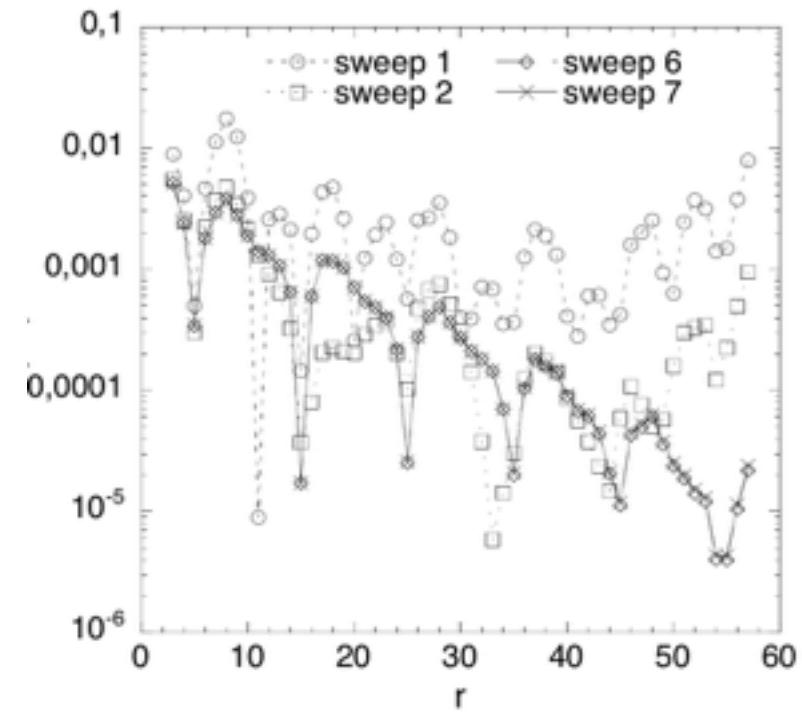
DMRG-approach

- Ground states to machine precision



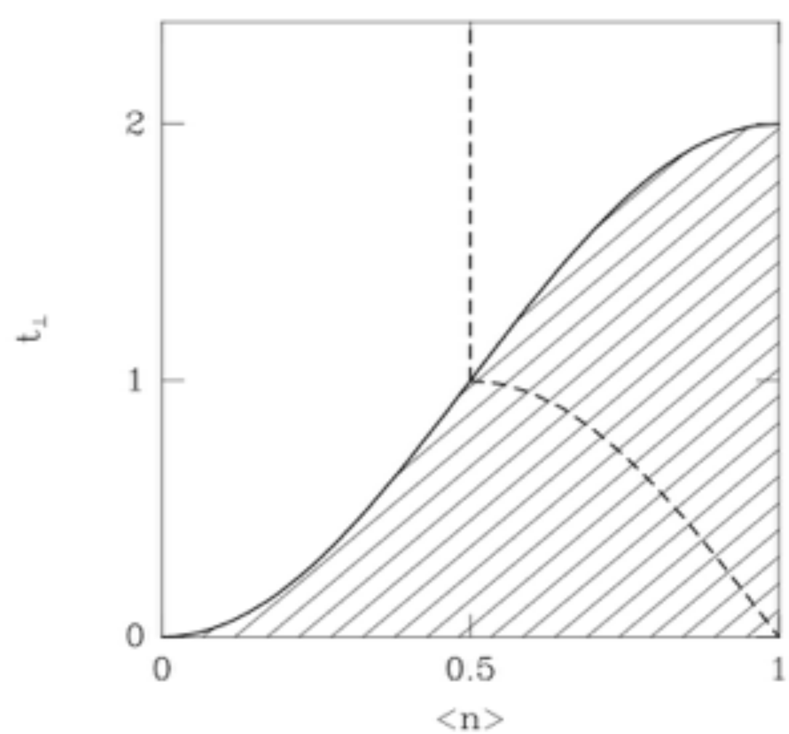
Phase diagram of Bose-Hubbard model

$$H = -\sum_{j=1}^{n-1}\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) - \frac{U}{2}\sum_{j=1}^{n} n_j(n_j - 1)$$



Plaquette current on a t-J-V-V ladder

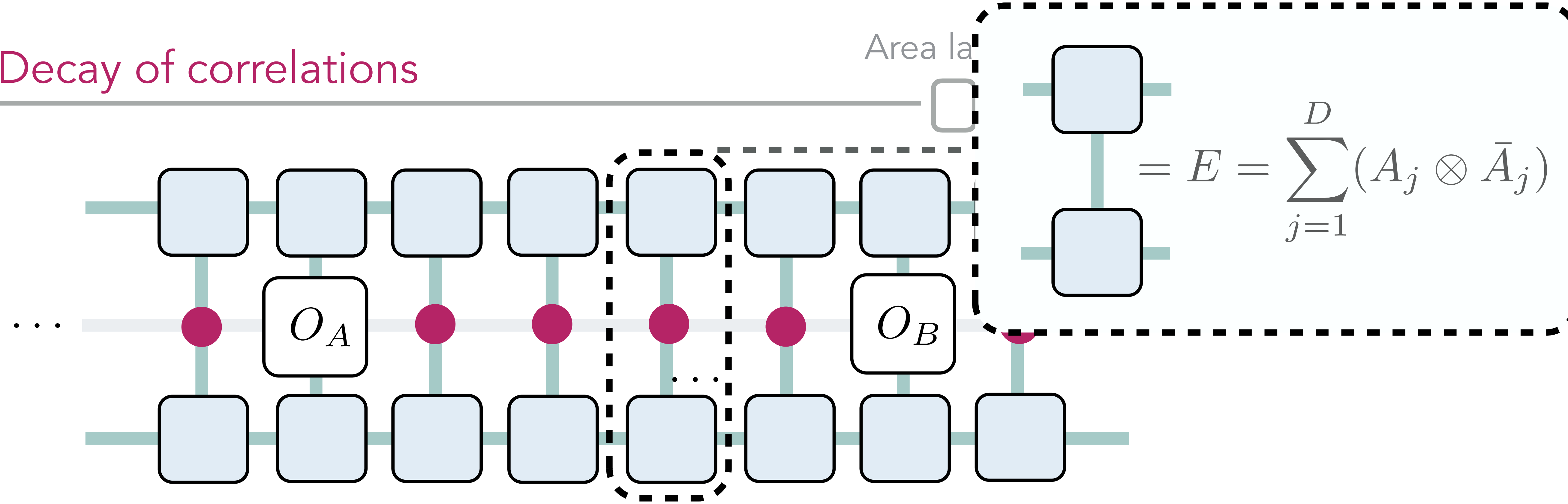
1992)



Phase diagram of a two-leg Hubbard ladder

White, Phys Rev Lett 69, 2863 (1992)
Ejima, Fehske, Gebhard, EPL 93, 30002 (2011)
Schollwoeck, Chakravarty, Fjaerestad, Marston, Troyer, Phys Rev Lett 90, 186401 (20C
Eisert, Mod Sim 3, 520 (2013)
Eisert, Cramer, Plenio, Rev Mod Phys 82, 277 (2010)
Barthel, Schollwoeck, White, Phys Rev B 79, 245101 (2009)
Schollwoeck, Ann Phys 326, 96 (2011)
Orus, Ann Phys 349, 17 (2014)

Landau, Vazirani, Vidick, Nature Physics 11, 566 (2015)



- So $\langle O_A O_B \rangle = \dfrac{\mathrm{tr}(E_{O_A} E^{\mathrm{dist}(A,B)-1} E_{O_B} E^{n-\mathrm{dist}(A,B)-1})}{\mathrm{tr}(E^n)}$
- Write powers of transfer operators as $E^k = \lambda_1 |r_1\rangle\langle l_1| + \sum_{k=2}^{D^2} \lambda_j^k |r_j\rangle\langle l_j|$
- Gives $\langle O_A O_B \rangle = \lambda_1 \langle l_1 | E_{O_A} | r_1 \rangle \langle l_1 | E_{O_B} | l_1 \rangle + \sum_{j=2}^{D^2} \lambda_j^{\mathrm{dist}(A,B)-1} \langle l_1 | E_{O_A} | r_j \rangle \langle l_j | E_{O_B} | l_1 \rangle$
- Correlation functions $|\langle O_A O_B \rangle - \langle O_A \rangle \langle O_B \rangle|$ decay exponentially in $\mathrm{dist}(A, B)$

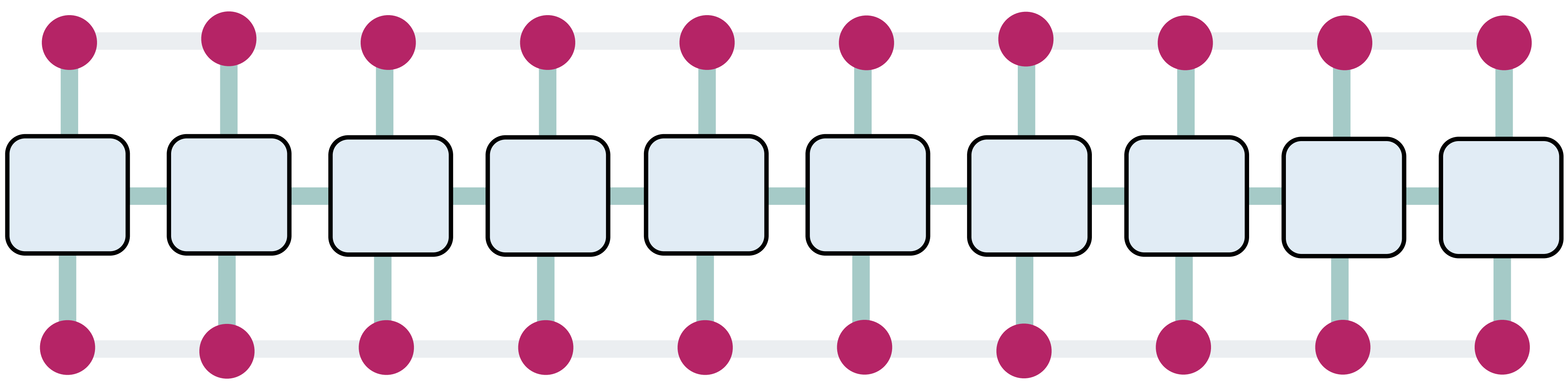
  on length scale $\xi^{-1} = -\log \dfrac{|\lambda_2|}{|\lambda_1|}$ ("inverse gap of transfer operator")

More of a good thing: Non-equilibrium, thermal and open systems, and many-body localisation

- Thermal states, open systems, mixed quantum states?

- **Matrix-product operators (MPO)**

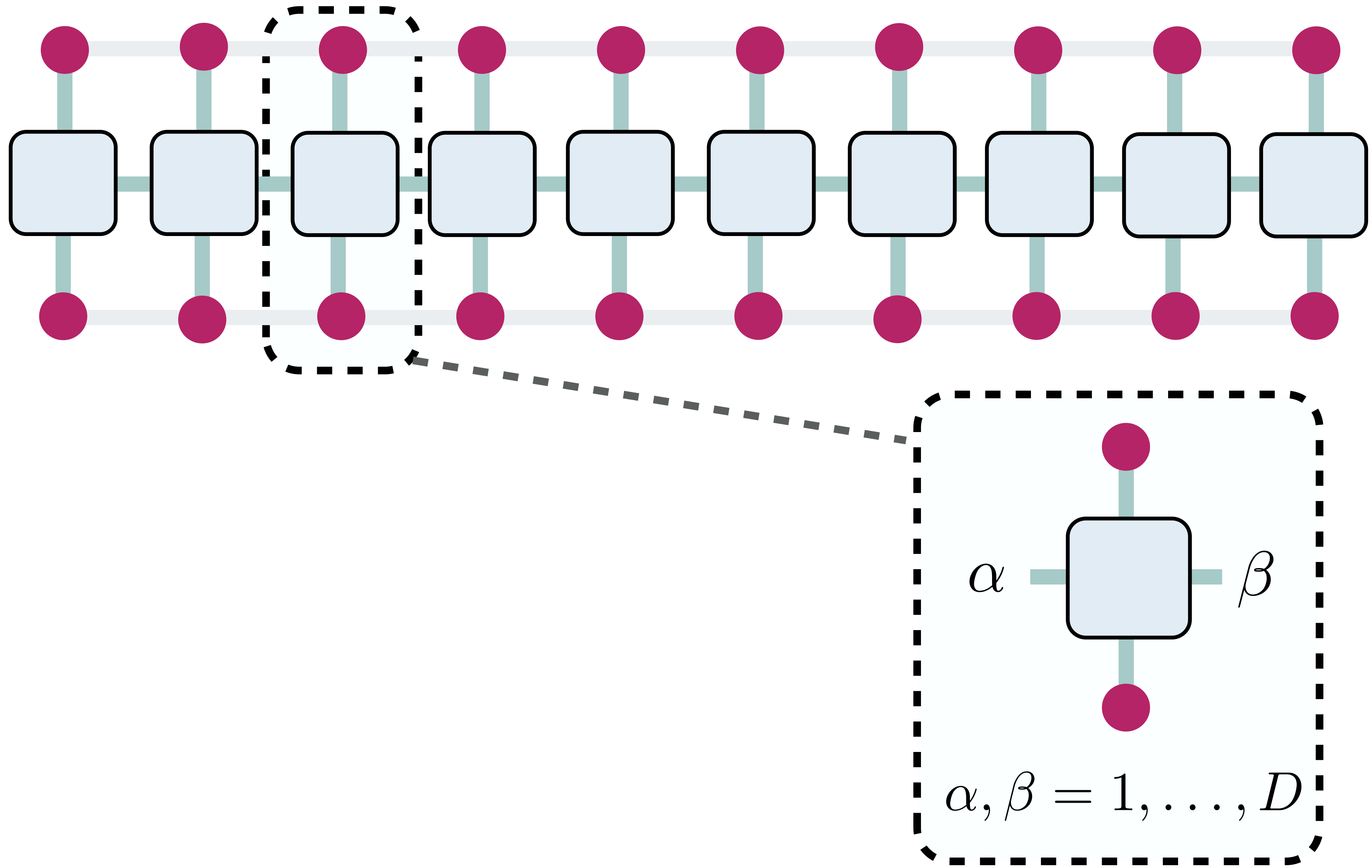
$\rho =$



- Thermal states, open systems, mixed quantum states?

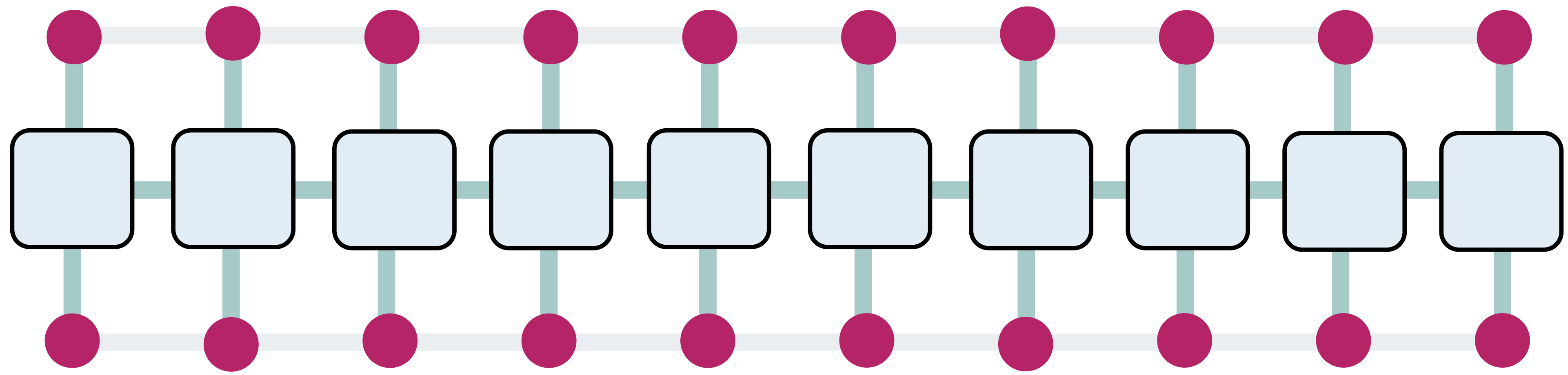
- **Matrix-product operators (MPO)**

$\rho =$



- Thermal states, open systems, mixed quantum states?

- **Matrix-product operators (MPO)**

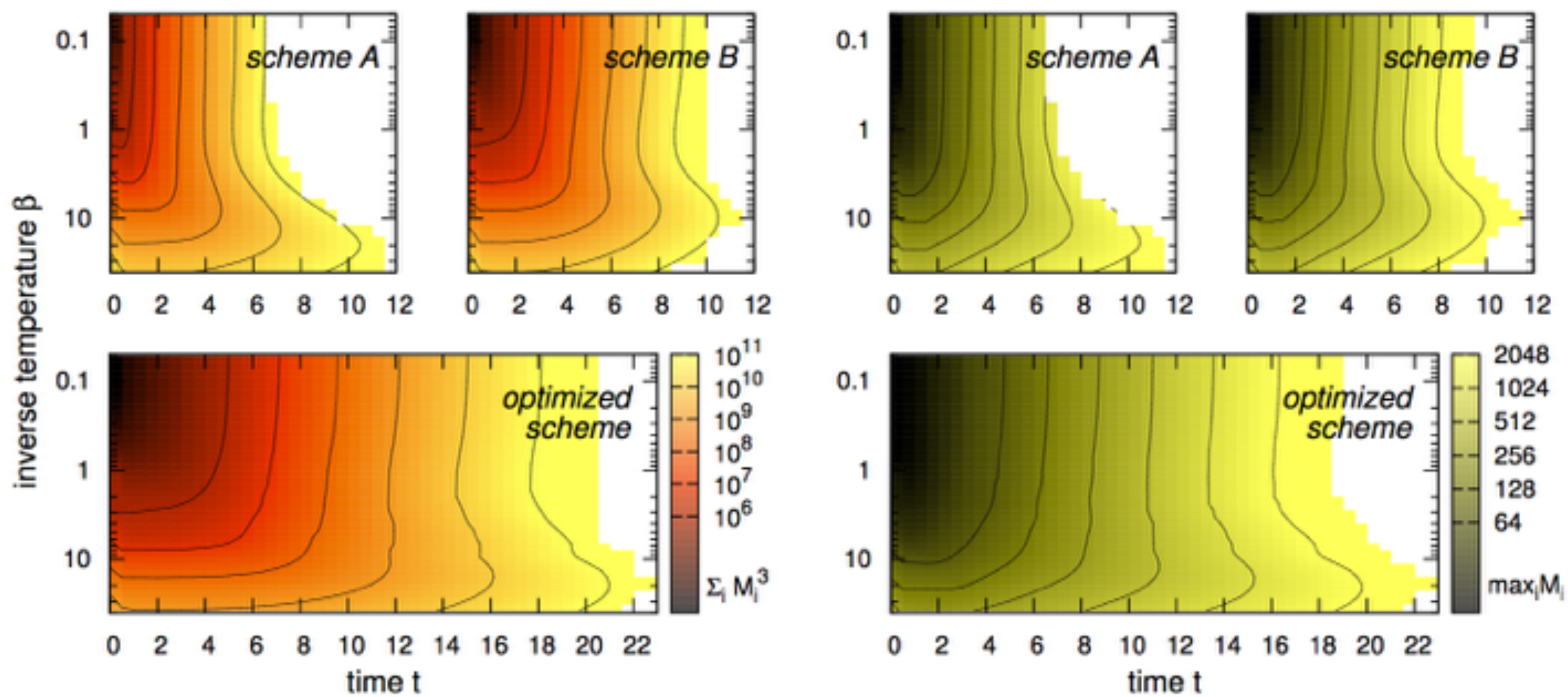
$\rho =$ 

- Very practical - theory less slightly well understood

- **Theorem:** Positivity of an MPO is undecidable

Kliesch, Gross, Eisert, Phys Rev Lett 113, 160503 (2014)

- **Matrix-product operators for thermal states**



Temperature dependence of 1D isotropic Heisenberg model

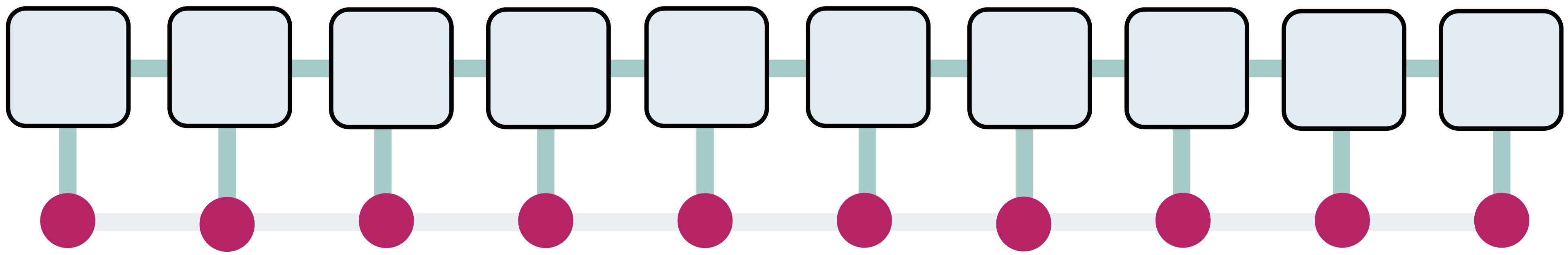
$$H = -\sum_{j=1}^{n-1}\left(\frac{J}{2}\left(S_j^+ S_{j+1} + S_j^- S_{j-1}^+\right) + J^z S_j^z S_{j+1}^z\right) - h\sum_{j=1}^{n} S_j^z$$

Verstraete, Garcia-Ripoll, Cirac, Phys Rev Lett 93, 207204 (2004)
Karrasch, Bardarson, Moore, Phys Rev Lett 108, 227206 (2012)

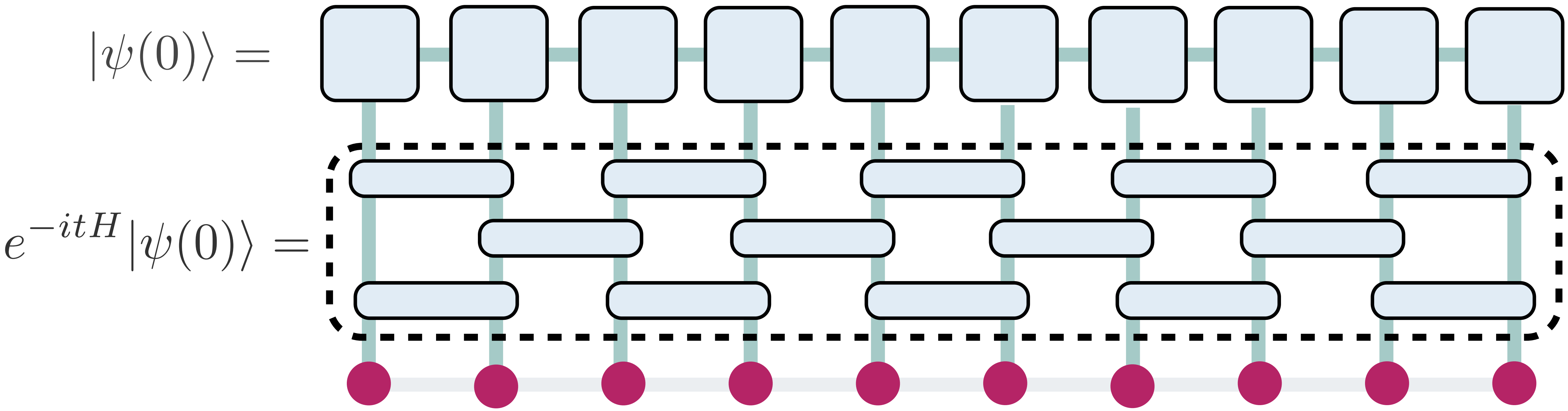
- Provably exist for **high temperature** states

Kliesch, Gogolin, Kastoryano, Riera, Eisert, Phys Rev X 4, 031019 (2014)
Ge, Molnár, Cirac, Phys Rev Lett 116, 080503 (2016)

- **Out-of-equilibrium dynamics:** Quenches $e^{-itH}|\psi(0)\rangle$

$|\psi(0)\rangle =$ 

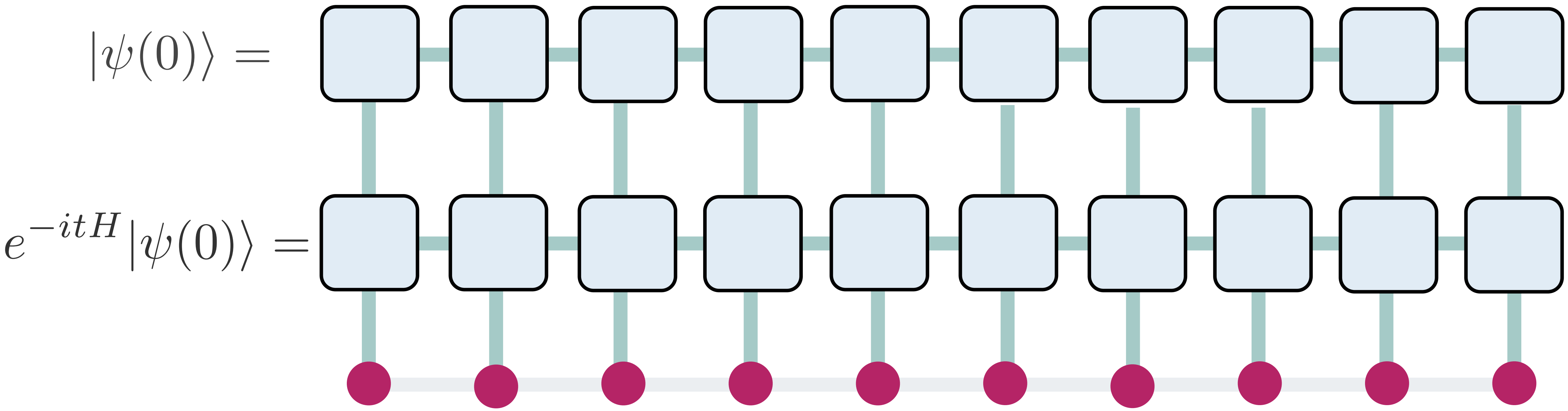
- **Out-of-equilibrium dynamics:** Quenches $e^{-itH}|\psi(0)\rangle$



Apply suitable Trotter formula

Schollwoeck, Ann Phys 326, 96 (2011)

- **Out-of-equilibrium dynamics:** Quenches $e^{-itH}|\psi(0)\rangle$



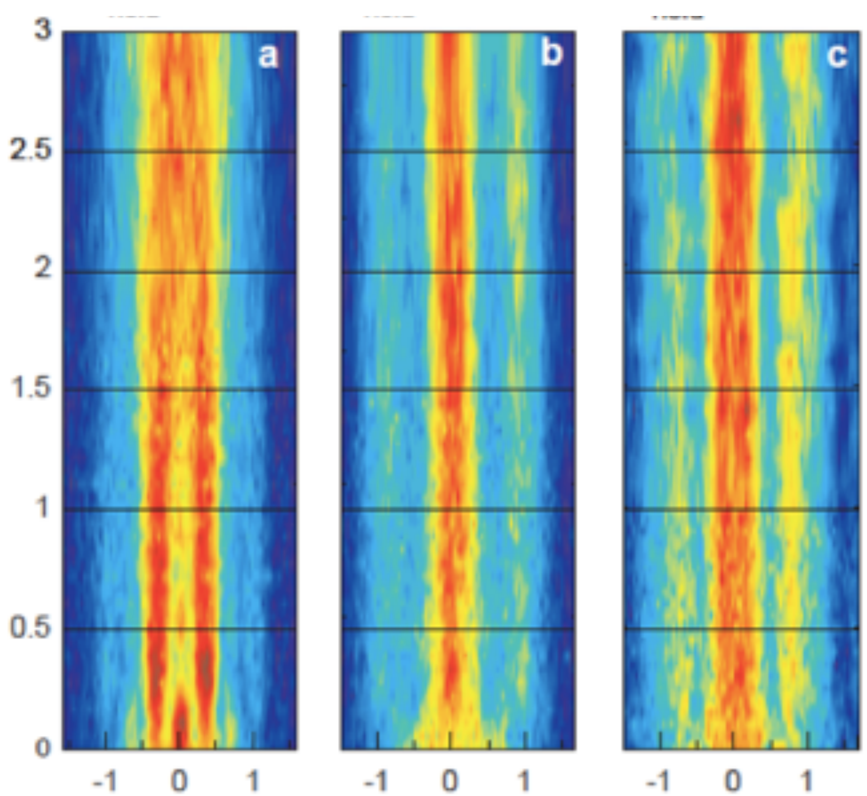
Contract to MPO

- Gives - for short times - an efficient algorithm for out-of-equilibrium

- **Out-of-equilibrium dynamics:** Quenches $e^{-itH}|\psi(0)\rangle$

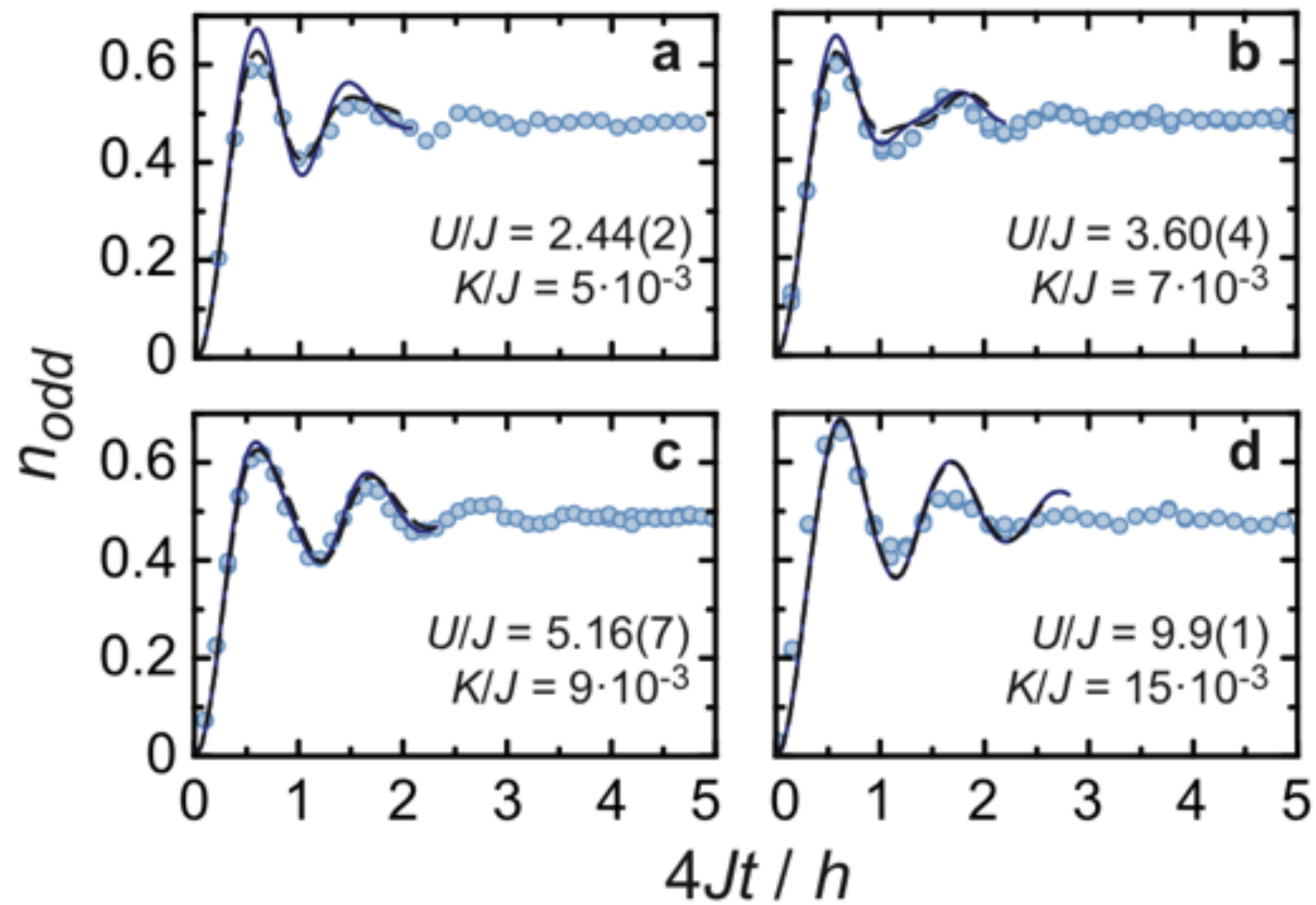
e^{-i}

- Quenched quantum many-body systems



Out of equilibrium Bose-Hubbard dynamics in momentum space

$$H = -\sum_{j=1}^{n-1}\left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) - \frac{U}{2}\sum_{j=1}^{n} n_j(n_j-1)$$



Dynamics of cold atoms in optical super-lattices compared with experimental data in quantum simulators

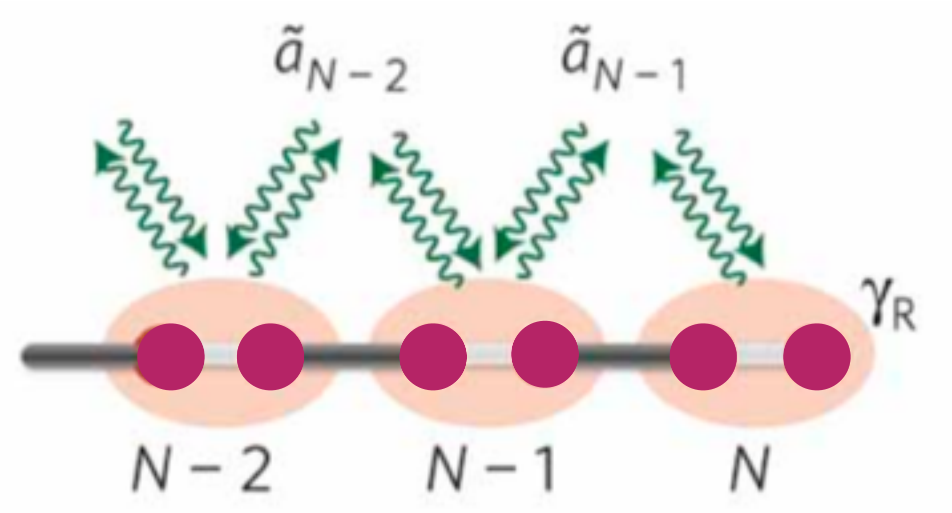
Trotzky, Chen, Flesch, McCullock, Schollwoeck, Eisert, Bloch, Nature Phys 8, 325 (2012)
Daley, Kollath, Schollwock, Vidal, J Stat Mech, P04005 (2004)

-

- **Open or driven quantum many-body systems**
- Describe environment by Markovian Lindblad quantum master equation

$$\partial_t \rho = -i[H,\rho] + \kappa \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right) =: \mathcal{L}(\rho)$$

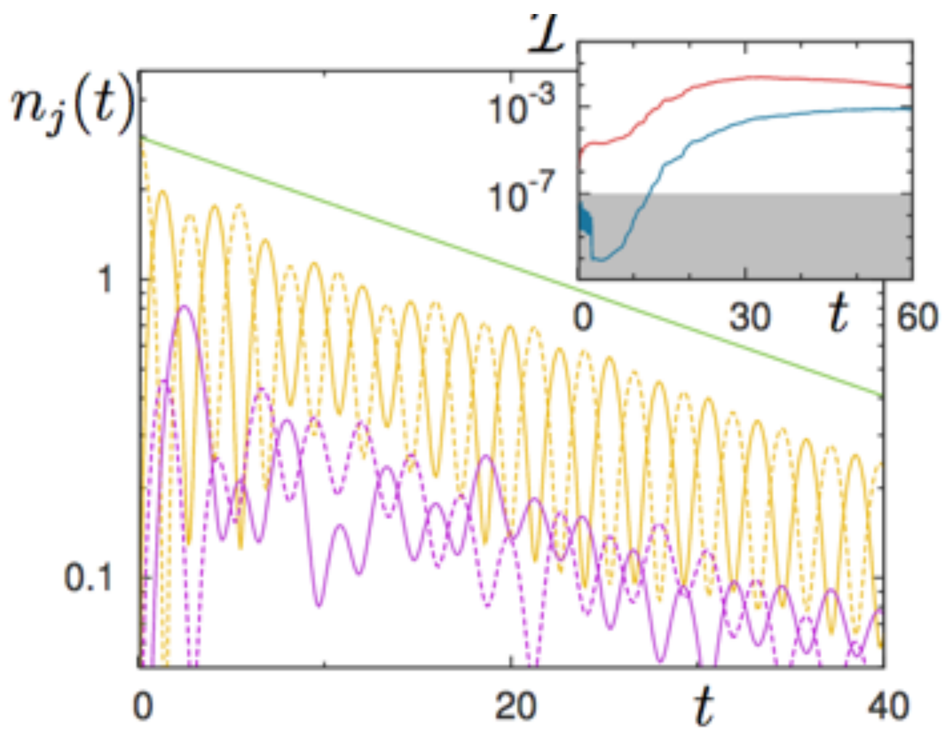
Coherent part Dissipative part



- Preparation of topologically non-trivial states as stationary states $\mathcal{L}(\rho) = 0$

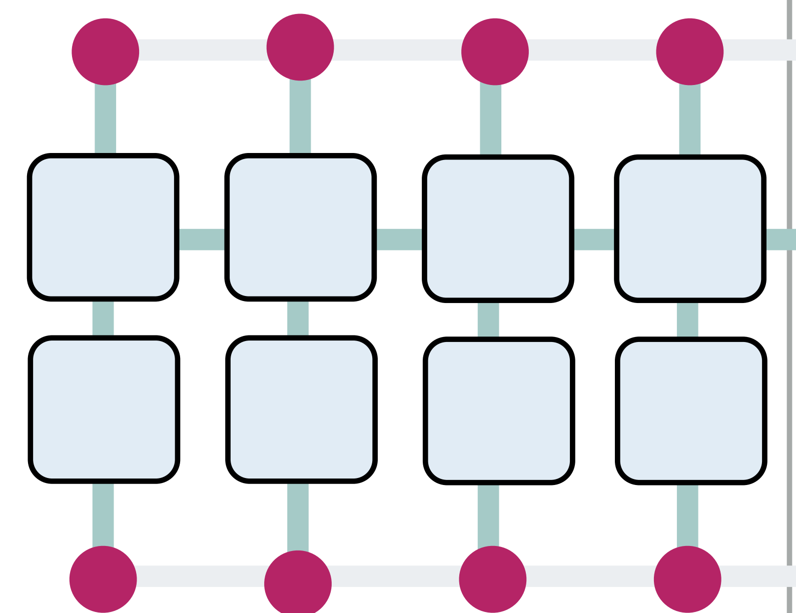
Diehl, Micheli, Kantian, Krais, Buechler, Zoller, Nature Phys 5, 878 (2008)
Verstraete, Wolf, Cirac, Nature Phys 5, 633 (2009)
Herold, Campbell, Eisert, Kastoryano, Nature P J Quant Inf (2015)

- Positive MPO for open quantum systems



Excitation population in a dissipative spin-cavity-model

$\rho =$



Positive matrix-product operator

$$\rho \geq 0$$

Verstraete, Garcia-Ripoll, Cirac, Phys Rev Lett 93, 207204 (2004)
Werner, Jaschke, Silvi, Kliesch, Calarco, Eisert, Montangero, Phys Rev Lett 116, 237201 (2016)
Zwolak, Vidal, Phys Rev Lett 93, 207205 (2004).

Diehl, Micheli, Kantian, Krais, Buechler, Zoller, Nature Phys 5, 878 (2008)
Verstraete, Wolf, Cirac, Nature Phys 5, 633 (2009)
Herold, Campbell, Eisert, Kastoryano, Nature P J Quant Inf (2015)

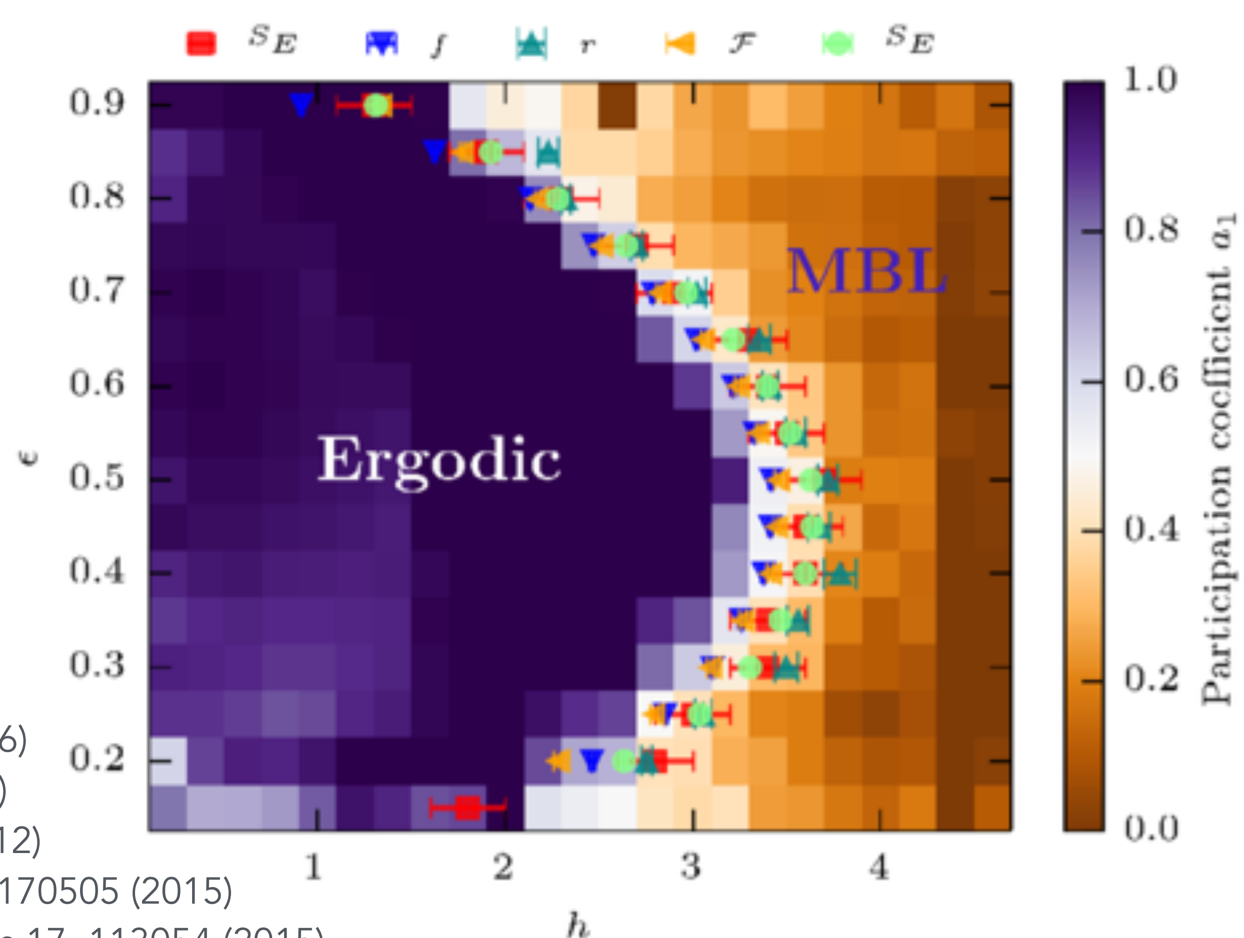
Many-body localisation

- **Many-body localisation:** Intriguing phenomenon in which disorder and interactions come together

$$H = -\sum_{j=1}^{n-1}\left(\frac{J}{2}\left(S_j^+ S_{j+1} + S_j^- S_{j-1}^+\right) + J^z S_j^z S_{j+1}^z\right) - \sum_{j=1}^{n} h_j S_j^z$$

Pal, Huse, Phys Rev B 82, 174411 (2010)
Bauer, Nayak, J Stat Mech P09005 (2013)
Basko, Aleiner, Altshuler, Ann Phys 321, 1126 (2006)
Znidaric, Prosen, Prelovsek, PRB 77, 064426 (2008)
Badarson, Pollmann, Moore, PRL 109, 017202 (2012)
Friesdorf, Werner, Scholz, Brown, Eisert, PRL 114, 170505 (2015)
Friesdorf, Werner, Goihl, Eisert, Brown, New J Phys 17, 113054 (2015)

- **Many-body localisation:** Intriguing phenomenon in which disorder and interactions come together
- Rich phenomenology:
  - Absence of thermalisation and transport
  - Logarithmic growth of entanglement entropies
  - Linearly many approx local constants of motion
  - Area laws and MPS eigenstates of excited states



Pal, Huse, Phys Rev B 82, 174411 (2010)
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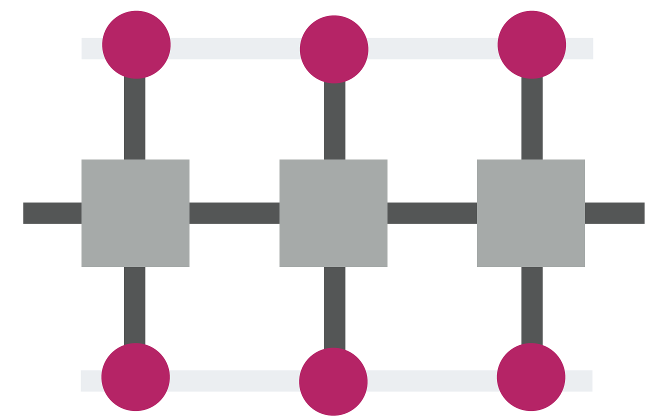
Badarson, Pollmann, Moore, PRL 109, 017202 (2012)
Friesdorf, Werner, Scholz, Brown, Eisert, PRL 114, 170505 (2015)
Friesdorf, Werner, Goihl, Eisert, Brown, New J Phys 17, 113054 (2015)

- All **eigenstates are MPS** follows from dynamical localisation

  Friesdorf, Werner, Scholz, Brown, Eisert, PRL 114, 170505 (2015)



- Can be used in **X-DMRG** and variants

  Khemani, Pollmann, Sondhi, Phys Rev Lett 116, 247204 (2016)
  Karrasch, Kennes, arXiv:1511:02205

- Diagonalisation to l-bit form via **quasi-local unitary**

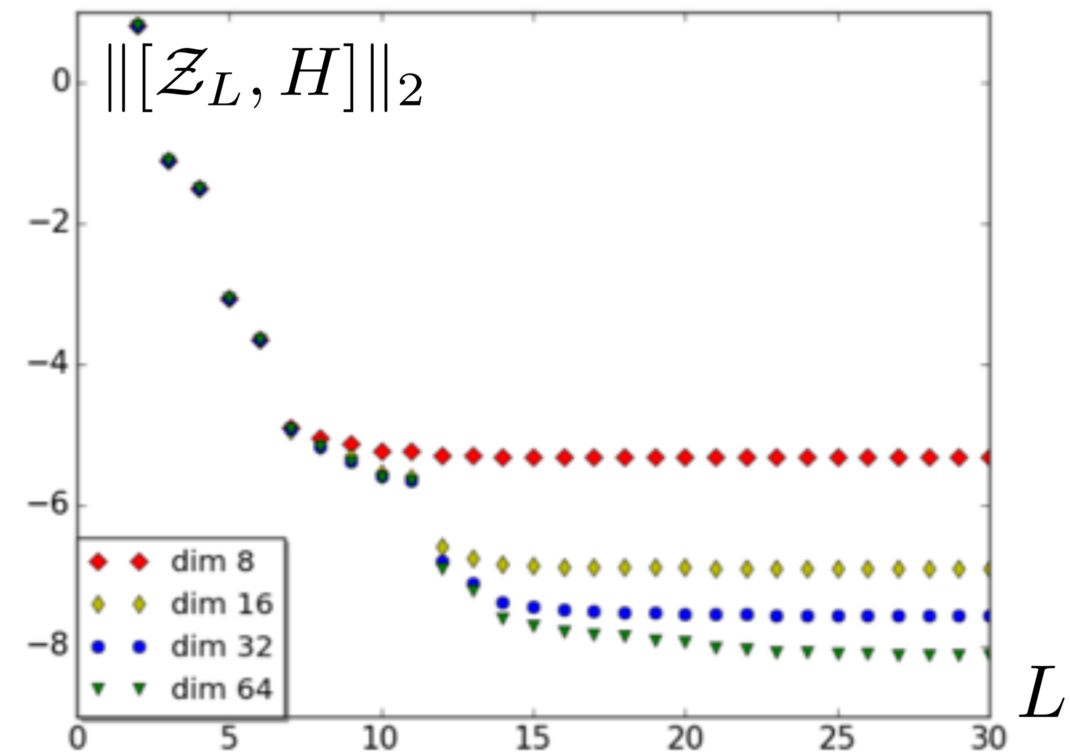
  Pollmann, Khemani, Cirac, Sondhi, Phys Rev B 94, 041116 (2016)
  Wahl, Pal, Simon, arXiv:1609.01552

- Finding **local constants of motion**:

  Minimise $\|[\mathcal{Z}, H]\|_2$

  subject to MPO bond-dimension and support constraints

  Kim, Bañuls, Cirac, Hastings, Huse, Phys. Rev. E 92, 012128 (2015)
  Nebendahl, Goihl, Brown, Werner, Eisert, in preparation (2016)



- Combine **Wegner-flow** and **MPO-simulations:** Full MPO representation

  Orus, Schmidt, Eisert, in preparation (2016)

Applications in quantum chemistry

- **Quantum chemistry:** Interacting fermions

$$H = \sum_{j=1}^{n} \left(-\frac{1}{2} \nabla_j^2 - \sum_I \frac{Z_I}{r_{j,I}} \right) + \sum_{j<k} \frac{1}{r_{j,k}}$$

- In second quantisation, long-ranged interacting model,

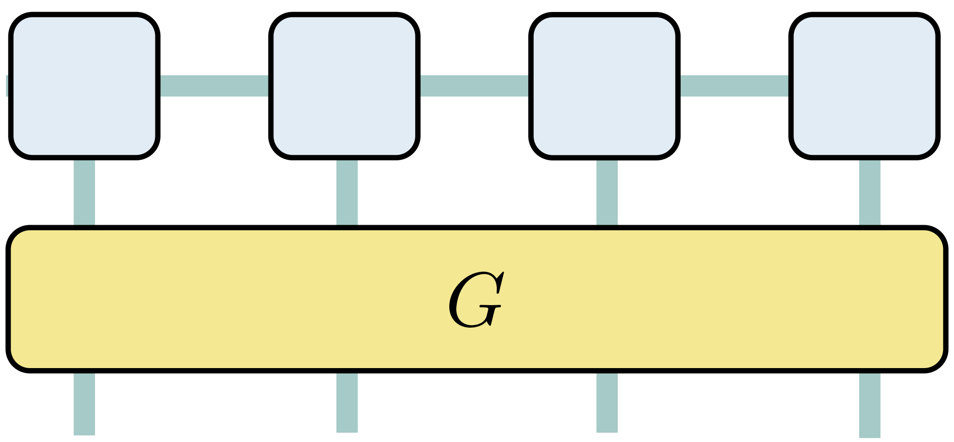
$$H = \sum_{j,k} h_{j,k} c_j^\dagger c_k + \frac{1}{2} \sum_{j,k,l,m} V_{j,k,l,m} c_j^\dagger c_k^\dagger c_l c_m$$

long-ranged DMRG methods can be applied

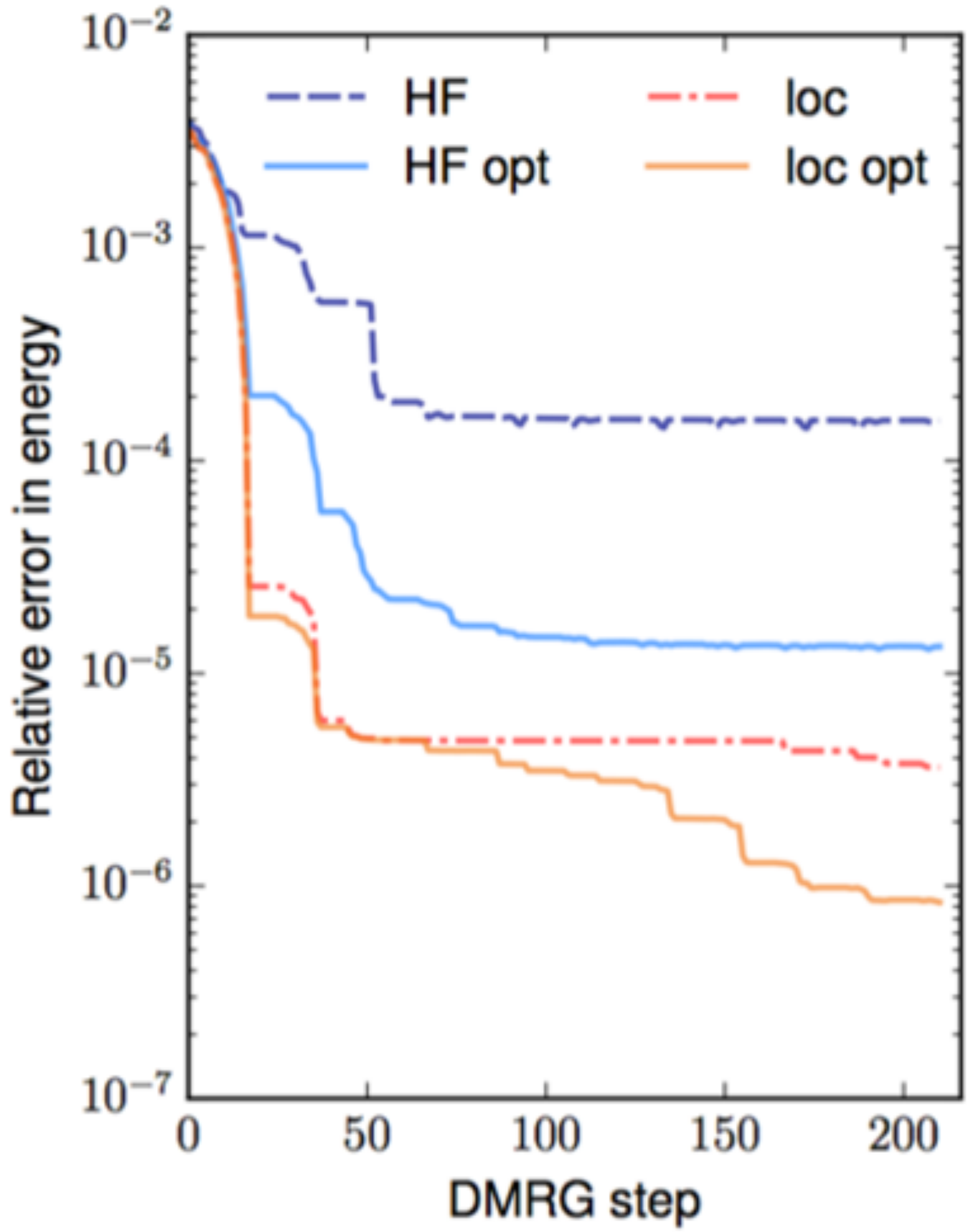
White, Martin, J Chem Phys 110, 4127 (1999)
Chan, Head-Gordon, J Chem Phys 116, 4462 2002)

Applications in quantum chemistry

- DMRG outperforms CI for strongly correlated models, if orbital optimisation is applied for (based on Renyi entanglement entropies)

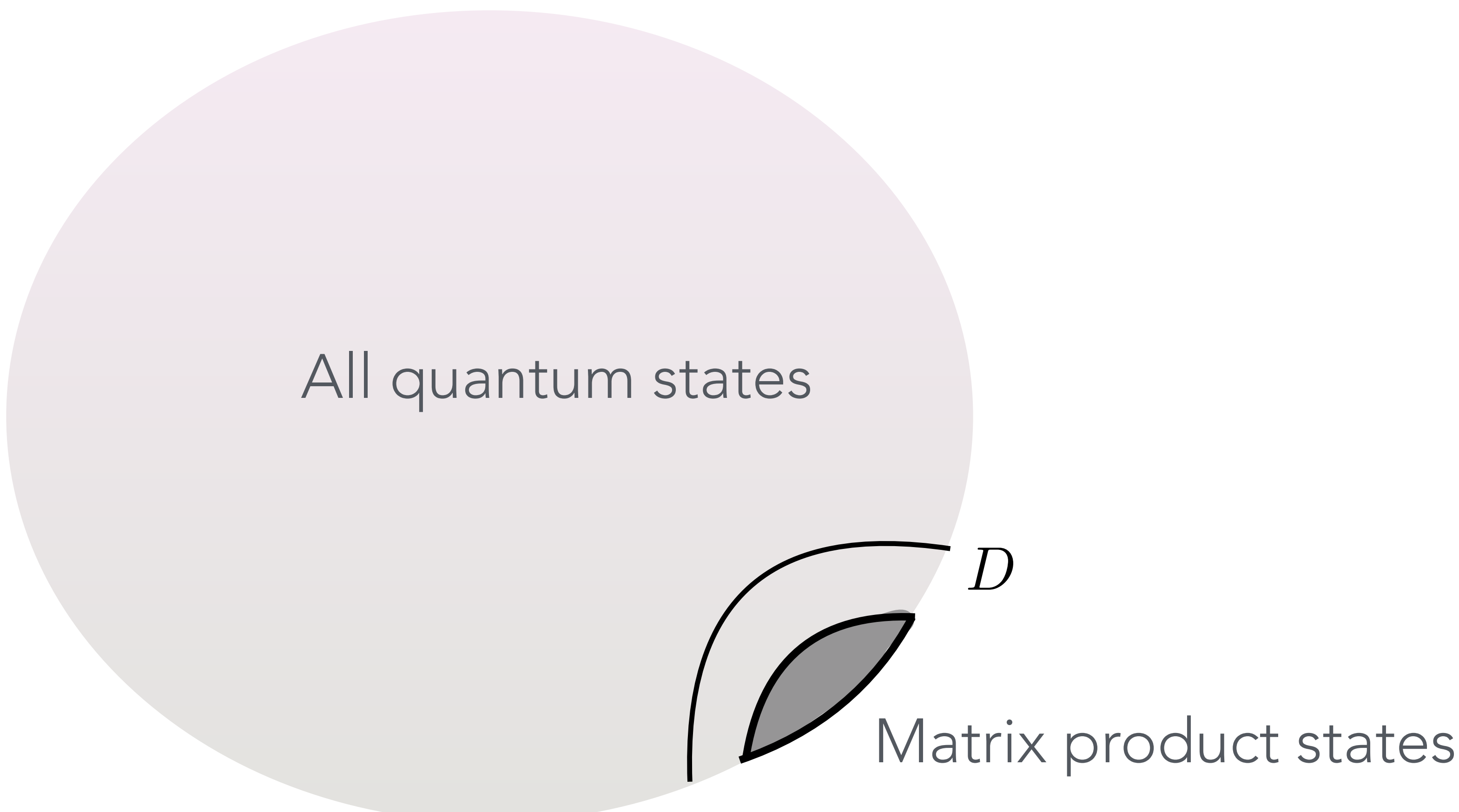


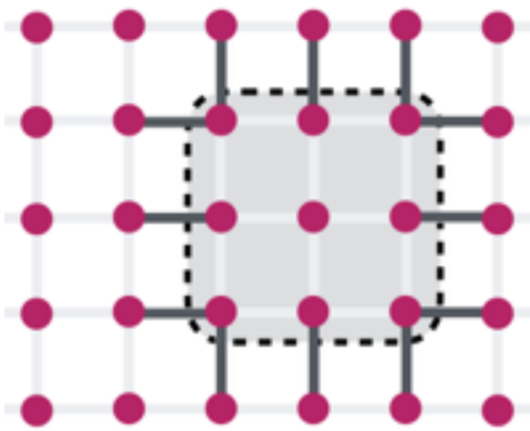
$$|\psi\rangle \mapsto G|\psi\rangle = \exp\left(\sum_{j,k} c_j^\dagger (\log U^\dagger)_{j,k} c_k\right)|\psi\rangle$$



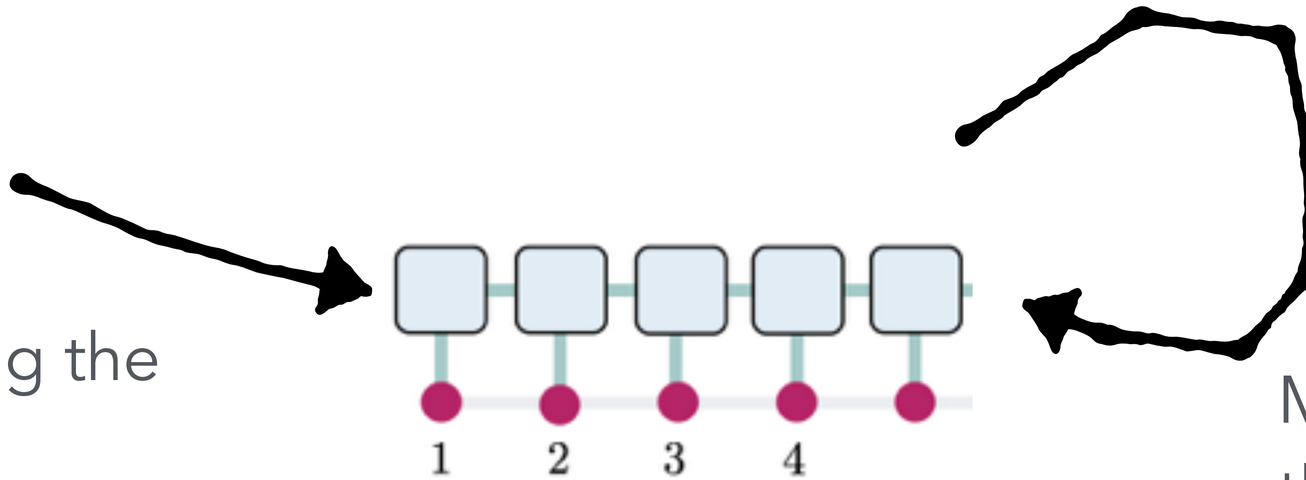
Krumnow, Veis, Legeza, Eisert, Phys Rev Lett 117 (2016)

- **Lesson so far:** Matrix-product states and operators versatile and powerful tool to capture 1D strongly correlated systems in the "physical corner"

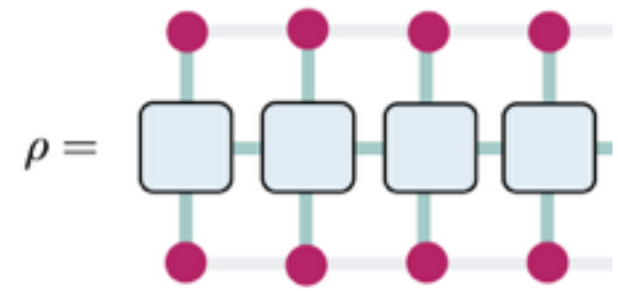




Area laws deliniating the "physical corner"



Matrix-product states for one-dimensional systems

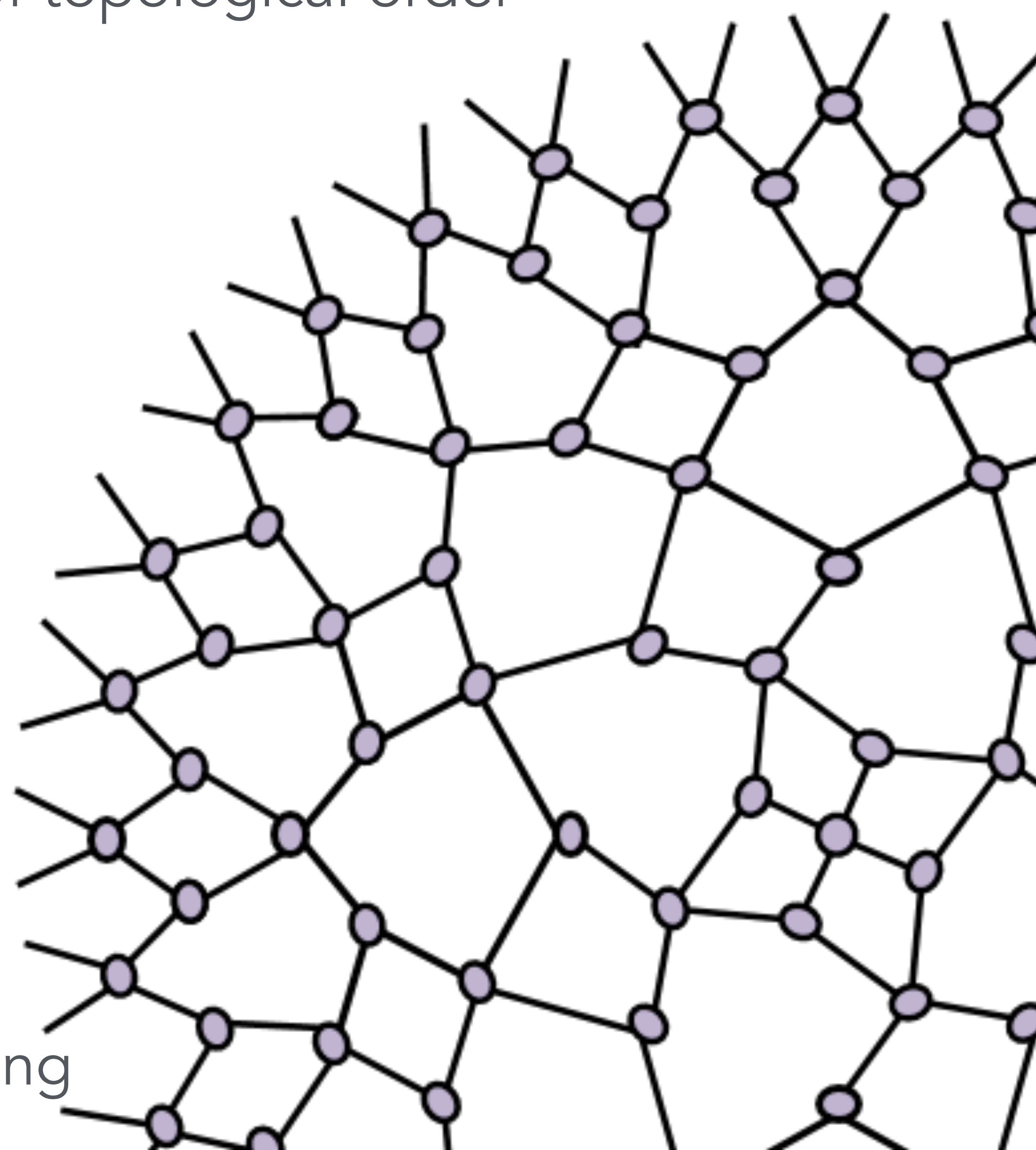


Matrix-product operators: thermal and open systems

Thanks for your attention!

Tensor network states

An entanglement based approach to numerical simulations of strongly correlated matter and analytical studies of topological order

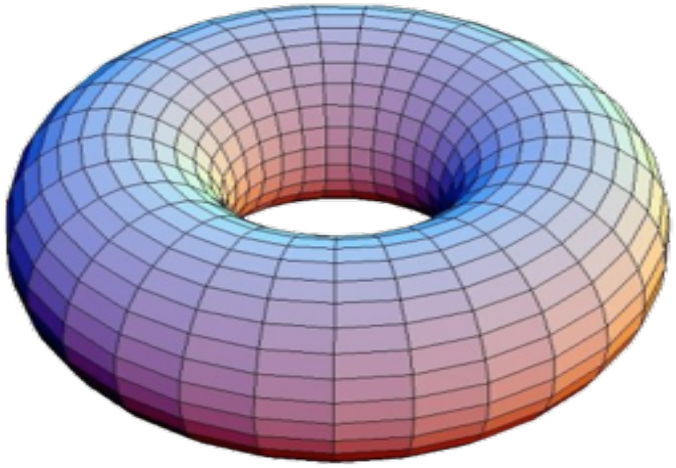


Jens Eisert, Freie Universität Berlin
The Capri Spring School 2017
Solid-state quantum information processing

- **This morning's message:** In 1D, MPS capture the "physical corner"

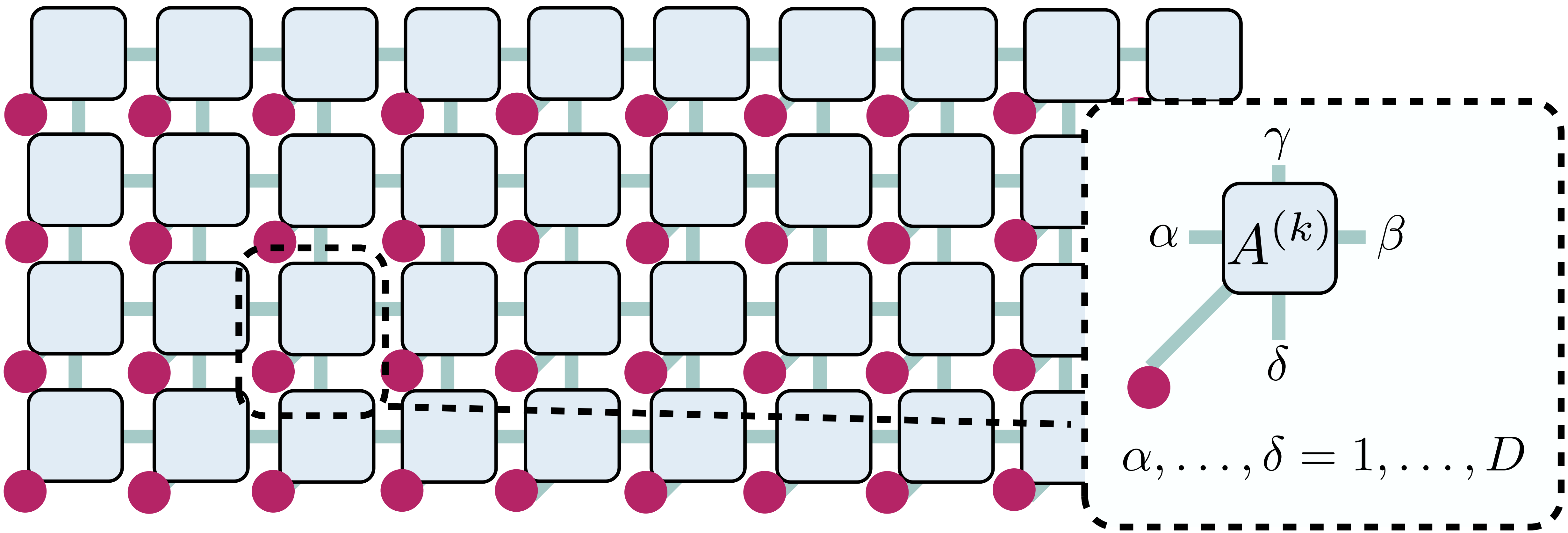
- **Now:** Asserted topics: 2D, quantum phases, MERA, quantum simulations

- **Tomorrow:** Topological order



Going higher-dimensional

Projected entangled pair states



- Idea for 2D systems: Projected entangled pair states (PEPS)
- Again, versatile numerical method

Verstraete, Cirac, cond-mat/0407066
Martin-Delgado, cond-mat/9610196

Projected entangled pair states

- PEPS still satisfy an area law for the **entanglement entropy** $S(\rho_A) = O(|\partial A|)$
- Note that the converse is strictly speaking not true

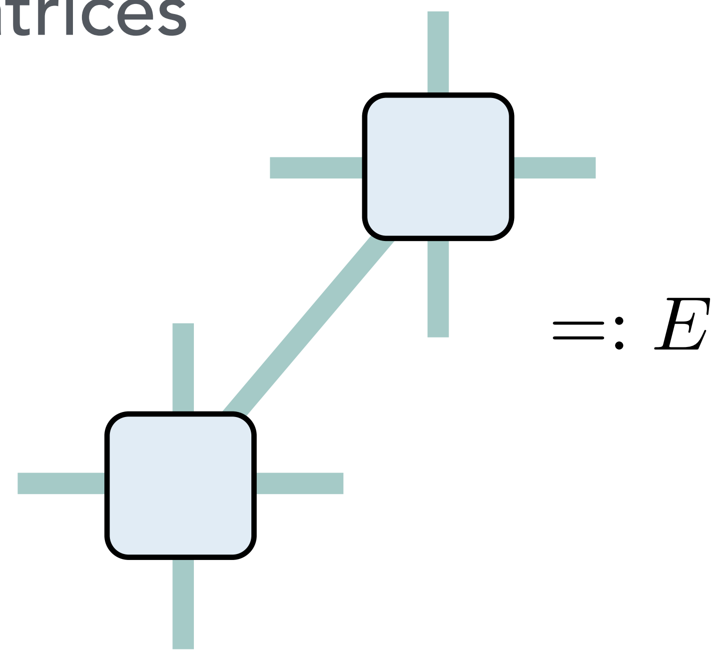
- **Theorem:** There exists translationally invariant states satisfying all area laws

$$S_\alpha(\rho_A) = O(|\partial A|)$$

for all $\alpha \geq 0$, yet they cannot be efficiently approximated to constant error in $\|.\|_1$-norm by any projected entangled pair state

Ge, Eisert, New J Phys 18, 083026 (2016)

- PEPS still satisfy an area law for the **entanglement entropy** $S(\rho_A) = O(|\partial A|)$
- PEPS still define **transfer matrices**



Projected entangled pair states

- PEPS still satisfy an area law for the **entanglement entropy** $S(\rho_A) = O(|\partial A|)$
- PEPS still define **transfer matrices**
- PEPS can have **algebraically** decaying correlations

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- PEPS still define **transfer matrices**
- PEPS can have **algebraically** decaying correlations
- PEPS can in practice be **efficiently contracted**
  - Cute twist:

> - **Theorem:** PEPS contraction is #P-complete
>
> Schuch, Wolf, Verstraete, Cirac, Phys Rev Lett 98, 140506 (2007)

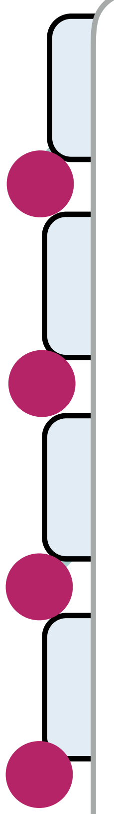
  - One cannot efficient compute expectation values in worst case complexity, which created a puzzle, but….

> - **Theorem:** PEPS that approximate ground states […] well, can be contracted in quasi-polynomial time
>
> Schwarz, Buerschaper, Eisert, arXiv:1606.06301

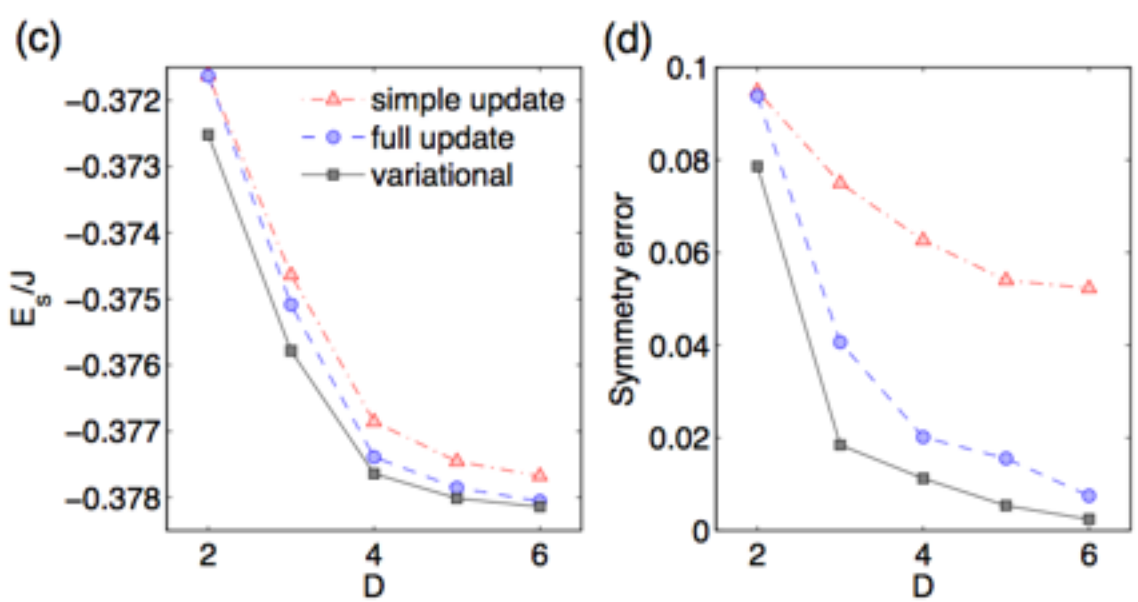
Projected entangled pair states

- PEPS still satisfy an area law for the **entanglement entropy** $S(\rho_A) = O(|\partial A|)$
- PEPS still define **transfer matrices**
- PEPS can have **algebraically** decaying correlations
- PEPS can in practice be **efficiently contracted**
- PEPS capture **topological order** and classify **phases of matter**

Projected entangled pair states

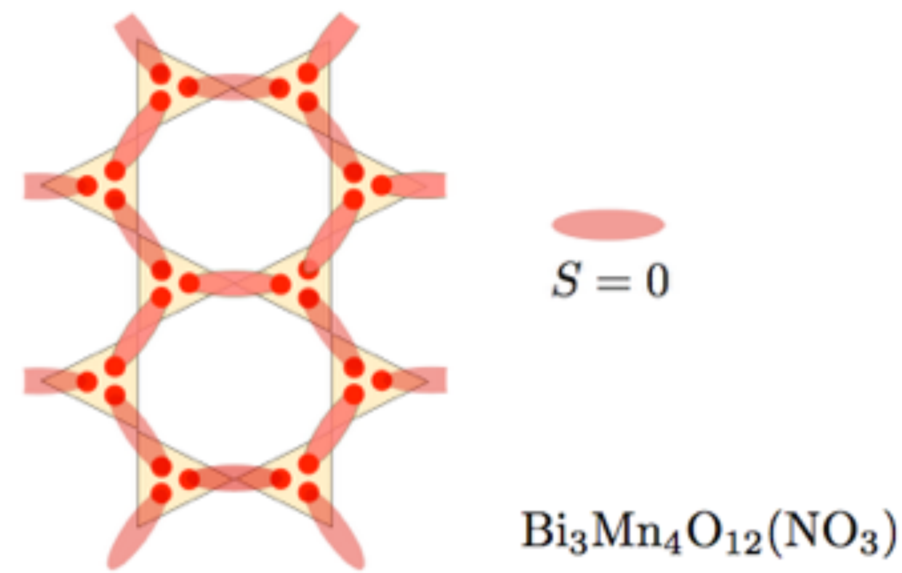


- Finite PEPS and iPEPS: Excellent numerical performance



Shustry-Sutherland model

Corboz, arXiv:1605.03006



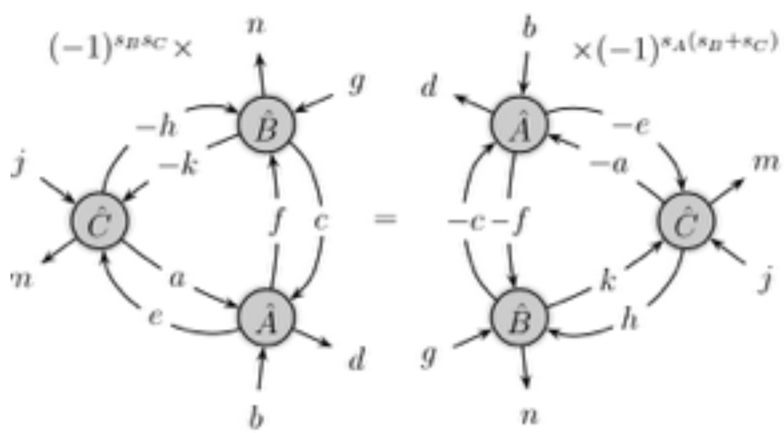
Spin-3/2 AKLT spin liquids

Lavoie at al, Nature Phys 6, 850 (2010)

Orus, Ann Phys 349, 17 (2014)

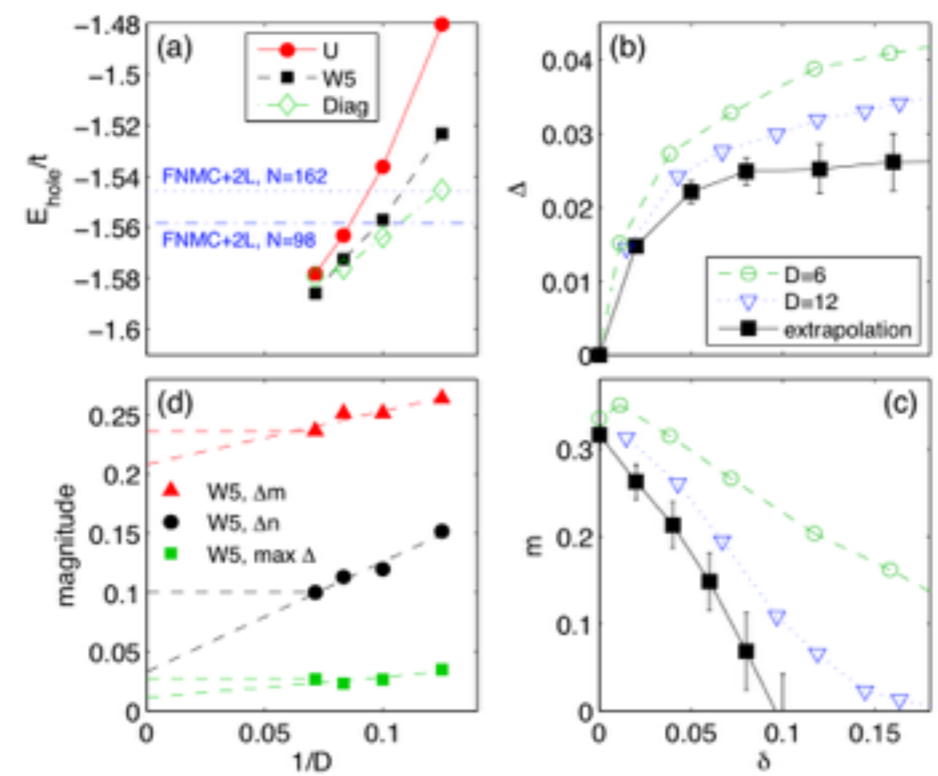
- Ide

- For fermions "sign-problem free"



Fermionic tensor networks

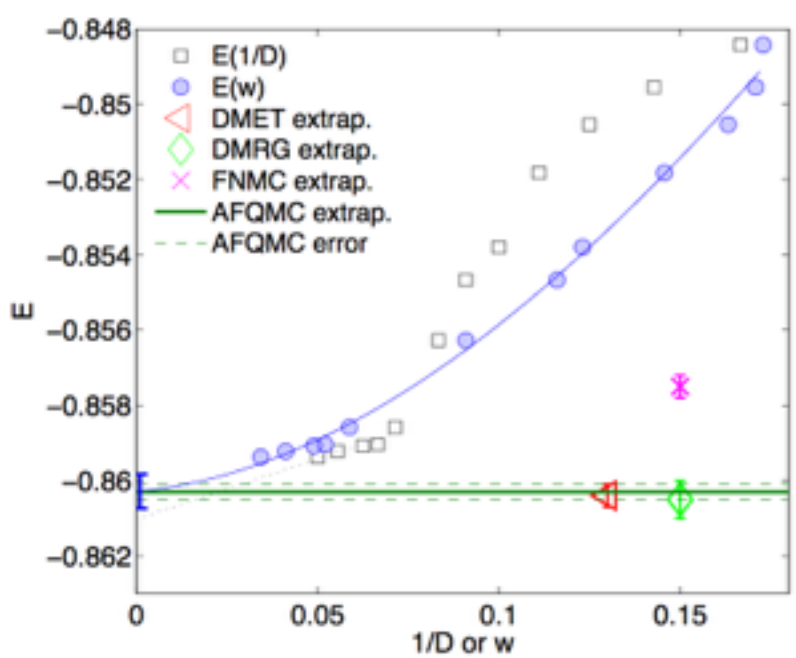
Pineda, Barthel, Eisert, Phys Rev A 81, 050303 (2010)
Corboz, Evenbly, Verstraete, Vidal, Phys Rev A 81, 010303 (2010)



Ground state energies
in the t-J model

Corboz, arXiv:1605.03006

- Ide



2D Hubbard model: In strongly correlated $U/t = 8,\ n = 0.875$ regime, iPEPS outperforms other state of the art algorithms in the ground state energy

Corboz, Phys Rev B 93, 045116 (2016)

The idea of a parent Hamiltonian

Aim of today

- Message from now on

  - The deliniation of the “physical corner”his allows for devising computational methods based on tensor networks.

Parent Hamiltonians

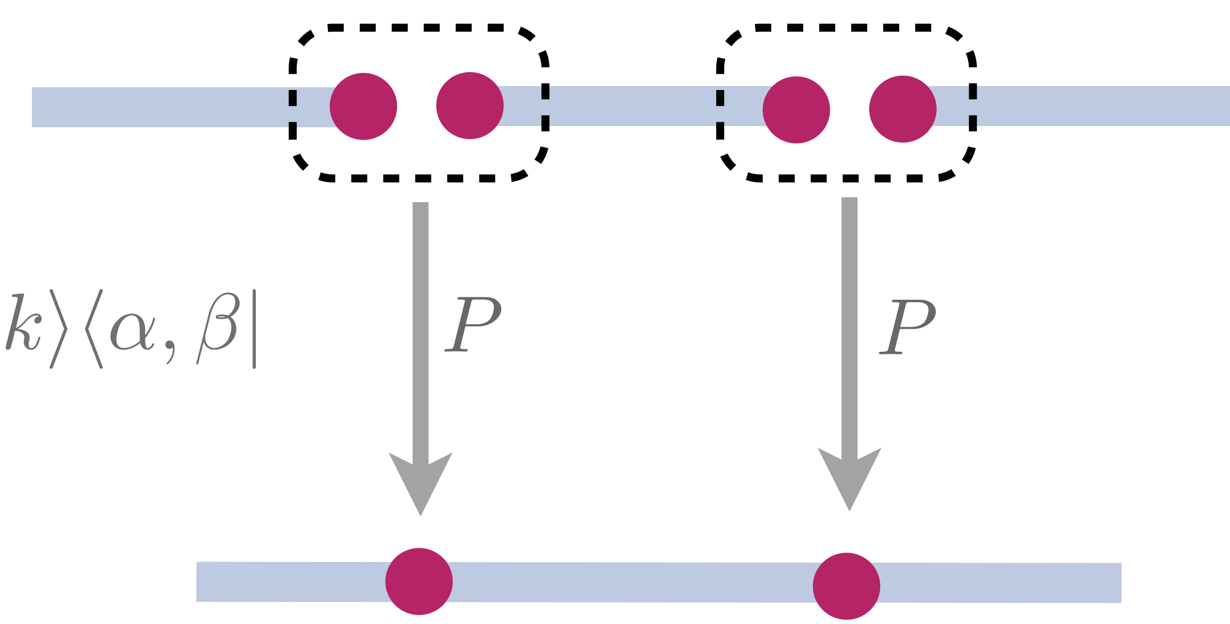
- Are there Hamiltonians that have exact MPS ground states?

Parent Hamiltonians

- Are there Hamiltonians that have exact MPS ground states?

  - As preparation: "PEPS projection"

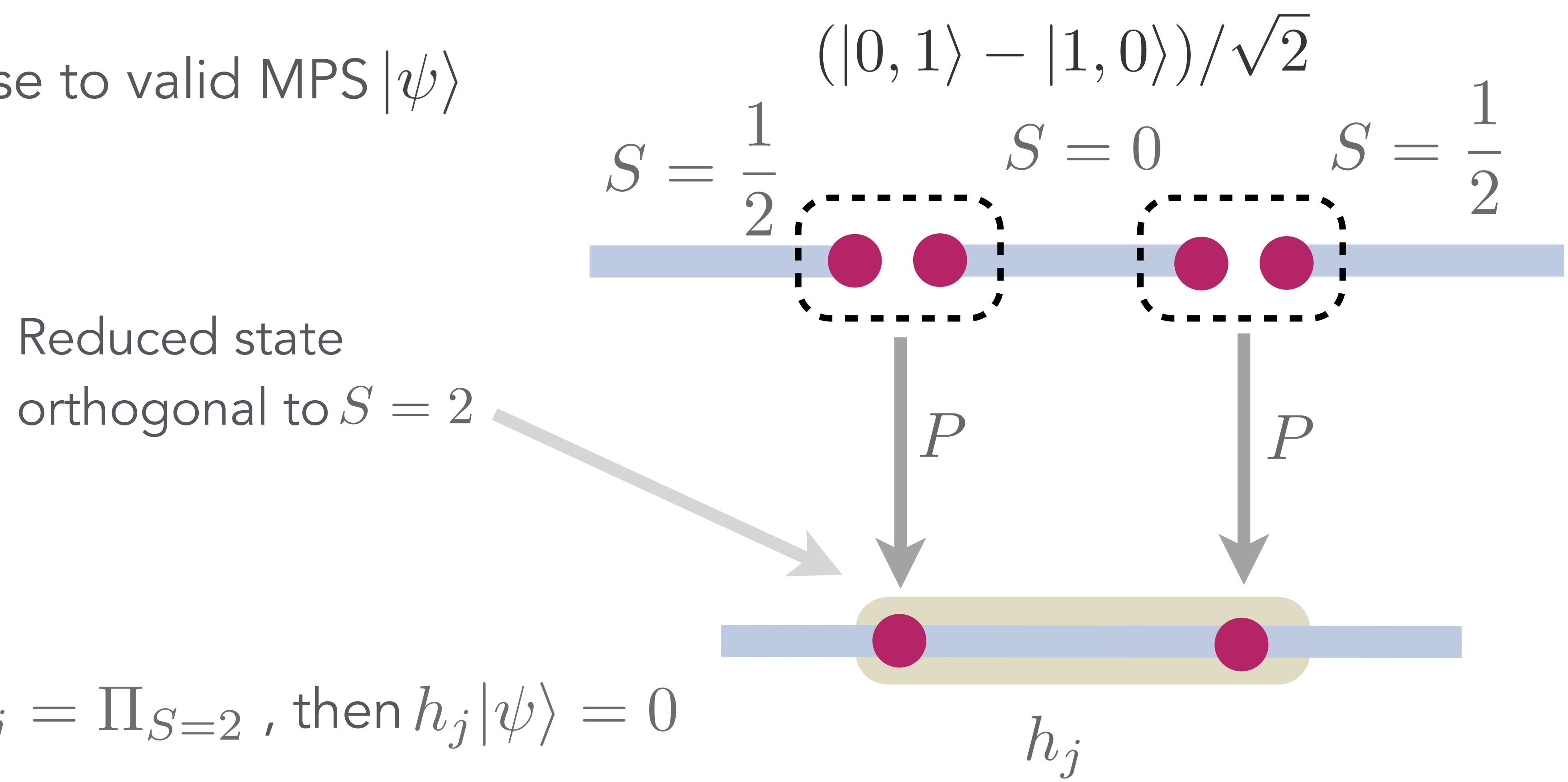
$$(|0,1\rangle - |1,0\rangle)/\sqrt{2}$$



$$P = \sum_{k=1}^{d} \sum_{\alpha,\beta=1}^{D} A_{\alpha,\beta;k} |k\rangle\langle\alpha,\beta|$$

- Are there Hamiltonians that have exact MPS ground states?

- Take physical dimension $d = 3$, a spin-1 model, and bond dimension $D = 2$
- In the PEPS picture take $P = \Pi_{S=1}(\mathbb{I} \otimes iY)$, where $\Pi_{S=1}$ is projection onto spin-1 subspace of two sites
- Gives rise to valid MPS $|\psi\rangle$

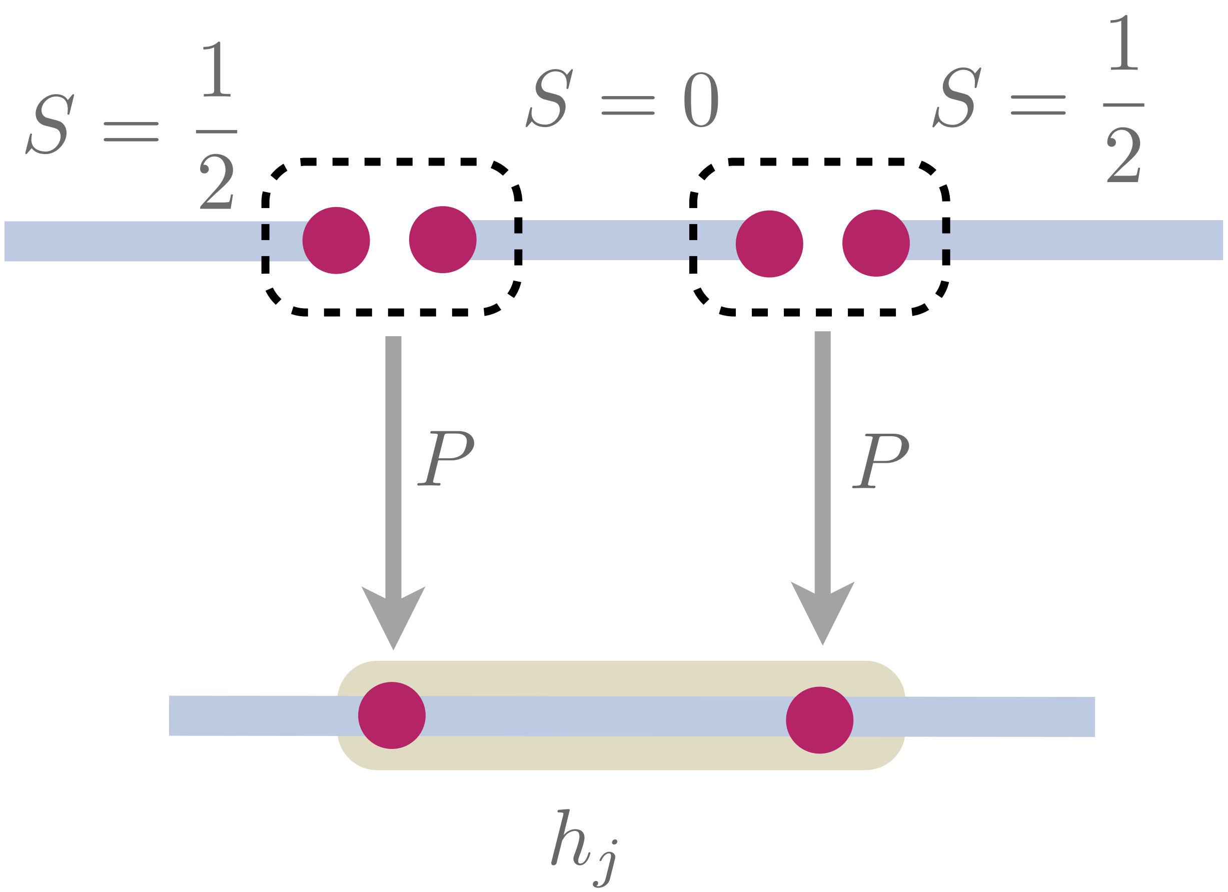


- Now $h_j = \Pi_{S=2}$, then $h_j|\psi\rangle = 0$

Affleck, Kennedy, Lieb, Tasaki, Phys Rev Lett 59, 799 (1987)

- Are there Hamiltonians that have exact MPS ground states?

  - But all h_j are positive, so $\langle\psi|H|\psi\rangle = \langle\psi|\sum_j h_j|\psi\rangle \geq 0$
  - That is, $|\psi\rangle$ must be a **ground state vector**



Affleck, Kennedy, Lieb, Tasaki, Phys Rev Lett 59, 799 (1987)

Parent Hamiltonians

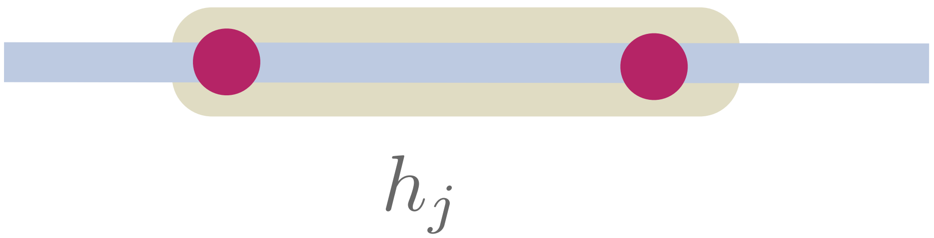
- Are there Hamiltonians that have exact MPS ground states?

  - But all h_j are positive, so $\langle\psi|H|\psi\rangle = \langle\psi|\sum_j h_j|\psi\rangle \geq 0$
  - That is, $|\psi\rangle$ must be a **ground state vector**

  - Famous AKLT (Affleck, Kennedy, Lieb, Tasaki) model

$$h_j = \frac{1}{2}S^{(j)} \cdot S^{(j+1)} + \frac{1}{6}(S^{(j)} \cdot S^{(j+1)})^2 + \frac{1}{3}$$

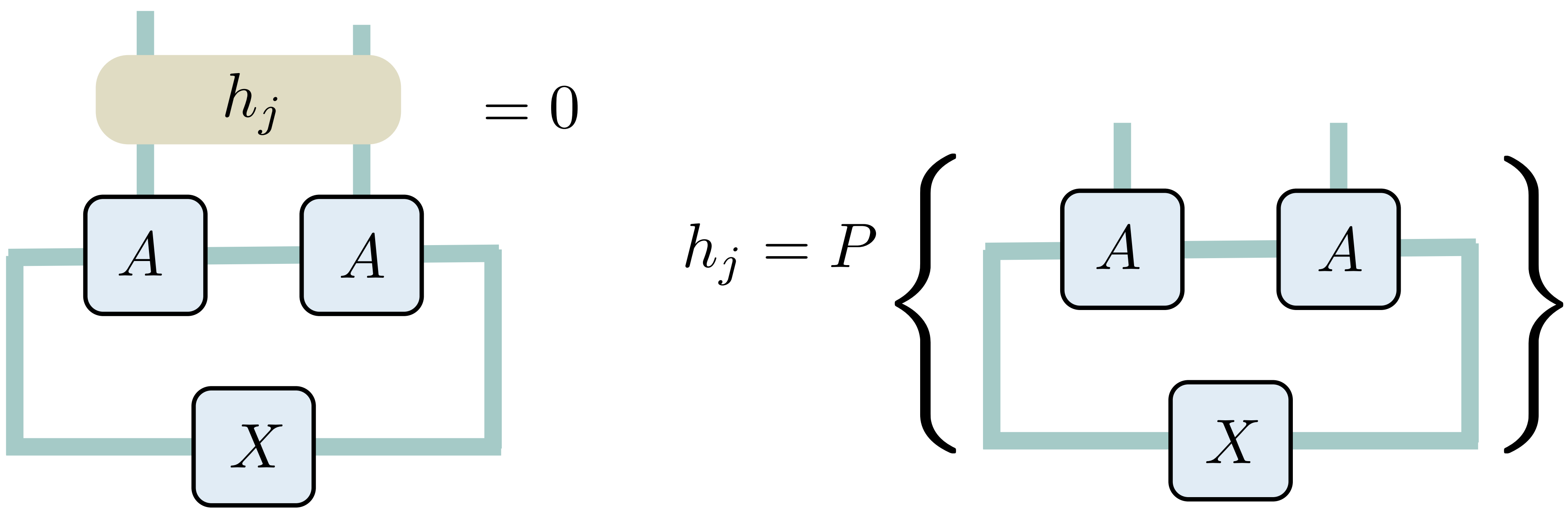
  - Resembles spin-1 Heisenberg model



Affleck, Kennedy, Lieb, Tasaki, Phys Rev Lett 59, 799 (1987)

Parent Hamiltonians

- Are there Hamiltonians that have exact MPS ground states?



- **Theorem:** All MPS and PEPS have frustration-free parent Hamiltonians

$$H = \sum_j h_j \,,\, h_j|\psi\rangle = 0$$

Injective tensor network states

- An MPS is injective if $P = \sum_{k=1}^{d} \sum_{\alpha,\beta=1}^{D} A_{\alpha,\beta;k} |k\rangle\langle\alpha,\beta|$ has a left inverse

- Intuitively, this means that we can achieve any action on the virtual indices by acting on the physical spins

- **Theorem:** All injective MPS and PEPS have frustration-free parents
$$H = \sum_j h_j \;,\; h_j|\psi\rangle = 0$$
to which they are the unique ground state

Injective tensor network states

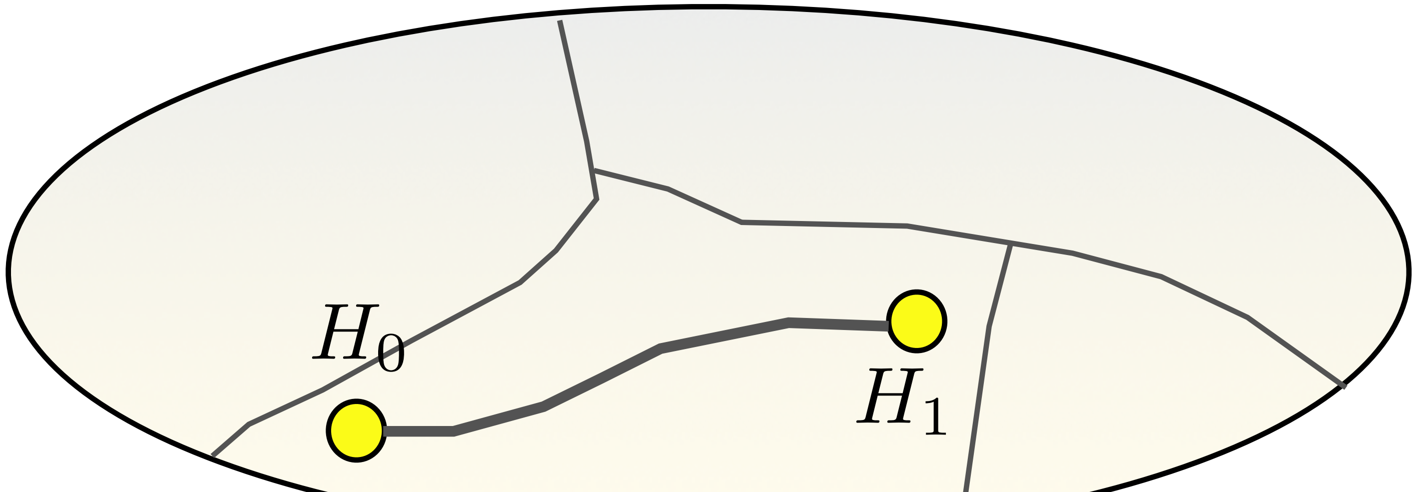
- Injective PEPS do not allow for degeneracies

- In the tensor network program, emphasis is on states, not Hamiltonians: The latter are reinserted by the concept of a parent Hamiltonian

Symmetries and phases of matter in 1D

Quantum phases

- The Hamiltonians H_0 and H_1 are in the same **phase**



- The Hamiltonians H_0 and H_1 are in the same **phase** if there ex. a k such that upon blocking of k sites, both H_0 and H_1 are two-local

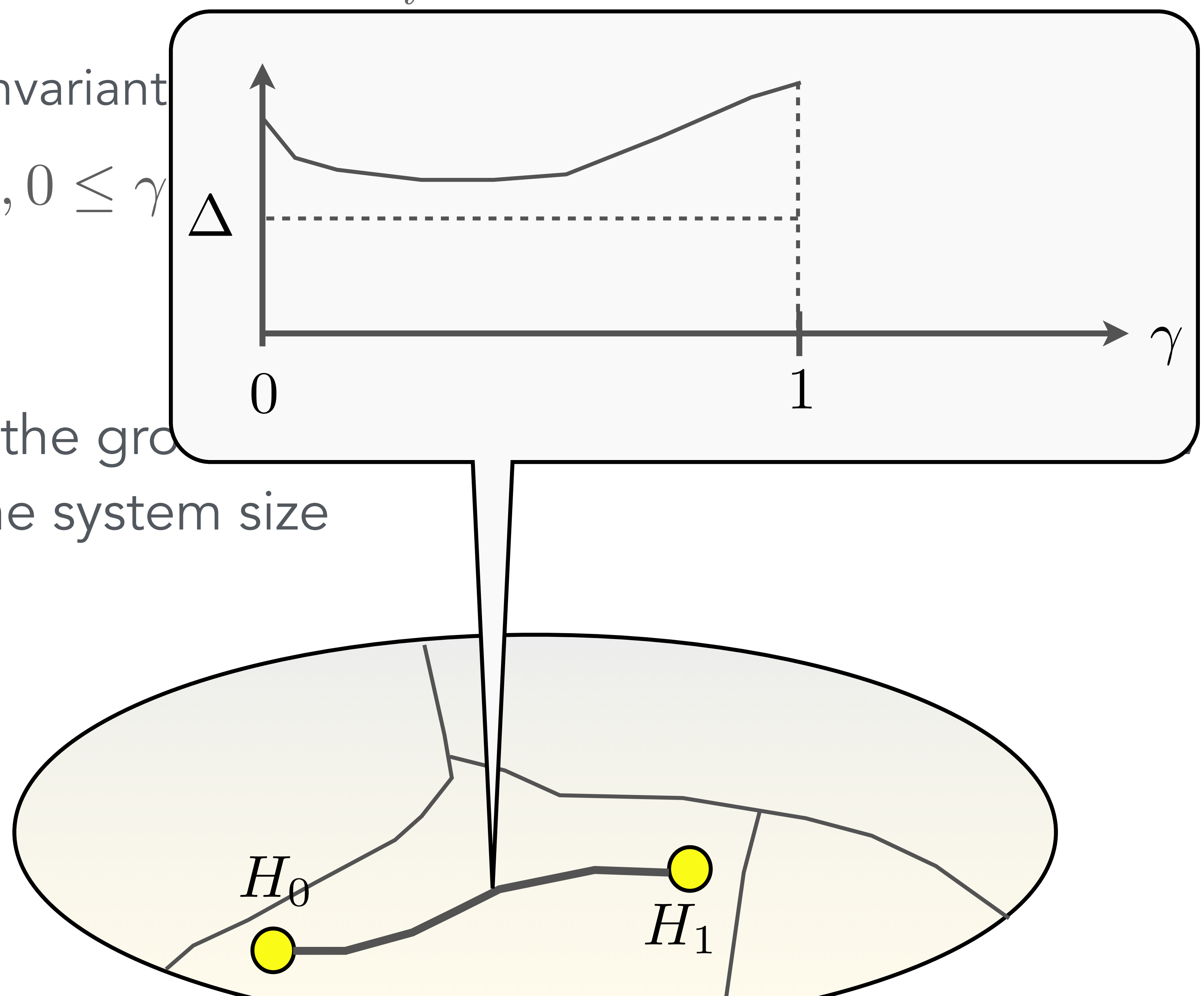
$$H_0 = \sum_i h_0(i, i+1) \qquad H_1 = \sum_i h_1(i, i+1)$$

- There exists a translationally invariant

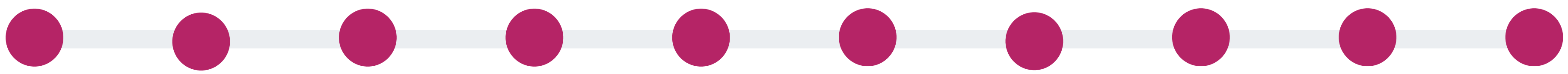
$$H_\gamma = \sum_i h_\gamma, 0 \leq \gamma$$

such that $||h_\gamma|| \leq 1$

- H_γ has a spectral gap above the gro
by $\Delta > 0$ independent of the system size



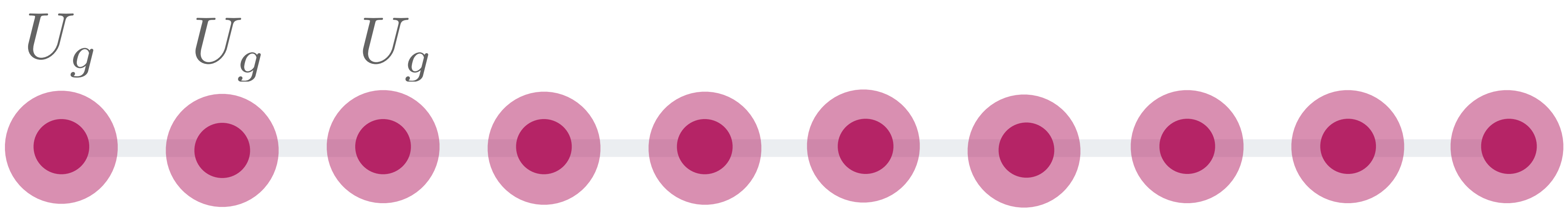
Quantum phases with no symmetries



- Phases in 1D are defined in terms of **ground state degeneracy**
- Specifically, every ground state of a non-degenerate Hamiltonian is in **same phase as trivial product state**

Schuch, Perez-Garcia, Cirac, Phys Rev B 84, 165139 (2011)
Pollmann, Turner, Berg, Oshikawa, Phys Rev B 81, 064439 (2010)
Chen, Gu, Wen, Phys Rev B 83, 035107 (2011)
Nietner, Krumnow, Bergholtz, Eisert, arXiv:1704.02992

Phases with symmetries

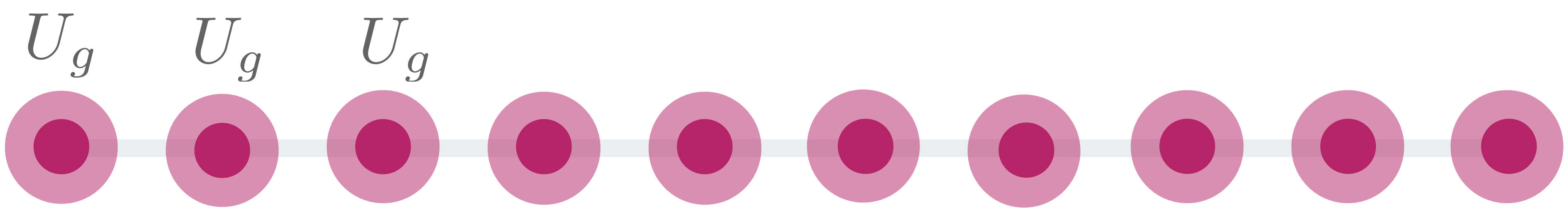


- A Hamiltonian acting on $\mathcal{H}^{\otimes n}$ has a local symmetry if there is a linear unitary representation U_g of some group $G \ni g$ acting on $\mathcal{H}$ such that

$$[H, U_g^{\otimes n}] = 0$$

Schuch, Perez-Garcia, Cirac, Phys Rev B 84, 165139 (2011)
Pollmann, Turner, Berg, Oshikawa, Phys Rev B 81, 064439 (2010)
Chen, Gu, Wen, Phys Rev B 83, 035107 (2011)
Nietner, Krumnow, Bergholtz, Eisert, arXiv:1704.02992

Phases with symmetries



- **Roughly:** Being in the same phase while preserving the local symmetry

- **Precisely:** H_0 and H_1 are in the same phase under the symmetry G if there ex. a phase gauge for U_g^0 and U_g^1 and a representation of G

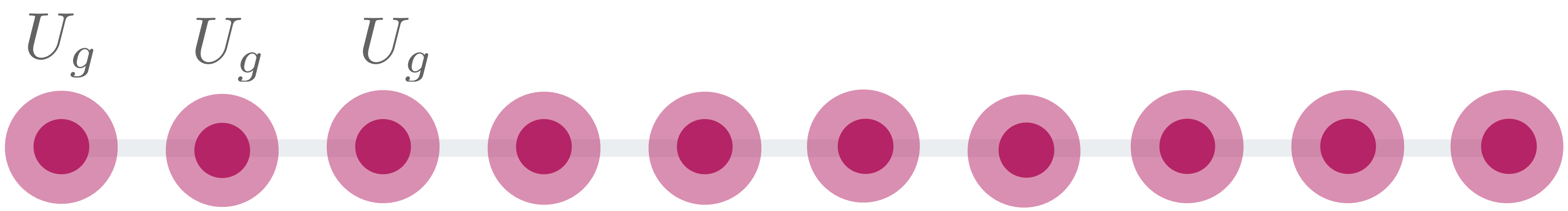
$$U_g = U_g^0 \oplus U_g^1 \oplus U_g^{\text{path}}$$

defined on $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}^{\text{path}}$ and an interpolating path $\gamma \mapsto H_\gamma$ with the above properties, such that $[H_\gamma, U_g^{\otimes n}] = 0$

and H_0 and H_1 are supported on $\mathcal{H}_0^{\otimes n}$ and $\mathcal{H}_1^{\otimes n}$, respectively

Schuch, Perez-Garcia, Cirac, Phys Rev B 84, 165139 (2011)
Pollmann, Turner, Berg, Oshikawa, Phys Rev B 81, 064439 (2010)
Chen, Gu, Wen, Phys Rev B 83, 035107 (2011)
Nietner, Krumnow, Bergholtz, Eisert, arXiv:1704.02992

Phases with symmetries

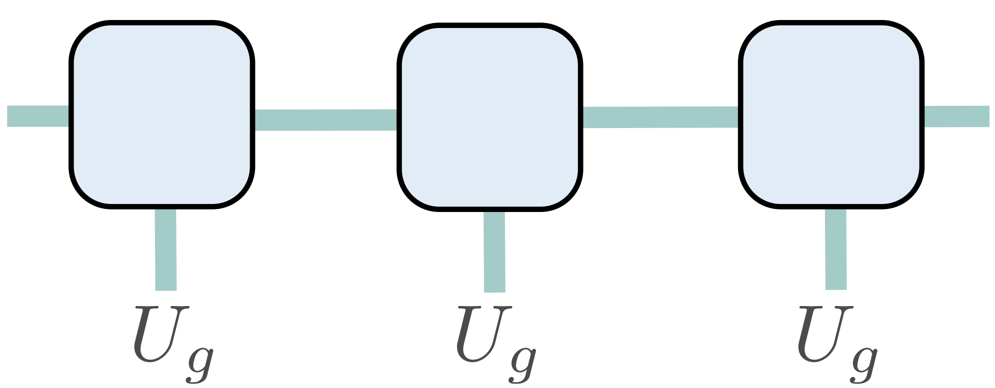


- Symmetry of state vectors

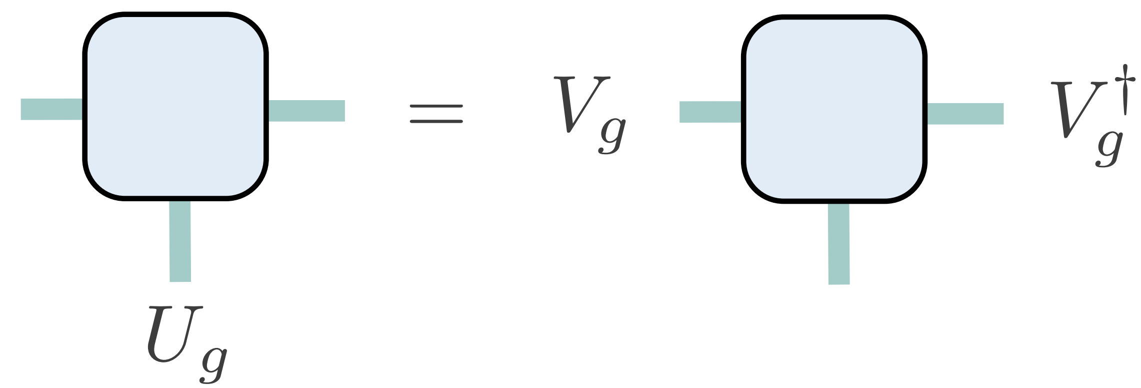
$$U_g \otimes \cdots \otimes U_g |\psi\rangle = e^{i\phi_g n} |\psi\rangle$$

- **Linear** representation

$$U_g U_h = U_{gh}$$

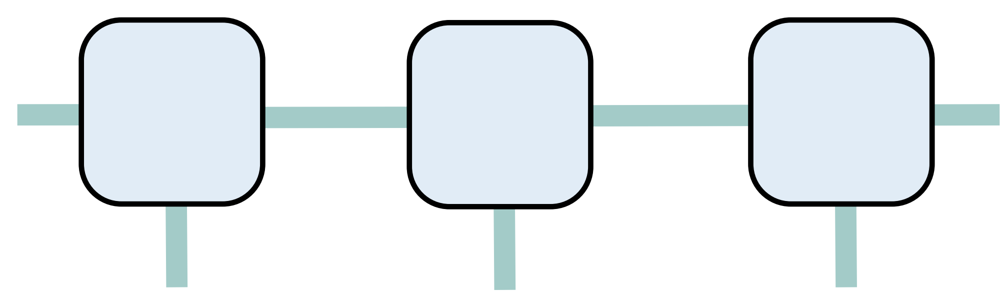


- **Symmetry of MPS** (exists standard form s.t.)



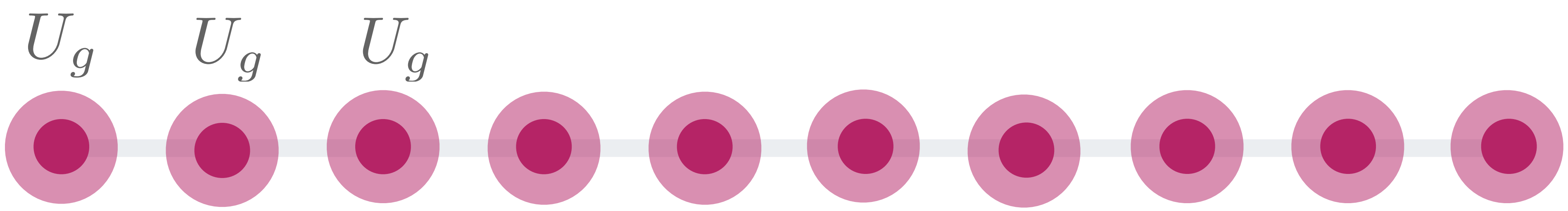
- V_g **projective** unitary representation of G

$$V_g V_h = e^{i\omega(g,h)} V_{gh}$$



Schuch, Perez-Garcia, Cirac, Phys Rev B 84, 165139 (2011)
Pollmann, Turner, Berg, Oshikawa, Phys Rev B 81, 064439 (2010)
Chen, Gu, Wen, Phys Rev B 83, 035107 (2011)
Nietner, Krumnow, Bergholtz, Eisert, arXiv:1704.02992

Phases with symmetries



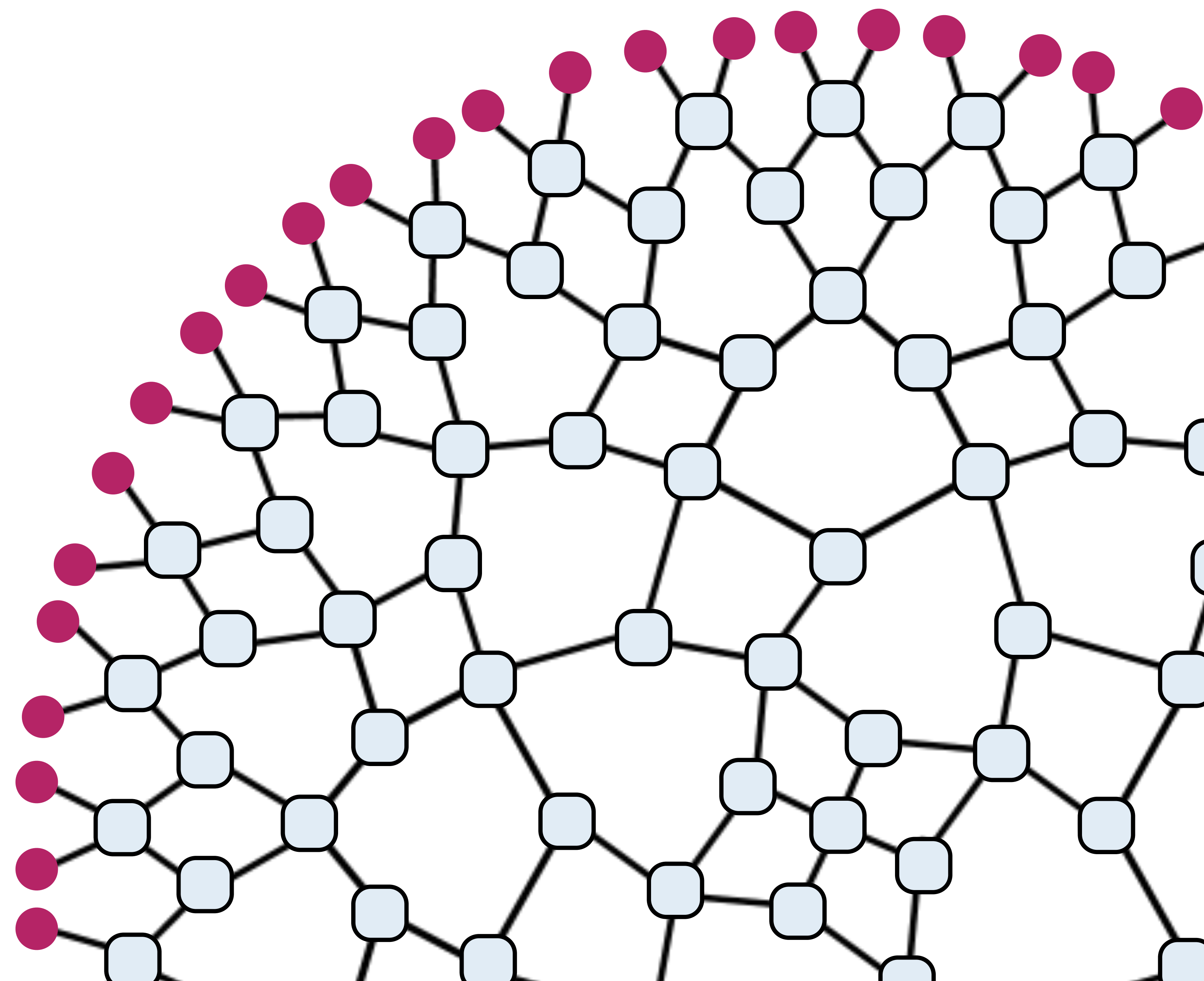
- **Non-degenerate ground states:**
- Phases are defined in terms of **2nd cohomology classes** of the projective representations of the group G

- **Degenerate ground states:**
- Phases are defined in terms of **2nd cohomology classes** of the induced projective representations of the group G

Schuch, Perez-Garcia, Cirac, Phys Rev B 84, 165139 (2011)
Pollmann, Turner, Berg, Oshikawa, Phys Rev B 81, 064439 (2010)
Chen, Gu, Wen, Phys Rev B 83, 035107 (2011)
Nietner, Krumnow, Bergholtz, Eisert, arXiv:1704.02992

MERA and the AdS-cft correspondence

- More elaborate tensor networks capture critical quantum systems

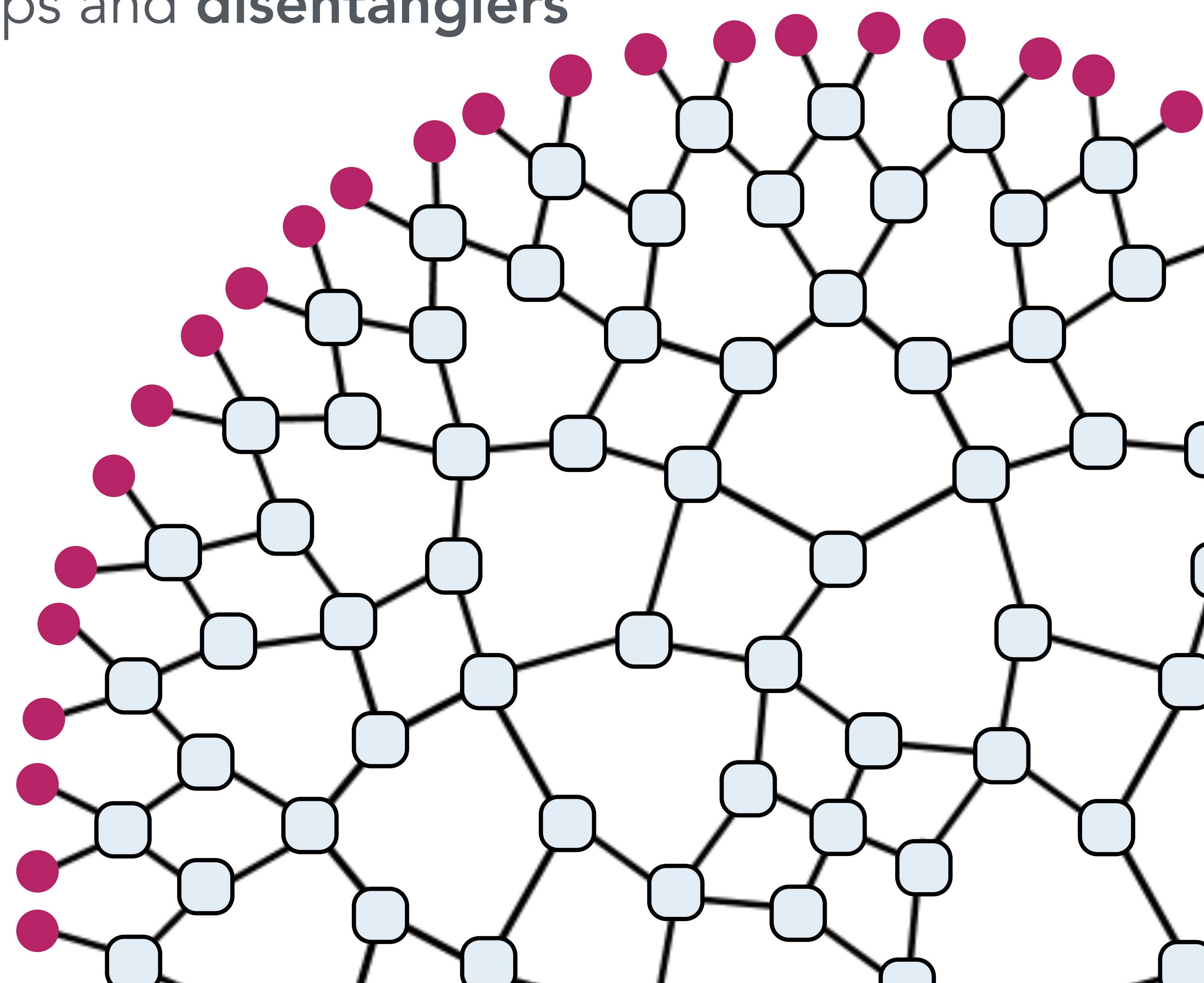


- More elaborate tensor networks capture critical quantum systems

- "Multi-scale entanglement renormalisation"

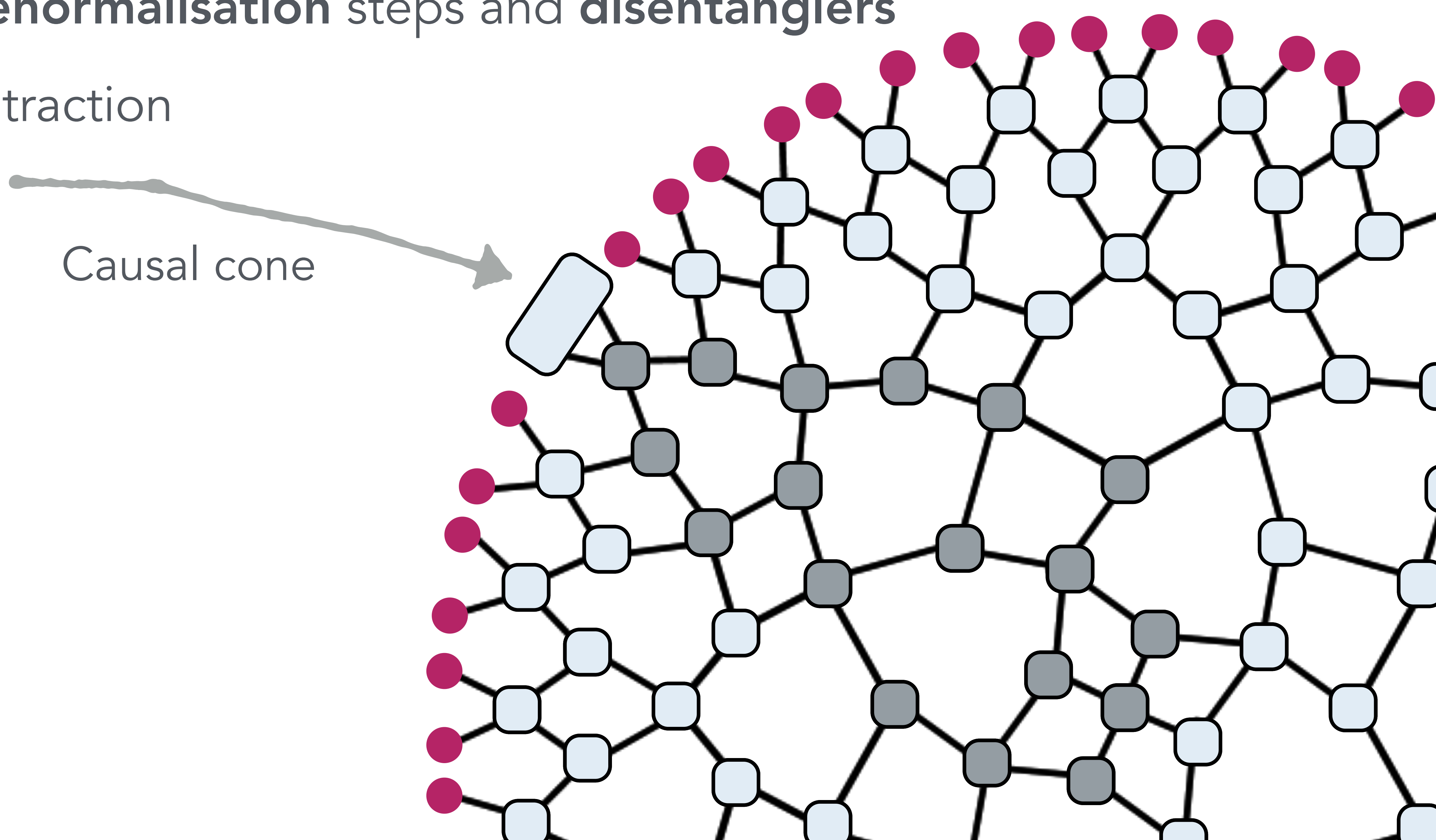
- Interlaced **renormalisation** steps and **disentanglers**

Vidal, Phys Rev Lett 101, 110501 (2008)



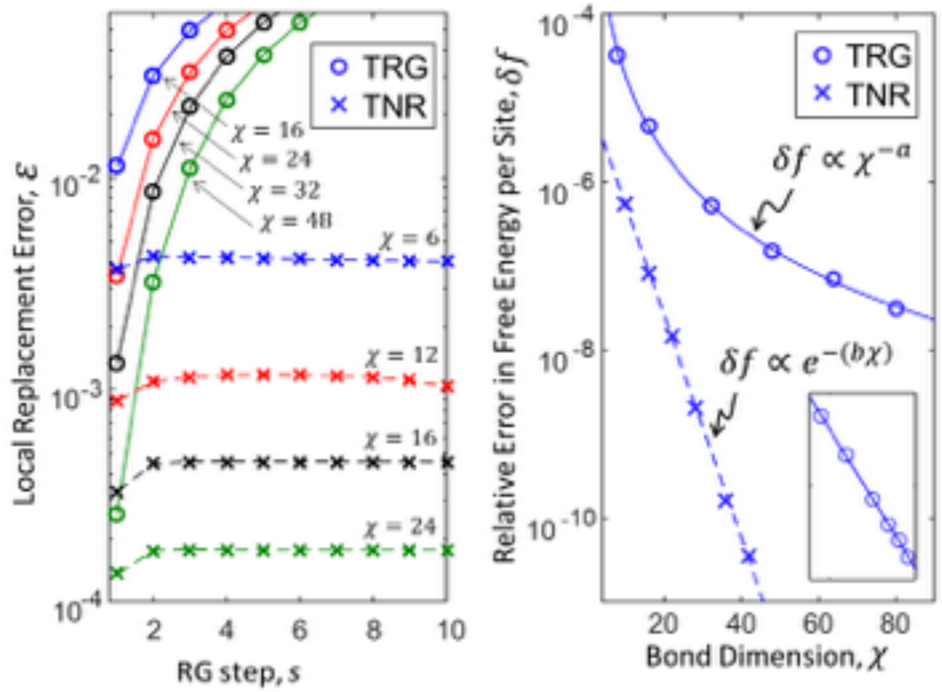
- More elaborate tensor networks capture critical quantum systems

- "Multi-scale entanglement renormalisation"
- Interlaced **renormalisation** steps and **disentanglers**
- Efficient contraction



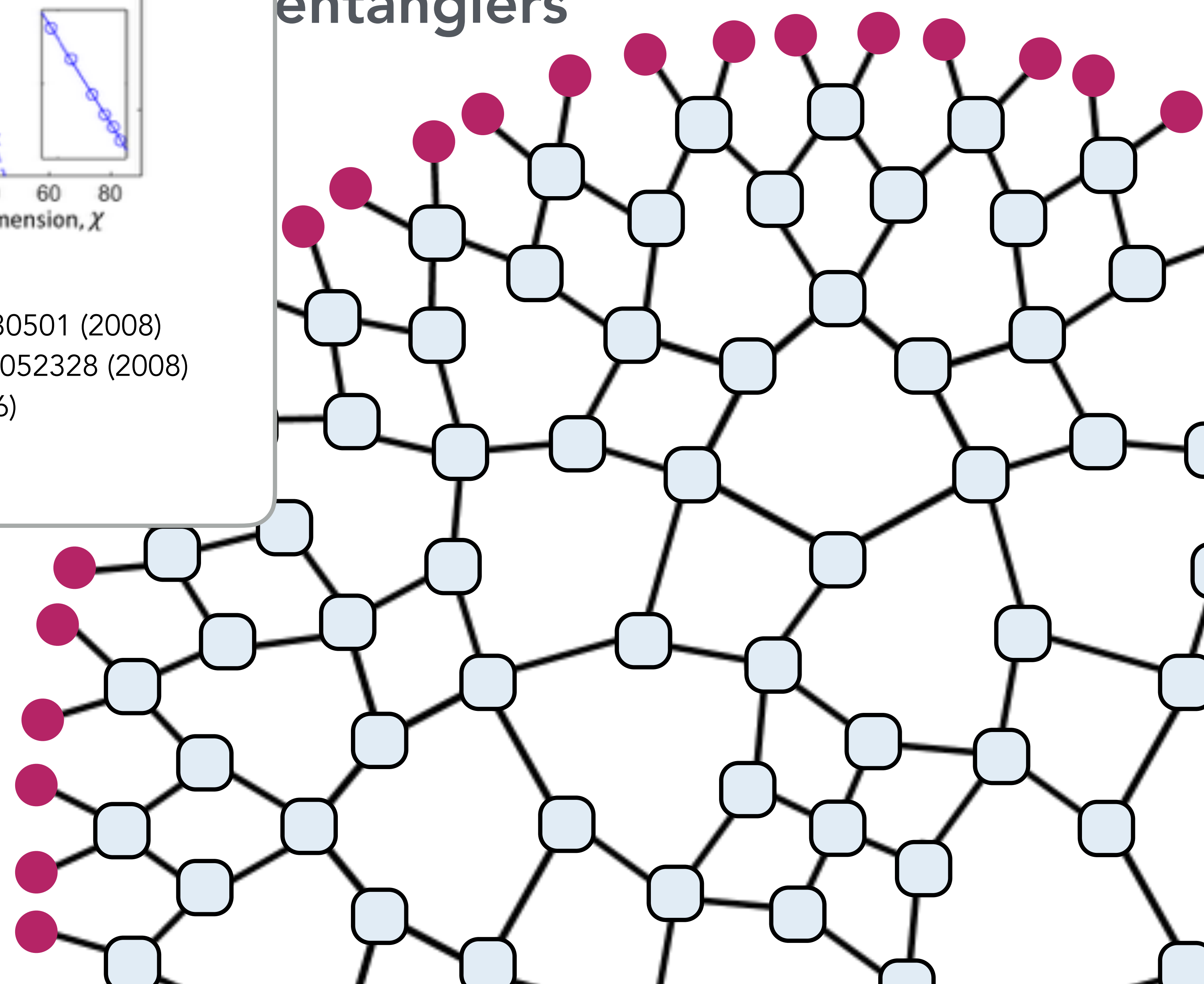
Multi-scale entanglement renormalisation

- MERA performance



Dawson, Eisert, Osborne, Phys Rev Lett 100, 130501 (2008)
Vidal, Rizzi, Montangero, Vidal, Phys Rev A 77, 052328 (2008)
Evenbly, Vidal, Phys Rev Lett 116, 040401 (2016)
Glen Evenbly, Phys Rev B 95, 045117 (2017)

- More ... critical quantum systems
- “Mul ... on”
- Interl ... **entanglers**
- Effici



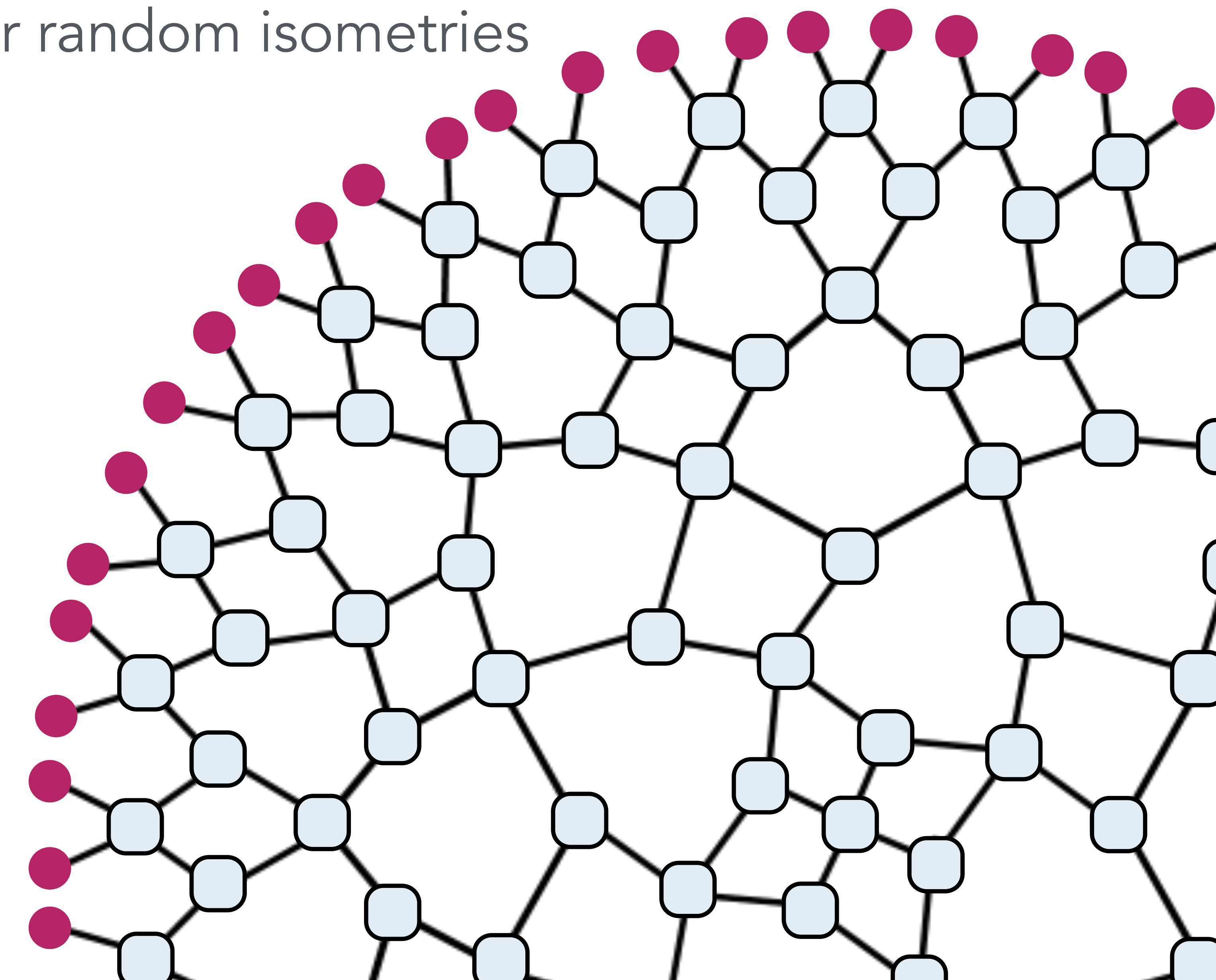
- More elaborate tensor networks capture critical quantum systems

- Seen as toy model for AdS-cft correspondence

Nozaki, Ryu, Takayanagi, JHEP10, 193 (2012)

- Random tensor networks: Haar random isometries

Qi, Yang, You, arXiv:1703.06533

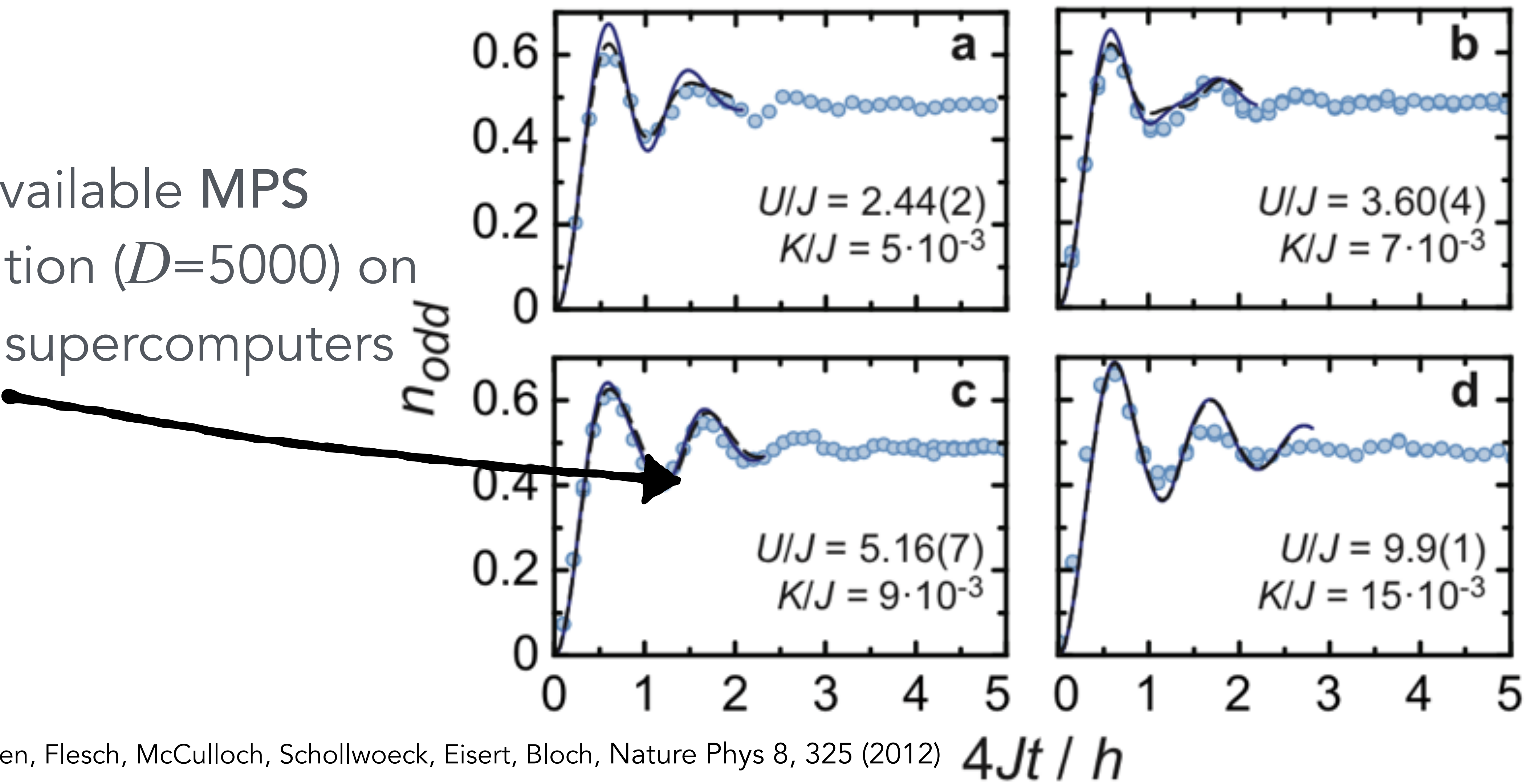


Three slides on quantum simulations (hommage to Enrique)

- Think of **quantum simulators** outperforming classical supercomputers

- Quenched cold atoms **many-body dynamics** outperforms classical supercomputers (10.000 atoms)

Best available **MPS** simulation (D=5000) on Jülich supercomputers



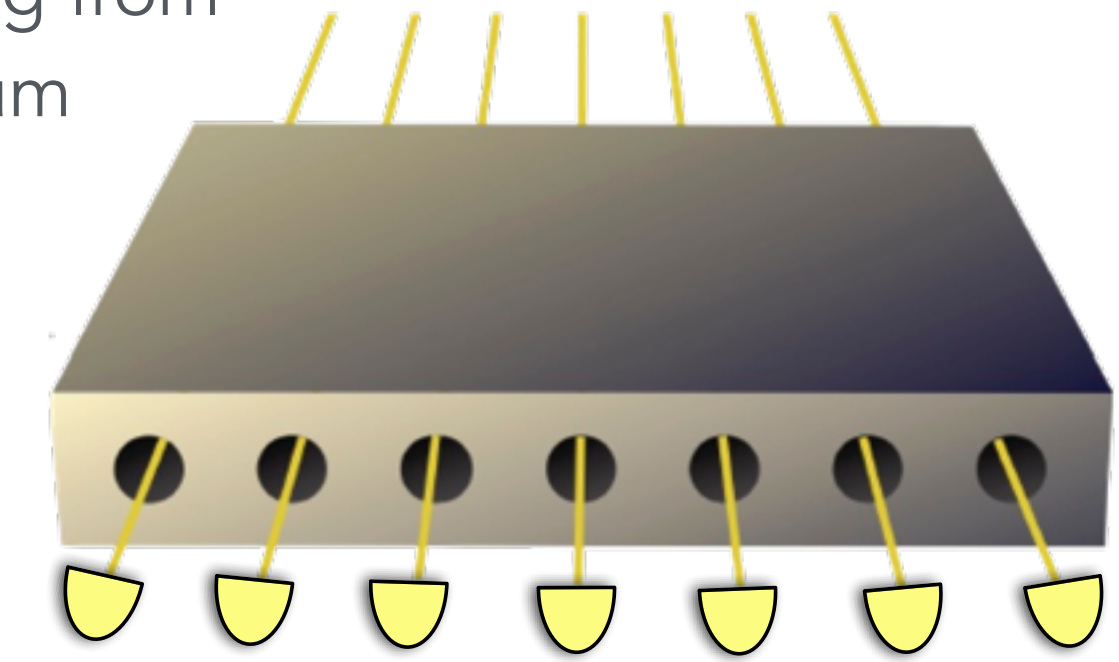
Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, Nature Phys 8, 325 (2012)

- **Devil's advocate:** But maybe there is a simple description?

- Think of **quantum simulators** outperforming classical supercomputers

- **Boson sampling with photons:** Sampling from a distribution close in 1-norm to quantum distribution is computationally hard

Aaronson, Arkhipov, Proceedings of ACM Symposium on the Theory of Computing, STOC (2011)

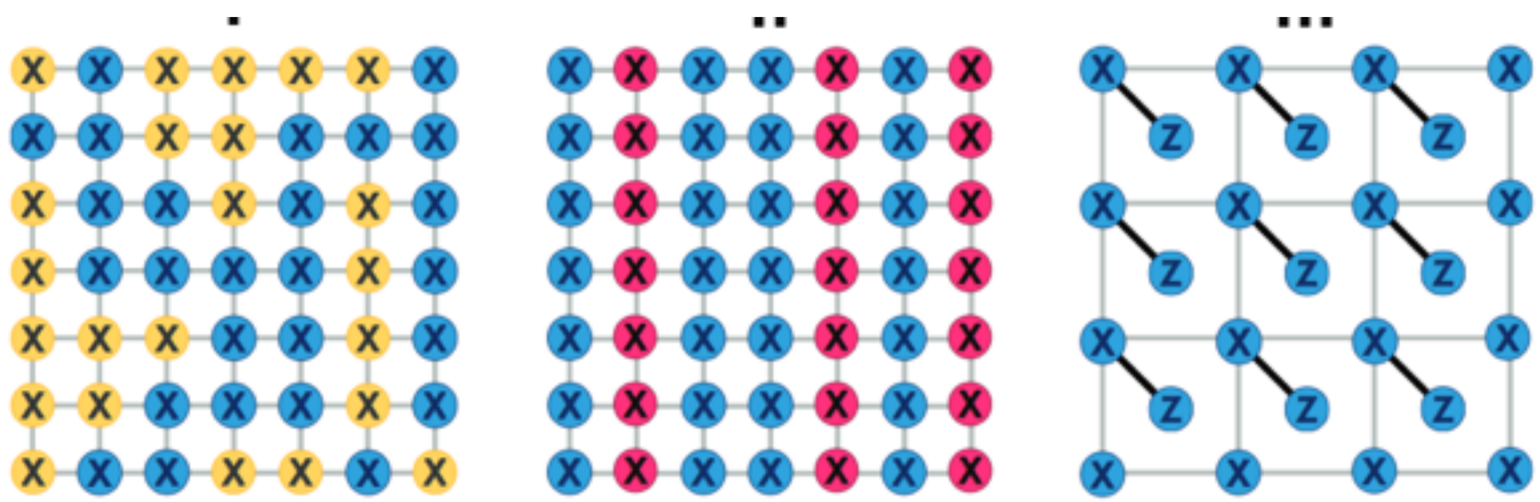


- Output cannot be distinguished from efficiently preparable distribution

Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Trevisan, Tulsiani, Vadhan, Proc IEEE Conf Comp Complex, 126 (2009)

- **Common prejudice: In order to verify a quantum simulation, one has to be able to classically keep track of it**

- Think of **quantum simulators** outperforming classical supercomputers



- (i) With disordered **initial state**, quenched **Ising dynamics**
- (ii) **non-adaptive local measurements** (50x50 lattice)

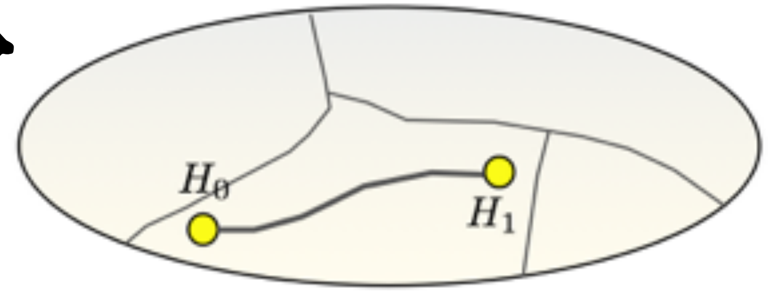
one can sample from **IQP circuits ("hard problem")**, but now one can also

- (iii) efficiently certify correctness of prepared state (PEPS)

Bermejo-Vega, Hangleiter, Schwarz, Raussendorf, Eisert, arXiv:1703.00466
Compare Boixo, Isakov, Smelyanskiy, Babbush, Ding, Jiang, Martinis, Neven, arXiv:1608.00263.

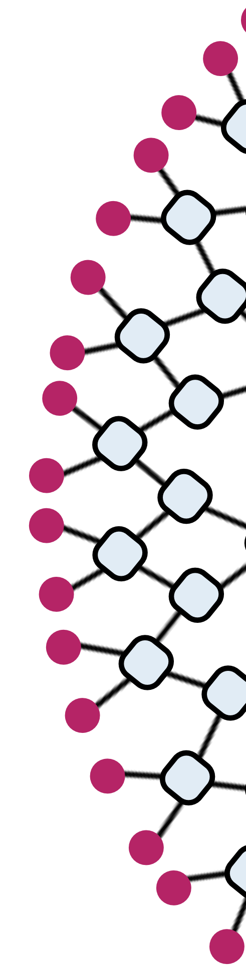
- **The correctness of quantum simulations can sometimes be certified, even if one cannot predict the outcome!**

Shifting emphasis from Hamiltonians to states

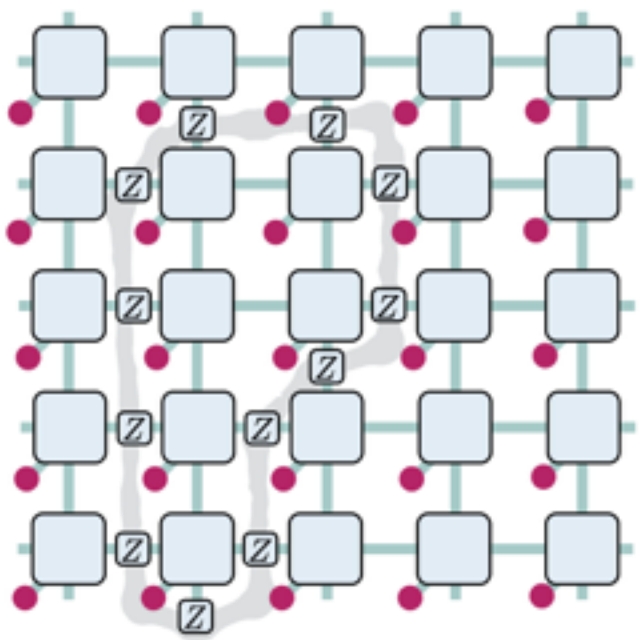


Classifying phases of matter

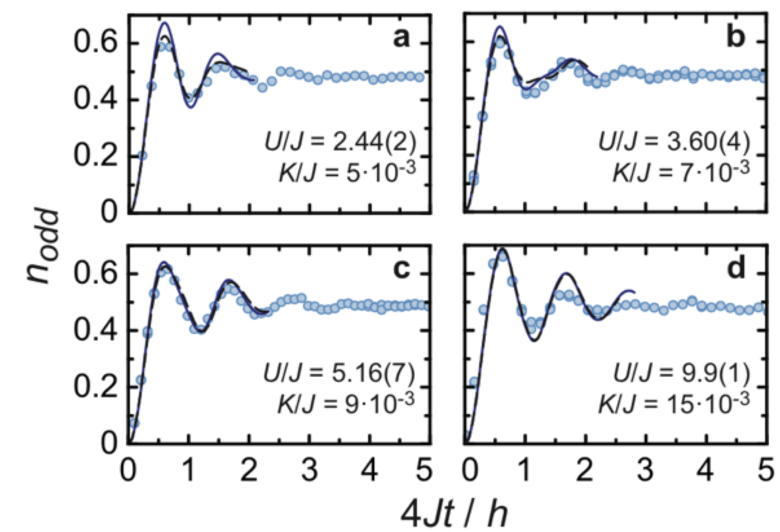
MERA



Sneak preview:



Capturing topological order

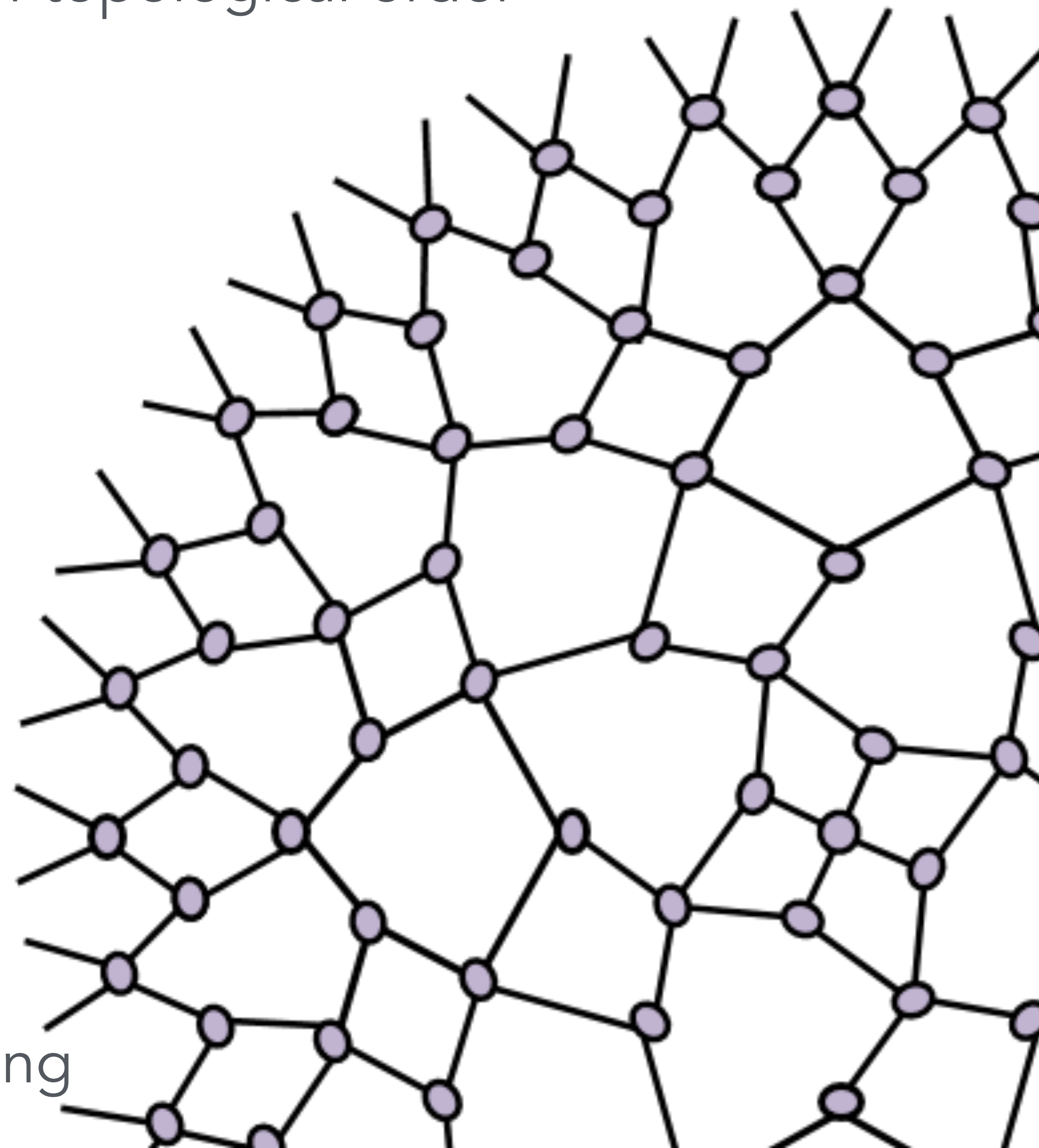


Quantum simulations

Thanks for your attention!

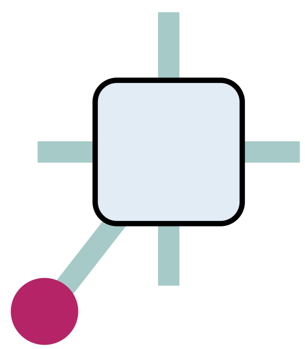
Tensor network states

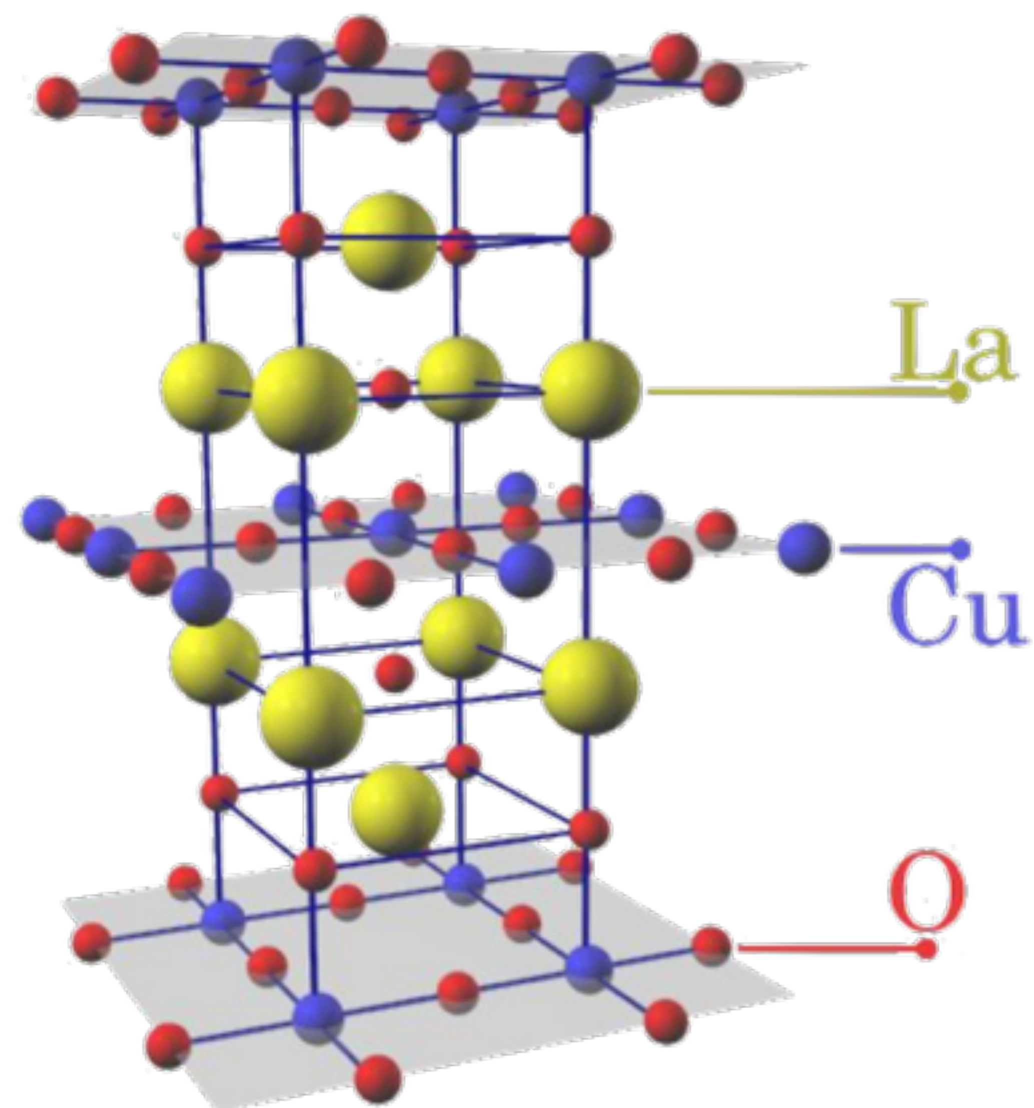
An entanglement based approach to numerical simulations of strongly correlated matter and analytical studies of topological order



Jens Eisert, Freie Universität Berlin
The Capri Spring School 2017
Solid-state quantum information processing

- Natural ground states of quantum many-body systems are very little entangled in a precise sense. This allows for computational methods based on tensor networks as well as new ways for their mathematical study."

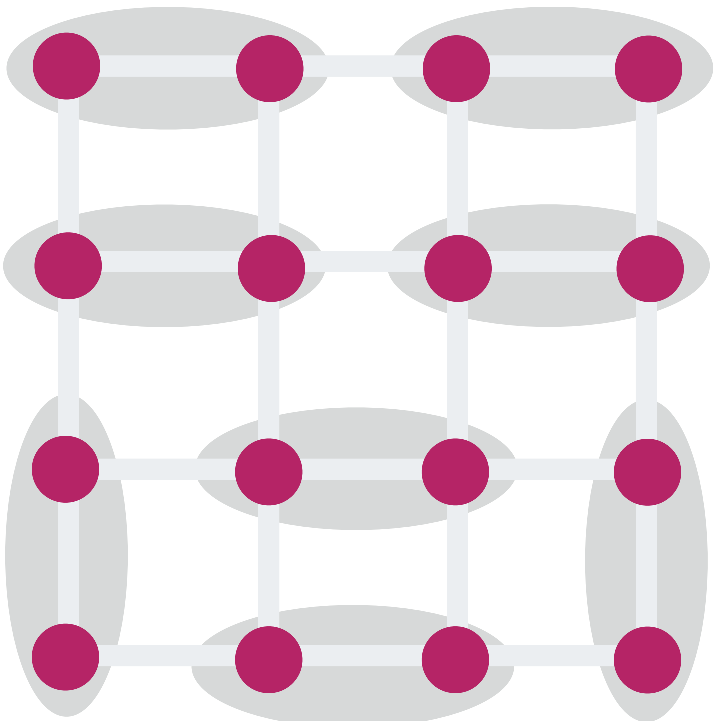




Topological order

A new type of order

- Fractional quantum Hall effect, spin liquids etc, see Alex's talk
- **A new kind of order: Topological order**
- Resonating valence bond states (RVB), quantum dimer models (QDM)



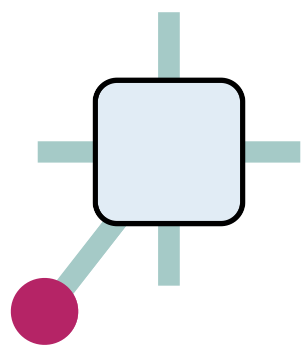
$(|0,1\rangle - |1,0\rangle)/\sqrt{2}$

Singlet

- Configuration: covering of the lattice

A new type of order

- Fractional quantum Hall effect, spin liquids etc, see Alex's talk
- **A new kind of order: Topological order**



A new type of order

- Definition of topological order

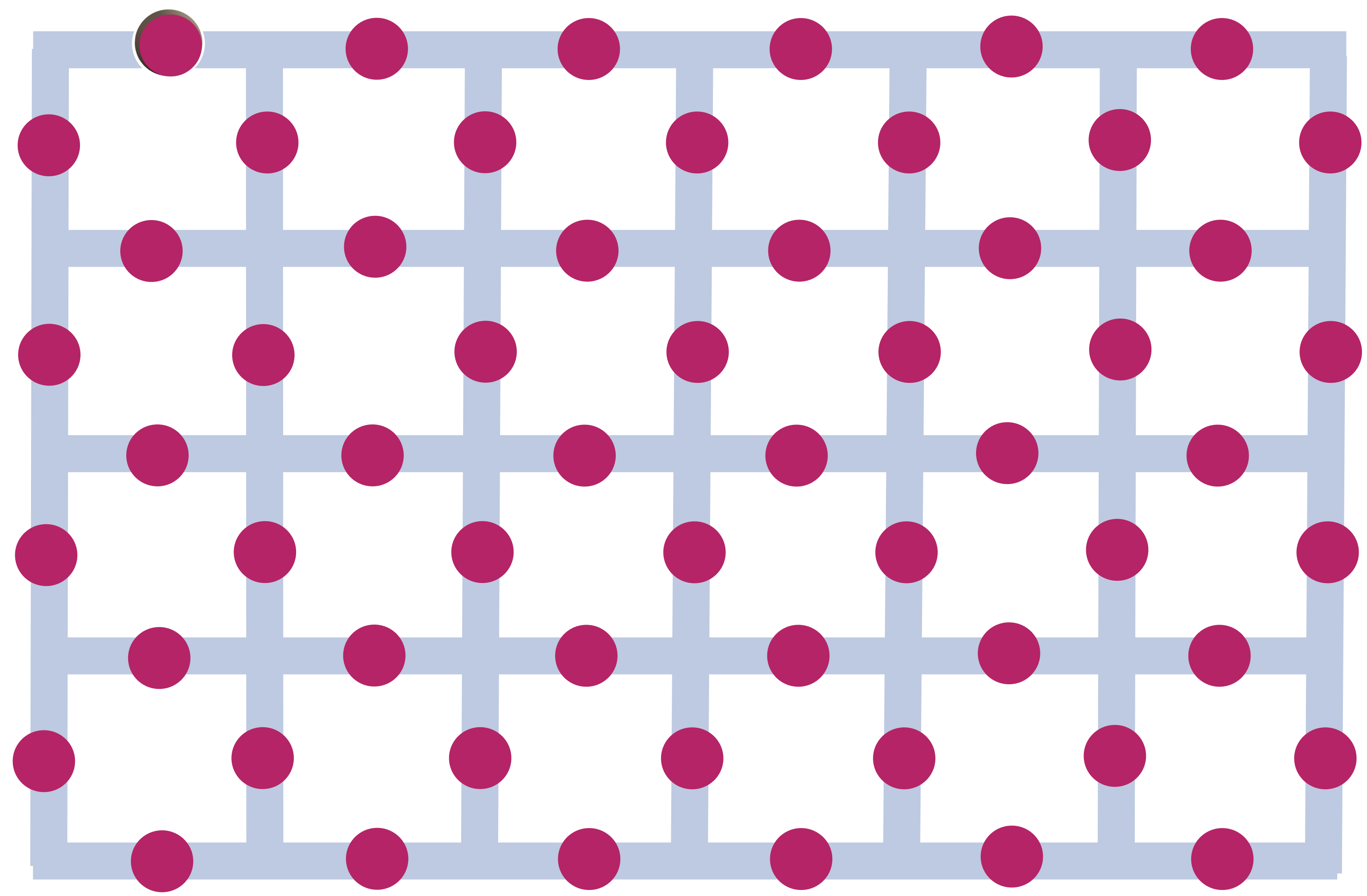
  - **Degeneracy** of the Hamiltonian constant and depends on **topology**
  - All GS are **locally indistinguishable** (no local order parameter)
  - To map between them, you need a **non-local operator**
  - Excitations behave like quasi-particles with **anyonic statistics**

A new type of order

- How can it be captured in PEPS?
- Injective PEPS do not do it, as unique GS, need a bit more…

Toric code as a paradigmatic example

Toric code



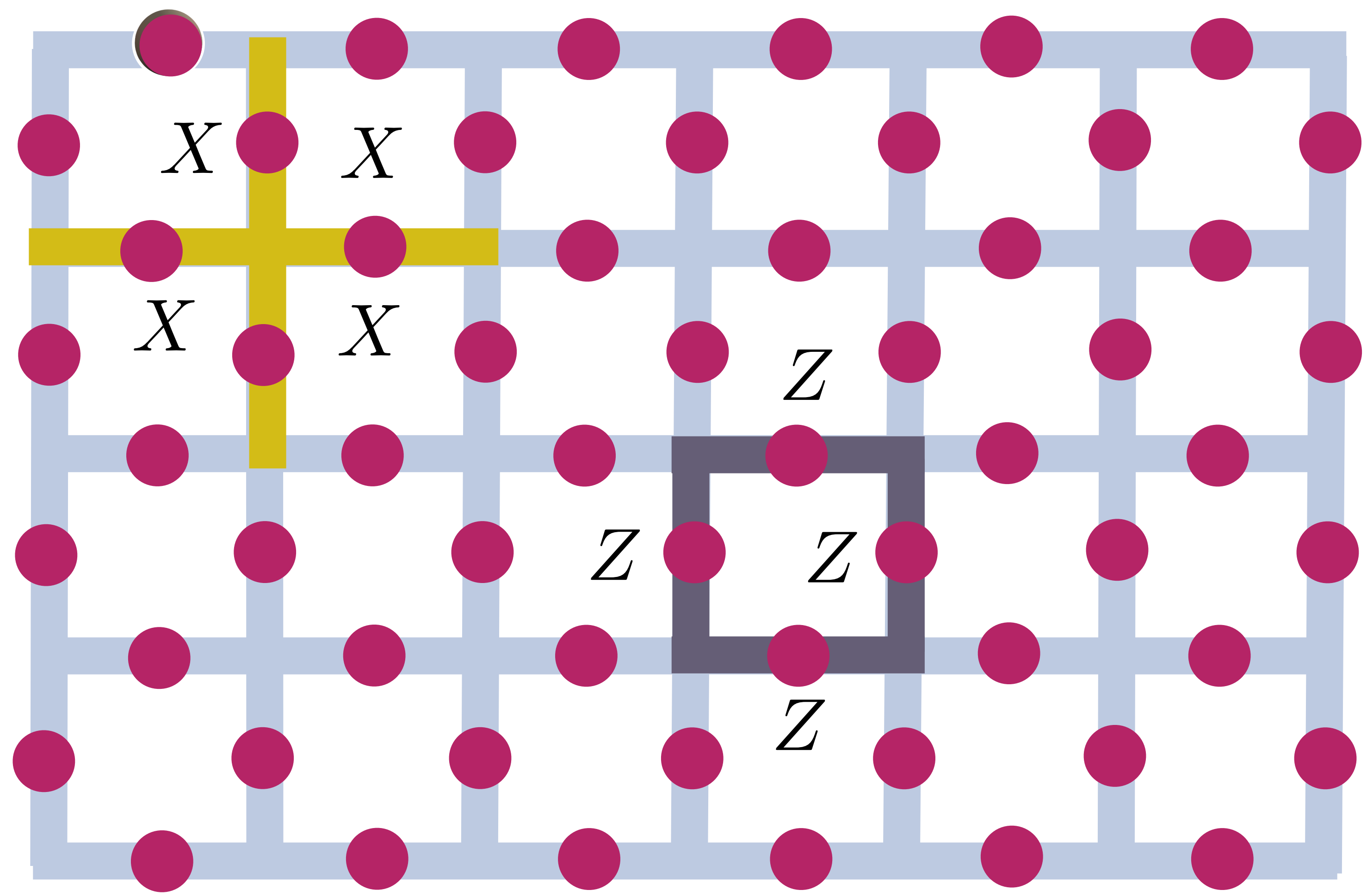
Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)

Toric code

- Spins on edges of cubic lattice
- Star operators $\prod_{j\in+} X_j$

  (flux at plaquette)
- Plaquette operators $\prod_{j\in\Box} Z_j$

  (charge at vertex)



- Hamiltonian

$$H = -J\sum_k \left[\prod_{j\in\Box_k} Z_j + \prod_{j\in+_k} X_j\right]$$

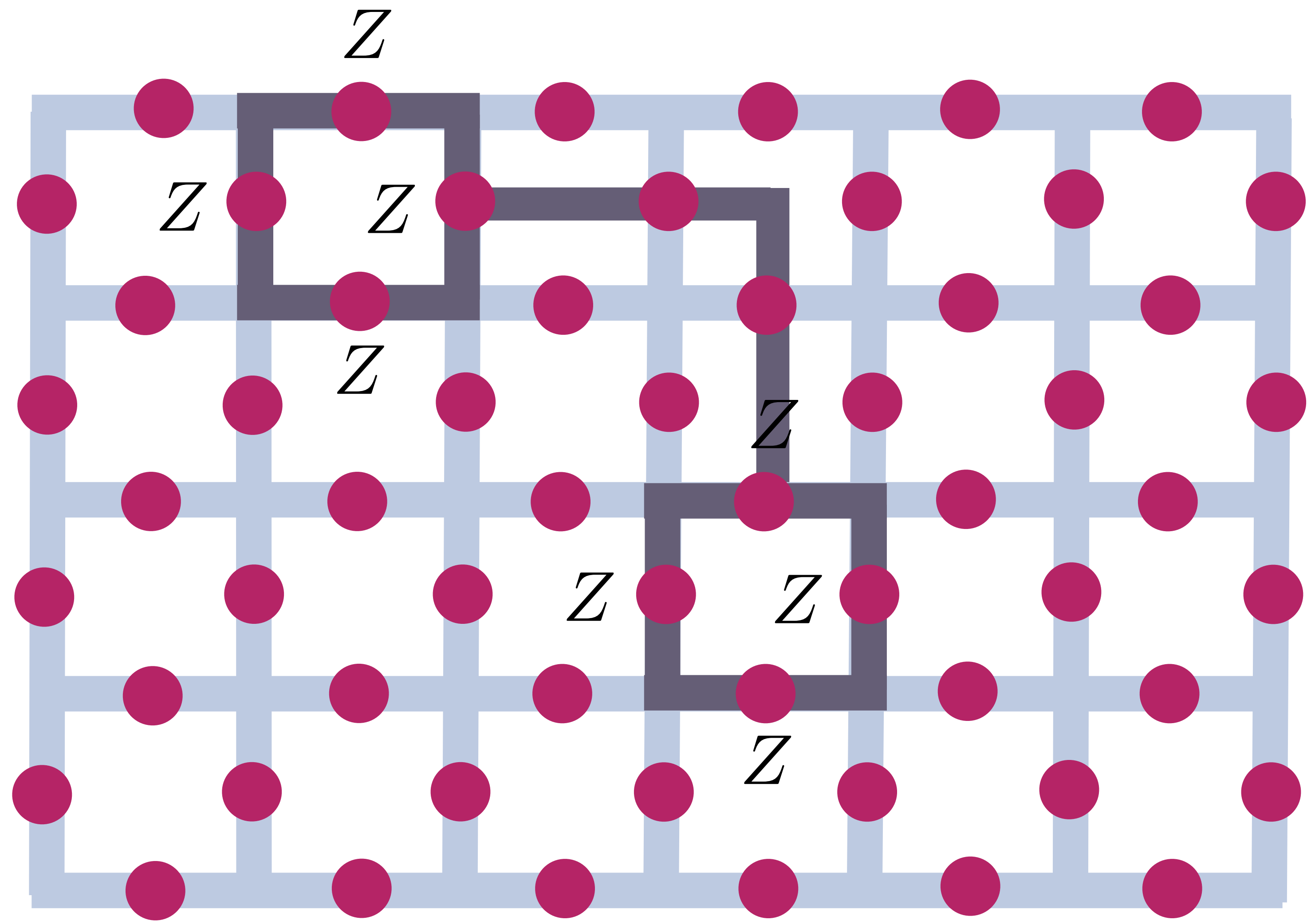
- Star and plaquette operators act trivially on ground state of Hamiltonian

$$\Box_j Z_j|\psi\rangle = +_j|\psi\rangle = |\psi\rangle$$

Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)

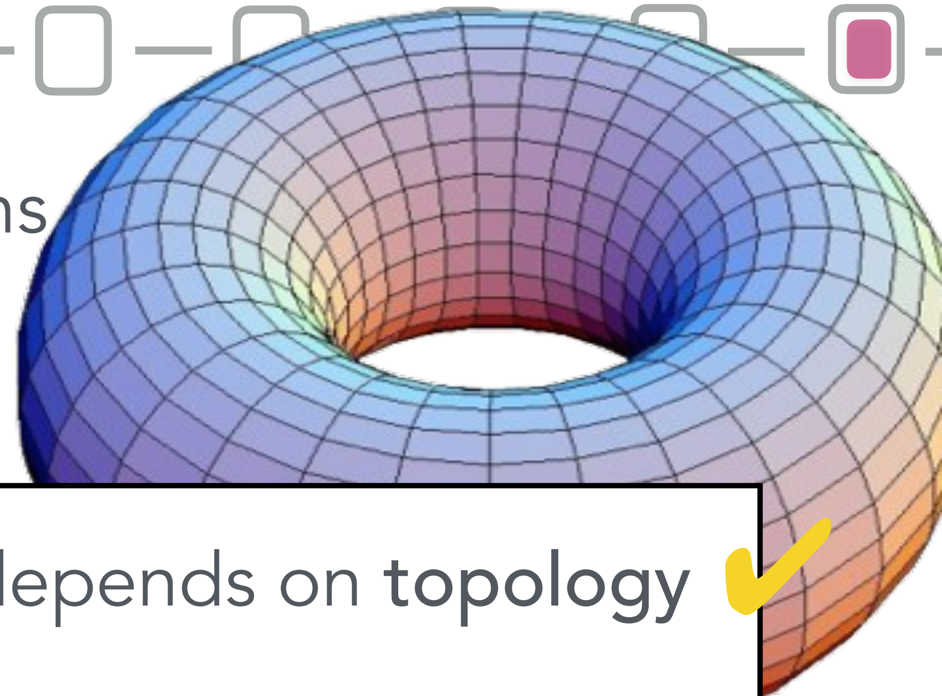
Toric code

- Can define **string operators**



Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)

Toric code



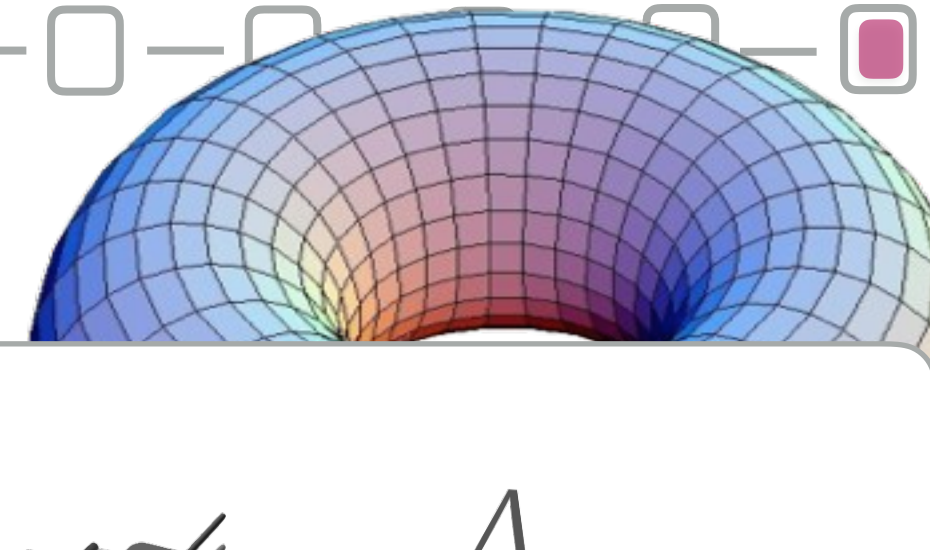
- Ground state formed by closed loop configurations
- Shows $\mathbb{Z}_2$-topological order

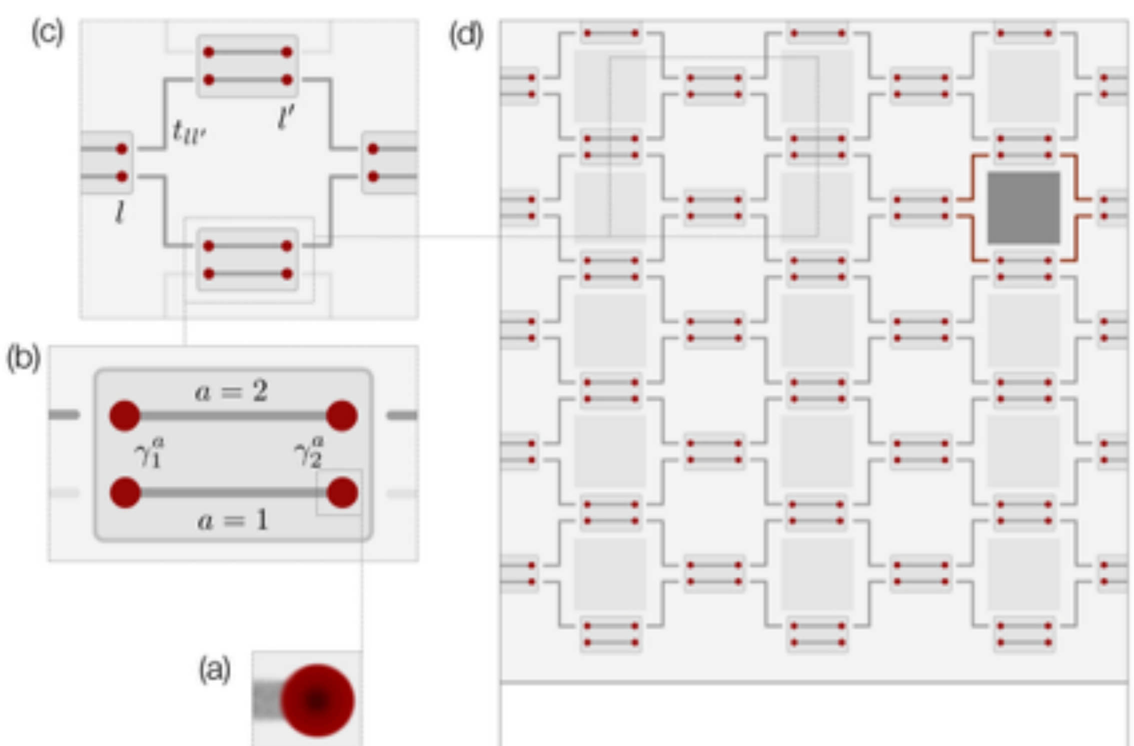
- **Degeneracy** of the Hamiltonian constant and depends on **topology** ✓
  (4 on the torus)
- All GS are **locally indistinguishable** (no local order parameter) ✓
- To map between them, you need a **non-local operator** ✓
  (Strings around the torus)
- Excitations behave like quasi-particles with **anyonic statistics** ✓
  (e- anyons on vertices
  m- anyons on plaquettes)

- Gapped, frustration-free Hamiltonian

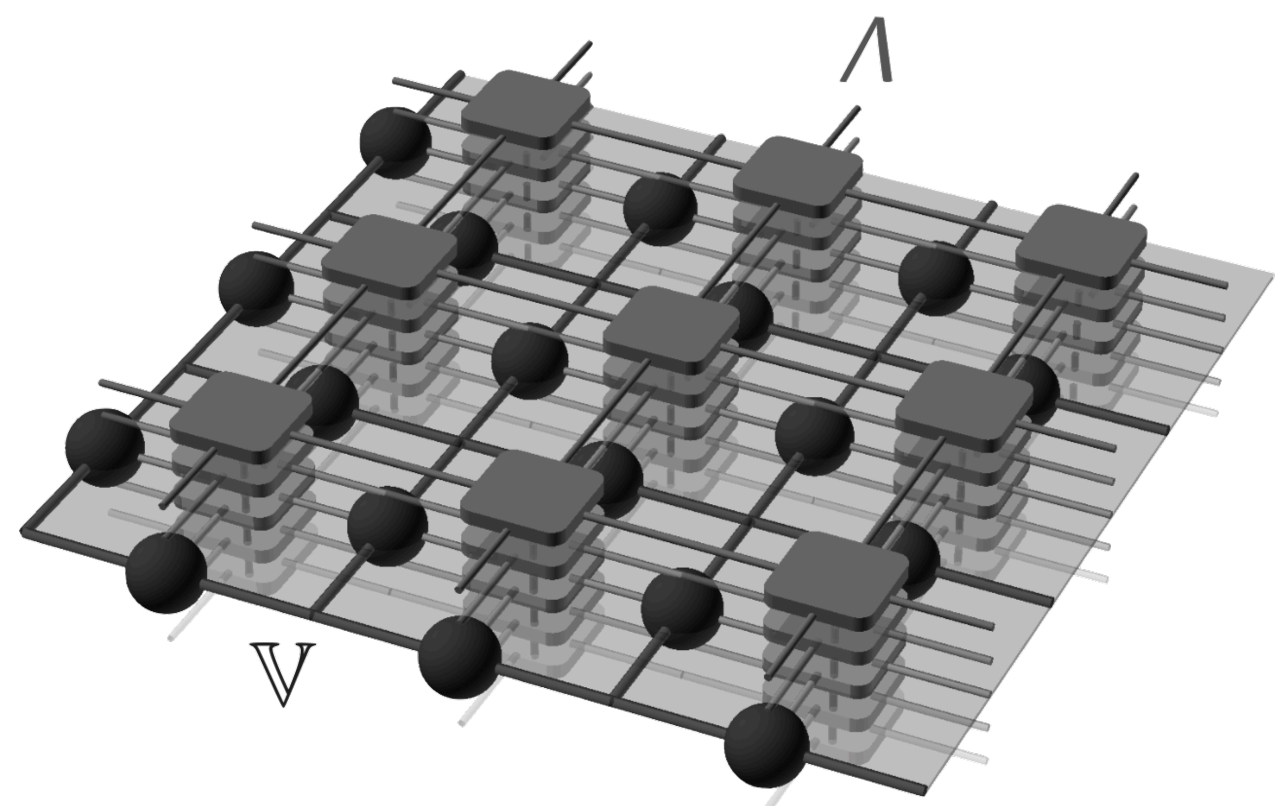
Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)

Toric code





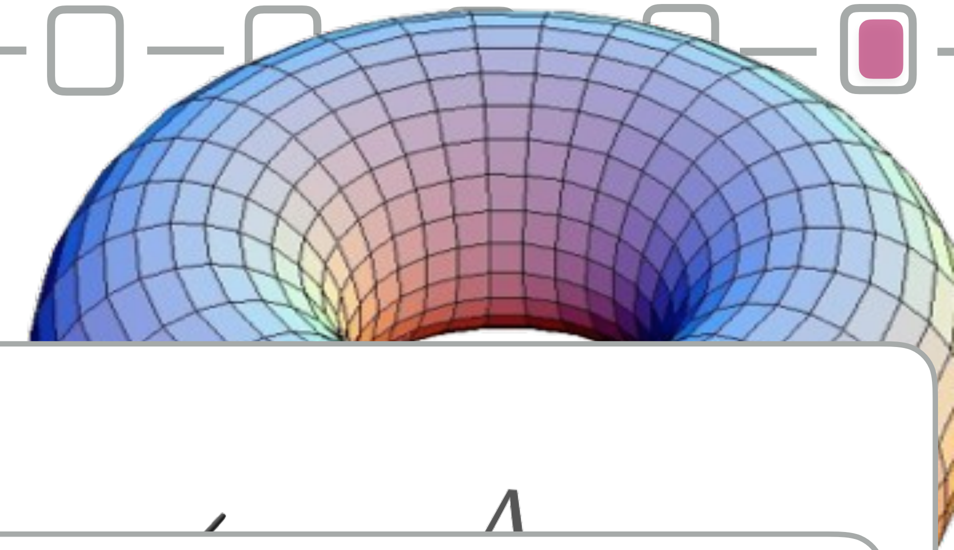
Landau, Plugge, Sela, Altland, Albrecht, Egger, Phys Rev Lett 116, 050501 (2016)

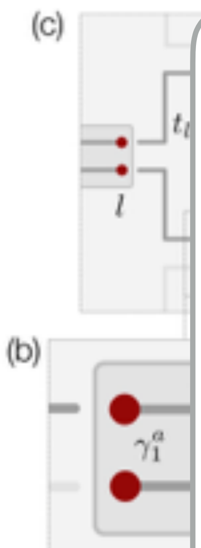


Herold, Campbell, Eisert, Kastoryano, Nature P J Quant Inf (2015)
Herold, Campbell, Kastoryano, Eisert, Phys Rev A (2016)

- **Topological quantum memory** protecting quantum information

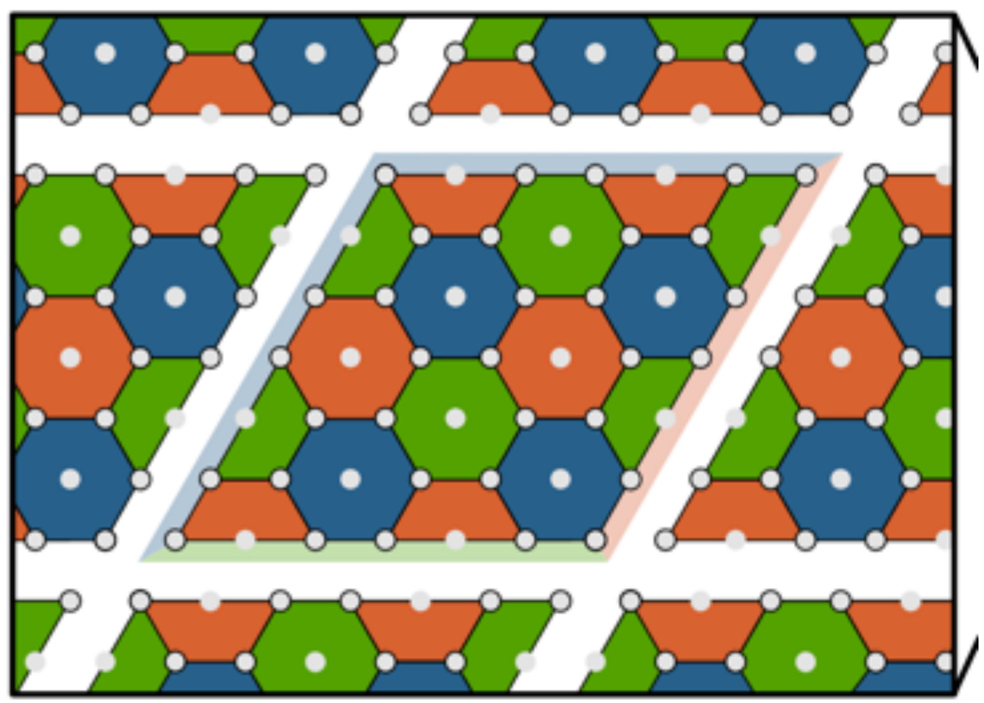
Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)





Landau, P
116, 0505

- **Realistic implementations of color codes:** Daniel yesterday



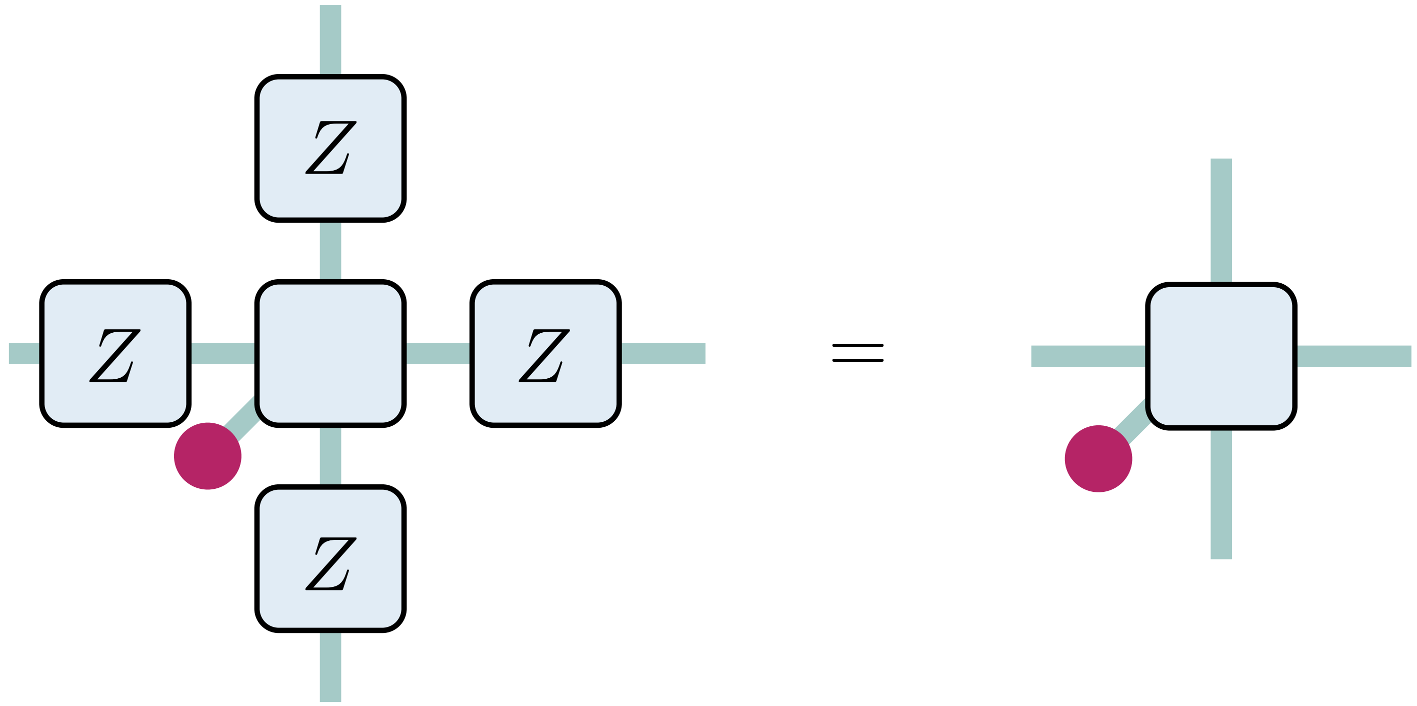
Litinski, Kesselring, Eisert, von Oppen, arXiv:1704.01589

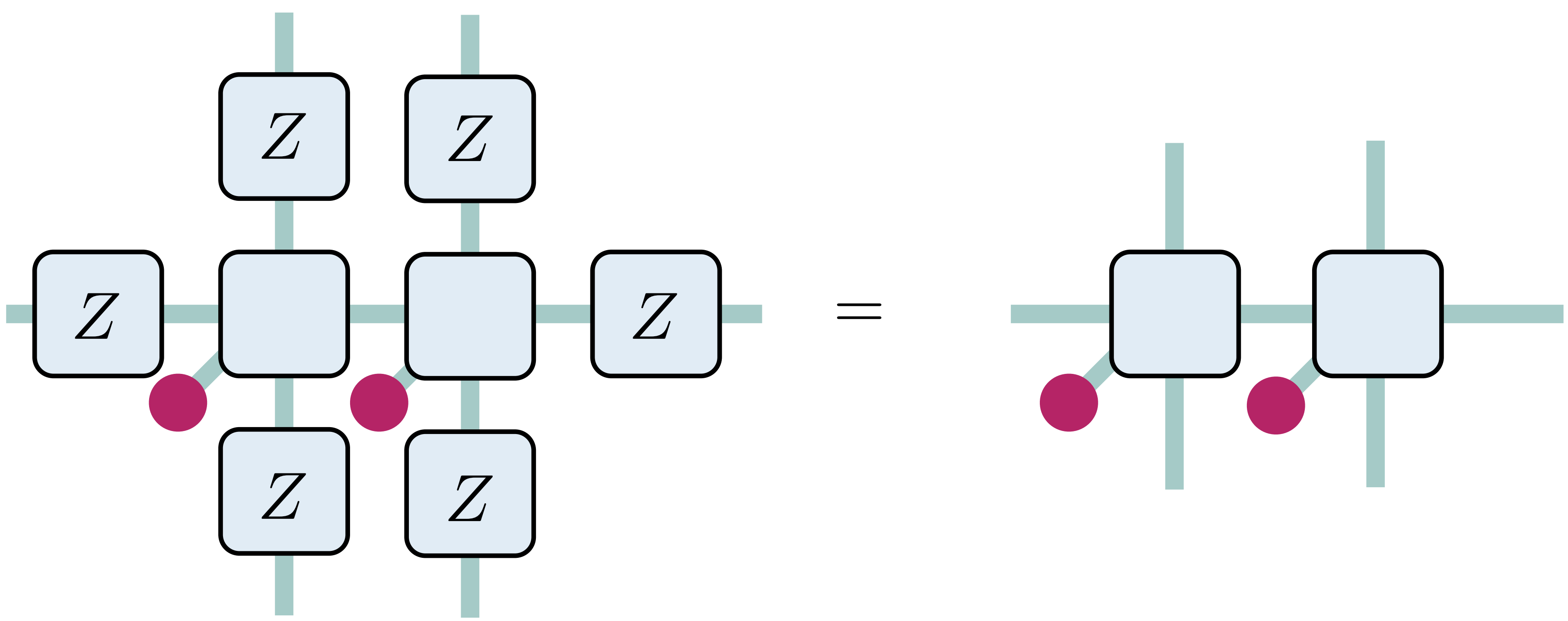
- **Topological quantum memory** protecting quantum information

Kitaev, quant-ph/9707021
Dennis, Kitaev, Landahl, Preskill, J Math Phys 43, 4452 (2002)

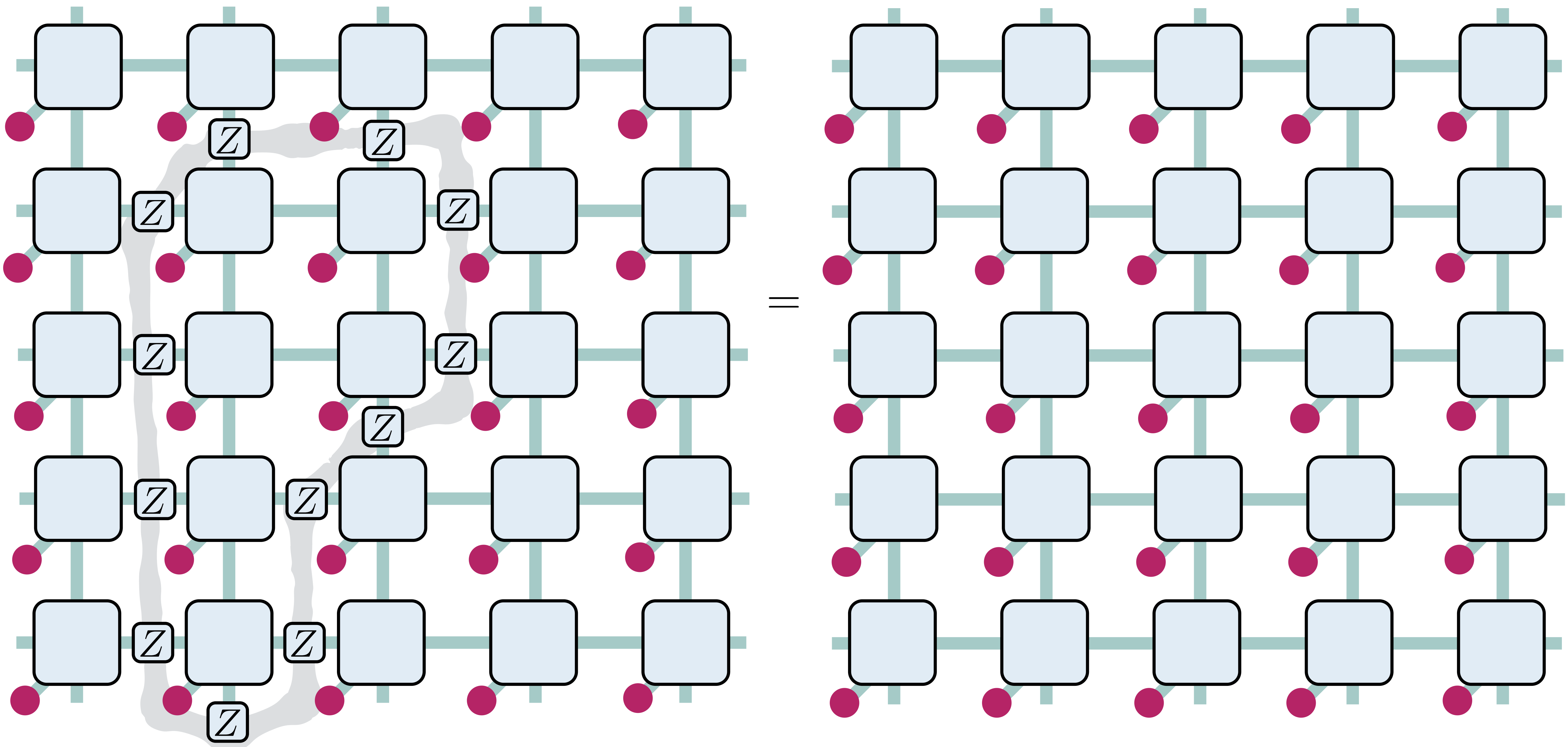
Topological order in PEPS

- Gauge symmetry:
- Let G be any finite group, e.g., $G = \mathbb{Z}_2 = \{1, Z\}$





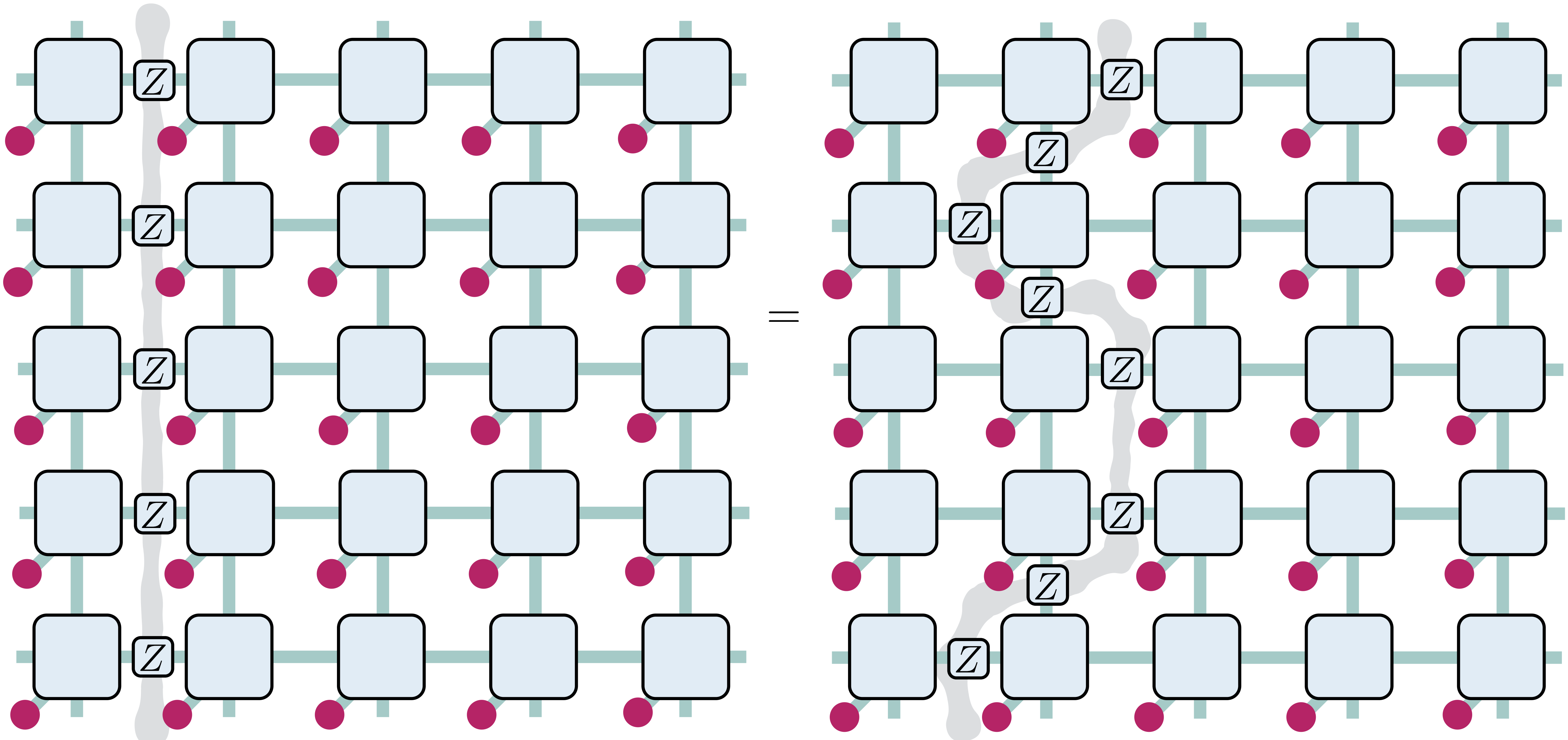
- Contractible loops of Z vanish



- What about loops that are non-contractible?

Topology in PEPS

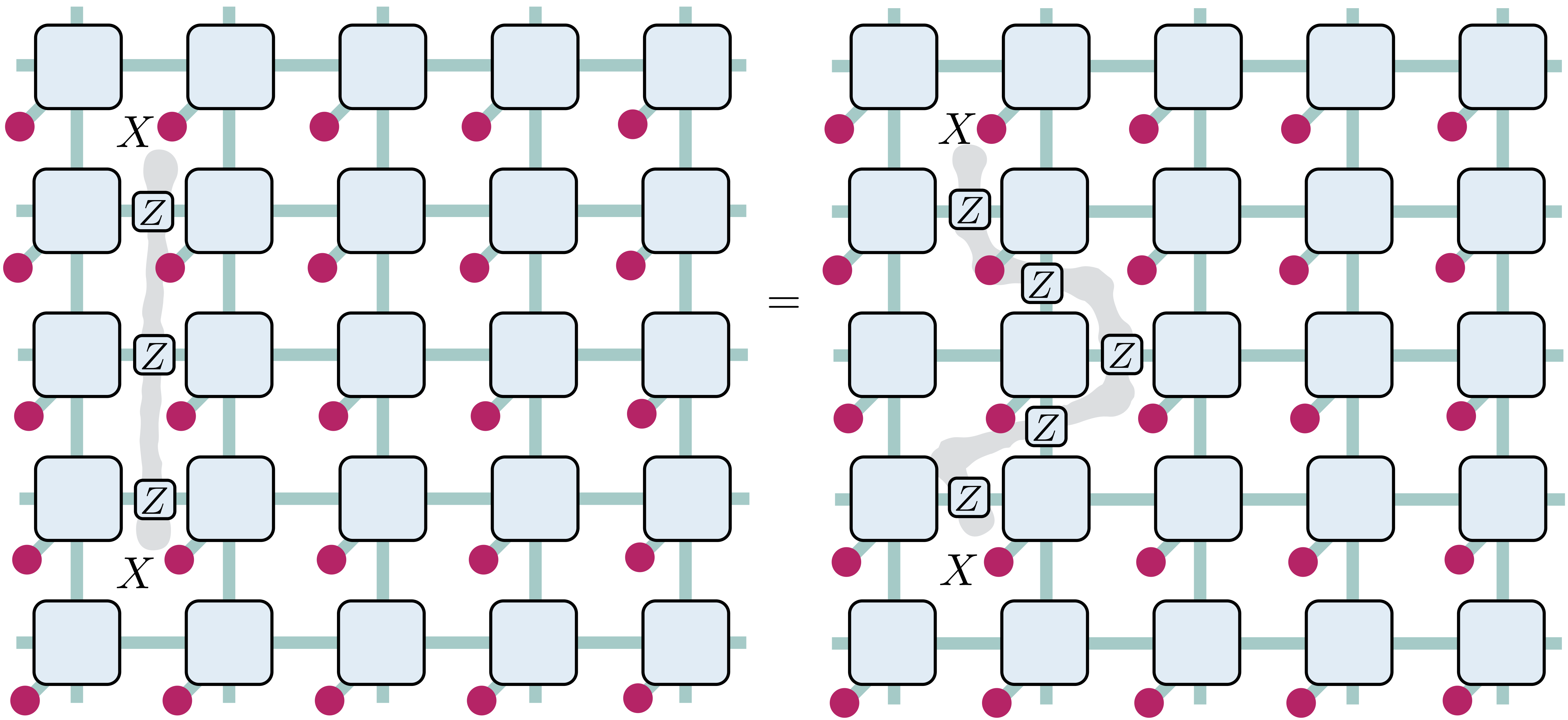
- Non-contractible loops can be arbitrarily deformed but they do not vanish



- New ground states of parent Hamiltonian (which are locally equal)

Excitations as open strings

- Open strings can be deformed, except from end points (quasi-particles)



- All of them have the same energy, they can move freely

G-injective and G-isometric PEPS

- Require less than full injectivity

- G**-injective PEPS:** Symmetry group G is acting on virtual indices and PEPS tensors are left-invariant on the G-invariant subspace

Schuch, Cirac, Perez-Garcia, Ann Phys 325, 2153 (2010)

G-injective and G-isometric PEPS

- Require less than full injectivity

- **G-injective PEPS:** Symmetry group G is acting on virtual indices and PEPS tensors are left-invariant on the G-invariant subspace
- **G-isometric PEPS:** All PEPS tensors are isometries

- For G- isometric PEPS, it is possible to unitarily transform between any two states in ground space by acting on two stripes wrapping around the torus

Schuch, Cirac, Perez-Garcia, Ann Phys 325, 2153 (2010)

G-injective and G-isometric PEPS

- Require less than full injectivity

- G**-injective PEPS:** Symmetry group G is acting on virtual indices and PEPS tensors are left-invariant on the G-invariant subspace
- G**-isometric PEPS:** All PEPS tensors are isometries

- For G- isometric PEPS, the states in the ground subspace cannot be distinguished by local operations (acting on topologically trivial region)

Schuch, Cirac, Perez-Garcia, Ann Phys 325, 2153 (2010)

G-injective and G-isometric PEPS

- Require less than full injectivity

- G-**injective PEPS:** Symmetry group G is acting on virtual indices and PEPS tensors are left-invariant on the G-invariant subspace
- G-**isometric PEPS:** All PEPS tensors are isometries

- For G- isometric PEPS, the **entanglement entropy** of any topologically trivial subregion is given by

$$S(\rho_A) = \log|G||\partial A| - \log|G|$$

- Here $-\log|G|$ is the topological correction to the area law

Schuch, Cirac, Perez-Garcia, Ann Phys 325, 2153 (2010)

Topological order

- We recover topological order

  - **Degeneracy** of the Hamiltonian constant and depends on **topology** ✓
  - All GS are **locally indistinguishable** (no local order parameter) ✓
  - To map between them, you need a **non-local operator** ✓
  - Excitations behave like quasi-particles with **anyonic statistics** ✓

A complete picture?

- Good enough to capture **toric code, quantum double models** etc

  Kitaev Ann Phys 303, 2 (2003)

- Take $G = S_3$, suitable for universal **topological quantum computation**
- Not capturing **string net models**

  Levin, Wen, Phys Rev B 71, 045110 (2005)

> - Can a complete understanding of topological order be achieved in terms of PEPS?

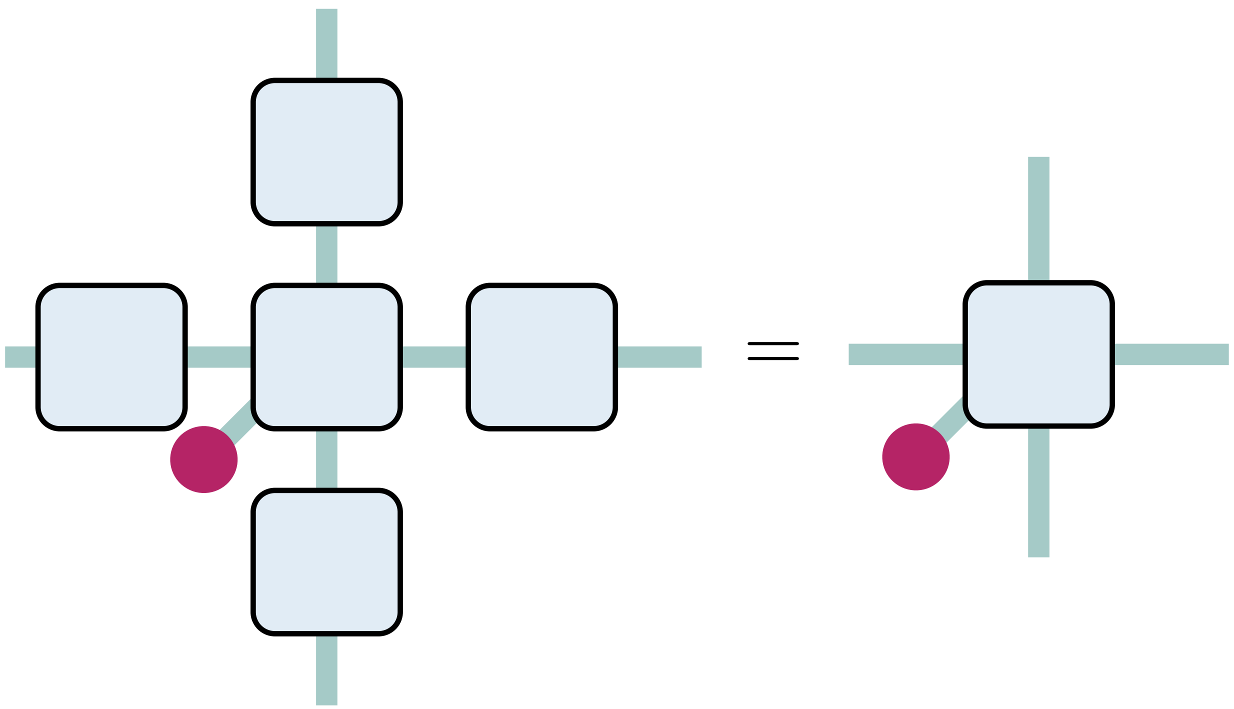
A complete picture?

- Can a complete understanding of topological order be achieved in terms of PEPS?

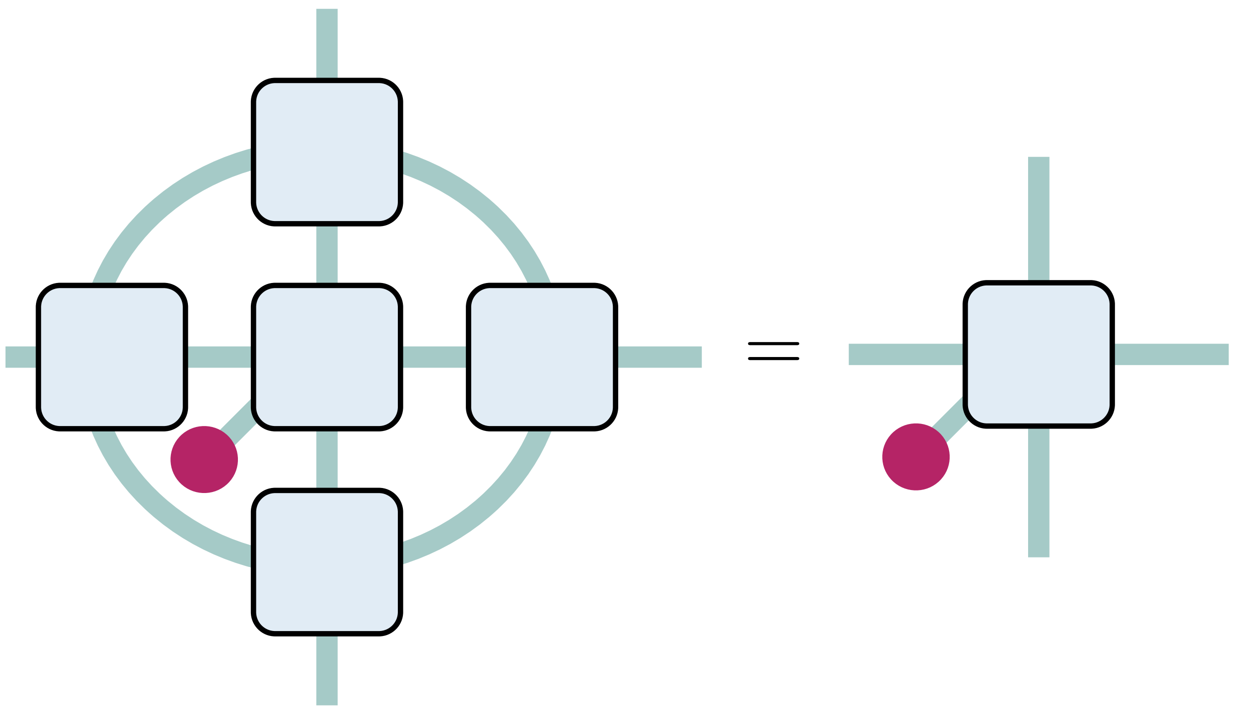
MPO-injective PEPS

Beyond G-injective PEPS

- Virtual symmetries



- G -symmetry

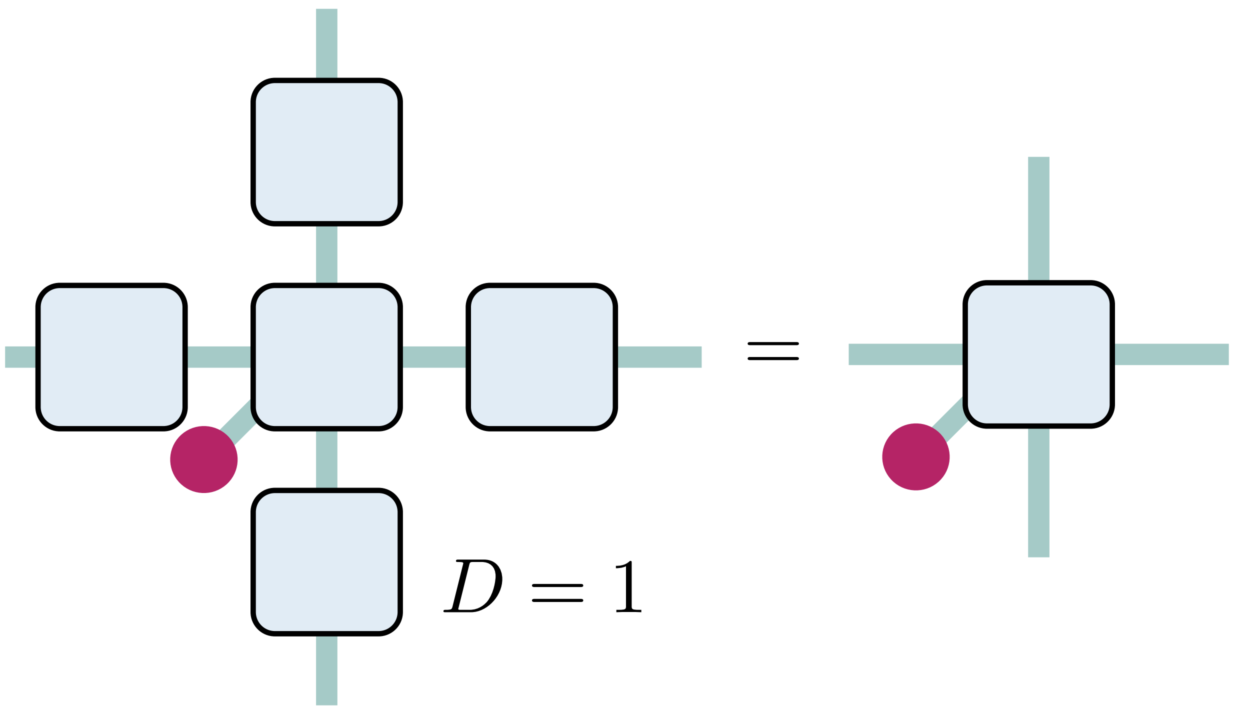


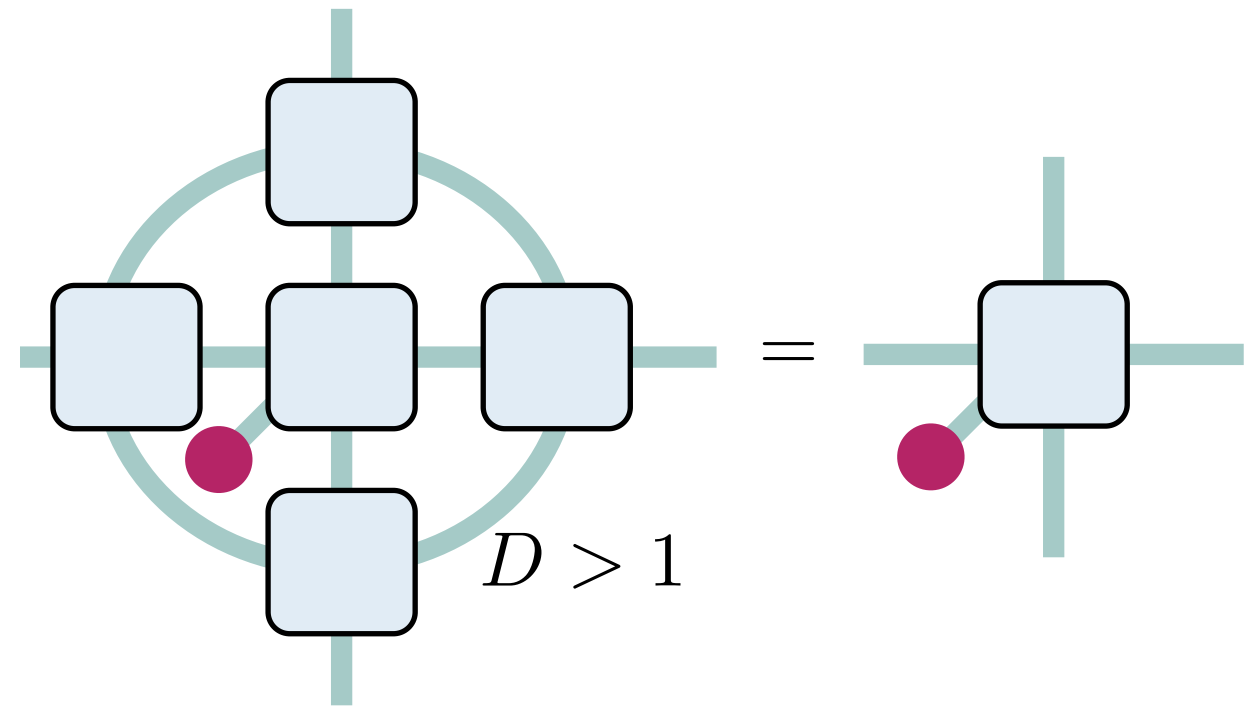
- MPO-symmetry

Şahinoğlu, Williamson, Bultinck, Mariën, Haegeman, Schuch, Verstraete, arXiv:1409.2150

Beyond G-injective PEPS

- Virtual symmetries



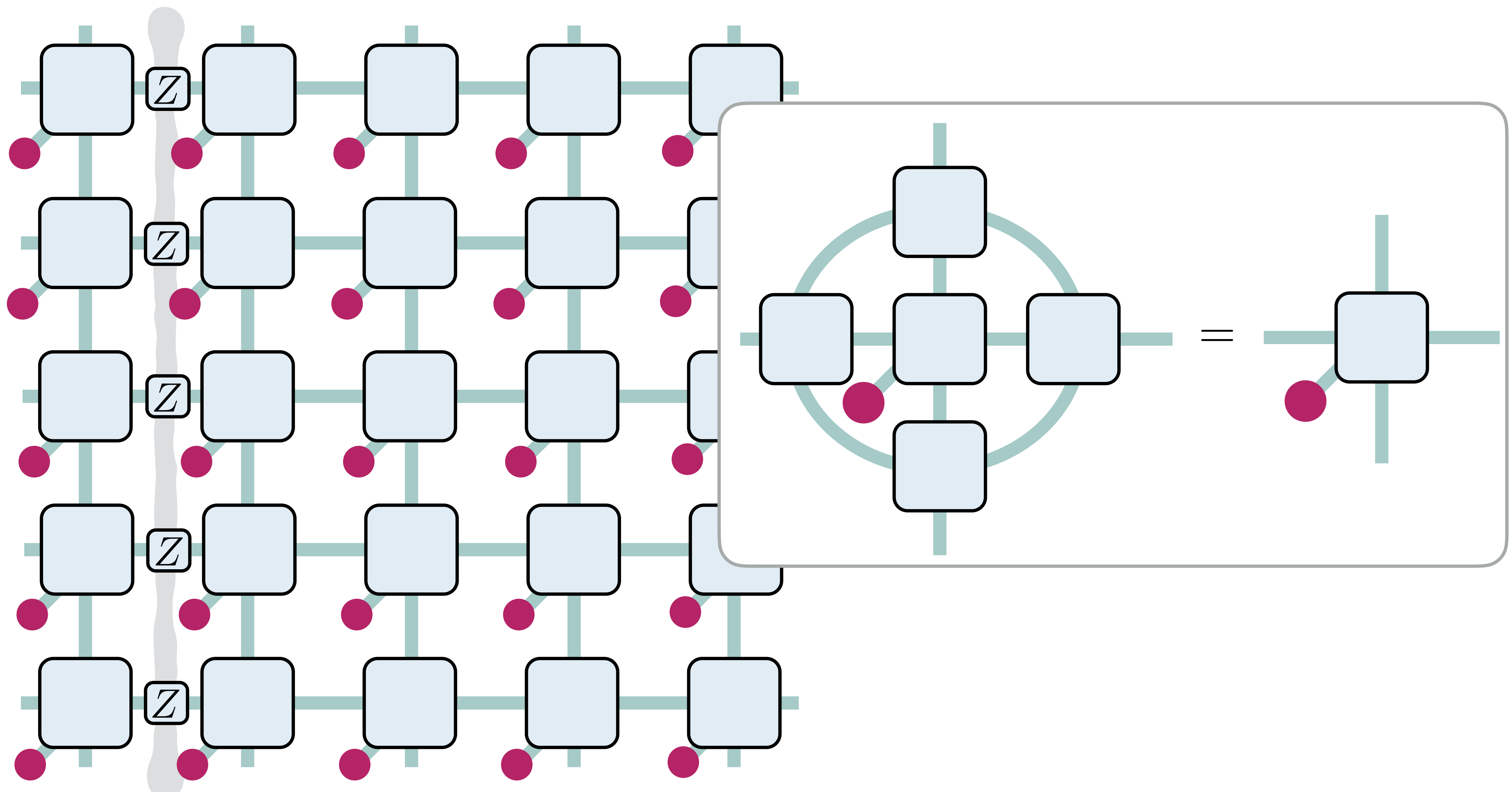


- G-**symmetry**
- Uncorrelated products
- Groups

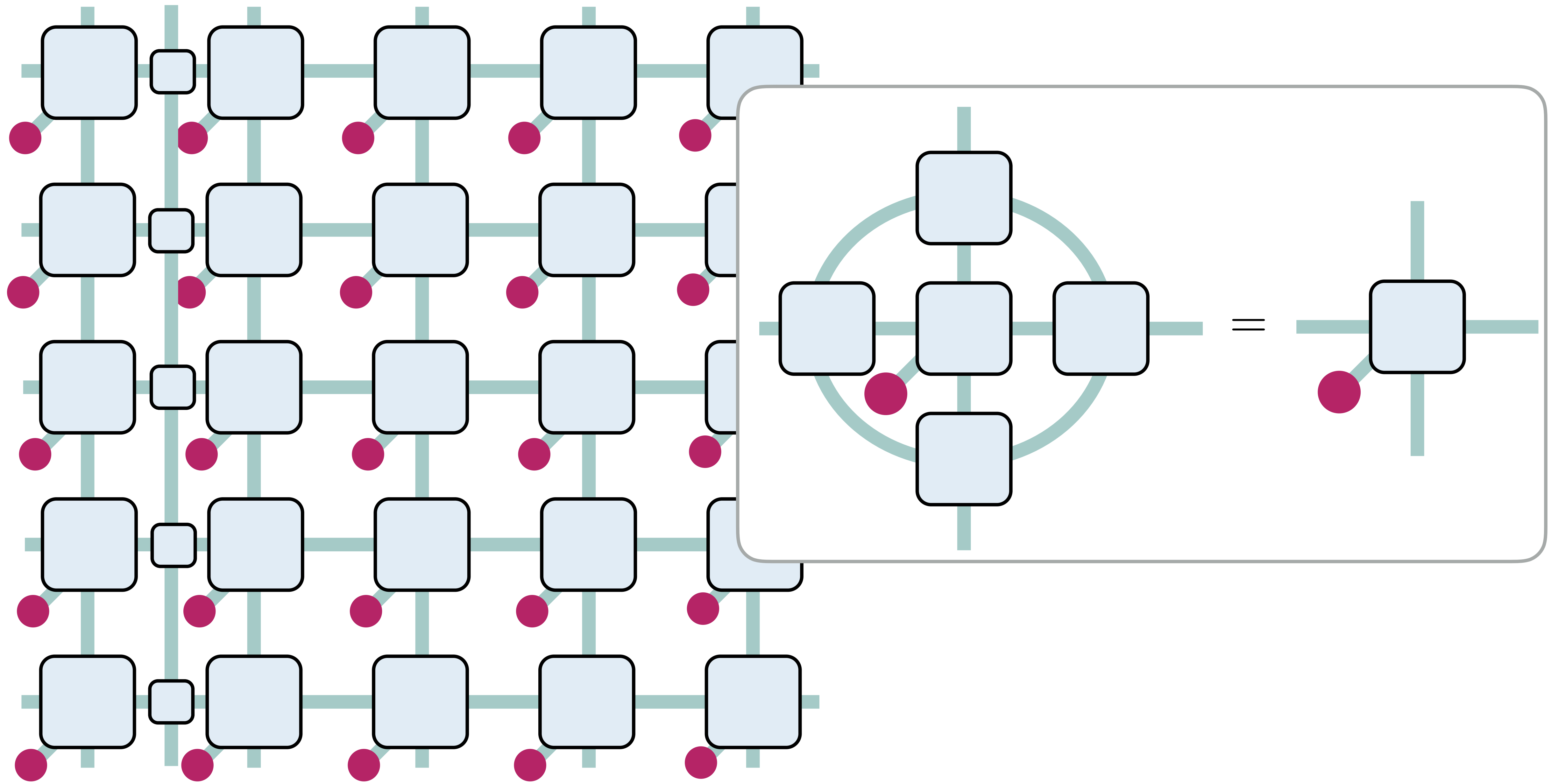
- **MPO-symmetry**
- Matrix-product operator
- Twisted groups and more

Şahinoğlu, Williamson, Bultinck, Mariën, Haegeman, Schuch, Verstraete, arXiv:1409.2150

Beyond G-injective PEPS



Beyond G-injective PEPS



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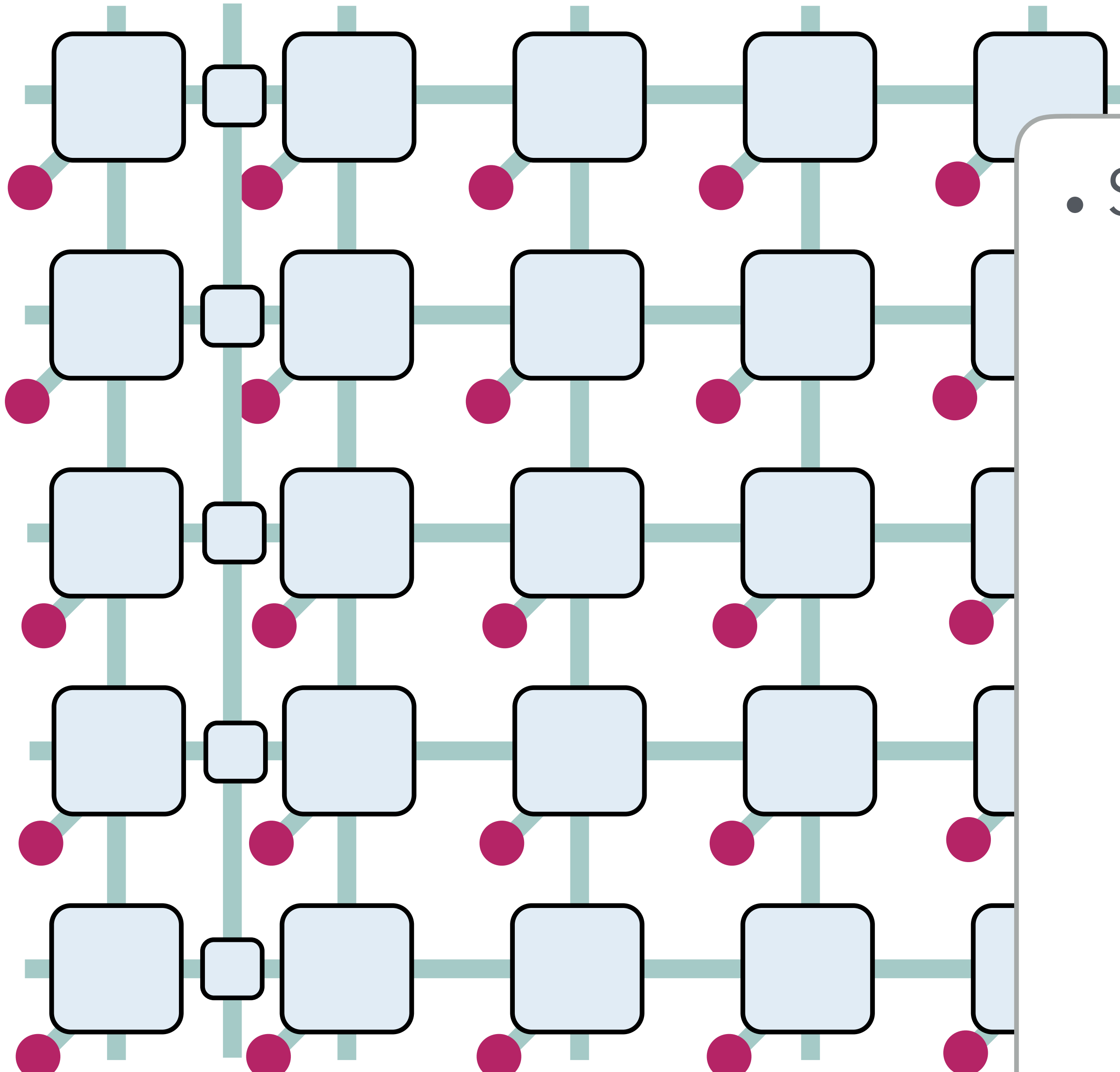
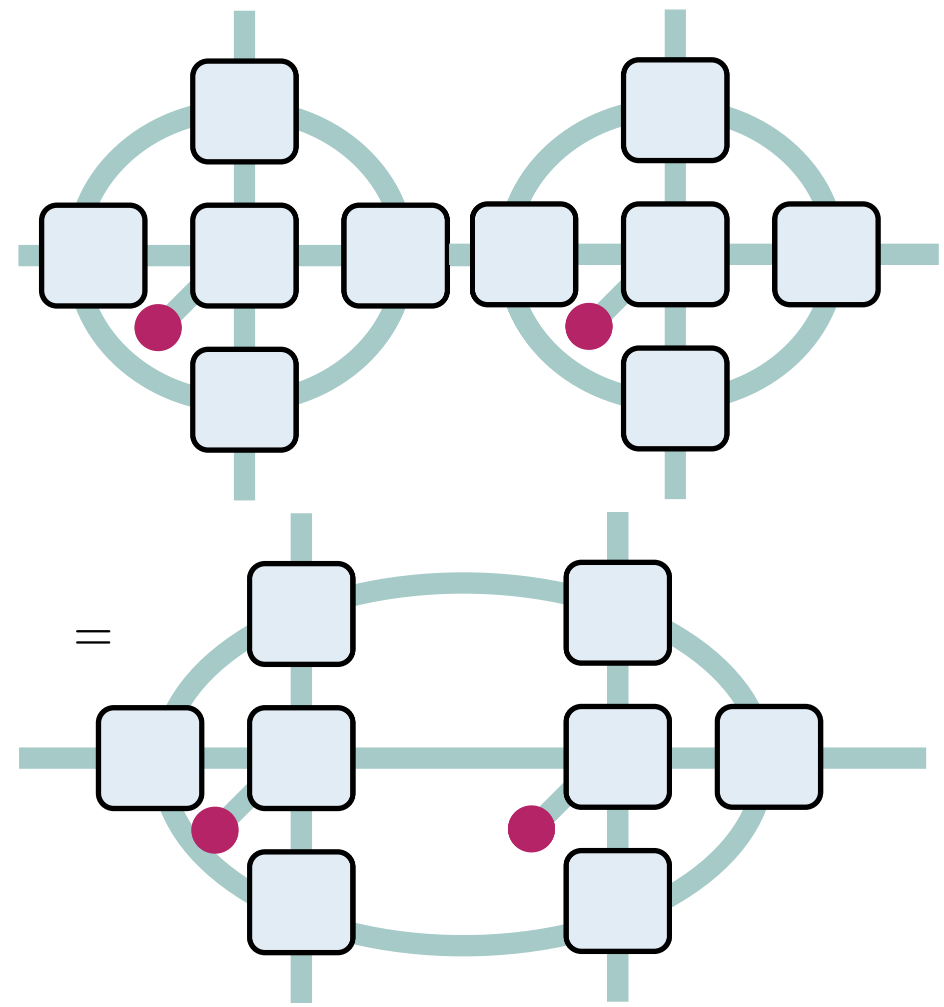
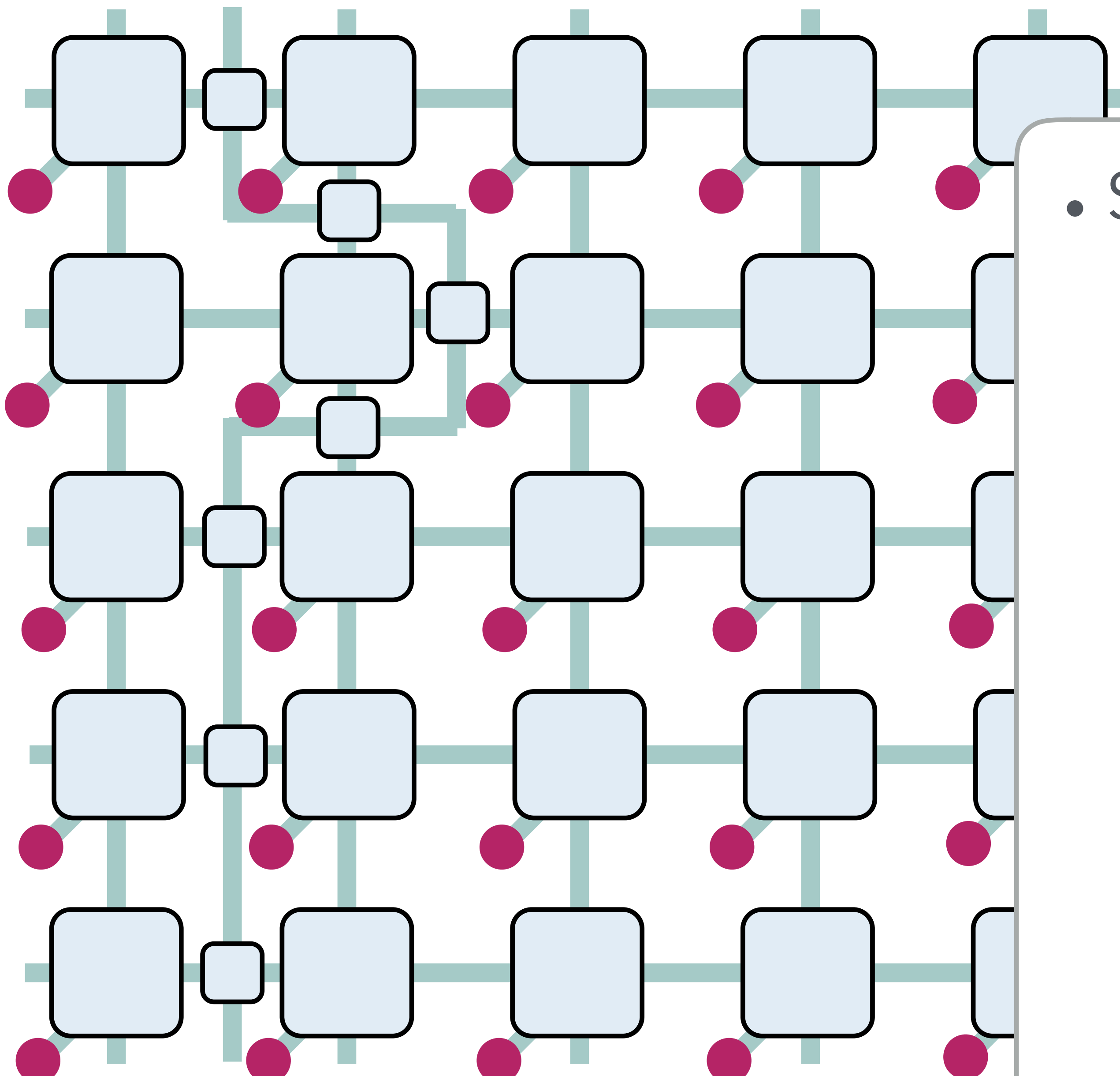


- Stable under concatenation



Beyond G-injective PEPS



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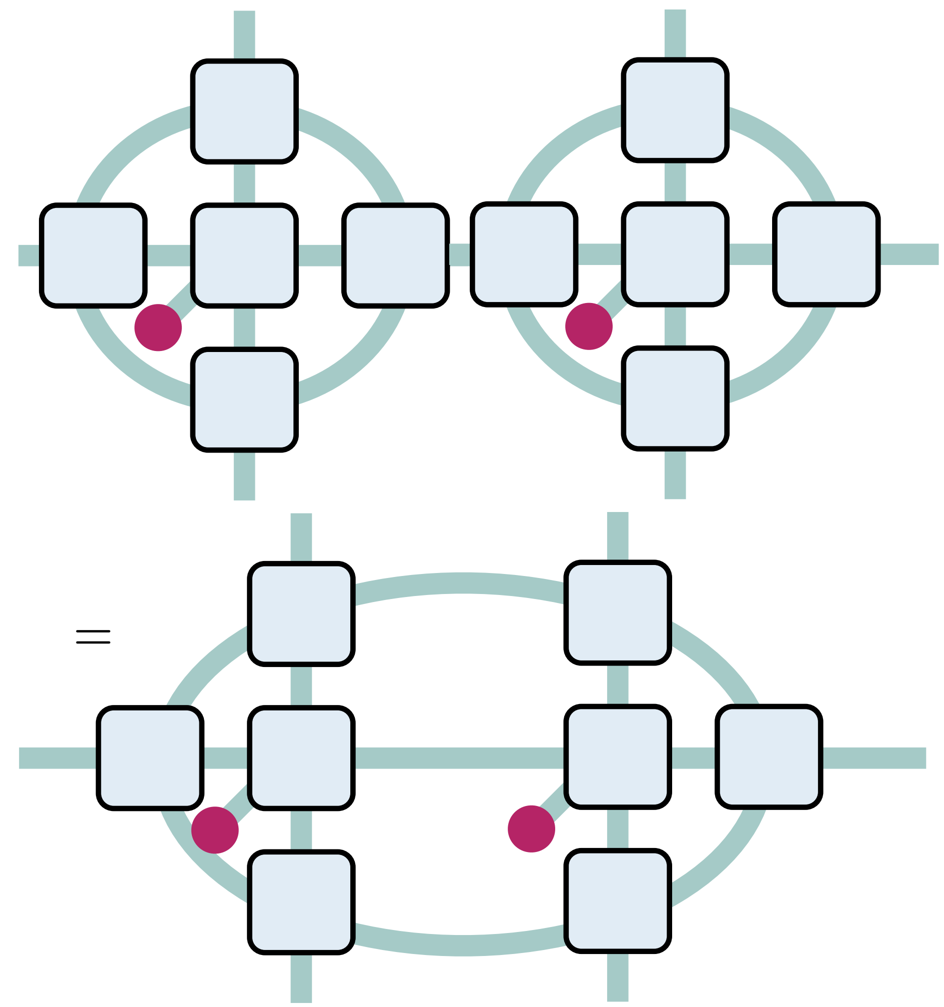
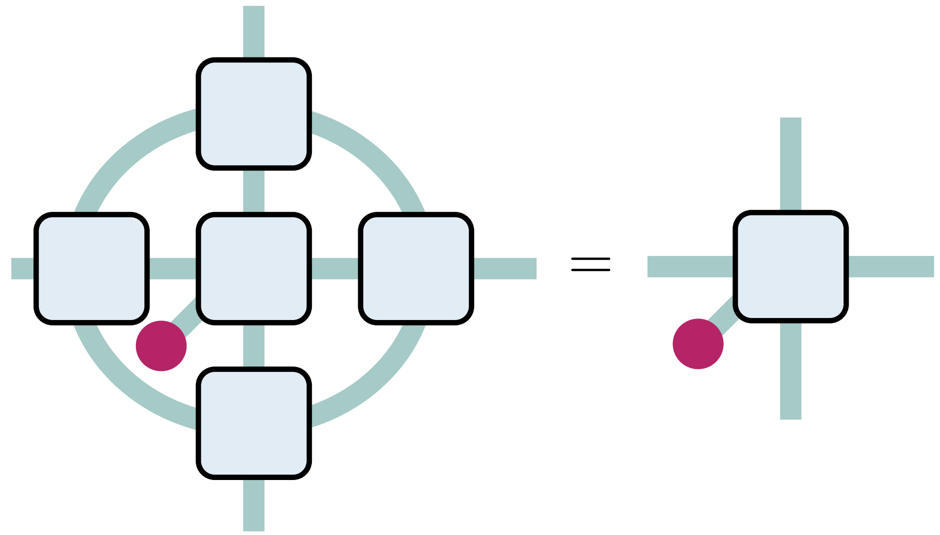


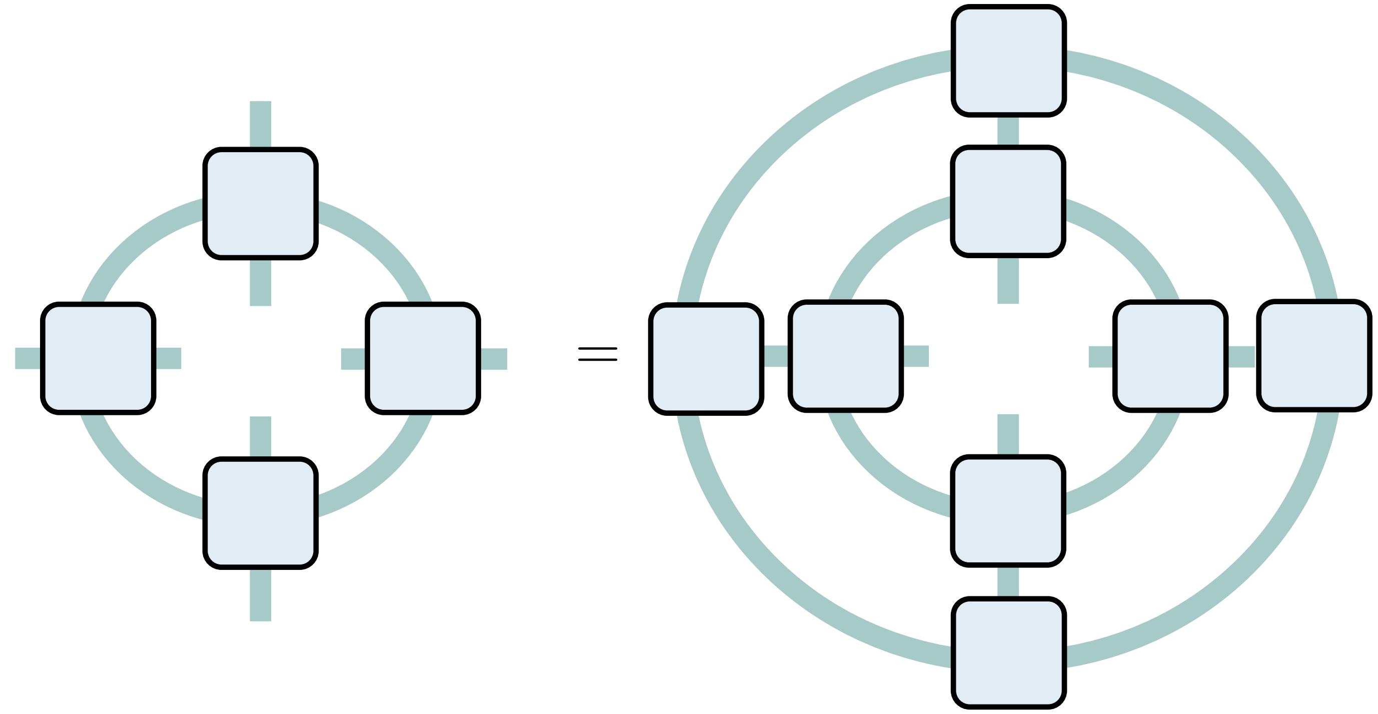
Axioms of MPO-injectivity

- MPO symmetry



Axioms of MPO-injectivity

- MPO symmetry
- MPO projector



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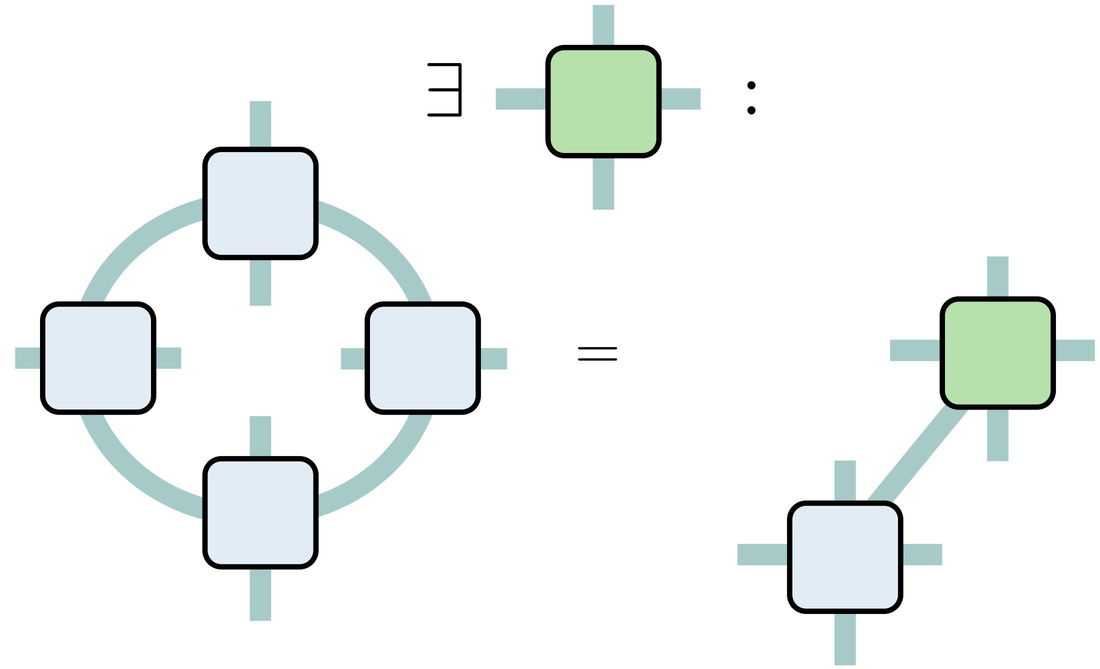
- MPO projector
- MPO injectivity



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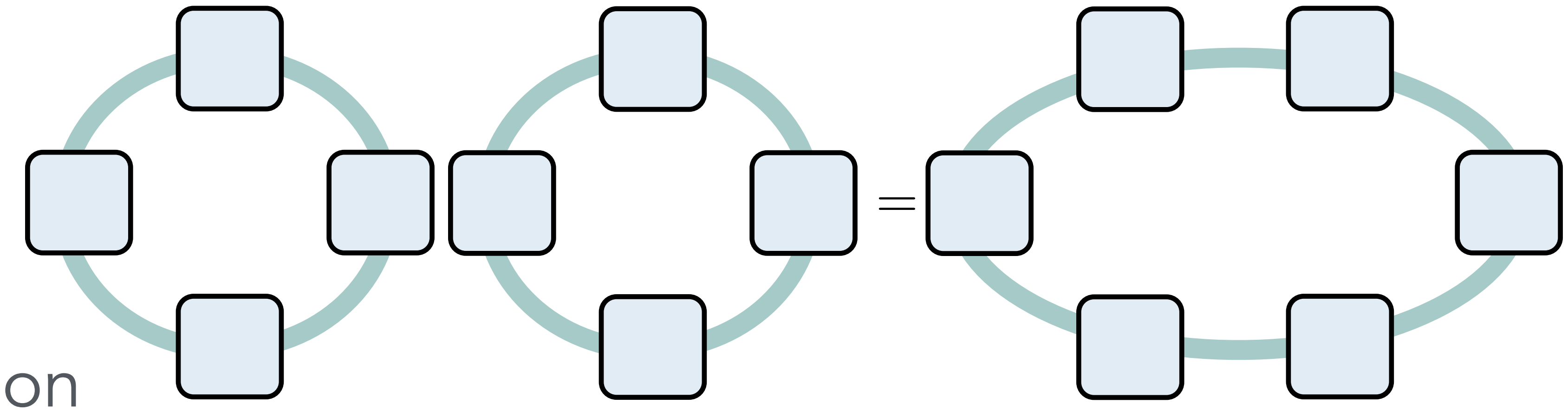
- Stability under concatenation



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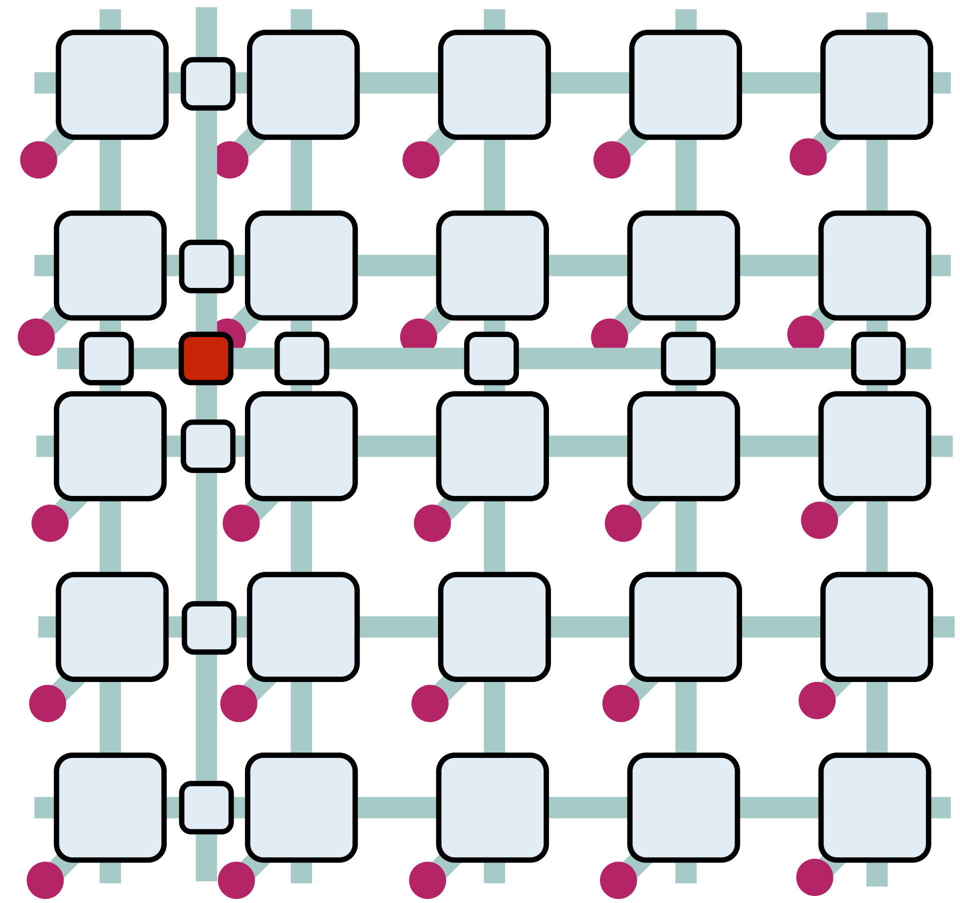

- **Can compute:**
  - Topological correction to area law

$$S(\rho_A) = c|\partial A| - \gamma$$

  - Ground state space

Şahinoğlu, Williamson, Bultinck, Mariën, Haegeman, Schuch, Verstraete, arXiv:1409.2150

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- **Can compute:**
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  - Anyonic statistics: S and T matrices

Şahinoğlu, Williamson, Bultinck, Mariën, Haegeman, Schuch, Verstraete, arXiv:1409.2150

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- **Can compute:**
    - Topological correction to area law
    $$S(\rho_A) = c|\partial A| - \gamma$$
    - Ground state space
    - Anyonic statistics: S and T matrices
    - Captures Levin-Wen string net models

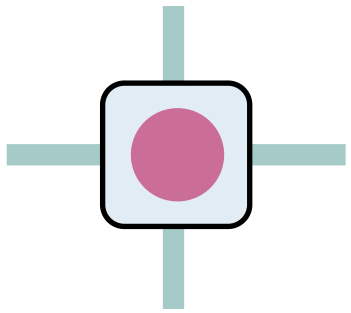
Levin, Wen, Phys Rev B 71, 045110 (2005)
Gu, Levin, Swingle, Wen, Phys Rev B 79, 085118 (2009).

Şahinoğlu, Williamson, Bultinck, Mariën, Haegeman, Schuch, Verstraete, arXiv:1409.2150

Towards tensor networks for fermionic systems

Axioms of MPO-injectivity

- **Tensors with physical fermions**
    - Book-keeping of the order (manual)



Wille, Buerschaper, Eisert, arXiv:1609.02574

Axioms of MPO-injectivity

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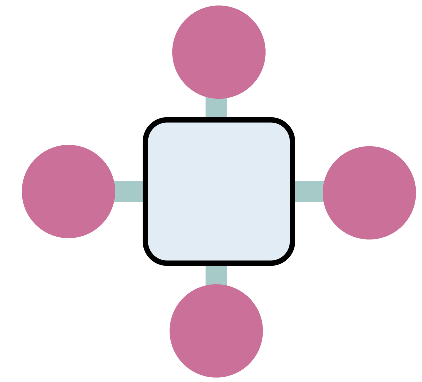
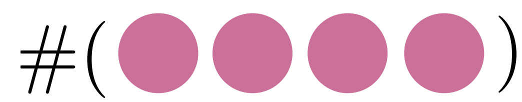


- **Add virtual fermions**
  - Book-keeping of the order (in-built)
    - Fermionic entangled pairs
    - Grassmann numbers

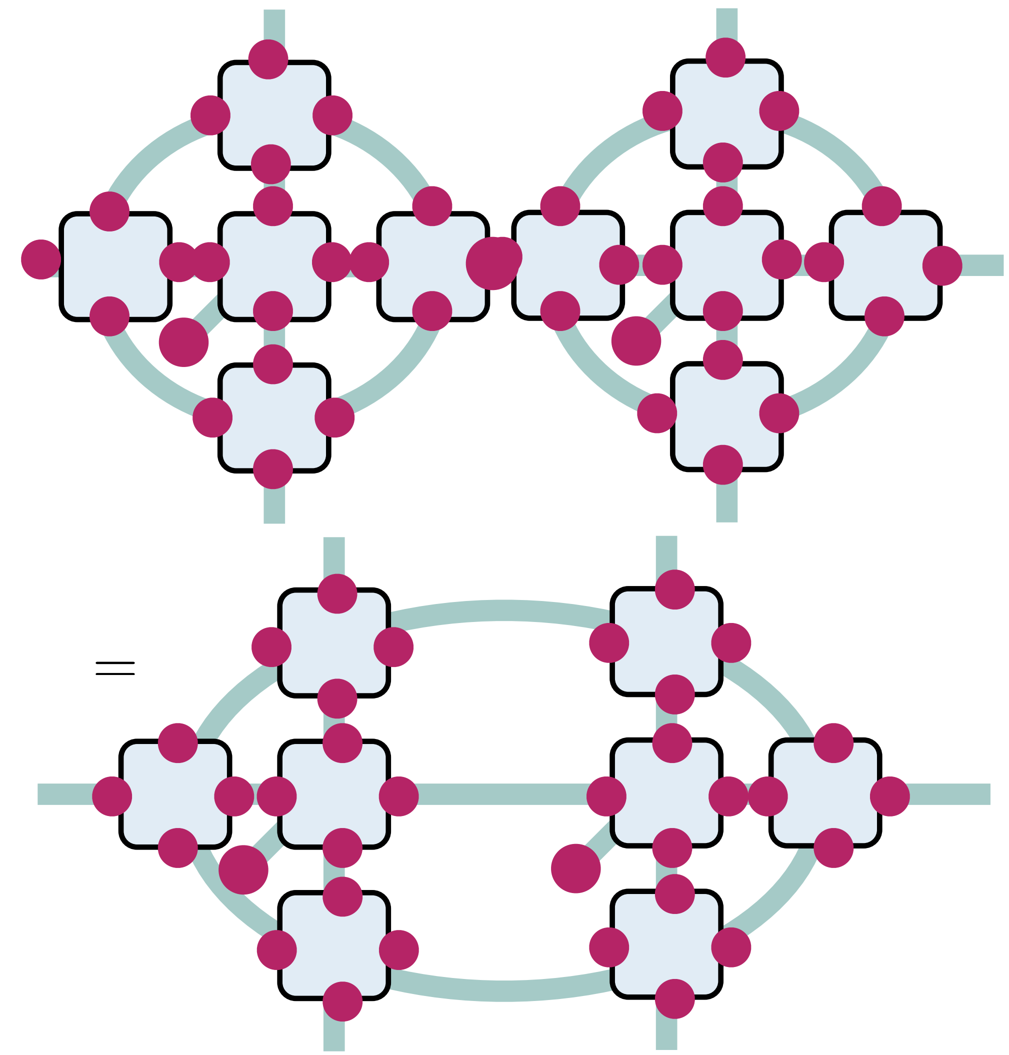


even

Wille, Buerschaper, Eisert, arXiv:1609.02574
Williamson, Bultinck, Haegeman, Verstraete, arXiv:1609.02897

Fermionic MPOs?

- Fermionic MPOs
- Axioms take analogous form
- Graded algebraic structure
- Axioms fulfillable?



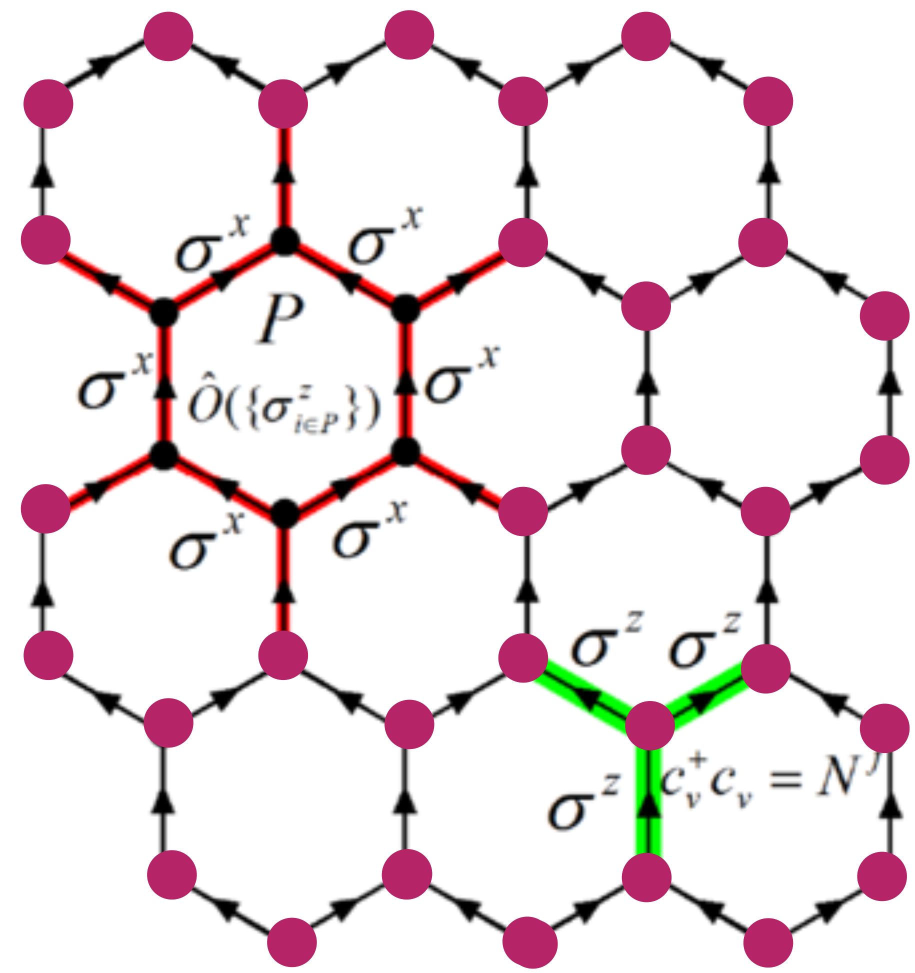
Wille, Buerschaper, Eisert, arXiv:1609.02574

- **Edges:** Spin 1/2
- **Vertices:** Fermions

$$H = \sum_v Q_v + \sum_p Q_p$$

$$Q_v = \frac{1}{2}(1 + \prod_{i \in v} \sigma_i^Z) F_v$$

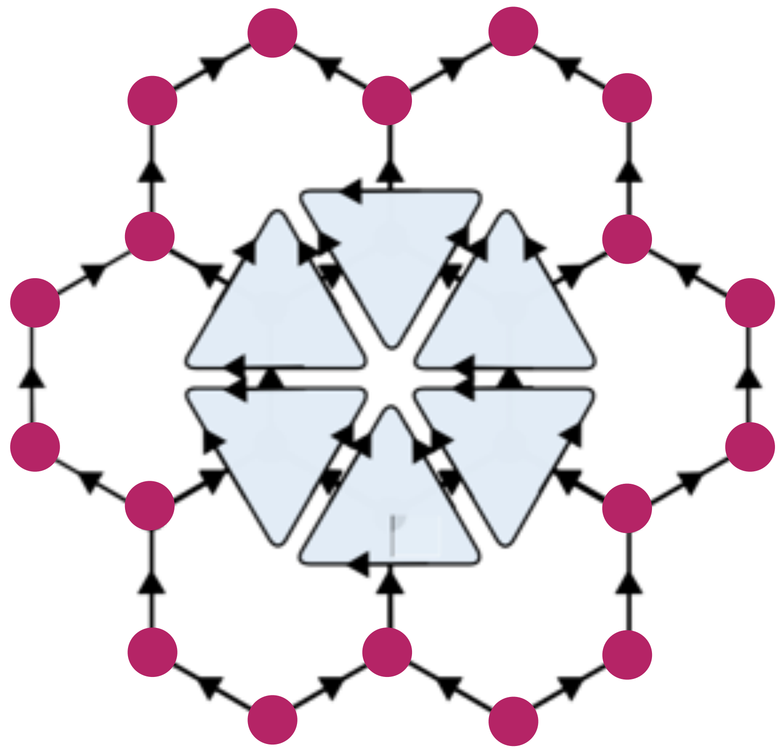
$$Q_p = \frac{1}{2}(1 + \prod_{i \in p} \sigma_i^X) F_p$$



Gu, Wang, Wen, Phys Rev B 90, 085140 (2014)

Fermionic MPO-injectivity

- Dual lattice
- Grassmann numbers



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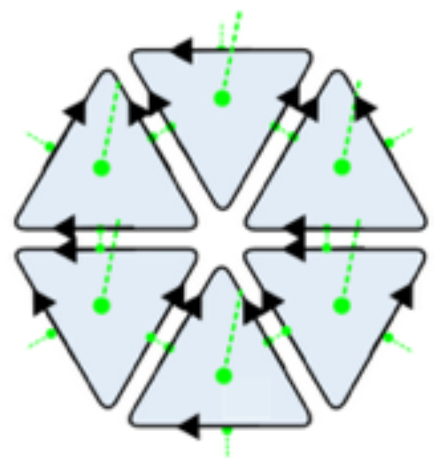
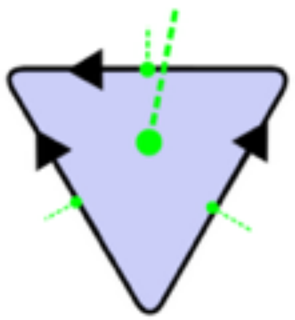


$$A = \sum A^{p_1 p_2 p_3 v_1 v_2 v_3}_{p f_1 f_2 f_3} \; \theta^p \theta^{f_1} \bar{\theta}^{f_2} \bar{\theta}^{f_3} |p_1, p_2, p_3\rangle\langle v_1, v_2, v_3|$$

Wille, Buerschaper, Eisert, arXiv:1609.02574

Fermionic MPO-injectivity

- Dual lattice
- Grassmann numbers
- Virtual symmetries with branching structure*



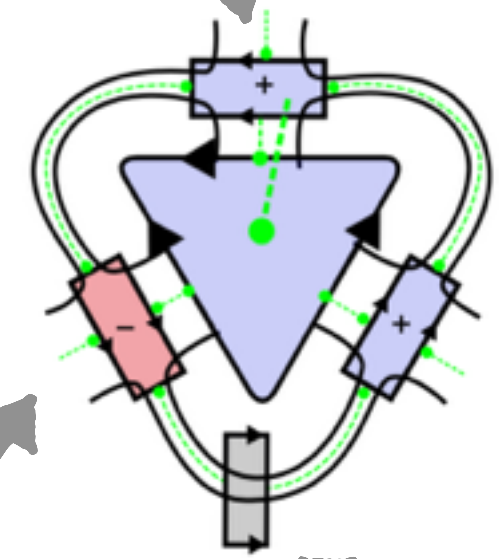
*Edges of PEPS tensor are oriented such that no cyclic orientation arises

Wille, Buerschaper, Eisert, arXiv:1609.02574

Fermionic MPO-injectivity

- Dual lattice
- Grassmann numbers
- Virtual symmetries with branching structure

T_+ at edges parallel to MPO direction



T_- at edges anti-parallel to MPO direction

Purely bosonic Y to ensure concatenation

- **Theorem:** Construction satisfies axioms

- Can compute properties, e.g., ground state degeneracy
- Interesting physical models?

Wille, Buerschaper, Eisert, arXiv:1609.02574

Twisted fermionic double models

- **Twisted fermionic quantum doubles** (instances of fermionic string nets)

- Graded group cohomology: Triple (G, s, ω)
  - **Group** G, defining bosonic degrees of freedom
  - **2-cocycle** $\mathcal{H}^2(G, \mathbb{Z}_2)$, governing coupling

$$s(a,b) + s(ab,c) + s(a,bc) + s(b,c) = 0$$

  - **Graded 3-cocycle** $\mathcal{H}_f^3(G, U(1), s)$

$$\omega(a,b,c)\omega(a,bc,d)\omega(b,c,d) = (-1)^{s(a,b)s(c,d)}\omega(ab,c,d)\omega(a,b,cd)$$

- Can all be shown to satisfy framework (tedious)
- **Fermionic toric code:** Simplest triple
  - $G = \mathbb{Z}_2$
  - $s(1,1) = 1$, $s = 0$ otherwise

Wille, Buerschaper, Eisert, arXiv:1609.02574
Bultinck, Williamson, Haegeman, Verstraete, arXiv:1610.07849

Consistent framework

- Consistent framework of topological PEPS for fermionic systems

Wille, Buerschaper, Eisert, arXiv:1609.02574
Williamson, Bultinck, Haegeman, Verstraete, arXiv:1609.0289

Fermionic PEPS and a crime story

- Consistent framework of topological PEPS for fermionic systems

Wille, Buerschaper, Eisert, arXiv:1609.02574
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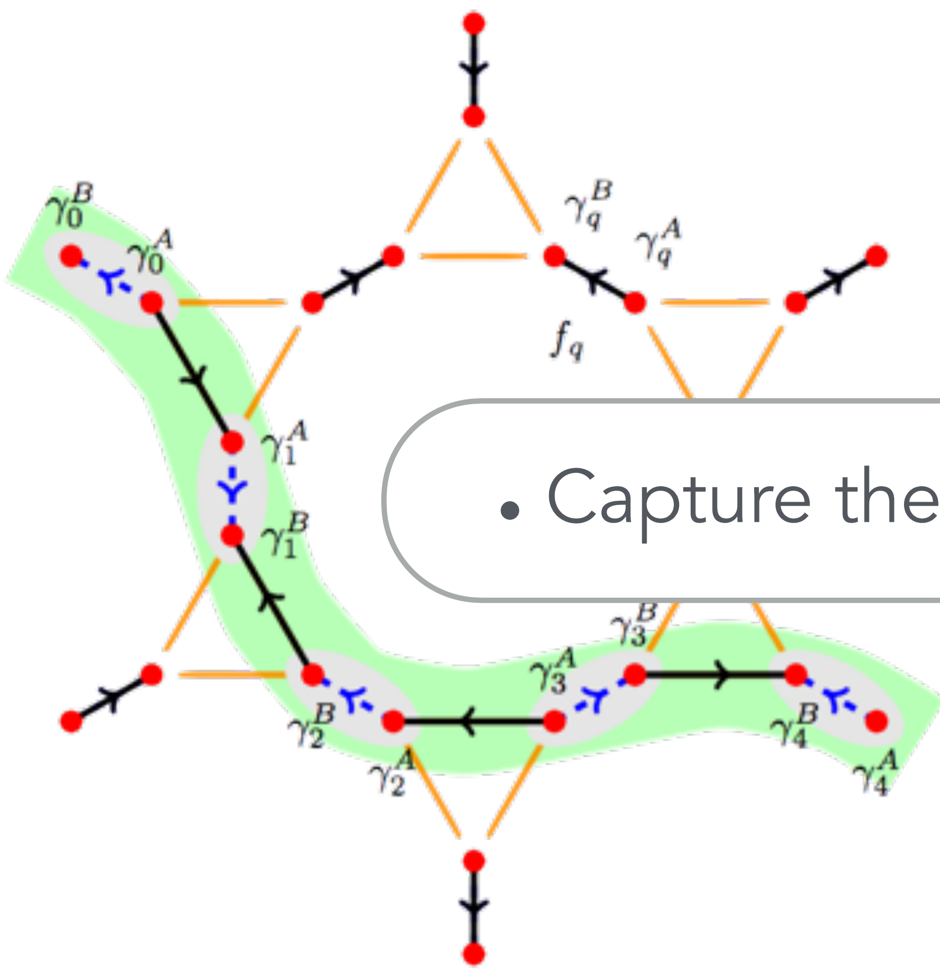

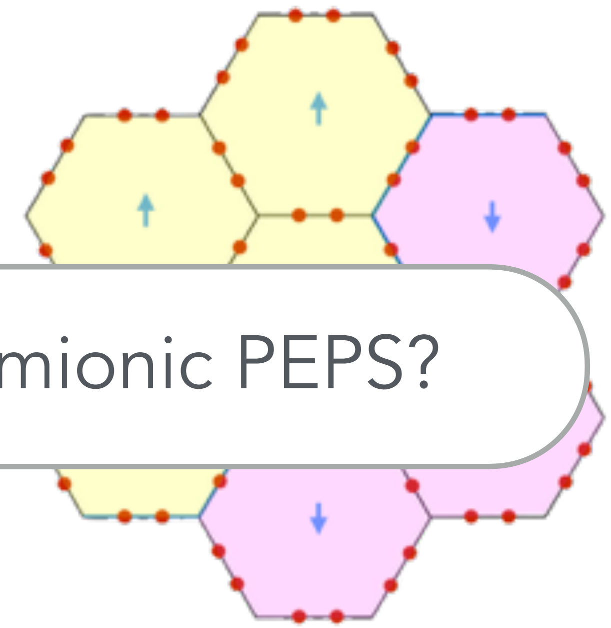

- Capture them as fermionic PEPS?

Ising anyons in frustration-free Majorana dimer models

Ware, Son, Cheng, Mishmash, Alicea, Bauer, arXiv:1605.06125

Discrete spin structures and commuting projector models for 2d fermionic symmetry protected topological phases

Tarantino, Fidowski, Phys Rev B 94, 115115 (2016)

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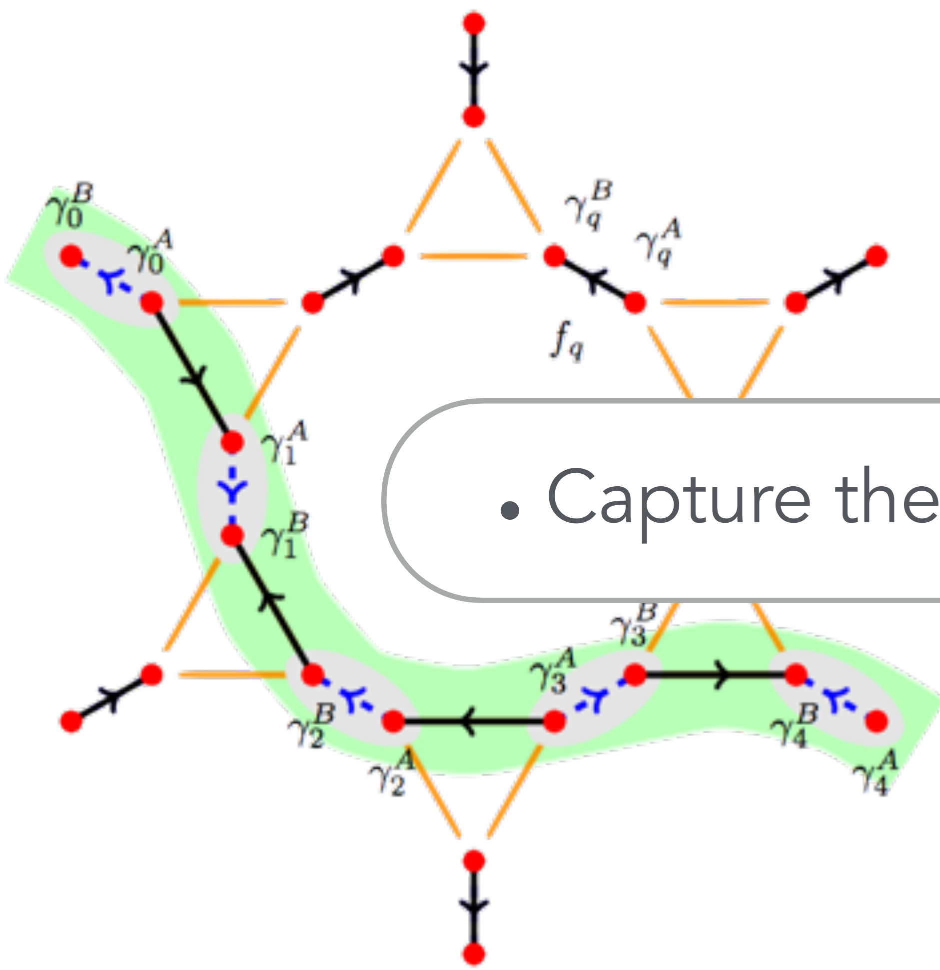

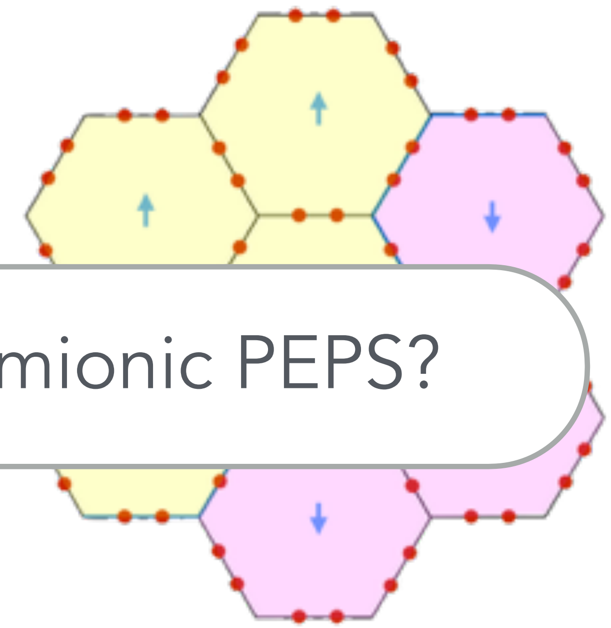

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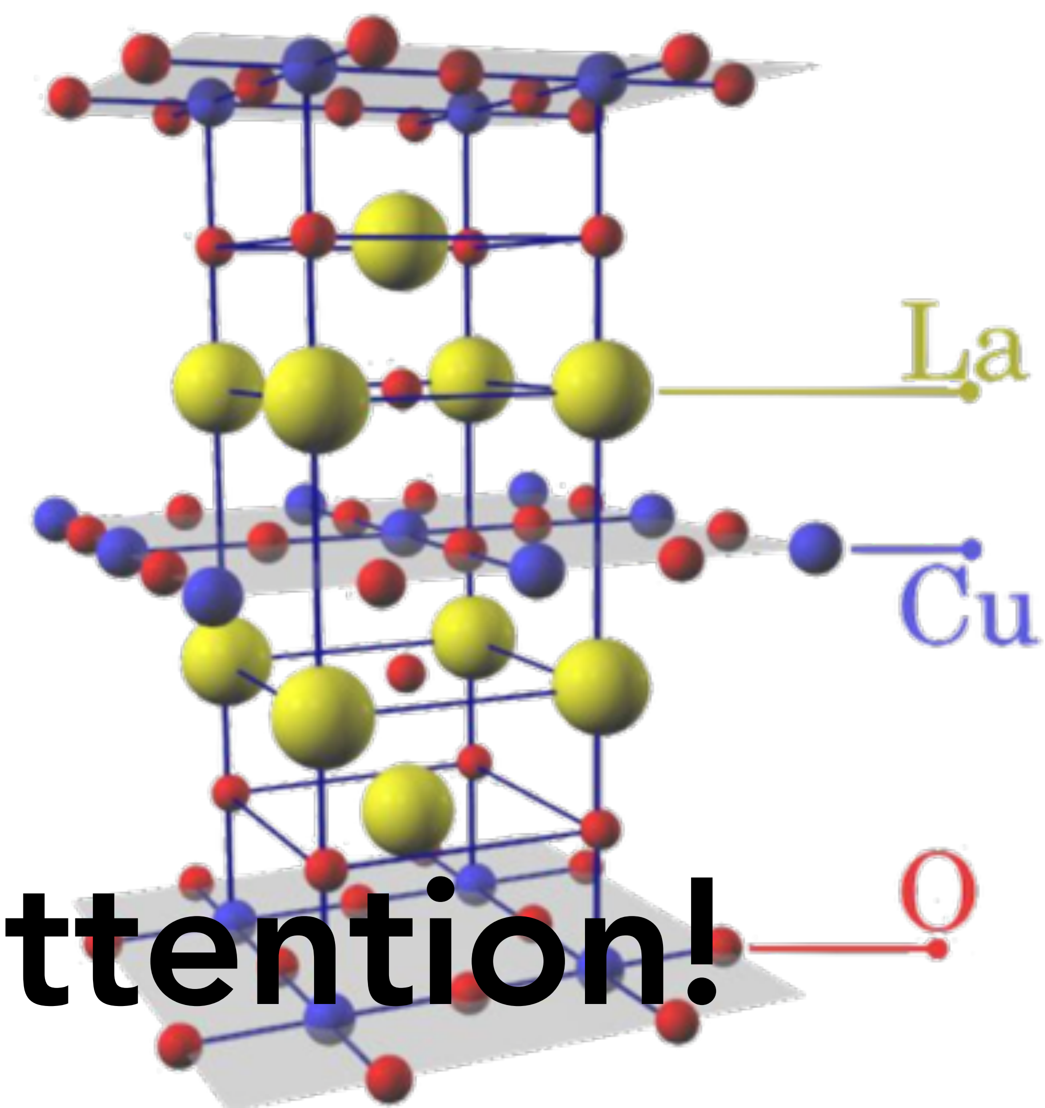
Summary

- Natural ground states of quantum many-body systems are very little entangled in a precise sense. This allows for computational methods based on tensor networks as well as new ways for their mathematical study."



Thanks for your attention!