Non-local transport of strongly-interacting fermions via topological and Shockley edge states

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Introduction

- Floquet engineered systems

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- Harper-Hofstadter model: M. Aidelsburguer et al., Nat. Phys. **11** 162-166 (2015)

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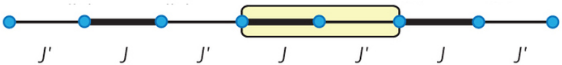

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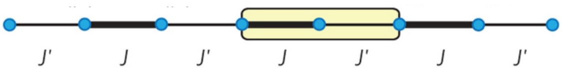

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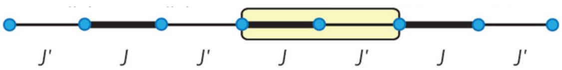

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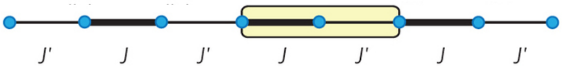

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If $\lambda < 1 \Rightarrow$ Topological phase (Edge states)

P. Delplace, PRB 84, 195452

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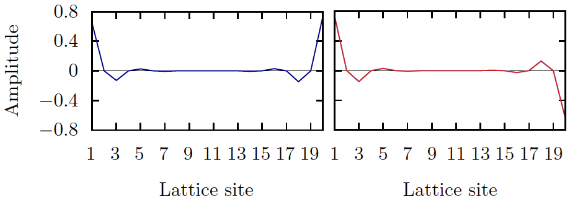

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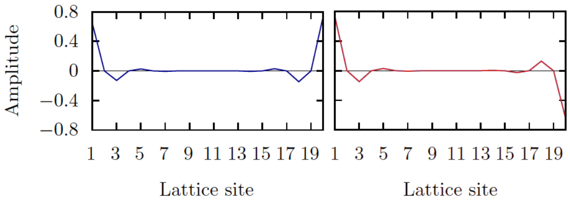

Edge states form a non-local two-level system

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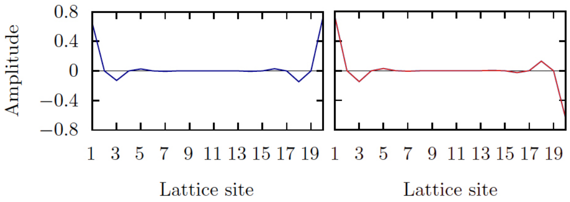

Edge states form a non-local two-level system $\Rightarrow$ **Long-range transfer**

SSH-Hubbard model

$$H = -J' \sum_{i=1,\sigma}^{M} c^\dagger_{2i\sigma} c_{2i-1\sigma} - J \sum_{i=1,\sigma}^{M-1} c^\dagger_{2i+1\sigma} c_{2i\sigma} + H.c. + U \sum_{i=1}^{2M} n_{i\uparrow} n_{i\downarrow}$$

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On-site interaction

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- Strongly interacting regime: $U > 4J$

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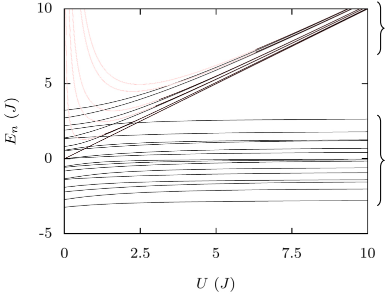
C. E. Creffield & G. Platero, PRB 69, 165312; PRL 105, 086804
K. Winkler et al., Nature 441, 853-856
S. Wall et al., Nat. Phys. 7, 114-118

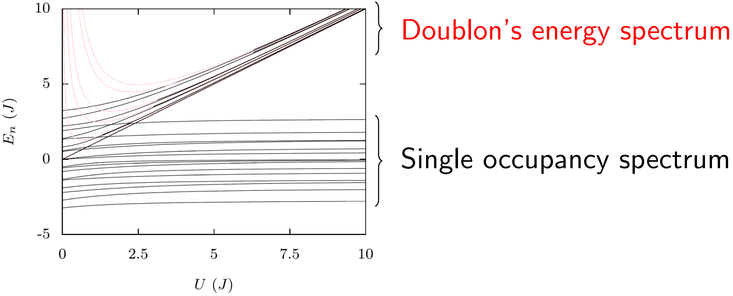
Schrieffer-Wolff transformation (SWT)



Doublon's energy spectrum

Single occupancy spectrum

Schrieffer-Wolff transformation (SWT)



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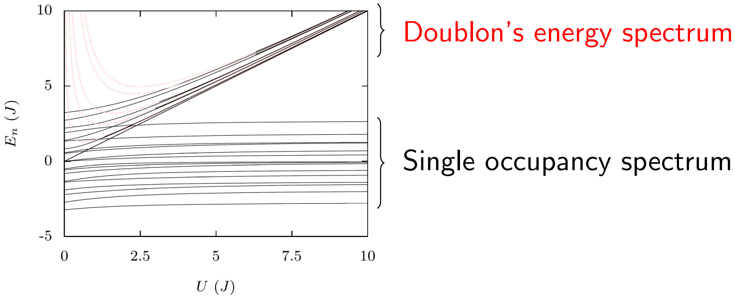

- We want to project out states we are not interested in
- Unitary transformation that block-diagonalizes the Hamiltonian, perturbatively in powers of J/U and J'/U up to first order.

Schrieffer-Wolff transformation (SWT)

$$H_{\text{eff}} = J'_{\text{eff}} \sum_{i=1}^{M} d_{2i}^{\dagger} d_{2i-1} + J_{\text{eff}} \sum_{i=1}^{M-1} d_{2i+1}^{\dagger} d_{2i} + H.c. + \sum_{i=1}^{2M} \mu_i n_i^d$$

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$\mu_i = U + \sum_{\langle i,j \rangle} J_{ij}^{\text{eff}}$ Sum over first neighbours of site i

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For a finite lattice:

$$\mu_i = \begin{cases} \mu_{\text{bulk}} = J'_{\text{eff}} + J_{\text{eff}} + U & \text{if} \quad 1 < i < 2M \\ \mu_{\text{edge}} = J'_{\text{eff}} + U & \text{if} \quad i \in \{1, 2M\} \end{cases}$$

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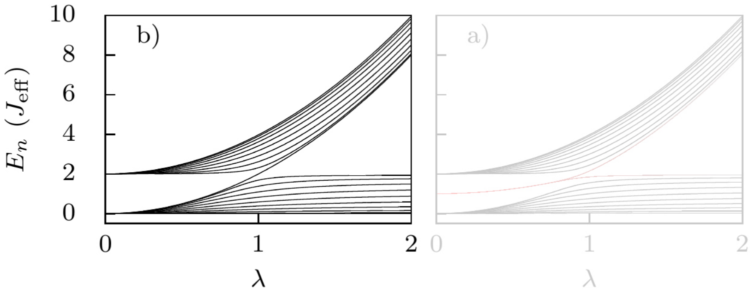
This forbids the presence of edge states for doublons

Topological long-range transfer

If we add a gate potential at the ends of the chain to compensate for the chemical potential difference, we recover the SSH model for doublons



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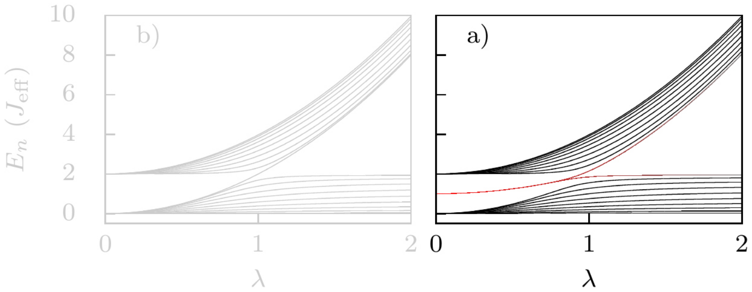

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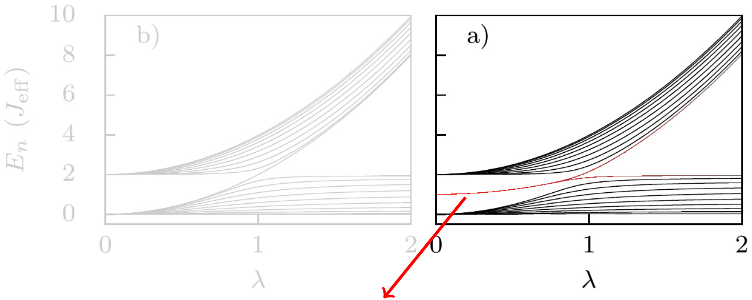
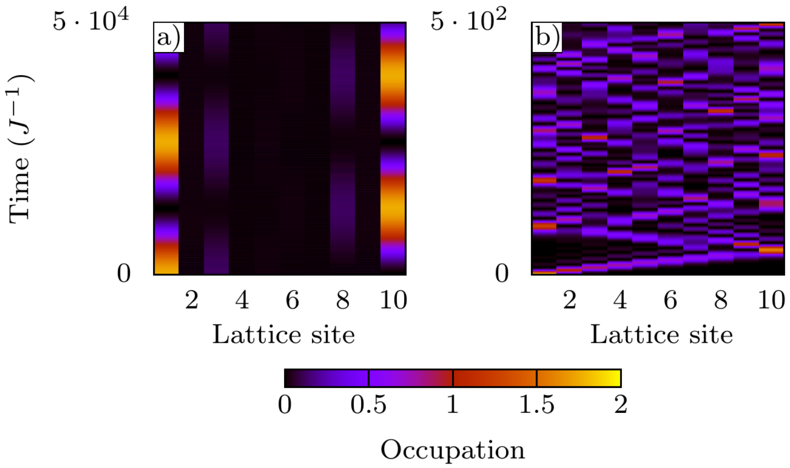


Topological edge states!

Topological long-range transfer



$U = 16J$; a) $\lambda = 0.5$, b) $\lambda = 1$

Floquet theory and HFE

$$H(t) = H_{hopp} + H_U + E\cos(\omega t)\sum_{i=1}^{2M} x_i(n_{i\uparrow} + n_{i\downarrow})$$

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For time-periodic Hamiltonians, $H(t+T) = H(t)$, the fundamental solutions to the Schrödinger equation are of the form:

$$\psi(\mathbf{x}, t) = e^{-i\epsilon_n t}\phi_n(\mathbf{x}, t)$$

$$U(t_2, t_1) = e^{-iK(t_2)}e^{-iH_{eff}(t_2-t_1)}e^{iK(t_1)}$$

$$H_{eff} = H^{[0]} + \frac{1}{\omega}H^{[1]} + \frac{1}{\omega^2}H^{[2]} + \cdots \quad (1)$$

M. Bukov, L. D'Alessio & A. Polkovnikov, Adv. Phys. 2015, Vol. 64, No. 2, 139-226

Floquet theory and HFE

$$H(t) = H_{hopp} + H_U + E\cos(\omega t)\sum_{i=1}^{2M} x_i(n_{i\uparrow} + n_{i\downarrow})$$

– Different effective Hamiltonians are obtained in the two regimes: $U \gg \omega$ and $\omega \gg U$

Effective Hamiltonian, regime $U \gg \omega > J, J'$

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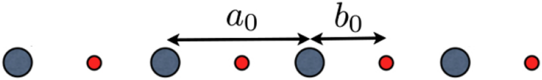

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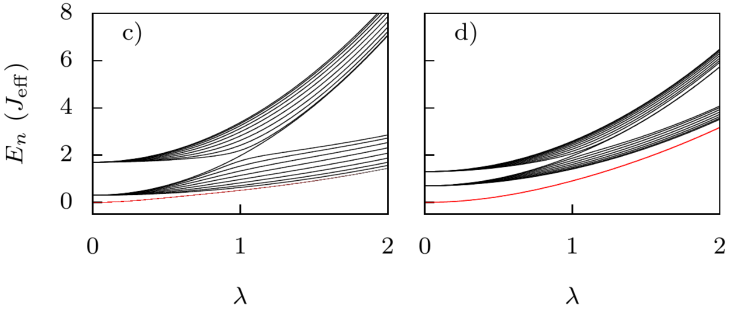
– The ac field allows to tune the ratio between hopping and $\Delta\mu$

$\Longrightarrow$ It is able to induce Shockley-like edge states

Effective Hamiltonian, regime $U \gg \omega > J, J'$

$b_0 = a_0/2$; c) $\mathcal{J}_0(Ea_0/\omega) = 0.7$, d) $\mathcal{J}_0(Ea_0/\omega) = 0.3$



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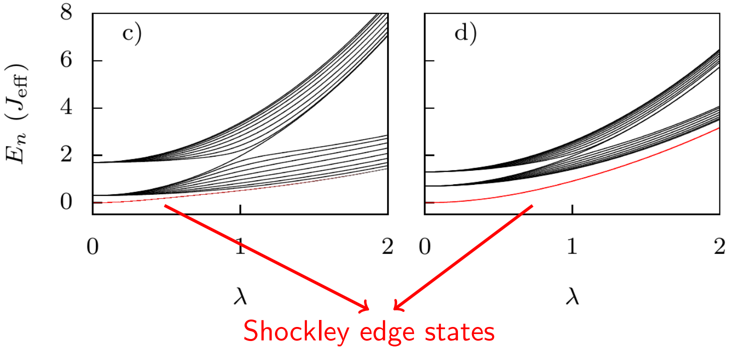

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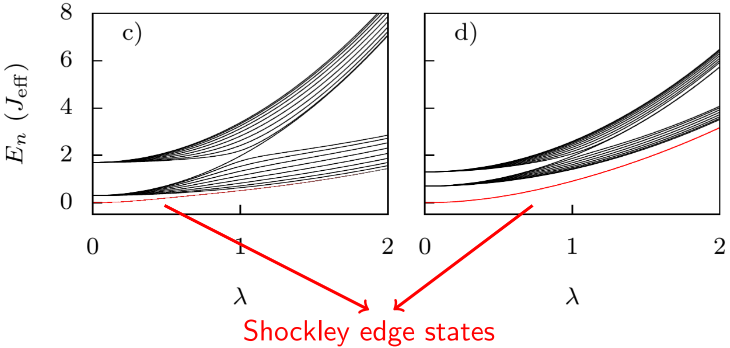

These edge states also form a non-local two level system and allow the long-range transfer

AC fields + gate potentials

SSH model for doublons with tunable hoppings:

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SSH model for doublons with tunable hoppings:

$\Longrightarrow$ Control over the topology of the system:

AC fields + gate potentials

SSH model for doublons with tunable hoppings:

$\implies$ Control over the topology of the system:

$$\left| \frac{\lambda^2 \mathcal{J}_0\left(\frac{2E}{\omega} b_0\right)}{\mathcal{J}_0\left(\frac{2E}{\omega}(a_0 - b_0)\right)} \right| < 1 \quad \Leftrightarrow \quad \mathcal{Z} = \pi$$

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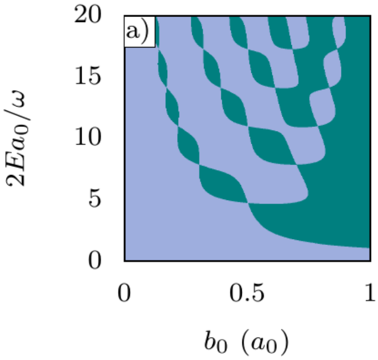

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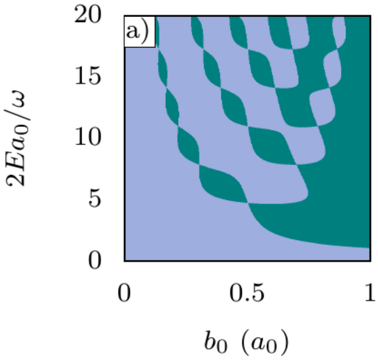

A. Gómez-León & G. Platero PRL **110**, 200403 (2013)

Conclusions

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  - Adding gate potentials $\rightarrow$ Topological transfer
  - Driving the system with an AC field $\rightarrow$ Shockley transfer
  - Combination of both $\rightarrow$ AC induced topological transfer

Thank you!