Bloch oscillations in the magnetoconductance of twisted bilayer graphene

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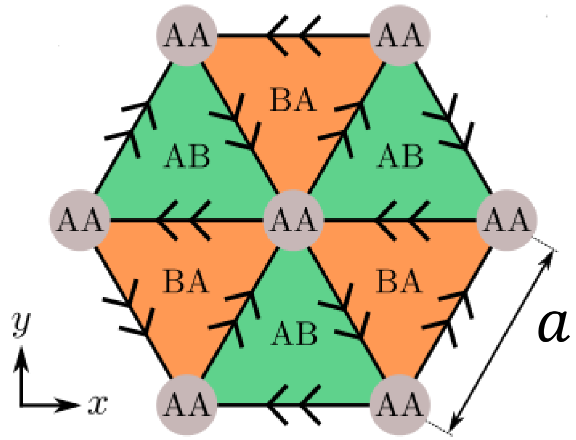
Minimally twisted bilayer graphene

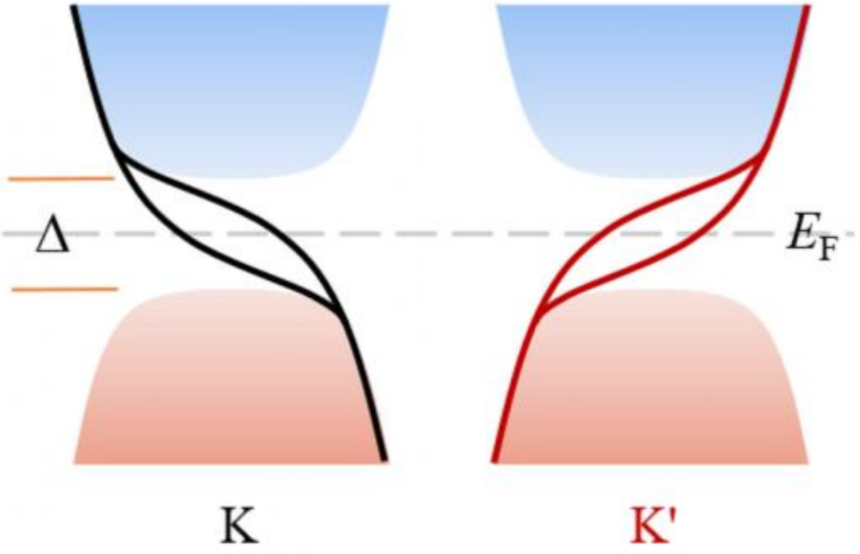
- Twist angle $\Theta \ll 1°$ $(\Theta \sim 0.1°)$
- Lattice relaxes to triangle domains of AB / BA stacking
- Topologically protected helical states in AB|BA domain walls
- Length of domain walls = Moire unit length 'a'

- Sh. Huang et. al., PRL 121, 037702 (2018)
- P. Rickhaus et. al., Nano Lett. 18, 6725 (2018)
- S. G. Xu, et. al., Nature Comm. 10, 4008 (2019)
- C. De Beule, et. al., PRL 125, 096402 (2020)
- …

Lattice



Band structure

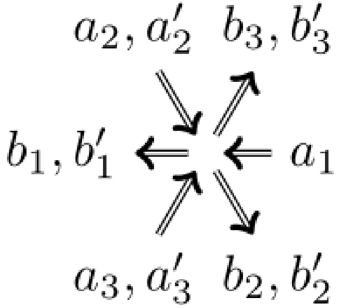


Network of 1-D channel in TBLG

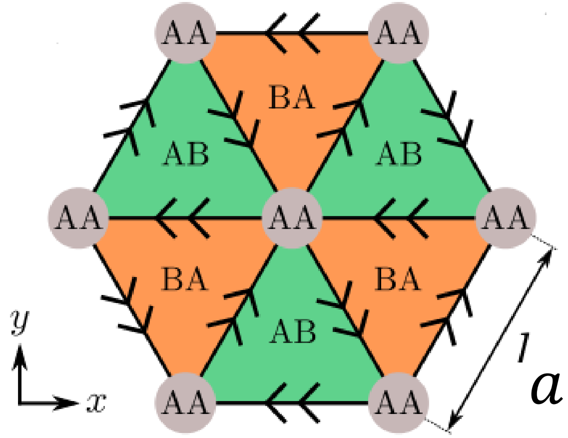
- Two helical modes per domain wall in one valley

$$S\cdot\{a_1,a_2,a_3,a_1',a_2',a_3'\}^\top=\{b_1,b_2,b_3,b_1',b_2',b_3'\}^\top,$$

- S-matrix description



$$S=\begin{pmatrix} S_1 & S_2 \\ S_2^\dagger & -S_1^\dagger \end{pmatrix},$$



C. De Beule, F. Dominguez, and P. Recher
PRL 125, 096402 (2020)

- The number of parameters can be reduced by taking equal intra-channel and inter-channel probabilities $P_{f1}=P_{f2}=\frac{1}{2}P_f$, $P_{d1}=P_{d2}=\frac{1}{4}(1-P_f)$ -> $\beta=\alpha+\pi/2$

$$S_1=e^{i\alpha}\sqrt{P_{d1}}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}+e^{i\beta}\mathbb{1}\sqrt{P_{f1}},$$

$$S_2=\sqrt{P_{d2}}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}-\mathbb{1}\sqrt{P_{f2}}.$$

- The parameter α governs the appearance of closed loops of scattering sequences

Quantum random walk description

- $\alpha = 0$ regime: quasi-1D quantum random walk
- 3 copies in $0°$, $\pm 120°$ directions
- Wave amplitudes of 2 modes form pseudospin degree

$$\psi = (\psi_+, \psi_-)$$

- Discrete evolution with time steps dt $= a/v$

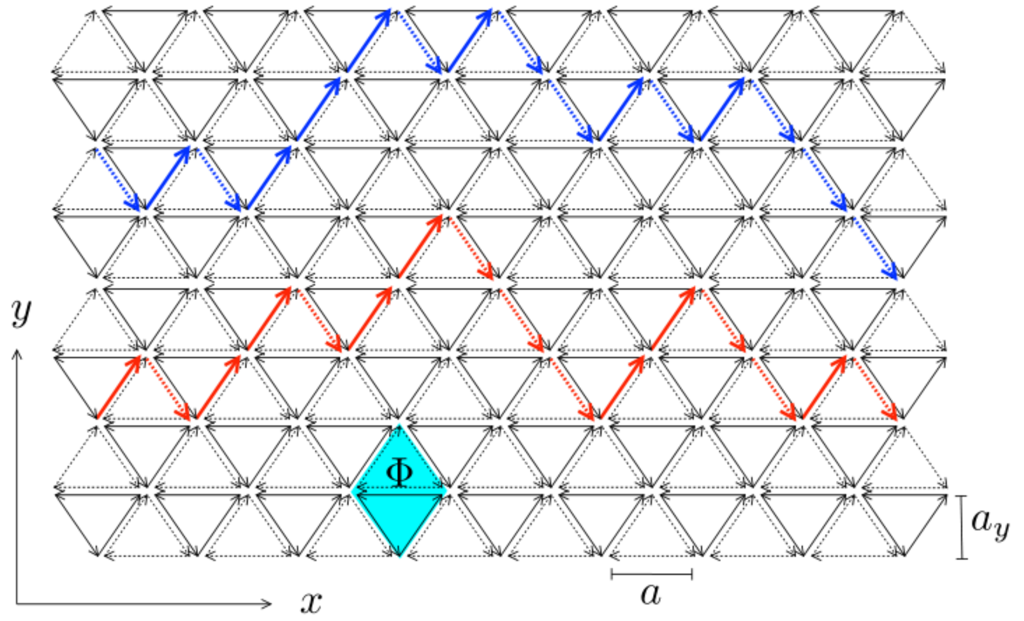
$$\psi_{t+t_0} = \mathcal{T} R \psi_t,$$

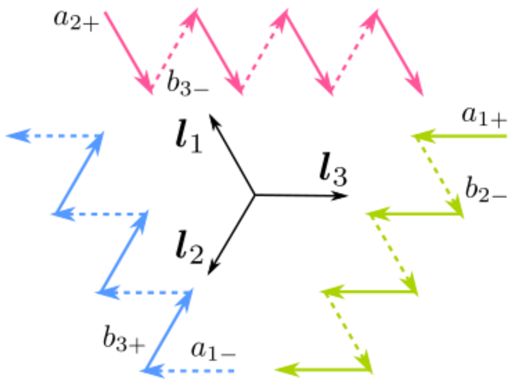
$$\mathcal{T}\psi(y) = (\psi_+(y - a_y), \psi_-(y + a_y)) = e^{-i a_y \hat{k}_y \sigma_z} \psi(y)$$

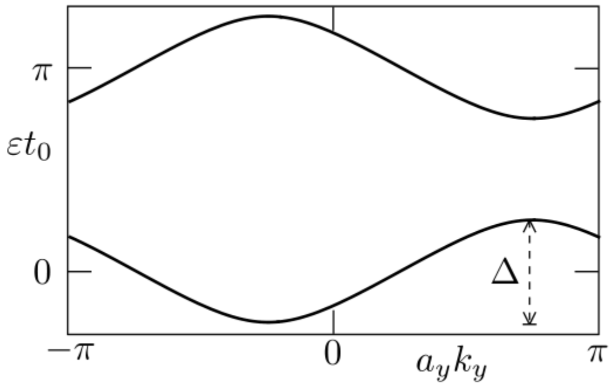
$$R = \begin{pmatrix} e^{i\pi/4}\sqrt{P_f} & \sqrt{1-P_f} \\ \sqrt{1-P_f} & -e^{-i\pi/4}\sqrt{P_f} \end{pmatrix}$$

- Eigenvalues of $\mathcal{T}R$ operator $e^{-i\varepsilon t_0}$

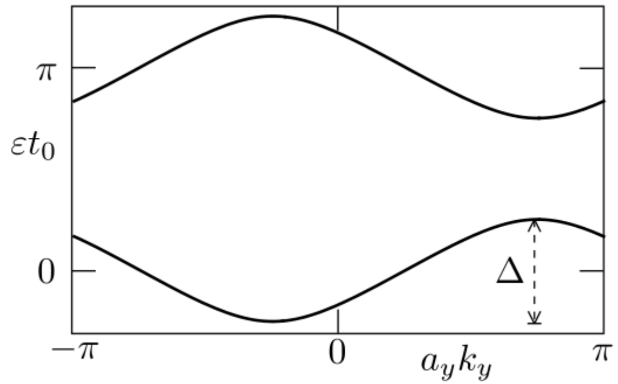
$$\varepsilon_\pm t_0 = \pm \arccos[\sqrt{P_f}\sin(a_y k_y - \pi/4)] + \pi/2$$







Perpendicular magnetic field <-> Bloch oscillations



- Perpendicular magnetic field

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \qquad \boldsymbol{A} = (-By, Ba/4, 0)$$

- Adds phase on each domain wall $-e\int \boldsymbol{A}\cdot d\boldsymbol{l}$

- Modification of quantum random walk

$$\Phi = Baa_y$$

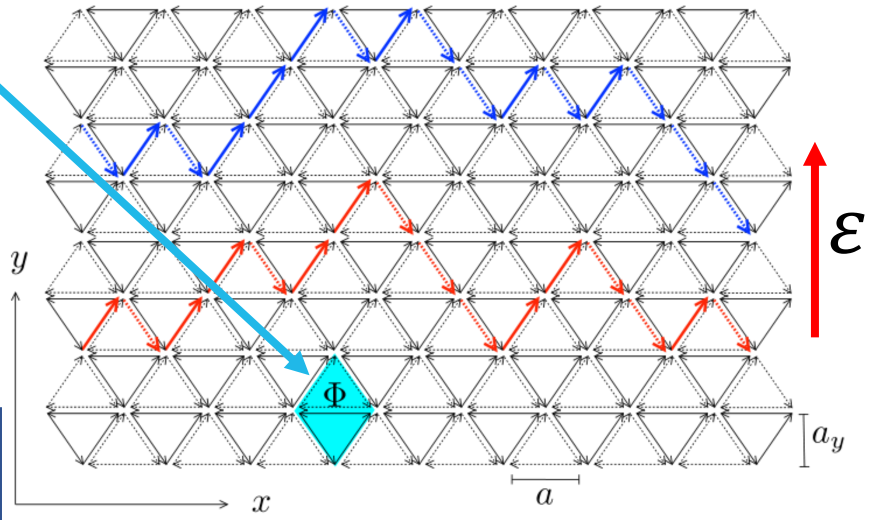
$$\psi_{t+t_0} = e^{i\phi\hat{y}/a_y}\mathcal{T}R\psi_t, \quad \phi = \pi\Phi/\Phi_0 \qquad \Phi_0 = h/e$$

- Comparing with eigenvalues of $\mathcal{T}R$ -- $e^{-i\varepsilon t_0}$

- Like adding electric field to Bloch band $\mathcal{E} \equiv Bv/2$

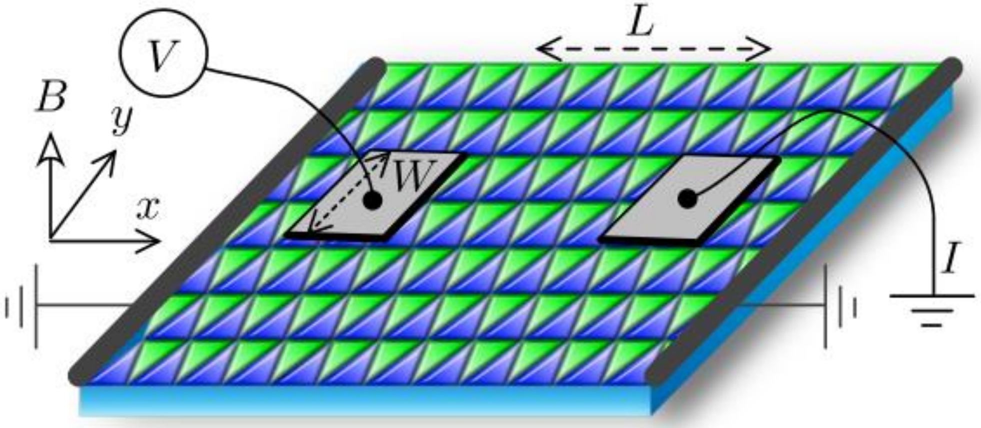
- $\varepsilon_\pm t_0 = \pm \arccos[\sqrt{P_f}\sin(a_y k_y - \pi/4)] + \pi/2$ -- $\phi\hat{y}/a_y$

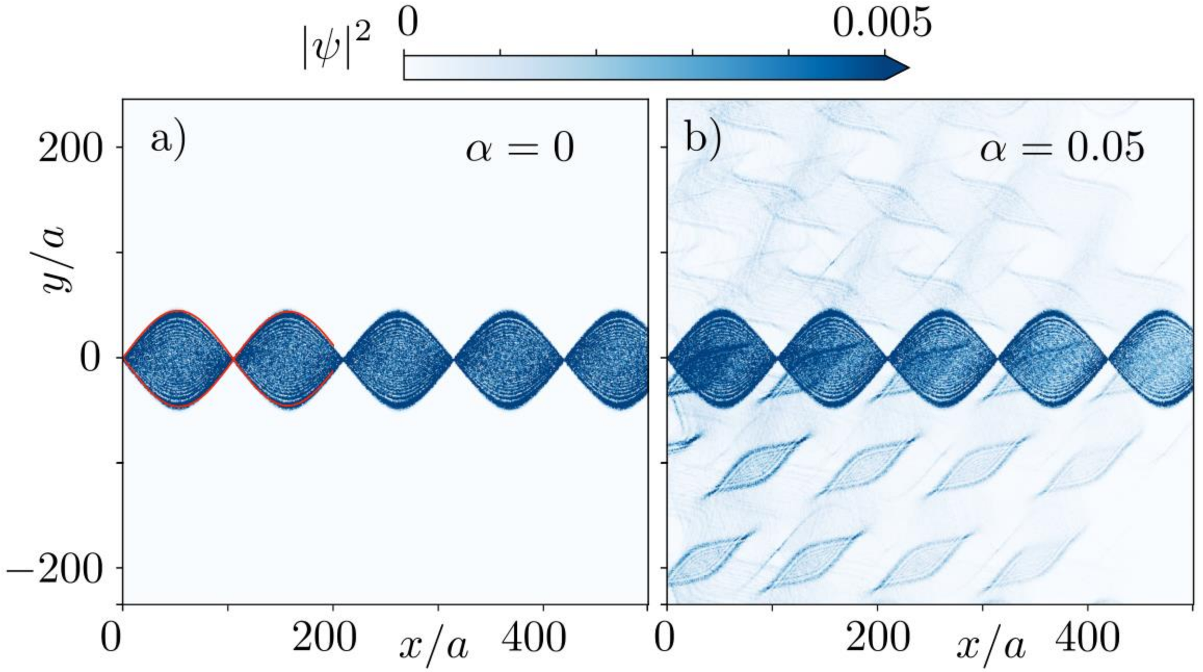
- Effective Bloch oscillations frequency $\omega_{\mathrm{B}} = a_y e\mathcal{E}/\hbar = \phi/t_0$



(Ref. T. Hartmann et al 2004 NJP 6 2, …)

Proposed setup and currents distribution in TBLG





At energy level E=0, $\phi = \frac{\pi\Phi}{\Phi_0} = 0.03$

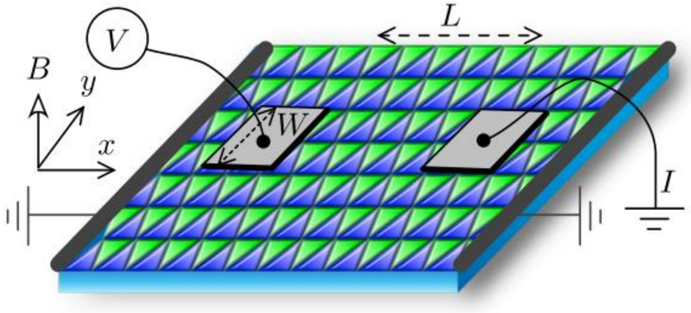
- Current is injected into left narrow contact
- Conductance is measured by right narrow contact -> **"Local conductance"** I = G V
- Changing magnetic field we would find different parts of oscillations – focused / not focused regime

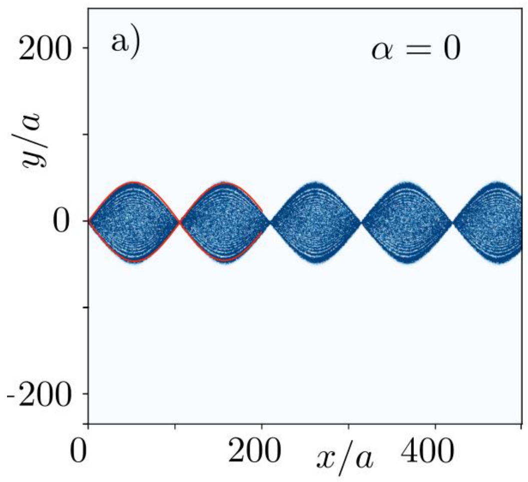
$$y_{\text{envelope}}(x) = \pm(2a_y/\phi)\arcsin\sqrt{P_f} \times \sin(\phi x/a) \qquad t \mapsto 2x/v$$

Oscillations in magnetoconductance

- Possibility to test in transport experiment: varying magnetic field, period of G oscillations depends on contact separation

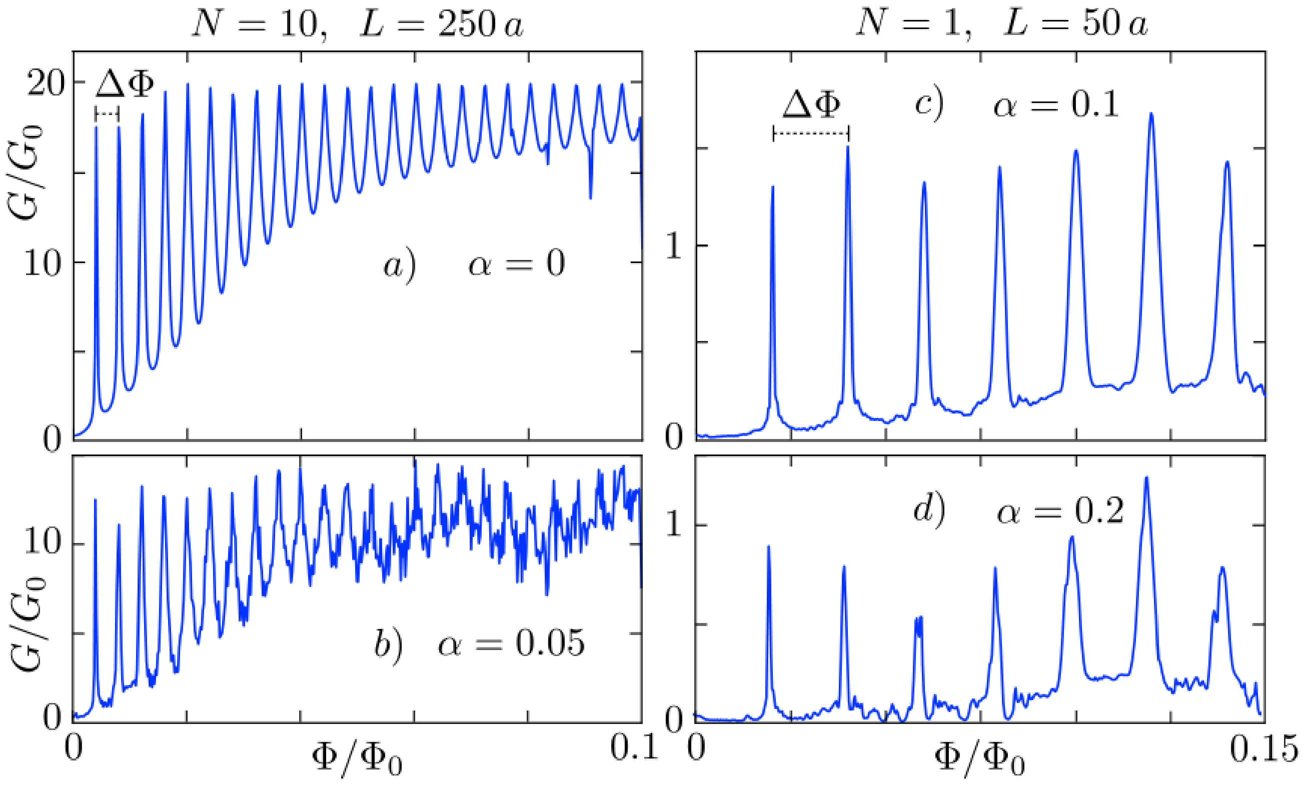
$$G = G_0 \sum_{n,m=1}^{8N} |t_{nm}|^2 \qquad G_0 = 2e^2/h \qquad \Delta\Phi = \Phi_0 \times a/L \Rightarrow \Delta B = (h/e)(a_y L)^{-1}$$





And changing magnetic flux

Injection into N domain walls



Conclusions

- Minimally twisted bilayer graphene -> conducting network of 1D channels -> quantum random walk
- Perpendicular magnetic field maps onto in-plane electric field
- Possibility to observe Bloch oscillations as **local conductance G oscillations**
- Stability with phenomenological parameters (α, P_f)
- Magnetic fields are L/a times smaller that for Aharonov – Bohm oscillations (approx 100 times)

- arXiv:2203.01858